Exploring a Mechanical Resonance System's Relations to the Harmonic Oscillator Model

Yamato Nakahara

April 7, 2017

In this report we investigated how well the model of a harmonic oscillator,

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + w_0^2 x = 0 \tag{1}$$

matched with the data obtained via on LabVIEW, with x being the displacement of the object undergoing oscillation, w_0 is the natural oscillation frequency and λ is the damping coefficient. The free decay is measured by pushing a saw blade attached to a brick and letting it decay naturally under different damping conditions. The forced oscillation is forced by a mechanical oscillator and is also used to find the resonance frequency of the oscillating system.The damping conditions were applied by placing a copper block above the accelerometer, and by covering different amounts of the accelerometer we set the different damping conditions.

First we examined the free decay of the oscillator to examine what the natural oscillation frequency of the system was and we found that by calculating the period of the system and using the equation 2πf= $\frac{1}{T}$ we found the natural oscillating frequency to be 70.7 rad/s. Then we set the accelerometer so that it was 7.8±.1cm from the brick and set the oscillation frequency of the mechanical vibrator to 12.17Hz, and found that the voltage of the accelerometer when the accelerometer is at rest is 1.5V. We then found the frequency of the system under random damping conditions with Figure 1 being an example of one of the tests. After five tests we found that the mean frequency of the system was 76.1 rad/s. Using Figure 1 as an example, we found the Q factor to be 63.33 using the equation

$$Q = \frac{w_0}{2\lambda} \tag{2}$$

which is expected to be the highest as it is the lowest damping condition. Also while testing the different damping conditions, we found that the system was undergoing underdamped decay at all damping conditions, including the maximum damping condition.

Next we moved onto analyzing the system when it is undergoing forced oscillation. First we found the resonance frequency of the system when the mechanical oscillator is touching the saw blade but not yet applying a force, and found the new resonance frequency to be around 91.58 rad/s. As seen in Figure 2, the fit of the data has the wrong initial amplitude as the mechanical oscillator acts as another unaccounted external damping force that damps the amplitude of the system as we began collecting the data.

We then put the system under the force of the mechanical oscillator to examine how it relates to the equation

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + w_0^2 x = \frac{F_0}{m}\exp(iwt) \tag{3}$$

and drove the oscillator at a constant voltage of 3V peak to peak and drove it at different frequencies to see how accurate our predicted resonance frequency was. As we wanted a moderate amount of points around the resonance frequency and a few points for from it, we drove the saw blade at 4Hz, 6Hz, 11Hz, 12Hz, 13Hz, 14Hz, 15Hz, 16Hz, 17Hz, 18Hz, 22Hz, and 24Hz.To measure the amplitudes of the accelerometer voltage and the mechanical oscillator's driving voltage, in addition to the phase shift between the mechanical oscillator and the accelerometer, we plotted the data onto one graph as seen in Figure 3. To measure the phase shift we took the difference of the positions of the peaks of the two waves and the amplitudes were found by placing the cursor at the top of the waves on IGOR. As seen by comparing Figure 3 and Figure 4, even at different damping conditions the resultant waves are just offset sine waves with differing amplitudes and wavelengths so the same measurement methods were used for all frequencies and all damping conditions.

After collecting our data we plotted our data and fit the data using the equation of the amplitude of a forced oscillating system,

$$A(w) = \frac{aw}{\sqrt[2]{(w_0^2 - w^2)^2 + 4\lambda w^2}} + \phi \tag{4}$$

where a is the scaling factor, w is the driving frequency, w_0 is the resonance frequency, ϕ is the phase shift, and λ is the damping coefficient. As seen in Figure 5, the low damping conditions result in a sharper and higher peak while adding more damping results in a wider and lower peak as the damping increases. The values we found through the fits for graph a) were, $a = 0.075 \pm 0.004 rad^2/s$, $w_0 = 14.58 \pm 0.02$ rad/s, $\lambda = 0.66 \pm 0.05 rad/s$, and $\phi = 1.46 \pm 0.01$ rad. Values for b) were, $a = 0.099 \pm 0.005 rad^2/s$, $w_0 = 14.85 \pm 0.04$ rad/s, $\lambda = 0.54 \pm 0.03 rad/s$, and $\phi = 1.423 \pm 0.018$ rad. Values for c) were, $a = 0.066 \pm 0.003 rad^2/s$, $w_0 = 15.13 \pm 0.05$ rad/s, $\lambda = 1.37 \pm 0.07 rad/s$, and $\phi = 1.461 \pm 0.006$ rad. Next we plotted the phase shift against the frequency and fit the plot with

$$\phi(w) = arctan \frac{-2\lambda w_0}{w_0^2 - w^2 + \phi} \tag{5}$$

using the atan2 function on IGOR instead of arctan. Unfortunately it appears that during our experiment we reversed our wires and had our phase shifts go from 0 to negative π instead of π to 0 as was expected as seen on Figure 6.

