**Data and Visualization**

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Table of Contents

[1 Business Understanding 3](#_Toc182403222)

[2 Data Preparation 4](#_Toc182403223)

[2.1 First k-means, Hierarchical, DBSCAN, and Fuzzy Clusterings 7](#_Toc182403224)

[2.2 Second k-means, Hierarchical, DBSCAN, and Fuzzy Clusterings 7](#_Toc182403225)

[2.3 Third k-means, Hierarchical, DBSCAN, and Fuzzy Clusterings 8](#_Toc182403226)

[2.4 Fourth k-means, Hierarchical, DBSCAN, and Fuzzy Clusterings 8](#_Toc182403227)

[3 Modeling 9](#_Toc182403228)

[3.1 K-means Clustering 11](#_Toc182403229)

[3.1.1 First k-means Clustering 11](#_Toc182403230)

[3.1.2 Second k-means Clustering 15](#_Toc182403231)

[3.1.3 Third k-means Clustering 20](#_Toc182403232)

[3.1.4 Fourth k-means Clustering 24](#_Toc182403233)

[3.2 Hierarchical Clustering 30](#_Toc182403234)

[3.2.1 Identifying the Linkage method 30](#_Toc182403235)

[3.2.2 First Hierarchical Clustering 30](#_Toc182403236)

[3.2.3 Second Hierarchical Clustering 36](#_Toc182403237)

[3.2.4 Third Hierarchical Clustering 42](#_Toc182403238)

[3.2.5 Fourth Hierarchical Clustering 47](#_Toc182403239)

[3.2.6 Supervised Cluster evaluation 53](#_Toc182403240)

[3.3 Density Based (DBSCAN) Clustering 54](#_Toc182403241)

[3.3.1 First Density Based Clustering 54](#_Toc182403242)

[3.3.2 Second Density Based Clustering 64](#_Toc182403243)

[3.3.3 Third Density Based Clustering 72](#_Toc182403244)

[3.3.4 Fourth Density Based Clustering 76](#_Toc182403245)

[3.3.5 Supervised Cluster evaluation 86](#_Toc182403246)

[3.4 Fuzzy Clustering 86](#_Toc182403247)

[3.4.1 First Fuzzy Clustering 87](#_Toc182403248)

[3.4.2 Second Fuzzy Clustering 93](#_Toc182403249)

[3.4.3 Third Fuzzy Clustering 99](#_Toc182403250)

[3.4.4 Fourth Fuzzy Clustering 105](#_Toc182403251)

[3.4.5 Supervised Evaluation 111](#_Toc182403252)

[4 Evaluation 112](#_Toc182403253)

[4.1 K-means clustering 112](#_Toc182403254)

[4.2 Hierarchical clustering 116](#_Toc182403255)

[4.3 Density Based clustering 117](#_Toc182403256)

[4.4 Fuzzy clustering 117](#_Toc182403257)

[5 List of References 117](#_Toc182403258)

[6 Appendix 118](#_Toc182403259)

[6.1 Graduate level additional work 118](#_Toc182403260)

# Business Understanding

A widespread disease COVID-19 (also known as coronavirus disease 2019) started at the end of 2019, and it quickly spread throughout the entire world impacting every aspect of human society. It was first identified in December 2019 in Wuhan district in China. Since then, there have been several kinds of studies conducted on the impact of this pandemic. The data on those studies are available for the general public. Among those datasets, we will be looking at four different datasets. The primary focus of the analysis is based on understanding the impact of this pandemic on various aspects of human society all around the world from 2019. These policies were primarily implemented to enforce “social distancing” amongst people. Social distancing involves measures taken to reduce close contact between individuals to slow the spread of infectious diseases such as COVID-19. By implementing measures like social distancing, mask-wearing, and hygiene practices, the goal is to spread out the number of cases over a longer period, resulting in a flatter curve.

This report tries to cluster various Texas counties (excluding counties from Austin, Dallas Fort Worth, San Antonio, and Houston) based on data available for various socio-economic factors such as income, age, working style, etc. Various clustering practices have been experimented to perform this cluster analysis. The report is primarily focused on providing various insights on statistics on COVID-19 confirmed cases and death rates to the Texas Department of State Health Services (DSHS). As the state-level health agency, DSHS is responsible for public health policy, disease surveillance, and health interventions in Texas. The analysis can provide insights into how socio-economic and health factors intersect to influence COVID-19 outcomes across different counties in Texas. This can help DSHS make more informed decisions about policy, resource allocation, and public health strategies. Using various clustering analysis techniques, we want to answer following questions:

* What counties does DSHS need to prioritize for targeted health interventions based on socio-economic and demographic factors?
* What factors (e.g., income, race, working style) contribute to higher vulnerability to COVID-19 cases and deaths in certain counties?
* Where are the counties, that need the highest state level assistance, located? Is there a certain pattern that can be observed on such demography based on various clustering techniques?
* Can we recognize counties that were more impacted by COVID based on income, race and the ability to work from home. Are the lower income communities impacted more than the higher income communities?

# Data Preparation

The dataset “COVID-19\_cases\_plus\_census” is extracted from USAFacts US Coronavirus Database (USAFacts).

The dataset has 259 feature columns and 3142 observations. These observations represent US COVID-19 cases and

death counts for all US states and counties. This is made available for the general public and is hosted in Google BigQuery. To maintain the focus of the analysis limited to specific aspects of the dataset, there were many feature columns removed or updated. For example, the variables providing information on the various income levels could be reduced to income ranges that would still provide enough insights on the financial status of the general public. Thus, the data of those variables were merged into four new income ranges: less than 50K, 50-100K, 100-150K, and 150K-above. This allowed us to drop the previous feature columns and reduce the dimension of the dataset. Following are some more feature processing performed on the dataset:

* 8 variables, containing information on certain percentage of income spent on rent, were merged into 2 new variables (i.e. rent\_under\_50\_percent and rent\_over\_50\_percent)
* variables containing various structures of families were dropped because they were less significant than people’s financial structures
* variables containing counts of males and females for various age groups were merged into new variables that contain counts of males and females for age groups 0-20, 21-49, and 50 above
* dropped variables that contained the counts of people belonging to multiple races and rather maintained emphasis on the counts of people belonging to only one specific race
* dropped variables that contained the counts of people holding various academic qualifications and rather maintained emphasis on income, race, working environment, gender and age
* grouped all the variables that contained the counts of people commuting using various methods and for certain time to work into simply a new group “commute” to have all the counts commuting to work

After all the feature processing completed, there were 3142 observations and 33 variables left in the dataset. The final dataset has no null values. There were also no duplicate observations found in the final dataset.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Feature** | **Statistics** |  | **Feature** | **Statistics** |  | **Feature** | **Statistics** |
| county\_name | Length:222  Class :character  Mode :character |  | white\_pop | Min. : 55  1st Qu.: 2580  Median : 7632  Mean : 18563  3rd Qu.: 18465  Max. :162449 |  | female\_0\_20 | Min. : 4.0  1st Qu.: 771.5  Median : 1880.5  Mean : 6437.0  3rd Qu.: 4783.8  Max. :159914.0 |
| male\_pop | Min. : 1  1st Qu.: 436  Median : 1077  Mean : 3619  3rd Qu.: 2544  Max. :107552 |  | black\_pop | Min. : 0.00  1st Qu.: 55.25  Median : 482.50  Mean : 3292.94  3rd Qu.: 2673.00  Max. :85901.00 |  | female\_21\_49 | Min. : 8.0  1st Qu.: 898.5  Median : 2184.0  Mean : 7743.7  3rd Qu.: 5924.0  Max. :164411.0 |
| female\_pop | Min. : 0.00  1st Qu.: 12.00  Median : 25.50  Mean : 77.73  3rd Qu.: 61.75  Max. :2018.00 |  | asian\_pop | Min. : 0.0  1st Qu.: 6.0  Median : 45.0  Mean : 556.8  3rd Qu.: 241.8  Max. :12857.0 |  | female\_50\_above | Min. : 23  1st Qu.: 1184  Median : 3010  Mean : 7664  3rd Qu.: 7389  Max. :136600 |
| hispanic\_pop | Min. : 12  1st Qu.: 1468  Median : 3700  Mean : 19338  3rd Qu.: 9827  Max. :770794 |  | amerindian\_pop | Min. : 0.0  1st Qu.: 5.0  Median : 30.5  Mean : 125.9  3rd Qu.: 113.5  Max. :2344.0 |  | unemployed\_pop | Min. : 0.0  1st Qu.: 121.2  Median : 382.0  Mean : 1197.7  3rd Qu.: 960.5  Max. :27566.0 |
| other\_race\_pop | Min. : 0.00  1st Qu.: 0.00  Median : 0.00  Mean : 31.38  3rd Qu.: 26.75  Max. :477.00 |  | median\_income | Min. :24794  1st Qu.:41537  Median :46412  Mean :47428  3rd Qu.:52295  Max. :80938 |  | employed\_pop | Min. : 39  1st Qu.: 2421  Median : 5354  Mean : 17677  3rd Qu.: 14372  Max. :341350 |
| income\_less\_50K | Min. : 9  1st Qu.: 1111  Median : 2740  Mean : 7589  3rd Qu.: 6524  Max. :148982 |  | income\_50K\_100K | Min. : 19.0  1st Qu.: 534.8  Median : 1427.5  Mean : 4176.5  3rd Qu.: 3627.0  Max. :74071.0 |  | commute | Min. : 66  1st Qu.: 4344  Median : 9925  Mean : 32754  3rd Qu.: 26827  Max. :647479 |
| income\_100K\_150K | Min. : 0.0  1st Qu.: 176.2  Median : 490.5  Mean : 1621.6  3rd Qu.: 1382.8  Max. :25925.0 |  | income\_150K\_more | Min. : 0.0  1st Qu.: 114.8  Median : 274.0  Mean : 984.0  3rd Qu.: 772.2  Max. :14222.0 |  | worked\_at\_home | Min. : 0.0  1st Qu.: 68.5  Median : 190.0  Mean : 598.5  3rd Qu.: 515.0  Max. :15026.0 |
| rent\_under\_50\_percent | Min. : 7.0  1st Qu.: 289.5  Median : 820.0  Mean : 3317.0  3rd Qu.: 2171.0  Max. :73181.0 |  | rent\_over\_50\_percent | Min. : 0.0  1st Qu.: 60.5  Median : 185.0  Mean : 1027.0  3rd Qu.: 599.0  Max. :19775.0 |  | walked\_to\_work | Min. : 0.0  1st Qu.: 47.0  Median : 101.5  Mean : 338.6  3rd Qu.: 242.2  Max. :6964.0 |
| median\_age | Min. :25.80  1st Qu.:34.67  Median :39.20  Mean :39.31  3rd Qu.:43.27  Max. :57.50 |  | male\_0\_20 | Min. : 3  1st Qu.: 837  Median : 2079  Mean : 6785  3rd Qu.: 5274  Max. :163853 |  | male\_21\_49 | Min. : 10  1st Qu.: 971  Median : 2600  Mean : 8189  3rd Qu.: 6300  Max. :167493 |
|  |  |  | male\_50\_above | Min. : 29  1st Qu.: 1112  Median : 2830  Mean : 6771  3rd Qu.: 6769  Max. :109200 |  |  |  |

