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Bài 1 $x(t) = 2\cos\left(\frac{\pi t}{3} + \frac{\pi}{4}\right) + \sin\left(\frac{\pi t}{4}\right)$

$= e^{j\left(\frac{\pi t}{3} + \frac{\pi}{4}\right)} + e^{-j\left(\frac{\pi t}{3} + \frac{\pi}{4}\right)} + \frac{1}{2j} e^{j\frac{\pi t}{4}} - \frac{1}{2j} e^{-j\frac{\pi}{4}t}$

$= e^{j\frac{\pi}{4}} \cdot e^{j\frac{\pi}{3}t} + e^{-j\frac{\pi}{4}} e^{-j\frac{\pi}{3}t} + \frac{1}{2j} e^{j\frac{\pi}{4}t} - \frac{1}{2j} e^{-j\frac{\pi}{4}t}$

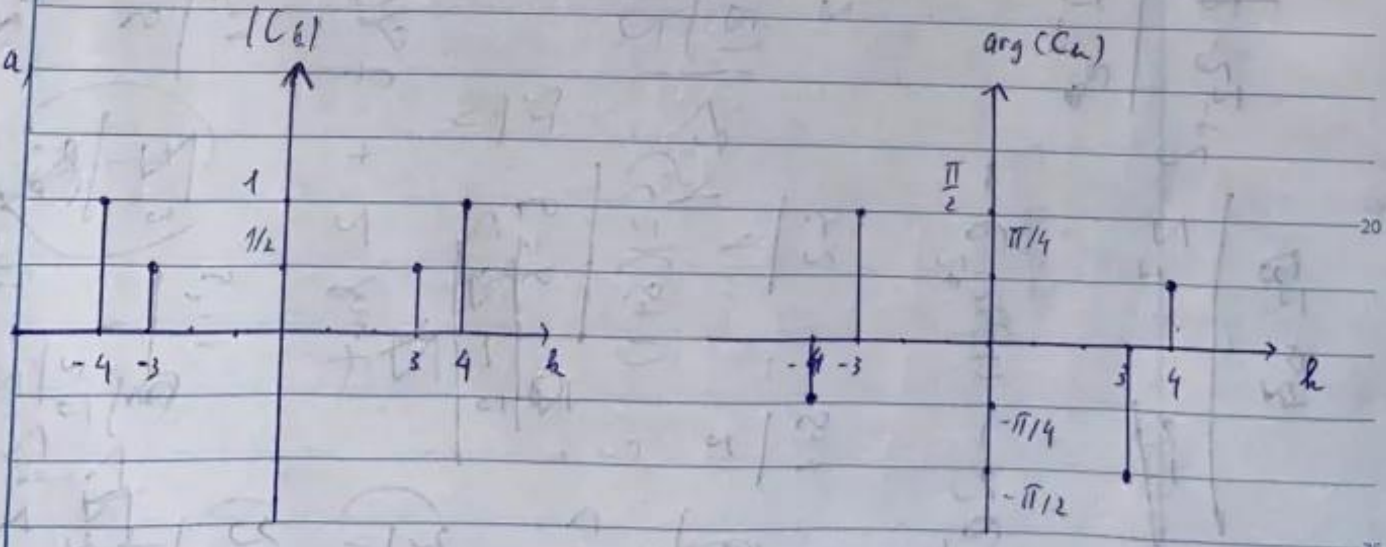
Xét $\omega_0 = \frac{\pi}{12}$

$\Rightarrow C_k = \begin{cases} e^{j\frac{\pi}{4}} & k = 4 \\ e^{-j\frac{\pi}{4}} & k = -4 \\ \frac{1}{2j} & k = 3 \\ \frac{-1}{2j} & k = -3 \end{cases} \Rightarrow |C_k| = \begin{cases} 1 & k = \pm 4 \\ \frac{1}{2} & k = \pm 3 \end{cases}$

$\left(\sqrt{Re^2(C_k) + Im^2(C_k)}\right)$

$\arg(C_k) = \begin{cases} \pi/4 & k = 4 \\ -\pi/4 & k = -4 \\ -\frac{\pi}{2} & k = 3 \\ \frac{\pi}{2} & k = -3 \end{cases}$ $\left(\arctan\frac{Im(C_k)}{Re(C_k)}\right)$

a)



b) $P = \sum_{k=-\infty}^{+\infty} |C_k|^2 = 1 + 1 + \frac{1}{4} + \frac{1}{4} = 2,5$

Bài 2: $y(n) - 4y(n-1) + 3y(n-2) = x(n) + 2x(n-1)$

a) $\lambda^n - 4\lambda^{n-1} + 3\lambda^{n-2} = 0$

$\Leftrightarrow \lambda^{n-2}(\lambda^2 - 4\lambda + 3) = 0$

$\Leftrightarrow \begin{bmatrix} \lambda_1 = 1 \\ \lambda_2 = 3 \end{bmatrix}$

$\Rightarrow y_N(n) = C_1 + C_2 \cdot 3^n$

$y(-1) = 1 \Rightarrow C_1 + \frac{1}{3}C_2 = 1$

$y(-2) = 0 \Rightarrow C_1 + \frac{1}{9}C_2 = 0$

$\Rightarrow \begin{cases} C_1 = \frac{-1}{2} \\ C_2 = \frac{9}{2} \end{cases}$

$\Rightarrow y_N(n) = \frac{-1}{2} + \frac{9}{2} \cdot 3^n$

b) $Y(z) - 4z^{-1}Y(z) + 3z^{-2}Y(z) = X(z) + 2z^{-1}X(z)$ $|z| > a$

$\Rightarrow Y(z)(1 - 4z^{-1} + 3z^{-2}) = X(z)(1 + 2z^{-1})$

$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1}}{1 - 4z^{-1} + 3z^{-2}}$ $|z| < 1$