By analyzing our results we found that the system follows the equations of the model of the harmonic oscillator as the freely decaying saw blade without the mechanical oscillator touching it as the fit for the decay of the free oscillation decayed as expected. Although our phase shifts were reversed, we also confirmed that the saw blade system undergoing forced oscillations follows the equation of the peak amplitude in addition to the resonance frequency. The Q factors we found using Eq. 2 with the damping coefficients from the forced oscillation were 86.96, 69.41, and 35.01 for the low, moderate, and high damping conditions respectively. These results reflect on the the sharpness of the peak in the amplitude fits in Figure 5. We also found that driving the system at a higher voltage causes problems in relation to the harmonic oscillating equation as the material of the saw blade allows it to bend at certain voltages and causes the oscillation to split into two modes.

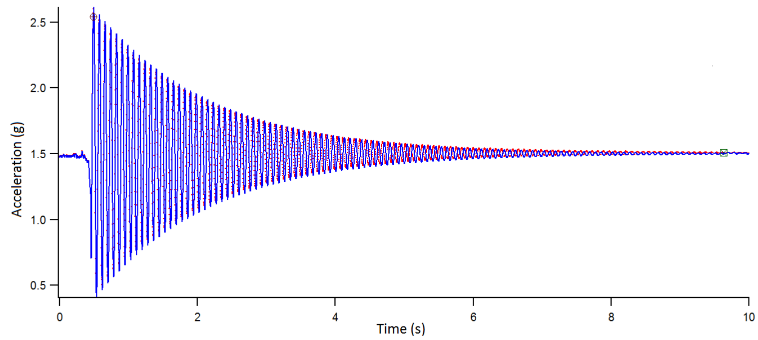

Figure 1: Figure of the natural decay at the lowest damping coefficient. The dots are the data points while the blue line is the fit found using the equation $v(t) = V_0 \exp(-\lambda t) \cos(w_1 t + \phi)$. From the fit we received values of 76 rad/s for the natural oscillation frequency, 0.6 for the λ damping coefficient and 0 for the phase shift as this is natural oscillation frequency.

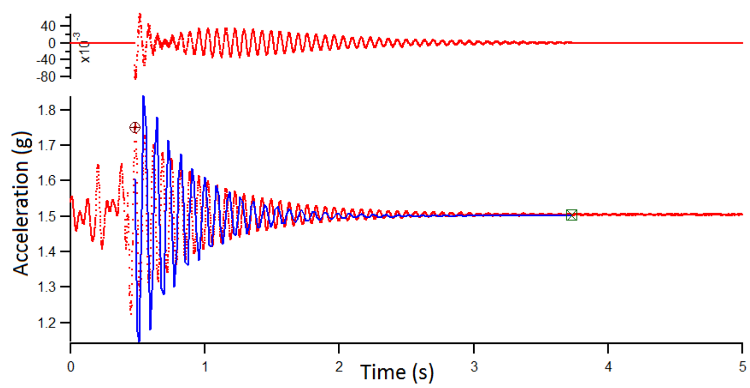

Figure 2: Figure displaying natural decay of the system while under the influence of an object touching the oscillator but not forcing it.The red dots are the data while the blue line is the fit using the same equation as Figure 1. The top graph is the residuals between the data and the fit. The values we received are $w = 93.19 \pm 0.03 rad/s$, $\lambda = 3.119 \pm 0.007 s^{-1}$, and $\phi = 19.90 \pm 0.02 rad$ where w is the oscillation frequency, λ is the damping coefficient and ϕ is the phase shift value.