Statistics of the dataset for all clusterings

Shown above are all the features that are used to perform all clusterings. The table also shows basic statistics of these features. Only the data points from Texas counties (excluding counties that belong to Dallas Fort Worth, Austin, San Antonio, and Houston) were selected for the dataset. Those counties that belonged to the four major cities were not taken into consideration because those counties hold the majority of the Texans which could skew our analysis. The objects for all clusterings chosen to cluster are the counties in Texas. Each county is represented by a row, and the clustering is based on features including the number of people in various racial groups and income ranges. Different sets of features for all clusterings were chosen because we are interested in understanding the clustering of people living in Texas counties based on their racial demographics and income levels. By clustering based on these features, we aim to identify patterns and groupings that can provide insights into the socio-economic and racial composition of different regions within these counties. This can help in policy making, resource allocation, and understanding demographic trends.

In this report, the several combinations of these features are selected to perform various types of clustering to understand clusterings of counties based on various demographic and socio-economic attributes. The dataset for each clustering was scaled to ensure that all of the datapoints lie within similar range. All of the features are on ratio scale because they represent counts of people from various counties that fall under these features. For k-means, DBSCAN, and fuzzy clusterings, we are using Euclidean distance for similarity/distance. This method was chosen because its simplicity, computational efficiency, and natural fit for geometric clusters. For the hierarchical clustering, we chose wards minimum variance method for similarity/distance. The agglomerative coefficient of the below methods was calculated, and the best method was chosen. The below table shows these methods and the corresponding coefficient for this dataset. Based on the Agglomerative Coefficient results, the maximum coefficient was for the Wards minimum variance method that the same is used for the clustering.

| **Linkage Method** | **Description** | **Agglomerative coefficient for this dataset** |
| --- | --- | --- |
| Complete linkage clustering: | It computes all pairwise dissimilarities between the elements in cluster 1 and the elements in cluster 2, and considers the largest value (i.e., maximum value) of these dissimilarities as the distance between the two clusters. It tends to produce more compact clusters. | 0.9422264 |
| Single linkage clustering | computes all pairwise dissimilarities between the elements in cluster 1 and the elements in cluster 2 and considers the smallest of these dissimilarities as a linkage criterion. It tends to produce long, “loose” clusters. | 0.8757889 |
| Mean or average linkage clustering: | It computes all pairwise dissimilarities between the elements in cluster 1 and the elements in cluster 2 and considers the average of these dissimilarities as the distance between the two clusters. | 0.9272677 |
| Ward’s minimum variance method: | It minimizes the total within-cluster variance. At each step the pair of clusters with minimum between-cluster distance are merged. | 0.9702966 |

## First k-means, Hierarchical, DBSCAN, and Fuzzy Clusterings

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Feature** | **Statistics** |  | **Feature** | **Statistics** |  | **Feature** | **Statistics** |
| white\_pop | Min. :-0.642387  1st Qu.:-0.554748  Median :-0.379400  Mean : 0.000000  3rd Qu.:-0.003394  Max. : 4.994073 |  | rent\_under\_50\_percent | Min. :-0.4068  1st Qu.:-0.3720  Median :-0.3069  Mean : 0.0000  3rd Qu.:-0.1408  Max. : 8.5854 |  | male\_21\_49 | Min. :-0.42670  1st Qu.:-0.37657  Median :-0.29161  Mean : 0.00000  3rd Qu.:-0.09854  Max. : 8.31051 |
| income\_100K\_150K | Min. :-0.49239  1st Qu.:-0.43887  Median :-0.34345  Mean : 0.00000  3rd Qu.:-0.07254  Max. : 7.37936 |  | commute | Min. :-0.4412  1st Qu.:-0.3834  Median :-0.3081  Mean : 0.0000  3rd Qu.:-0.0800  Max. : 8.2967 |  |  |  |

Statistics of the dataset for first clustering (k-means, hierarchical, DBSCAN, Fuzzy)

Shown above are the features related to white male aged 21-49. These men make 100-150k and spent under 50% on their rents, and they also commute to work. We will be using these features for clustering the counties using various clustering methods to answer the questions outlined in the business understanding section.

## Second k-means, Hierarchical, DBSCAN, and Fuzzy Clusterings

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Feature** | **Statistics** |  | **Feature** | **Statistics** |  | **Feature** | **Statistics** |
| white\_pop | Min. :-0.642387  1st Qu.:-0.554748  Median :-0.379400  Mean : 0.000000  3rd Qu.:-0.003394  Max. : 4.994073 |  | income\_100K\_150K | Min. :-0.49239  1st Qu.:-0.43887  Median :-0.34345  Mean : 0.00000  3rd Qu.:-0.07254  Max. : 7.37936 |  | rent\_under\_50\_percent | Min. :-0.4068  1st Qu.:-0.3720  Median :-0.3069  Mean : 0.0000  3rd Qu.:-0.1408  Max. : 8.5854 |
| male\_21\_49 | Min. :-0.42670  1st Qu.:-0.37657  Median :-0.29161  Mean : 0.00000  3rd Qu.:-0.09854  Max. : 8.31051 |  | commute | Min. :-0.4412  1st Qu.:-0.3834  Median :-0.3081  Mean : 0.0000  3rd Qu.:-0.0800  Max. : 8.2967 |  |  |  |

Statistics of the dataset for second clustering (k-means, hierarchical, DBSCAN, Fuzzy)

Shown above are the features related to population of Hispanic men aged 21-49 making 100-150K, commuted to work, and spent more than 50% on rent. We will be using these features for clustering the counties using various clustering methods to answer the questions outlined in the business understanding section.

## Third k-means, Hierarchical, DBSCAN, and Fuzzy Clusterings

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Feature** | **Statistics** |  | **Feature** | **Statistics** |  | **Feature** | **Statistics** |
| hispanic\_pop | Min. :-0.2510  1st Qu.:-0.2321  Median :-0.2031  Mean : 0.0000  3rd Qu.:-0.1235  Max. : 9.7590 |  | income\_50K\_100K | Min. :-0.48025  1st Qu.:-0.42067  Median :-0.31755  Mean : 0.00000  3rd Qu.:-0.06348  Max. : 8.07375 |  | female\_21\_49 | Min. :-0.40366  1st Qu.:-0.35719  Median :-0.29011  Mean : 0.00000  3rd Qu.:-0.09495  Max. : 8.17512 |
| worked\_at\_home | Min. :-0.40128  1st Qu.:-0.35535  Median :-0.27388  Mean : 0.00000  3rd Qu.:-0.05596  Max. : 9.67399 |  |  |  |  |  |  |

Statistics of the dataset for third clustering (k-means, hierarchical, DBSCAN, Fuzzy)

Shown above are the features related to population of Hispanic women aged 21-49 making 50-100K and worked from home. We will be using these features for clustering the counties using various clustering methods to answer the questions outlined in the business understanding section.

## Fourth k-means, Hierarchical, DBSCAN, and Fuzzy Clusterings

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Feature** | **Statistics** |  | **Feature** | **Statistics** |  | **Feature** | **Statistics** |
| hispanic\_pop | Min. :-0.2510  1st Qu.:-0.2321  Median :-0.2031  Mean : 0.0000  3rd Qu.:-0.1235  Max. : 9.7590 |  | income\_50K\_100K | Min. :-0.48025  1st Qu.:-0.42067  Median :-0.31755  Mean : 0.00000  3rd Qu.:-0.06348  Max. : 8.07375 |  | male\_21\_49 | Min. :-0.42670  1st Qu.:-0.37657  Median :-0.29161  Mean : 0.00000  3rd Qu.:-0.09854  Max. : 8.31051 |
| commute | Min. :-0.4412  1st Qu.:-0.3834  Median :-0.3081  Mean : 0.0000  3rd Qu.:-0.0800  Max. : 8.2967 |  | rent\_over\_50\_percent | Min. :-0.3775  1st Qu.:-0.3553  Median :-0.3095  Mean : 0.0000  3rd Qu.:-0.1573  Max. : 6.8913 |  |  |  |

Statistics of the dataset for fourth clustering (k-means, hierarchical, DBSCAN, Fuzzy)

Shown above are the features related to population of Hispanic men aged 21-49 making 50-100K, spent more than 50% on rent, and commuted to work. We will be using these features for clustering the counties using various clustering methods to answer the questions outlined in the business understanding section.

