ASSIGNMENT - 2

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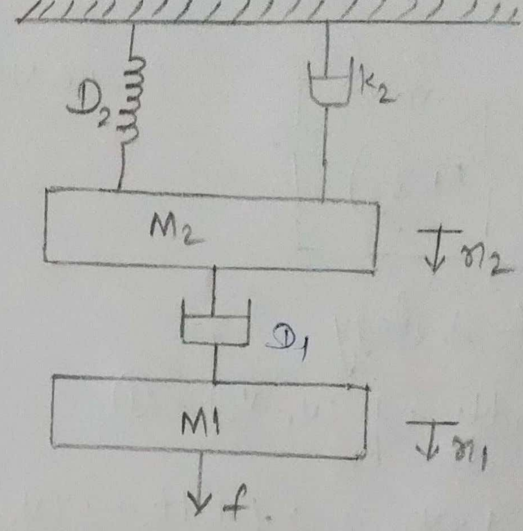
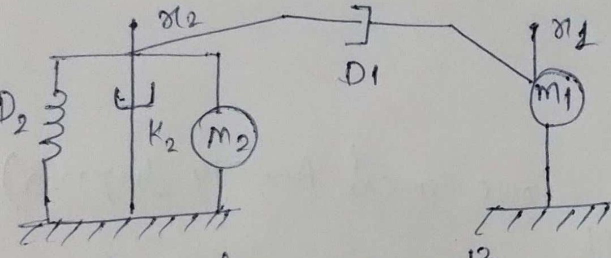
Quesⁿ → 1. Determine system equations of the system shown below. Draw force-voltage and force current analogies:



Solⁿ →



⇒ nodal analysis of x_1 : $f = m_1\frac{d^2x_1}{dt^2} + D_1\frac{d}{dt}(x_1 - x_2)$

⇒ nodal analysis of x_2 : $m_2\frac{d^2x_2}{dt^2} + K_2\frac{dx_2}{dt} + D_2x_2 + D_1\frac{d}{dt}(x_2 - x_1) = 0$

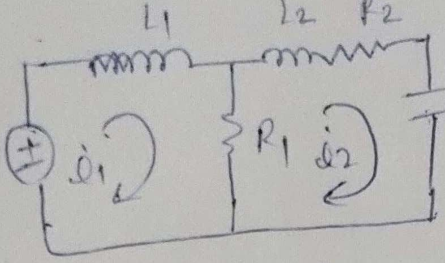
eq (1) ← $f = m_1\frac{d^2x_1}{dt^2} + D_1\frac{d}{dt}(x_1 - x_2)$ — (1)

eqⁿ $m_2\frac{d^2x_2}{dt^2} + K_2\frac{d}{dt}x_2 + D_2x_2 + D_1\frac{d}{dt}(x_2 - x_1) = 0$ — (2)

⇒ force - voltage analogy →

eqⁿ (1) : $V = L_1\frac{di_1}{dt} + R_1(i_1 - i_2)$

eqⁿ (2) : $L_2\frac{di_2}{dt} + R_2i_2 + \frac{1}{C}\int i_2\,dt + R_1(i_2 - i_1) = 0$

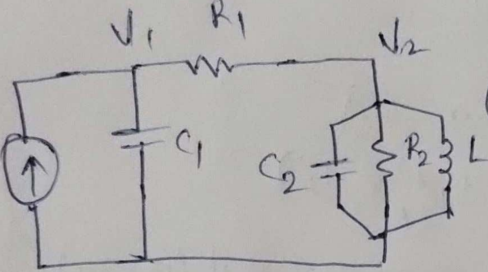


(force voltage Analogy diagram)

- Force current Analogy

eq(1) $I(t) = C_1 \frac{dV_1}{dt} + \frac{1}{R_1}(V_1 - V_2)$ — (1)

eq(2) $C_2 \frac{dV_2}{dt} + \frac{1}{R_2} \times V_2 + \frac{1}{L}\int V_2 dt + \frac{1}{R_1}(V_2 - V_1) = 0$ — (2)



(force current Analogy diagram)

Q → (2) show derivation of output response expression of a 2^{nd} order system with unit ramp input

Ans → In second order: $\frac{Y(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