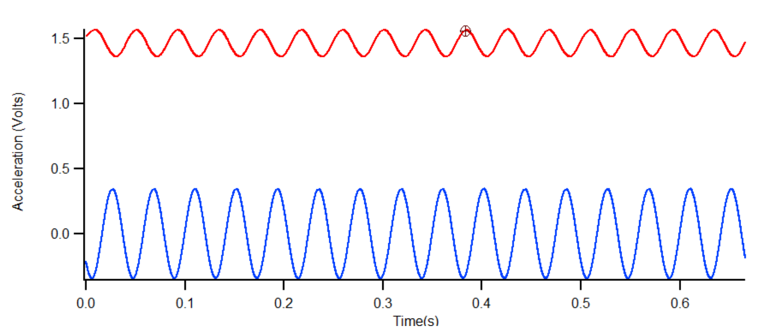

Figure 3: Plot of the measured voltage of the accelerometer and the measured voltage of the mechanical oscillator against time while driving it at 24Hz under moderate damping conditions.. Red line is the accelerometer data and the blue line is the mechanical oscillator data.

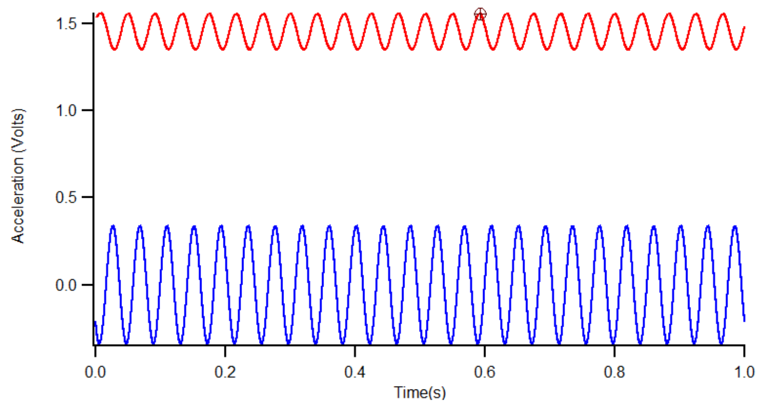

Figure 4: Plot of the measured voltage of the accelerometer and the measured voltage of the mechanical oscillator against time while driving it at 24Hz under the maximum damping conditions.. Red line is the accelerometer data and the blue line is the mechanical oscillator data.