# Modeling

The below table provides a summary of the Clusterings discussed in this report

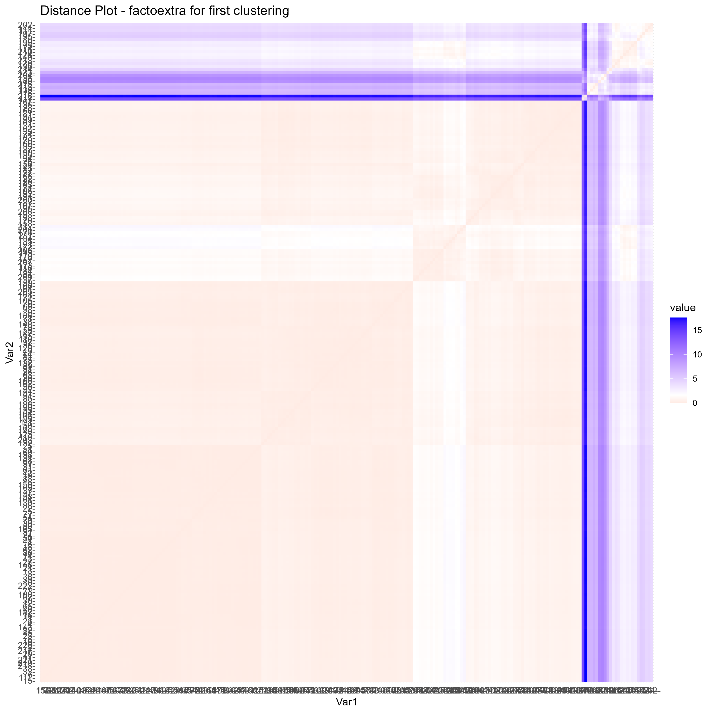
| **Clustering** | **Features Selected** | **Unsupervised**  **Evaluation** | **Supervised Evaluation** | **Similarity/Distance method** |
| --- | --- | --- | --- | --- |
| k-means clustering (first clustering) | * white\_pop, * income\_100K\_150K, * rent\_under\_50\_percent, * male\_21\_49, commute | * Average Silhouette Width * Dunn Index * Pearson gamma * Within cluster sum of squares * Elbow Method * Gap Statistic | Purity  Entropy | Euclidean distance |
| k-means clustering (second clustering) | * hispanic\_pop, * income\_50K\_100K, * female\_21\_49, * worked\_at\_home | * Average Silhouette Width * Dunn Index * Pearson gamma * Within cluster sum of squares * Elbow Method * Gap Statistic | Purity  Entropy | Euclidean distance |
| k-means clustering (third clustering) | * income\_100K\_150K, * rent\_under\_50\_percent, * male\_50\_above, * commute | * Average Silhouette Width * Dunn Index * Pearson gamma * Within cluster sum of squares * Elbow Method * Gap Statistic | Purity  Entropy | Euclidean distance |
| k-means clustering (fourth clustering) | * hispanic\_pop, * income\_50K\_100K, * male\_21\_49, * commute, * rent\_over\_50\_percent | * Average Silhouette Width * Dunn Index * Pearson gamma * Within cluster sum of squares * Elbow Method * Gap Statistic | Purity  Entropy | Euclidean distance |
| Hierarchical Clustering  (first clustering) | * white\_pop, * income\_100K\_150K, * rent\_under\_50\_percent, * male\_21\_49, commute | * Gap Statistic * Average Silhouette Width | Purity | Wards minimum variance method |
| Hierarchical Clustering  (second clustering) | * hispanic\_pop, * income\_50K\_100K, * female\_21\_49, * worked\_at\_home | * Gap Statistic * Average Silhouette Width | Purity | Wards minimum variance method |
| Hierarchical Clustering  (third clustering) | * income\_100K\_150K, * rent\_under\_50\_percent, * male\_50\_above, * commute | * Gap Statistic * Average Silhouette Width | Purity | Wards minimum variance method |
| Hierarchical Clustering  (fourth clustering) | * hispanic\_pop, * income\_50K\_100K, * male\_21\_49, * commute, * rent\_over\_50\_percent | * Gap Statistic * Average Silhouette Width | Purity | Wards minimum variance method |
| Density based Clustering  (first clustering) | * white\_pop, * income\_100K\_150K, * rent\_under\_50\_percent, * male\_21\_49, commute | * Dunn Index * Average Silhouette Width | Purity | Euclidean distance |
| Density based Clustering  (second clustering) | * hispanic\_pop, * income\_50K\_100K, * female\_21\_49, * worked\_at\_home | * Dunn Index * Average Silhouette Width | Purity | Euclidean distance |
| Density based Clustering  (third clustering) | * income\_100K\_150K, * rent\_under\_50\_percent, * male\_50\_above, * commute | * Dunn Index * Average Silhouette Width | Purity | Euclidean distance |
| Density based Clustering  (fourth clustering) | * hispanic\_pop, * income\_50K\_100K, * male\_21\_49, * commute, * rent\_over\_50\_percent | * Dunn Index * Average Silhouette Width | Purity | Euclidean distance |
| Fuzzy Clustering  (first clustering) | * white\_pop, * income\_100K\_150K, * rent\_under\_50\_percent, * male\_21\_49, commute | * Davies-Bouldin index | Purity | Euclidean Distance |
| Fuzzy Clustering  (second clustering) | * hispanic\_pop, * income\_50K\_100K, * female\_21\_49, * worked\_at\_home | * Davies-Bouldin index | Purity | Euclidean Distance |
| Fuzzy Clustering  (third clustering) | * income\_100K\_150K, * rent\_under\_50\_percent, * male\_50\_above, * commute | * Davies-Bouldin index | Purity | Euclidean Distance |
| Fuzzy Clustering  (fourth clustering) | * hispanic\_pop, * income\_50K\_100K, * male\_21\_49, * commute, * rent\_over\_50\_percent | * Davies-Bouldin index | Purity | Euclidean Distance |

Clusterings evaluation and similarity/distance methods

## K-means Clustering

### First k-means Clustering

Before performing k-means clustering with certain number of clusters, we calculated the Hopkins statistic. The value was 0.99 which is close to 1 indicating that the dataset had a strong clustering tendency. Additionally, we also created following plots to visualize distance matrix of the dataset. It’s a heatmap of the distances between observations in the dataset. These plots help us to understand the structure of the data which helps us to identify any natural groupings or patterns in the dataset. This gives us a round idea of how many clusters might be appropriate based on the distributions of the distances. Based on the factoextra plot, we can say that we might need 3 clusters in the k-means clustering for this dataset.

