The Viscosity of Fluids

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In this report we examine how the speed of a sphere falling through a viscous liquid depends on the size of the sphere and then we use those results to find the viscosity of the liquid. For our experiments we based our procedure on the following equation:

$$v = \frac{2gr^2}{9\eta}(\rho_s - \rho_l) \tag{1}$$

where v is velocity, g is the acceleration of gravity which we assume is 9.81m/s, r is the radius of the sphere, η is the viscosity of the liquid, ρ_s is the density of the spheres, and ρ_l is the density of the liquid.

In our experiments we used glycerol as our liquid and performed our experiments at room temperature which was at 24 ± 2 °C. It is important to note that the room temperature constantly fluctuated which is the source of the uncertainty in the temperature. We used a scale to measure the nine different sized spheres and measured the radius with a vernier caliper. We used a one litre graduated cylinder which measured in marks of 10mL which gives us a 5mL uncertainty, and we found the weight of the glycerol by filling the comparing the weight of the cylinder with and without the glycerol. Using this information we divided the masses by the volumes to find the densities for the spheres and glycerol which were 7455±8 kg/m^3 and 1256±6 kg/m^3 respectively, where the uncertainties come from the uncertainty in the vernier caliper and the scales.

Using a stopwatch, we began our data collection by filling the glycerol to around the one litre mark and set three markers at the 800mL, 600mL, and 200mL mark, which we refer to as points A, B, and C respectively. With this measurement method our uncertainties were caused by the human visual reaction time which is 0.3s. Referring to Fig. 1, we can see that the difference in velocities between AB and BC are quite noticeable, so we determined that the spheres will reach terminal velocity by the time it reaches point B. As it can be seen on Fig. 1 the trials were only taken up to box 7 as the speed of sphere passing the points were too fast for us to reliably time it with stopwatches. Due to the speed of the spheres, we began to suspect that the larger spheres were no longer undergoing laminar flow, and confirmed our suspicions using the Reynold's Number equation:

$$R = \frac{2rv}{\eta}\rho_l \tag{2}$$

and we found that from box 6 and onwards the Reynold's number was above one, which means that the data can not be analyzed.

To decrease the uncertainty in timing, we switched to recording the experimental process to more accurately find the time between points B and C. Due to the frame rate of the video, the uncertainty in the time measurements are now 0.05s. With our new data we once again used Eq. 2 and found that box 5 also had a Reynold number above 1, which means that we can safely ignore the results of box 5. We then plotted our results using Eq. 4, and plotted the velocity of the spheres against the radius squared of the spheres, which can be seen in Fig. 3. In our initial fit, we found that the viscosity of the glycerol was 0.78±0.02Ns/m^2. We then took into account the finite size effect using the equation:

$$\eta' = \frac{\eta}{(1 - \frac{r}{Rc})^{\alpha}} \tag{3}$$

where Rc is the radius of the cylinder which we found to be 2.86±0.05cm, r is the radius of the spheres, η is the initial value of viscosity, and α is a scaling constant that is between 2.2 and 2.4. We then use the new corrected viscosity in the equation:

$$v = \frac{2gr^2}{9\eta'}(\rho_s - \rho_l) \tag{4}$$

and we fit the new equation using an α of 2.2 as can be seen in Fig. 4 to find the corrected viscosity to be 0.69±0.02Ns/m^2. When changing the value of α to 2.3 and 2.4 the value of the corrected viscosity remains within 0.69±0.02Ns/m^2. Comparing Fig. 3 and Fig. 4, we can see that the corrected fit gets within all of the error bars while the uncorrected fit is within the first three points but does not agree with the fourth point as it should.

When calculating the overall uncertainty in the viscosity by taking into account the uncertainties with our measurement instruments, we found that the uncertainty in time had the most impact. By lowering that uncertainty by measuring with a video recording we can be more confident on the accuracy of our uncertainty in the viscosity. The value for the viscosity we received corresponds to a viscosity we would receive if the glycerol contained 1.5% water and the glycerol was at a temperature 25 degrees Celsius. The glycerol we used had a minimum water percentage of 0.5% and the temperature was within our uncertainty bounds so this result reasonably agrees with what we expect.

FIGURES

	trial 1		trial 2		trial 3					
box	AB	BC	AB	BC	AB	BC	AVG AB	Avg BC	Vel AB	Vel BC
1	3.07	7.89	3.73	7.38	3.22	6.45	3.27889	5.87667	0.02196	0.0245
2	2.05	4.25	2.14	3.91	1.92	4.02	1.909	3.30333	0.03772	0.04359
3	1.43	2.68	1.37	2.62	1.34	2.66	1.37	2.16	0.05255	0.06667
4	1.09	1.81	1.07	1.74	0.99	1.9	1.05	1.71667	1.71667	0.08388
5	0.75	1.47	0.81	1.38	0.74	1.38	0.76667	1.4	1.4	0.10286
6	0.56	1.21	0.53	1.21	0.54	1.16	0.54333	1.1	1.1	0.13091
7	0.48	1.03	0.47	1.02	0.43	1.1	0.46	0.9	0.9	0.16

FIG. 1. Table of trials where box 1 contains the smallest spheres and the size of the spheres increase as the box number increases up to box 9. In each trial the time taken between the sphere passing through points A, B, and C are recorded. AVG AB and Avg BC are the average times between the points and Vel AB and Vel BC are the calculated velocities of the spheres. The distance between A and B is 7.20±0.05cm and the distance between B and C is 14.40±0.05cm.