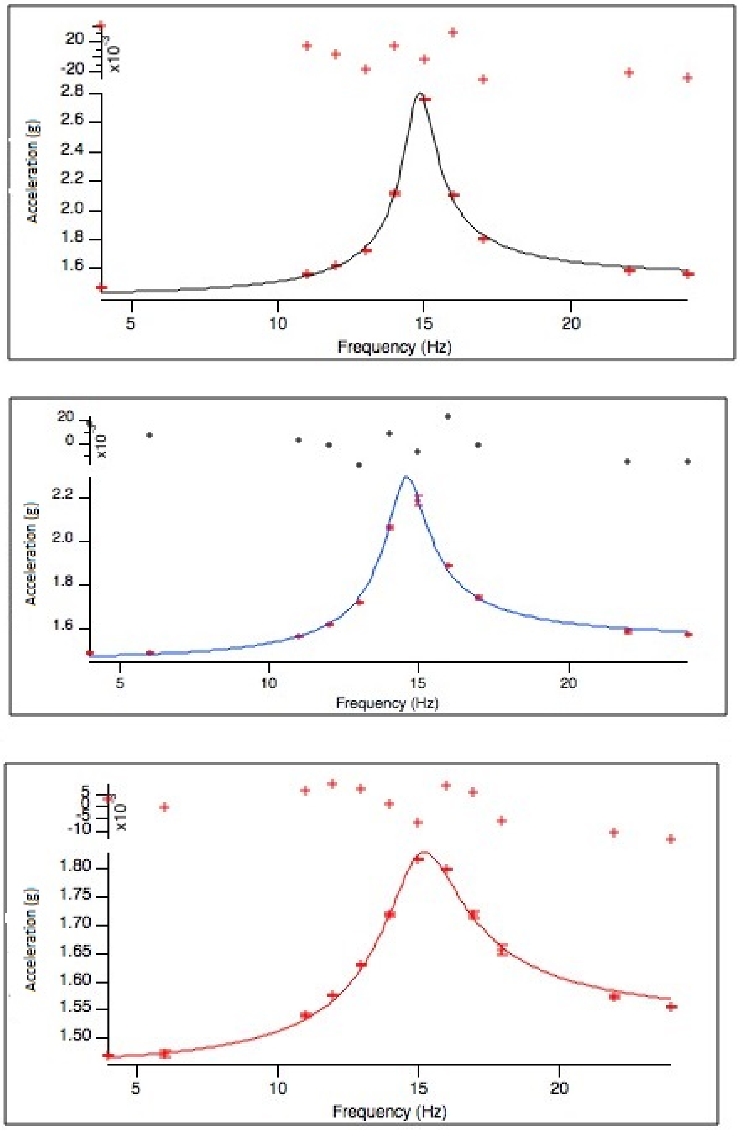

Figure 5: Plots of acceleration for an accelerometer undergoing forced oscillations against the frequency.The Y-Axis is Acceleration (Volts) while the X-Axis is Frequency(Hz). The blue solid line is the curve fit and the red markers on the graph are the collected data points along with their respective standard deviations on each point. The standard deviation were found by taking the standard deviations of the individual values used in Eq. 4 and combined the errors. Above the graph are the residuals from the differences between the fit line and the actual data points.From top to bottom the plots show the lowest damping condition, the moderate damping condition and the maximum damping condition.

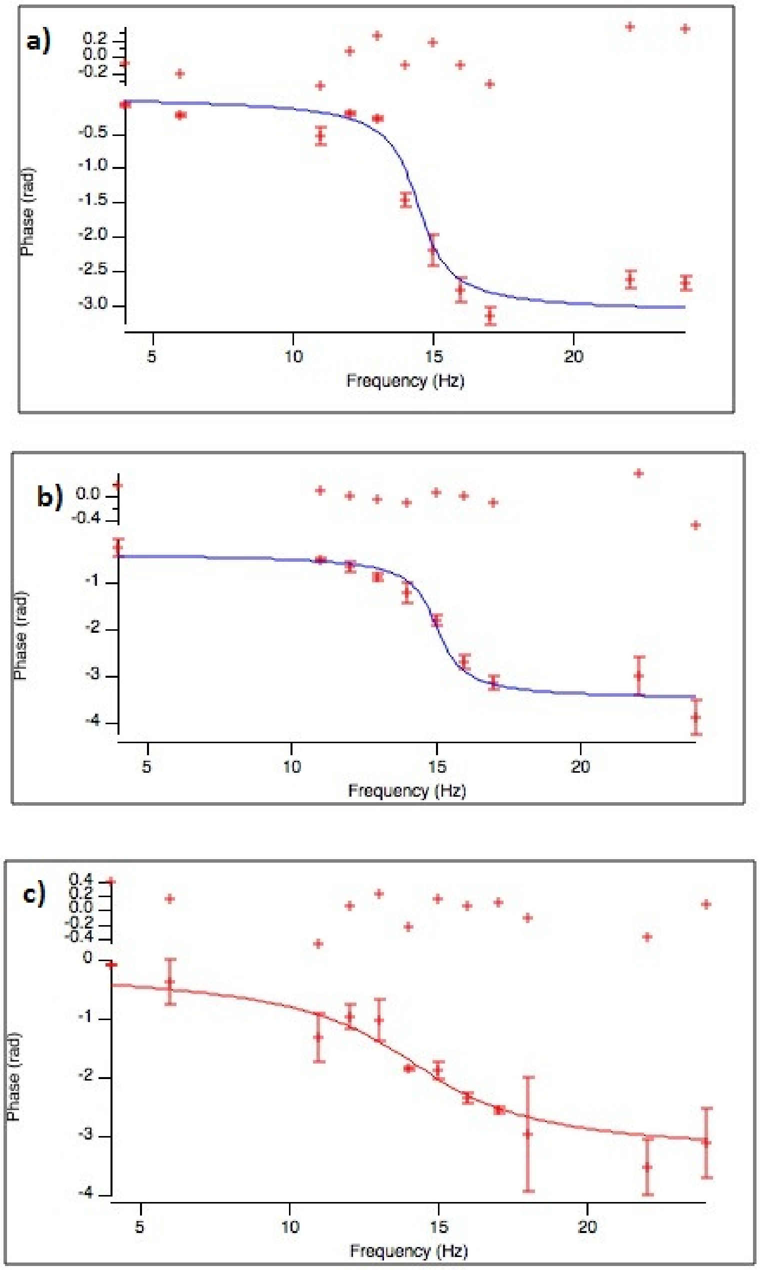

Figure 6: Plots of the phase shift against the frequency. The dots are the data points with their respective standard deviations found by combining the individual standard deviation from Eq. 6, and the solid line is the fit from using Eq. 6. The plot above the graph is the residual plot between the fit and the data points. a) is low damping, b) is moderate damping and c) is high damping.