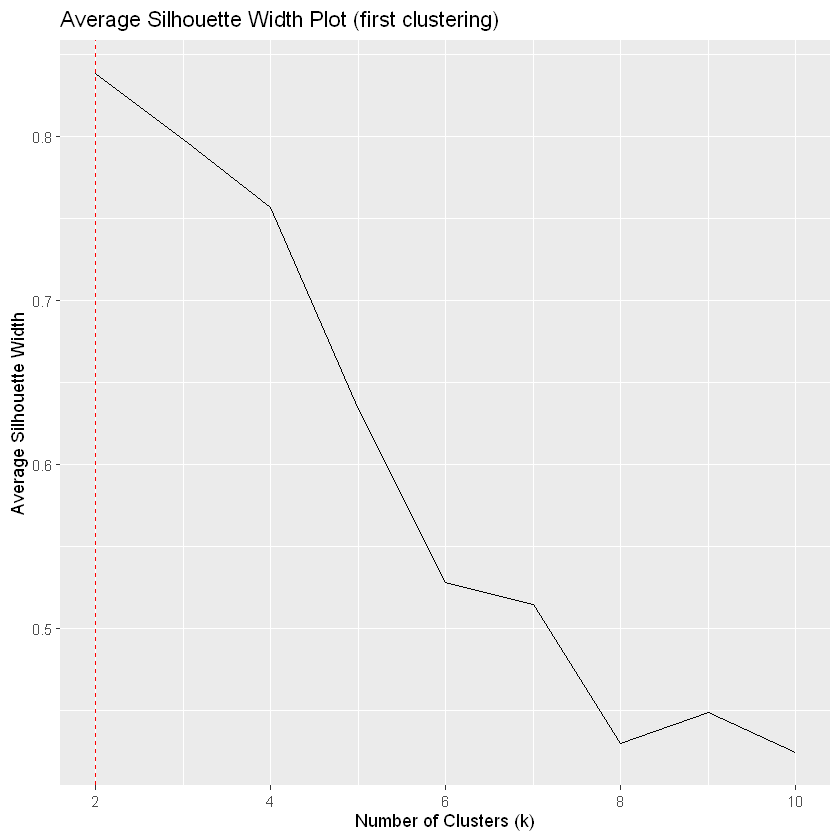


Distance matrix (left) and factoextra plot (right) for first k-means clustering

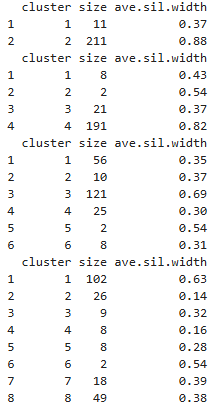
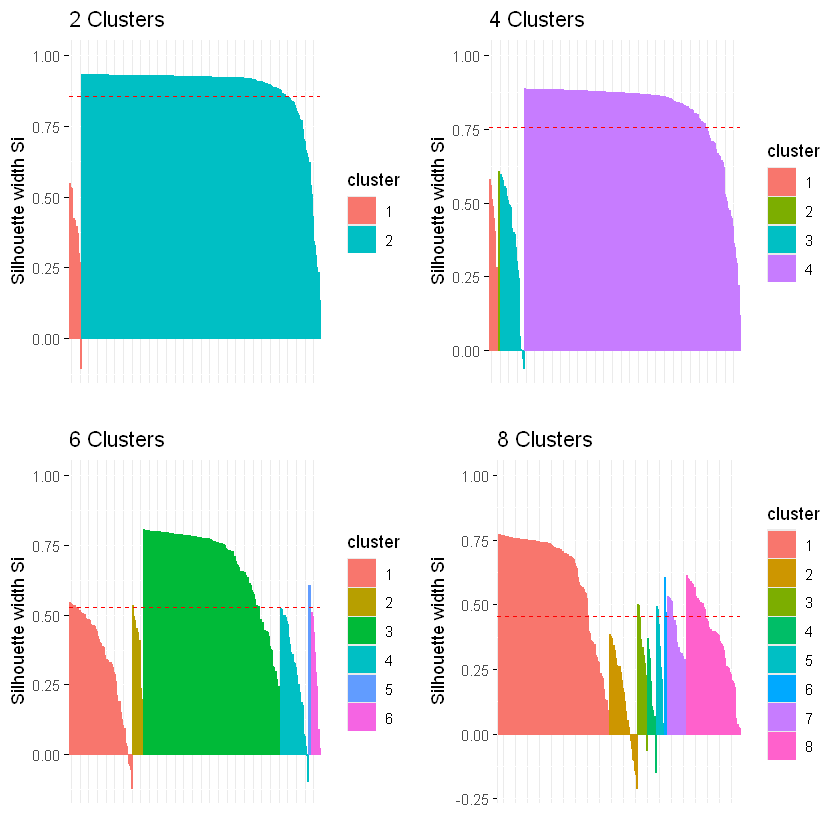
**Determine suitable number of clusters**

**Average Silhouette Width**

To determine suitable number of clusters for the dataset, we started with analyzing the average silhouette width for various number of clusters. Based on the plot below, we can see that the highest value is at 2 clusters, and the value starts to decrease rapidly with a greater number of clusters. Thus, based on this assessment, we just need 2 clusters for the dataset.



Average silhouette width plot for first k-means clustering



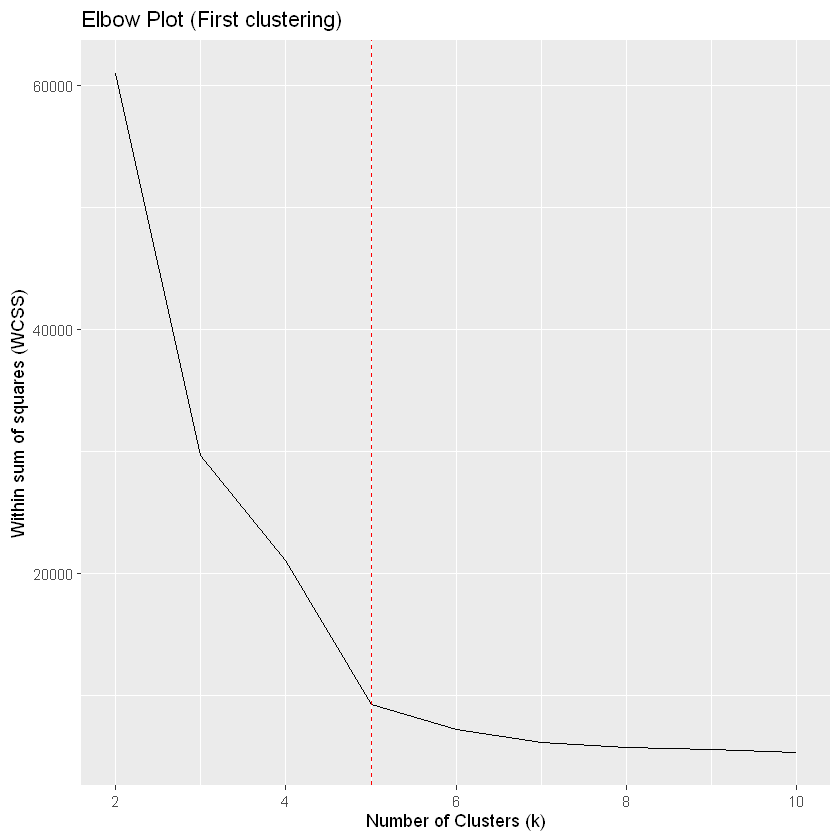
Silhouette plots for various number of clusters for first k-means clustering

Next, we look at analyzing the silhouette widths and plots for various number of clusters. Based on the above plot, 4 clusters appear to be promising, with two clusters having reasonably high silhouette widths (0.54 and 0.82), but the other two clusters have lower silhouette widths, reducing the overall quality of clustering. Also, the first and second clusters have very few datapoints. 2 Clusters seems to be the optimal choice as it has one cluster with a very high silhouette width (0.88). Although the other cluster has a lower silhouette width (0.37), the simplicity of having fewer

clusters often provide more meaningful insights without overcomplicating the interpretation.

**Elbow Method: Within-Cluster Sum of Square**

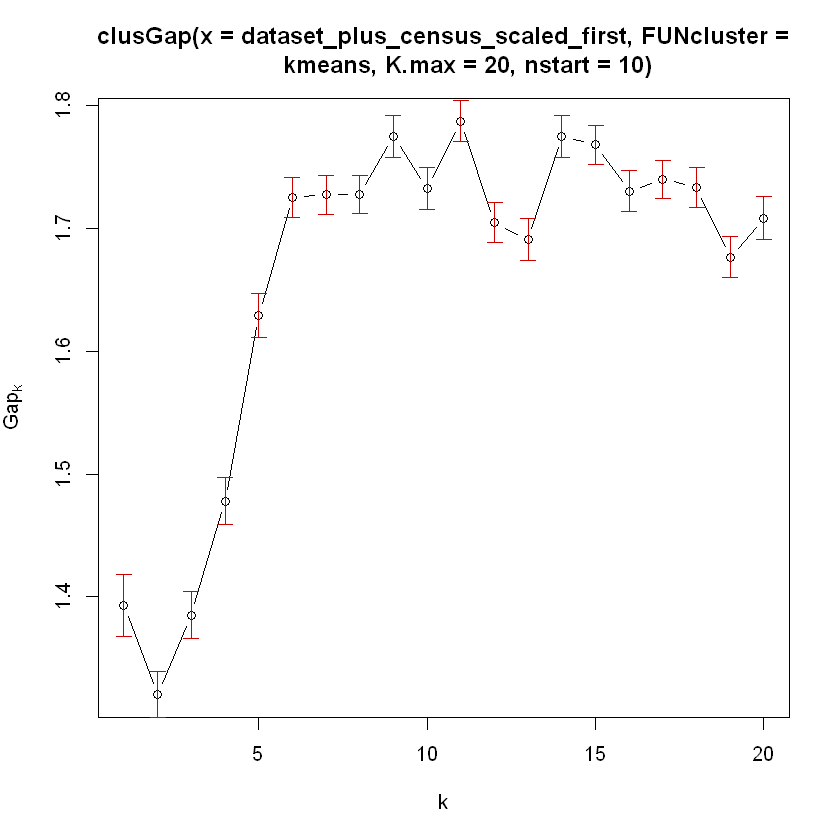
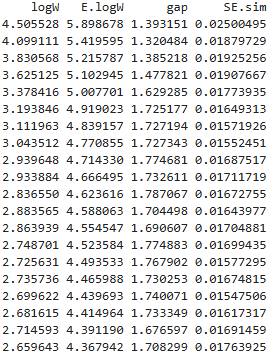
Next, we used the Elbow method to visually identify the optimum number of clusters. Based on the elbow plot below, we can see that the within sum of squares value starts to decrease not so significantly after 5 clusters. Since the elbow point (or knee) indicates a point where adding more clusters does not significantly improve the compactness of the clusters, suggesting that the optimal number of clusters has been reached, we can say that the optimum number of clusters is 5.



Elbow plot for first k-means clustering

**Gap Statistic**

Next, we used gap statistic method to find optimum number of clusters. It compares the total within-cluster variation for different numbers of clusters with their expected values under null reference distributions of the data. Based on the plot below, we can see that the max gap value is 1.787 at 11 clusters. The threshold using 1 standard error (SE) is 1.77033945 (i.e. 1.787067 - 0.01672755). The small k value such the Gap(k) is within 1 SE of the highest gap value is 1.774681 at k=9 indicating that the optimum number of clusters is 9.