Box	BC (s)	BC (s)	BC (s)	AvgBC	VelBC	Reynolds
1	5.5	5.41	5.51	5.47333	0.02631	0.10379
2	3.1	3.2	3.2	3.16667	0.04547	0.23798
3	2.1	2.1	2.1	2.1	0.06857	0.44167
4	1.5	1.57	1.61	1.56	0.09231	0.7209
5	1.1	1.2	1.11	1.13667	0.12669	1.14238

FIG. 2. Table of the new data collected by using the video recording measurement method. Each measurement has an uncertainty of 0.5s due to the frame rate of the video. Each of the BC (s) columns refer to a different trial than the previous column. The meaning of BC and AvgBC are the same as in Fig. 1.

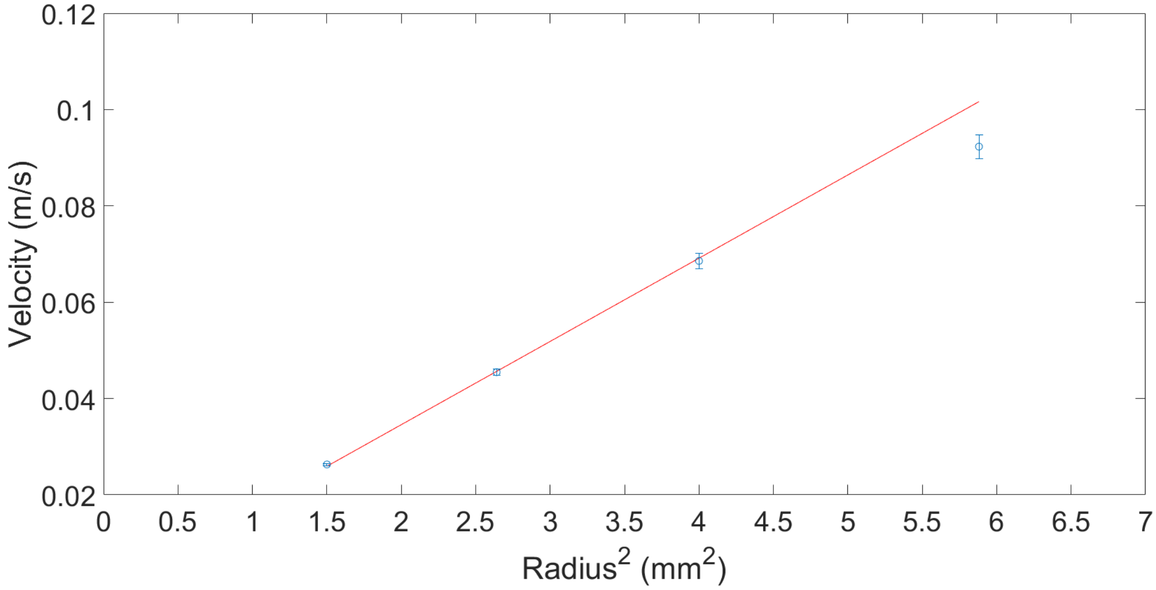

FIG. 3. Plot of the terminal velocity of the spheres against the radius squared of the spheres. The red line is the fit to the curve and the blue markers are the data points with their respective uncertainties attached. This fit is using the equation that assumes the sphere is dropping in an unlimited amount of liquid.

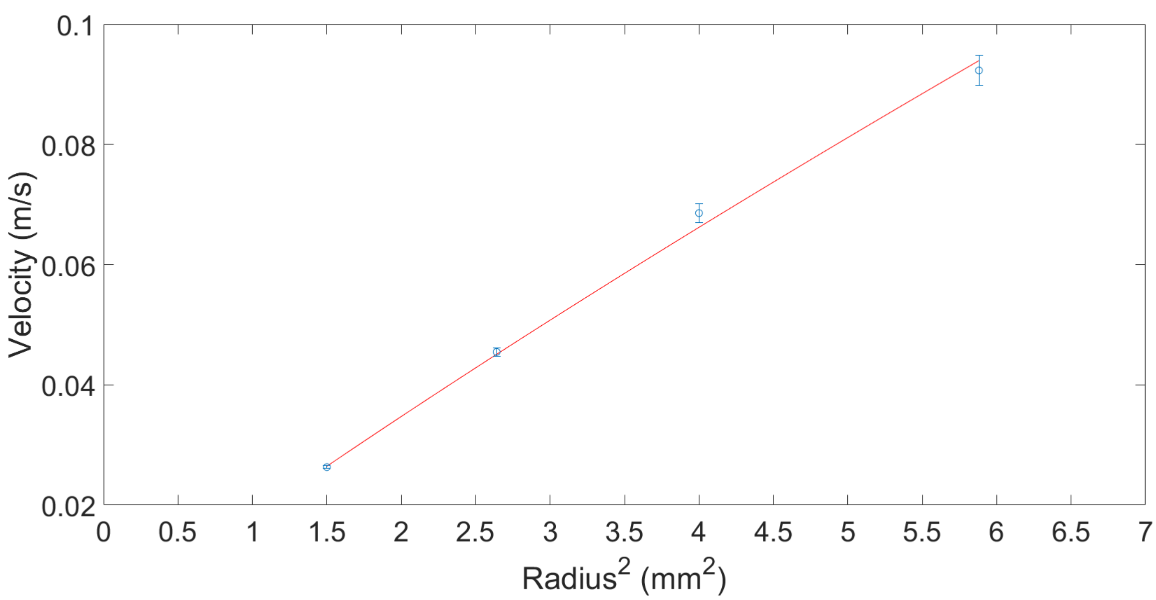

FIG. 4. Plot of the terminal velocity of the spheres against the radius squared of the spheres. The red line is the fit to the curve and the blue markers are the data points with their respective uncertainties attached. This fit takes into account that the liquid is enclosed by the graduated cylinder