$\text{Re}(s) < 0$

Các điểm cực $z = 1$ và $z = 3$ không nằm trong đường tròn đơn vị nên hệ thống không ổn định

c) $H(z) = \frac{-3}{2} \cdot \frac{z}{z-1} + \frac{5}{2} \cdot \frac{z}{z-3}$

$\Rightarrow h(n) = \frac{-3}{2}u(n) + \frac{5}{2} \cdot 3^n u(n)$

Do hệ thống không ổn định $\Rightarrow$ không tồn tại $H(\Omega)$

d) $x(n) = \left(\frac{1}{2}\right)^n u(n) \Rightarrow X(z) = \frac{z}{z - \frac{1}{2}}$

$\Rightarrow Y(z) = X(z)H(z) = \frac{z^3 + 2z^2}{(z-1)(z-3)(z-\frac{1}{2})}$

$= \frac{-3z}{z-1} + \frac{3z}{z-3} + \frac{z}{z-\frac{1}{2}}$

$\Rightarrow y(n) = -3u(n) + 3^{n+1}u(n) + \left(\frac{1}{2}\right)^n u(n)$

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$x_1(n) = 3.\left(\frac{1}{2}\right)^n u(n-1)$

$Z\{u(n)\} = \frac{z}{z-1} \rightarrow Z\{u(n-1)\} = \frac{1}{z-1}$

$\Rightarrow Z\left\{3\left(\frac{1}{2}\right)^n u(n-1)\right\} = \frac{3}{2z-1} = X_1(z)$

$\Rightarrow Y(z) = X_1(z)\,H(z) = \frac{1,5z^2+3z}{(z-1)(z-3)(z-\frac{1}{2})}$

$= \frac{-9}{2}.\frac{z}{z-1} + \frac{3}{2}.\frac{z}{z-3} + 3.\frac{z}{z-\frac{1}{2}}$

$\rightarrow y(n) = \frac{-9}{2}u(n) + \frac{3^{n+1}}{2}u(n) + 3\left(\frac{1}{2}\right)^n u(n)$

3,

a) $H(s) = \frac{1-s}{2+3s+s^2} = \frac{Y(s)}{X(s)}$

$\Rightarrow Y(s)(2+3s+s^2) = X(s)(1-s)$

$\Leftrightarrow 2Y(s) + 3sY(s) + s^2Y(s) = X(s) - sX(s)$

$\Rightarrow$ pt vp:

$2y(t) + 3\frac{dy(t)}{dt} + \frac{d^2y(t)}{dt^2} = x(t) - \frac{dx(t)}{dt}$

Các điểm cực $s=-1$ và $s=-2$ đều <0

$\Rightarrow$ hệ thống ổn định

b) ta có $H(s) = \frac{1-s}{2+3s+s^2} = \frac{2}{s+1} - \frac{3}{s+2}$

$\rightarrow h(t) = (2.e^{-t} - 3e^{-2t})u(t)$

Hệ thống ổn định $\Rightarrow H(\omega) = Hs\Big|_{s=j\omega} = \frac{2}{j\omega+1} - \frac{3}{j\omega+2}$

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c) $x(t) = \cos(2t)\,u(t)$

$\rightarrow X(s) = \dfrac{s}{s^2+4}$

$Y(s) = X(s)\,H(s) = \dfrac{s-s^2}{(2+3s+s^2)(s^2+4)} = \dfrac{s-s^2}{(s+1)(s+2)(s-2j)(s+2j)}$

$= \dfrac{-2}{5}\cdot\dfrac{1}{s+1} + \dfrac{3}{4}\cdot\dfrac{1}{s+2} - \dfrac{7}{20}\cdot\dfrac{s}{s^2+4} + \dfrac{1}{20}\cdot\dfrac{2}{s^2+4}$

$\rightarrow y_1(t) = \dfrac{-2}{5}e^{-t}u(t) + \dfrac{3}{4}e^{-2t}u(t) - \dfrac{7}{20}\cos(2t)u(t) + \dfrac{1}{20}\sin(2t)u(t)$

$x(t) = \cos(2t) \quad -1$

$\rightarrow X(s) = \dfrac{s}{s^2+4} - \dfrac{1}{s} = \dfrac{4}{s(s^2+4)}$

$\Rightarrow Y(s) = \dfrac{4s-4}{(s+1)(s+2)(s^2+4)s}$

$\Rightarrow Y(s) = \dfrac{s-s^2}{(2+3s+s^2)(s^2+4)} + \dfrac{s-1}{s(s+1)(s+2)}$

$\rightarrow y_2(t) = y_1(t) - \dfrac{1}{2}u(t) + 2e^{-t}u(t) - \dfrac{3}{2}e^{-4}u(t)$