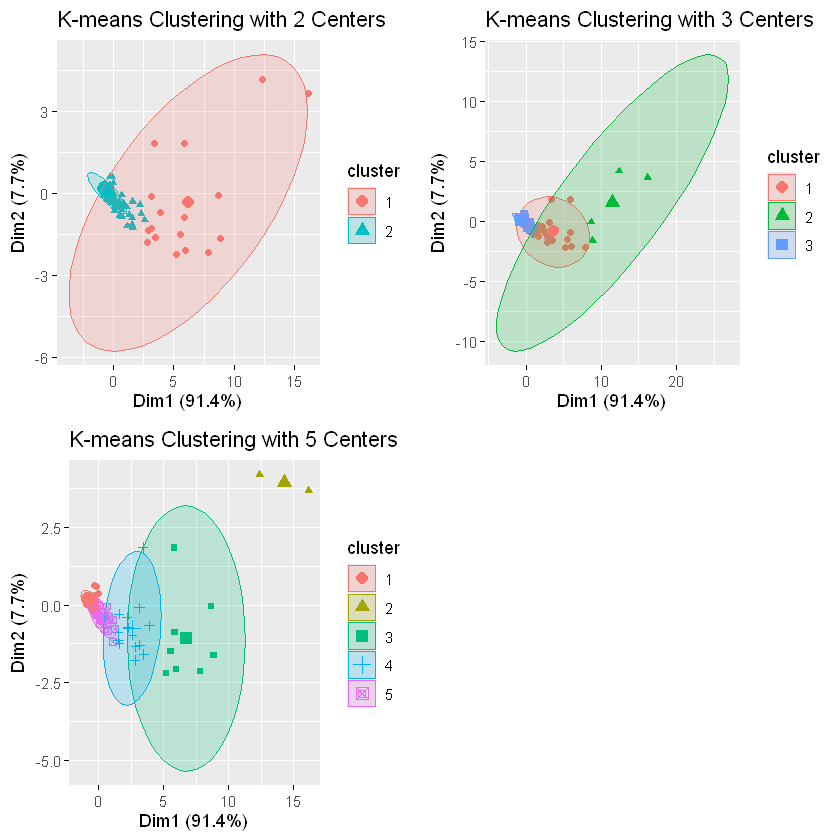
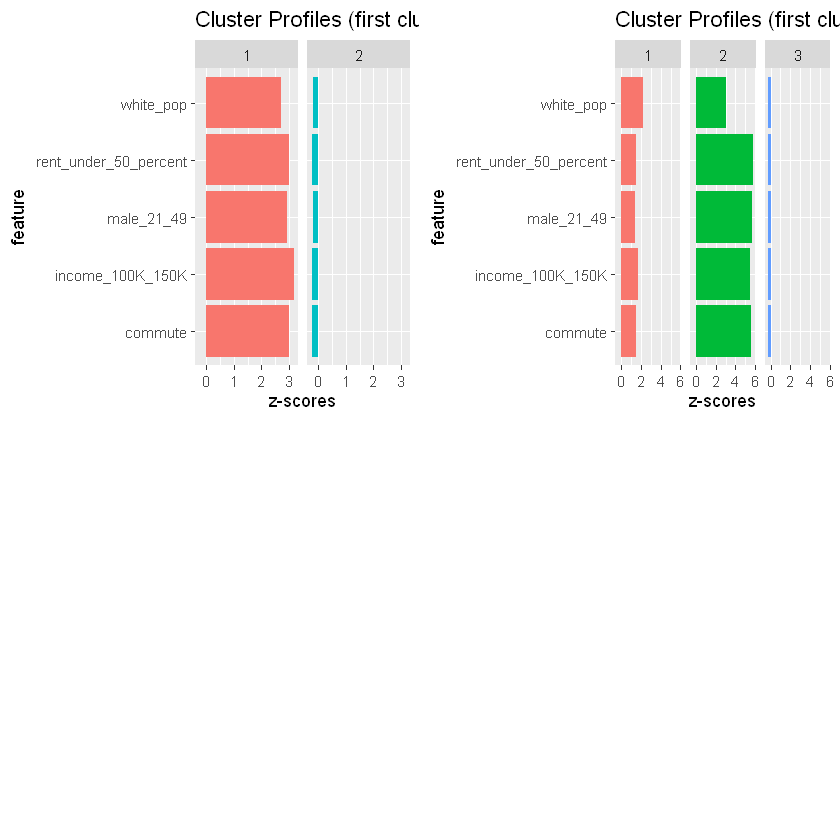
Gap statistic for first k-means clustering

**Unsupervised Evaluation**

Next, we computed various metrics for different number of clusters as shown below. Looking at the sum of squares, we see that the value starts to decrease not so significantly after 5 clusters supporting the observation made on elbow method. Looking at the average silhouette width, we can see that it is max at 2 clusters. The pearson gamma values also indicate that the optimum number of clusters is 2. The dunn index also indicates that 2 clusters are needed. While increasing the number of clusters reduces the within-cluster sum of squares, it also leads to a decrease in the average silhouette width, pearson gamma, and dunn index, indicating lower clustering quality. Thus, we can finally say that the optimum number clusters for the dataset is 2.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Metric** | **2 clusters** | **3 clusters** | **5 clusters** | **6 clusters** |
| within.cluster.ss | 397.8061 | 215.1592 | 82.48649 | 71.37958 |
| avg.silwidth | 0.838251 | 0.798405 | 0.633407 | 0.52843 |
| pearsongamma | 0.806838 | 0.771368 | 0.560038 | 0.399343 |
| dunn | 0.042062 | 0.035202 | 0.024113 | 0.014214 |

Unsupervised evaluation of first k-means clustering with different number of clusters

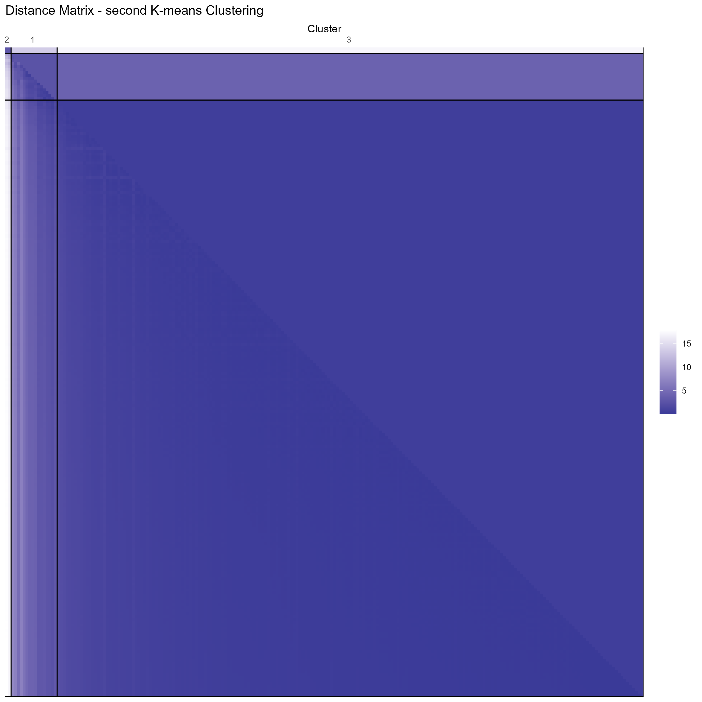
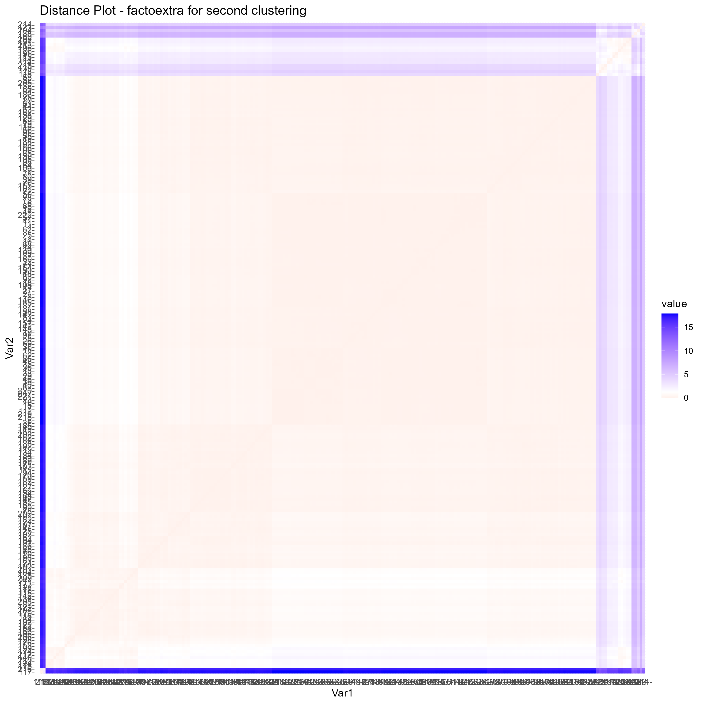
 

Visualization for first k-means clustering

To further validate the above conclusion, we used visualization of the clustering as shown above. Looking at the clusters on the left, we can see that we only need two clusters. When an additional cluster is added, the clusters don’t properly represent the datapoints. Some of the datapoints that belong to cluster 1 can also be part of cluster 3. Similarly, looking at the plot on the right, we can see that the third cluster has almost same distribution for all the features indicating that it does identify any significant variance of the dataset. It also supports that there are only 2 clusters needed for this dataset. Therefore, based on all of the assessments, we can say that 2 is the optimum number of clusters for the dataset.

### Second k-means Clustering

Before performing k-means clustering with certain number of clusters, we calculated the Hopkins statistic. The value was 0.99 which is close to 1 indicating that the dataset had a strong clustering tendency. Additionally, we also created following plots to visualize distance matrix of the dataset. Based on the factoextra plot, we can say that we might need 2 clusters in the k-means clustering for this dataset.