Now, input is unit ramp, $r(t) = t.u(t)$

$\therefore, R(s) = 1/s^2$

$\therefore\ Y(s) = \frac{1}{s^2} \times \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ — (1)

$\therefore$ By Partial fraction

$Y(s) = \frac{A}{s} + \frac{B}{s} + \frac{(s+D}{s^2 + 2\xi\omega_n s + \omega_n^2}$ — (2)

$\omega_n^2 = A\cancel{s}(s^2 + 2\xi\omega_n s + \omega_n^2) + B(s^2 + 2\xi\omega_n s + \omega_n^2) + (s+D)s^2$

$\omega_n^2 = (A+C)s^3 + (2\xi\omega_n A + B + D)s^2 + (A\omega_n^2 + 2\xi\omega_n)s + B\omega_n^2$

Solving equating coefficients of s^3

$A + C = 0$ — (3)

equating coefficient of s^2 $\boxed{B = 1}$ → from 6

$2\xi\omega_n A + B + D = 0$ — (4) $\boxed{C = \frac{2\xi}{\omega_n}}$ → from 3

equating coefficient of s

$A\omega_n^2 + 2\xi\omega_n B = 0$ — (5) $\boxed{A = -\frac{2\xi}{\omega_n}}$ → from 5

equating absolute terms

$B\omega_n^2 = \omega_n^2$ — (6) $\boxed{D = -1 + 4\xi^2}$ from (4)

Putting them in (2)

$$Y(s) = -\frac{2\xi}{\omega_n s} + \frac{1}{s^2} + \frac{\frac{2\xi s}{\omega_n} - 1 + 4\xi^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (\text{for } K=1)$$

$$= \frac{1}{s^2} - \frac{2\xi}{\omega_n s} + \frac{2\xi}{\omega_n}\left[\frac{s + \xi\omega_n - \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}\right) + \frac{4\xi^2 - 1}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$$Y(s) = \frac{1}{s^2} - \frac{2\xi}{\omega_n s} + \frac{2\xi}{\omega_n}\left(\frac{s + \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}\right) - \frac{2\xi}{\omega_n}\frac{\omega_n\xi}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$$+ \frac{4\xi^2 - 1}{(\xi\omega_n + s)^2 + \omega_n^2(1-\xi^2)}$$

$$Y(s) = \frac{1}{s^2} - \frac{2\xi}{\omega_n s} + \frac{2\xi}{\omega_n}\cdot\frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)} + \frac{2\xi^2 - 1}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

Put $\omega_d^2 = \omega_n^2(1-\xi^2)$

$$Y(s) = \frac{1}{s^2} - \frac{2\xi}{\omega_n s} + \frac{2\xi}{\omega_n}\frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + \omega_d^2} + \frac{2\xi^2 - 1}{\omega_d}\cdot\frac{\omega_d}{(s+\xi\omega_n)^2 + \omega_d^2} \quad \text{— (7)}$$

Inverse laplace of (7)

$$y(t) = t - \frac{2\xi}{\omega_n} + \frac{2\xi}{\omega_n} e^{-\xi\omega_n t} \cos\omega_d t + \frac{2\xi^2-1}{\omega_d} e^{-\xi\omega_n t} \sin\omega_d t$$

$$= t - \frac{2\xi}{\omega_n} + \frac{2\xi}{\omega_n}\frac{\sqrt{1-\xi^2}}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cos(\omega_d t) + \frac{2\xi^2-1}{\omega_d} e^{-\xi\omega_n t} \sin\omega_d t]$$

$$= t - \frac{2\xi}{\omega_n} + \frac{e^{-\xi\omega_n t}}{\omega_n\sqrt{1-\xi^2}}\left[2\xi\sqrt{1-\xi^2}\cos\omega_d t + (2\xi^2-1)\sin\omega_d t\right]$$

Put $2\xi\sqrt{1-\xi^2} = \sin\phi$, $2\xi^2-1 = \cos\phi$

$$x(t) = t - \frac{2\xi}{\omega_n} + \frac{e^{-\xi\omega_n t}}{\omega_n\sqrt{1-\xi^2}}\left[\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t\right]$$

$$= t - \frac{2\xi}{\omega_n} + \frac{e^{-\xi\omega_n t}}{\omega_n\sqrt{1-\xi^2}}\sin(\omega_d t + \phi)$$