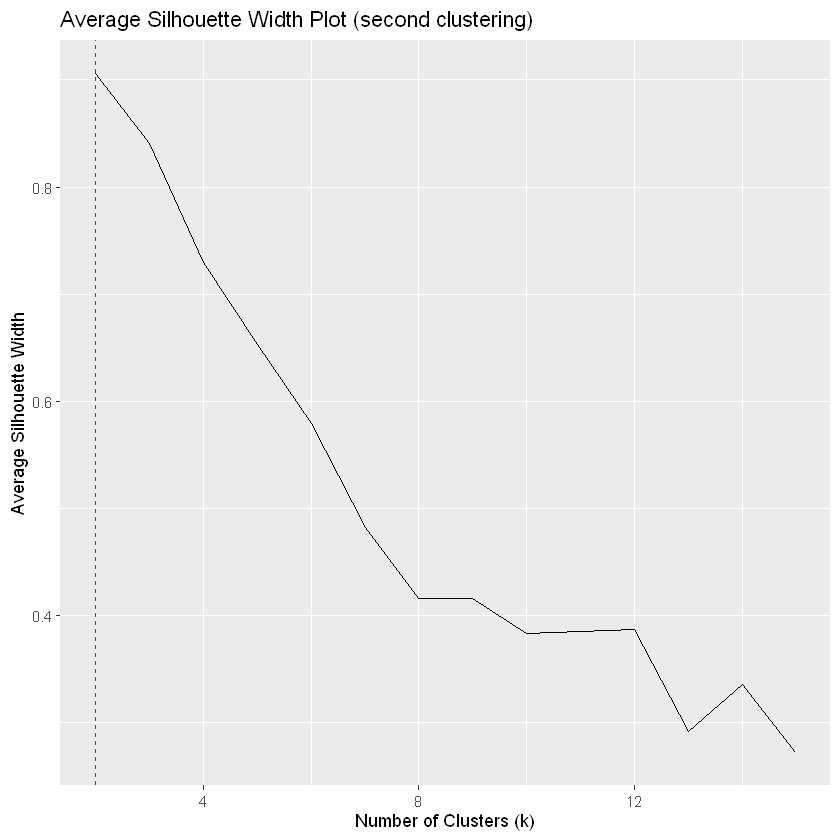
 

Distance matrix (left) and factoextra plot (right) for second k-means clustering

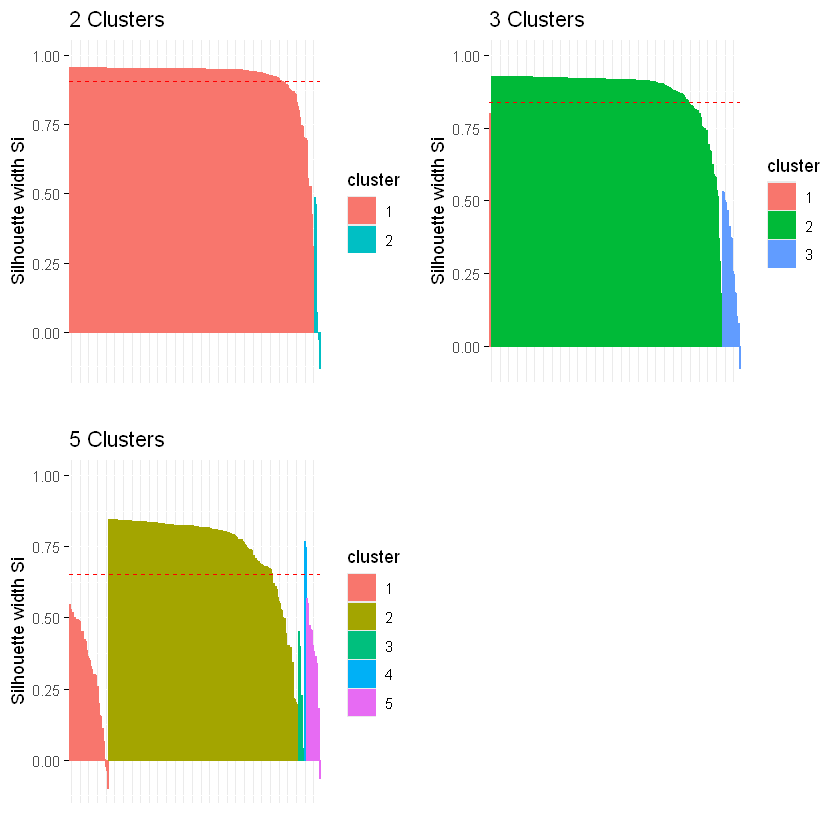
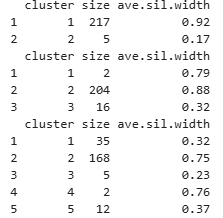
**Determine suitable number of clusters**

**Average Silhouette Width**

To determine suitable number of clusters for the dataset, we started with analyzing the average silhouette width for various number of clusters. Based on the plot below, we can see that the highest value is at 2 clusters, and the value starts to decrease rapidly with a greater number of clusters. Thus, based on this assessment, we just need 2 clusters for the dataset.



Average silhouette width plot for second k-means clustering

Silhouette plots for various number of clusters for second k-means clustering

Next, we look at analyzing the silhouette widths and plots for various number of clusters. Considering both the

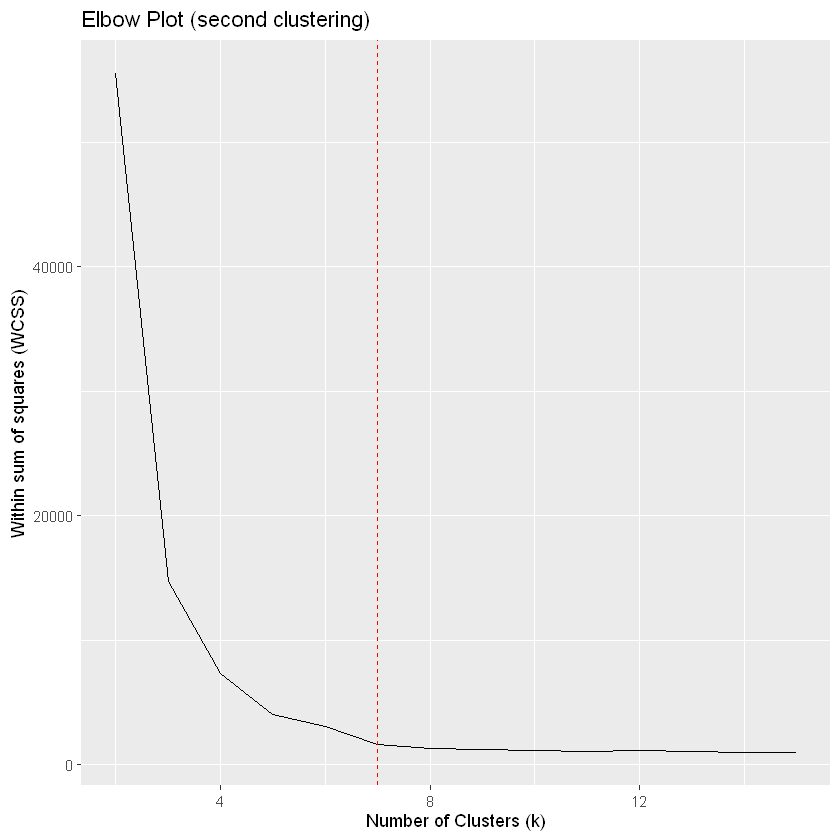
silhouette width and the distribution of cluster sizes, the clustering with 2 clusters seems to be the most appropriate

choice. Cluster 1 has a very high silhouette width (0.92), indicating well-separated clusters. While Cluster 2 is small,

the overall simplicity and high silhouette width for the main cluster make this a reasonable choice.

**Elbow Method: Within-Cluster Sum of Square**

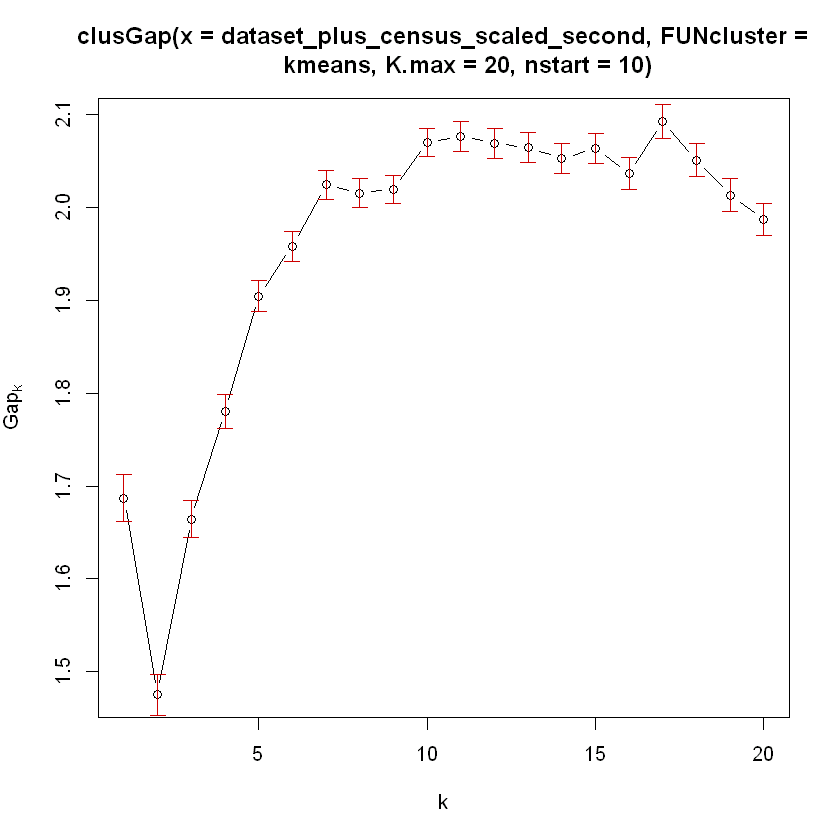
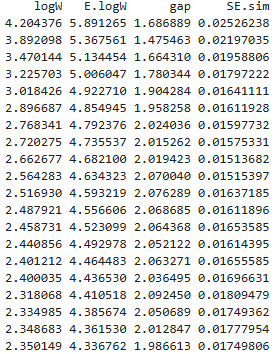
Next, we used the Elbow method to visually identify the optimum number of clusters. Based on the elbow plot below, we can see that the within sum of squares value starts to decrease not so significantly after about 7 clusters. Since the elbow point (or knee) indicates a point where adding more clusters does not significantly improve the compactness of the clusters, suggesting that the optimal number of clusters has been reached, we can say that the optimum number of clusters is 7.



Elbow plot for second k-means clustering

**Gap Statistic**

Next, we used gap statistic method to find optimum number of clusters. Based on the plot below, we can see that the max gap value is 2.09 at 17 clusters. So the optimum value of k using the 1-SE rule is 11 indicating that the optimum number of clusters is 11.

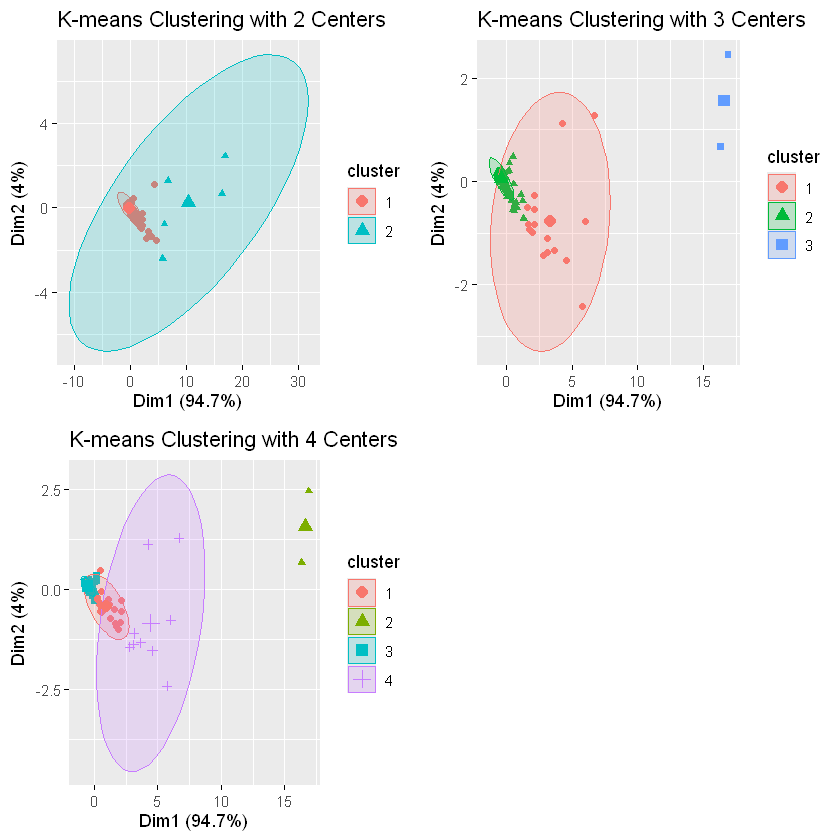
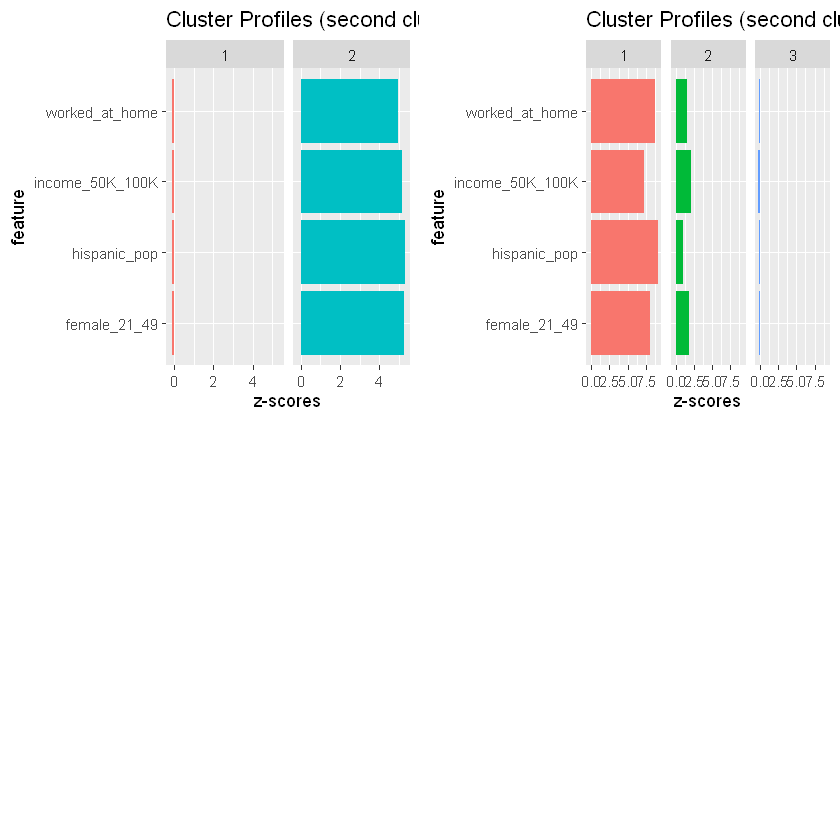
Gap statistic for second k-means clustering

**Unsupervised Evaluation**

Next, we computed various metrics for different number of clusters as shown below. All of the of the metrics are the highest for 2 clusters indicating that the optimum number of clusters is 2 for this dataset.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Metric** | **2 clusters** | **3 clusters** | **4 clusters** | **6 clusters** |
| within.cluster.ss | 336.9456 | 103.8761 | 64.20614 | 38.06195 |
| avg.silwidth | 0.9055904 | 0.8404076 | 0.729722 | 0.579127 |
| pearsongamma | 0.8086453 | 0.7104225 | 0.5720946 | 0.4022496 |
| dunn | 0.1275271 | 0.07974147 | 0.03998538 | 0.01211304 |

Unsupervised evaluation of second k-means clustering with different number of clusters

Visualization for second k-means clustering

To further validate the above conclusion, we used visualization of the clustering as shown above. Looking at the clusters on the left, we can see that we only need two clusters. When an additional cluster is added, the clusters don’t properly represent the datapoints. The third cluster seems to cluster outliers. Similarly, looking at the plot on the right, we can see that the third cluster has almost same distribution for all the features indicating that it does identify any significant variance of the dataset. It also supports that there are only 2 clusters needed for this dataset. Therefore, based on all of the assessments, we can say that 2 is the optimum number of clusters for the dataset.

### Third k-means Clustering

Before performing k-means clustering with certain number of clusters, we calculated the Hopkins statistic. The value was 0.99 which is close to 1 indicating that the dataset had a strong clustering tendency. Additionally, we also created following plots to visualize distance matrix of the dataset. Based on the factoextra plot, we can say that we might need 3 clusters in the k-means clustering for this dataset.

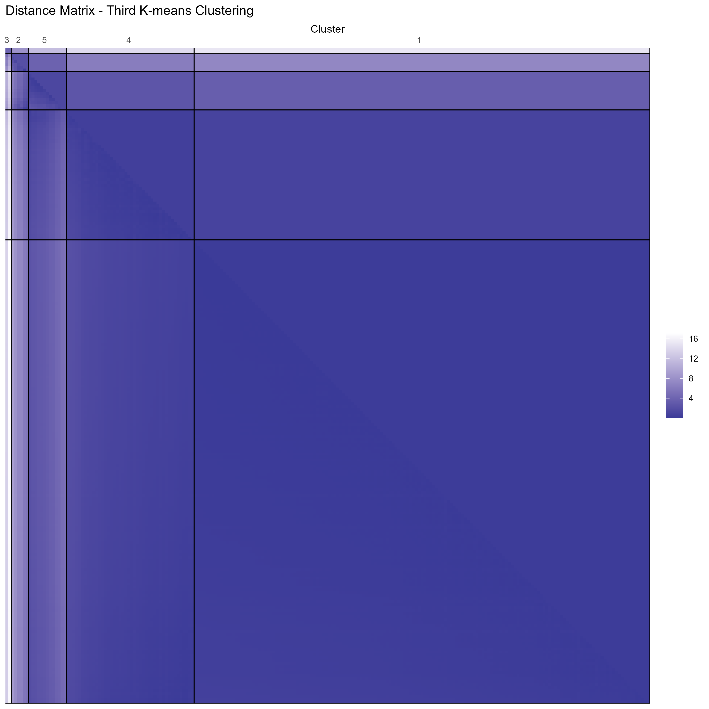
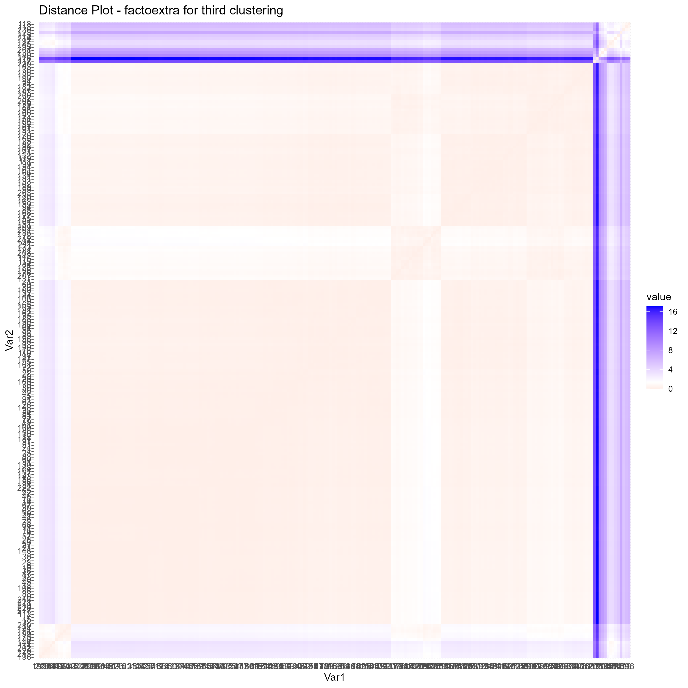
 

Fig 13. Distance matrix (left) and factoextra plot (right) for third k-means clustering

**Determine suitable number of clusters**

**Average Silhouette Width**

To determine suitable number of clusters for the dataset, we started with analyzing the average silhouette width for various number of clusters. Based on the plot below, we can see that the highest value is at 2 clusters, and the value starts to decrease rapidly with a greater number of clusters. Thus, based on this assessment, we just need 2 clusters for the dataset.