So

$$\boxed{x(t) = t - \frac{2\xi}{\omega_n} + \frac{e^{-\xi\omega_n t}}{\omega_n\sqrt{1-\xi^2}}\sin\left[\left(\omega_n\sqrt{1-\xi^2}\right)t + \phi\right]}$$

where $\phi = \cos^{-1}(2\xi^2-1)$ or $\phi = \tan^{-1}\left(\frac{2\xi\sqrt{1-\xi^2}}{2\xi^2-1}\right)$

Q→3 find static error coefficient K_p, K_v and K_a for

(I) $G(s) = \frac{10}{s^2}$ and $H(s) = 0.7$

(II) $G(s) = \frac{5}{s^2+3s+5}$ and $H(s) = 0.6$

(III) $G(s) = \frac{10(1+0.5s)(1+0.8s)}{s^2(s^2+3s+5)}$ and $H(s) = 0.8$

Solⁿ - Position error coefficient $(K_p) = \lim_{s\to 0} G(s)H(s)$

Velocity error coefficient $(K_v) = \lim_{s\to 0} s\,G(s)H(s)$

Acceleration error coefficient $(K_a) = \lim_{s\to 0} s^2 G(s)H(s)$

(I) $G(s)H(s) = \frac{7}{s^2}$

$$K_p = \lim_{s \to 0} \frac{7}{s^2} = \infty$$

$$K_v = \lim_{s \to 0} s \cdot \frac{7}{s^2} = \infty$$

$$K_a = \lim_{s \to 0} s^2 \cdot \frac{7}{s^2} = 7$$

(II) $G(s)H(s) = \frac{3}{s^2+3s+5}$

$$K_p = \lim_{s \to 0} \frac{3}{s^2+3s+5} = 0.6$$

$$K_v = \lim_{s \to 0} s \cdot \frac{3}{s^2+3s+5} = 0$$

$$K_a = \lim_{s \to 0} s^2 \cdot \frac{3}{s^2+3s+5} = 0$$

(III) $G(s)H(s) = \frac{8(1+0.5s)(1+0.8s)}{s^2(s^2+3s+5)}$

$$K_p = \lim_{s \to 0} \frac{8(1+0.5s)(1+0.8s)}{s^2(s^2+3s+5)} = \infty$$

$$K_v = \lim_{s \to 0} s \cdot \frac{8(1+0.5s)(0.8s+1)}{s^2(s^2+3s+5)} = \infty$$

$$K_a = \lim_{s \to 0} \frac{s^2(8)(0.5s+1)(1+0.8s)}{s^2(s^2+3s+5)} = 1.6$$

Q (4) A system has $G(s) = \frac{20}{s^2+5s+5}$ and unity feedback

Find (I) ω_n (II) ξ (III) ω_d (IV) T_d (V) T_r (VI) T_p (VII) M_p (VIII) T_s

characteristic equation of system

$$1 + G(s) = 0$$

$$1 + \frac{20}{s^2+5s+5} = 0, \quad s^2+5s+5=0, \quad s^2+$$

on comparing with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

so $\omega_n^2 = 25 \Rightarrow \boxed{\omega_n = 5 \text{ rad/sec}}$

$$\xi = \frac{5}{2\omega_n} = 0.5 \Rightarrow \boxed{\xi = 0.5}$$

$$\left(\omega_d = \omega_n\sqrt{1-\xi^2} = 5\sqrt{1-0.5^2} = 5 \times 0.866 = 4.33 \text{ rad/sec}\right)$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{4.33} = 0.725 \text{ sec}, \quad \boxed{T_p = 0.725 \text{ sec}}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{-0.5\pi/0.866} = 0.163$$

$$\boxed{M_p = 16.3\%}, \quad \theta = \cos^{-1}(0.5) = 1.047$$

$$T_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - 1.047}{4.33} \Rightarrow \boxed{T_r = 0.483 \text{ sec}}$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{2.5} = 1.6$$

$$\boxed{T_s = 1.6 \text{ sec}}$$

$$T_D = \frac{1 + 0.7\xi}{\omega_n} = \frac{1 + 0.7 \times 0.5}{5} = 0.27 \text{ sec}$$

$$\boxed{T_D = 0.27 \text{ sec}}$$