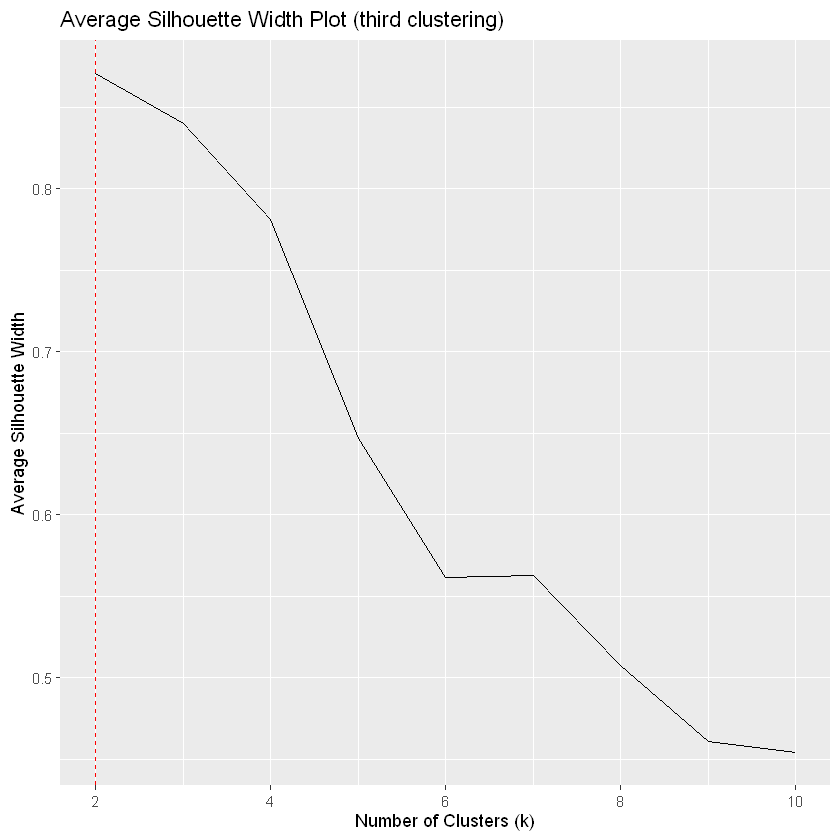


Fig 14. Average silhouette width plot for third k-means clustering

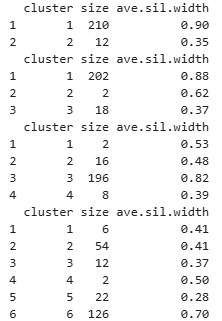
 

Fig 15. Silhouette plots for various number of clusters for third k-means clustering

Next, we look at analyzing the silhouette widths and plots for various number of clusters. For the clustering with 2

clusters, the average silhouette width for Cluster 1 is very high (0.90), indicating well-separated and compact

clusters. Cluster 2 has a significantly lower silhouette width (0.35), suggesting that it may not be as well-separated or compact. However, the overall silhouette width is strongly influenced by Cluster 1's high value. As we add more

clusters, we can see that the added clusters only cluster very few datapoints. At the same time, the average

silhouette widths also don’t improve much. The second cluster of the clustering with clusters, although smaller and with a lower silhouette width, does not significantly detract from the overall quality of the clustering compared to

higher cluster numbers with more uneven and lower silhouette widths. Therefore, 2 clusters appear to be the

optimum choice.

**Elbow Method: Within-Cluster Sum of Square**

Next, we used the Elbow method to visually identify the optimum number of clusters. Based on the elbow plot below, we can see that the within sum of squares value starts to decrease not so significantly after about 7 clusters. Since the elbow point (or knee) indicates a point where adding more clusters does not significantly improve the compactness of the clusters, suggesting that the optimal number of clusters has been reached, we can say that the optimum number of clusters is 7.



Fig 16. Elbow plot for third k-means clustering

**Gap Statistic**

Next, we used gap statistic method to find optimum number of clusters. Based on the plot below, we can see that the max gap value is at k = 13. So the optimum value of k using the 1-SE rule is 11 indicating that the optimum number of clusters is 11.

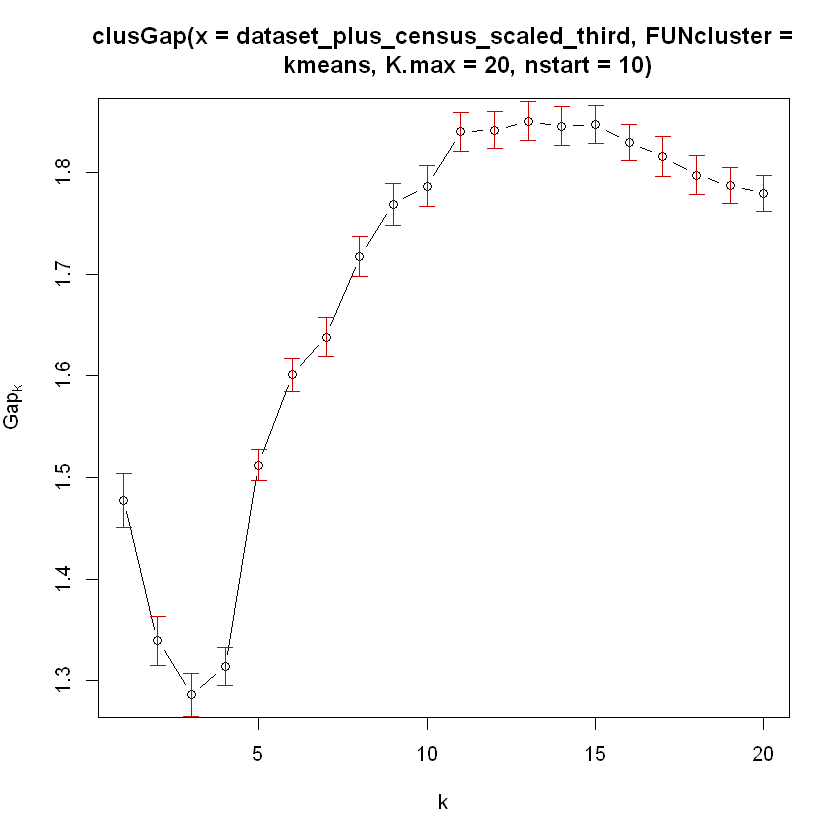
 

Fig 17. Gap statistic for third k-means clustering

**Unsupervised Evaluation**

Next, we computed various metrics for different number of clusters as shown below. 2 clusters have the highest average silhouette width and Pearson gamma, indicating well-defined and compact clusters. However, 3 clusters have the highest Dunn index, suggesting better separation between clusters. Given the strong performance in both silhouette width and Pearson gamma, 2 clusters appear to be the optimal number of clusters. We further validated this determination by visually analyzing the clustering with various clusters.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Metric** | **2 clusters** | **3 clusters** | **5 clusters** | **6 clusters** |
| within.cluster.ss | 283.8065 | 129.9854 | 46.36307 | 39.78291 |
| avg.silwidth | 0.8703733 | 0.8400603 | 0.6465164 | 0.5617945 |
| pearsongamma | 0.8150192 | 0.7699686 | 0.4882054 | 0.3815488 |
| dunn | 0.04766757 | 0.08112688 | 0.01734194 | 0.007537748 |

Table 8. Unsupervised evaluation of third k-means clustering with different number of clusters

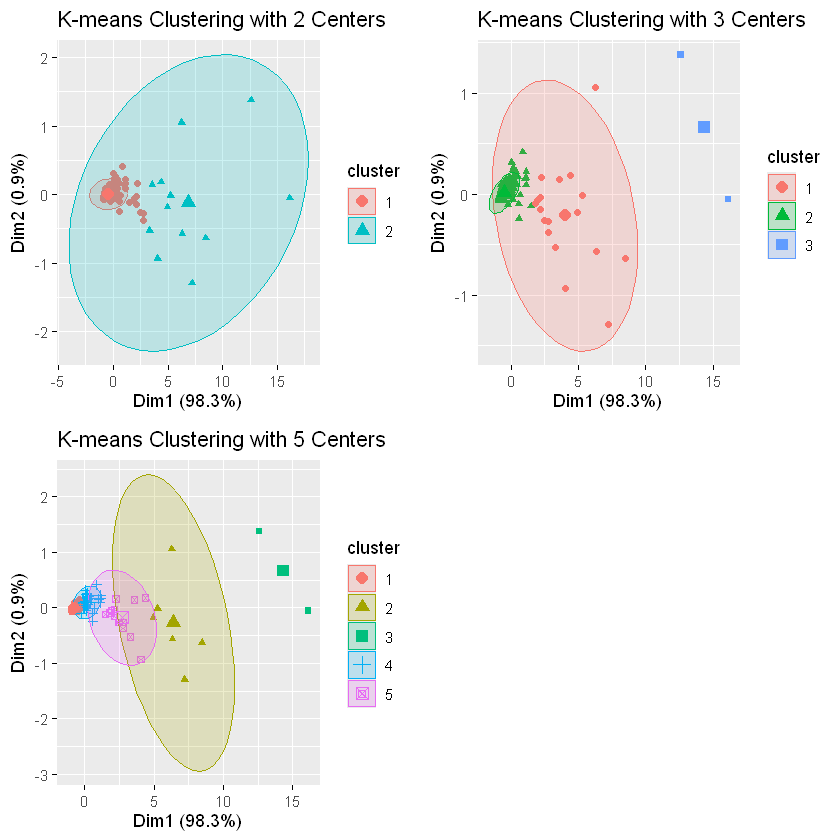
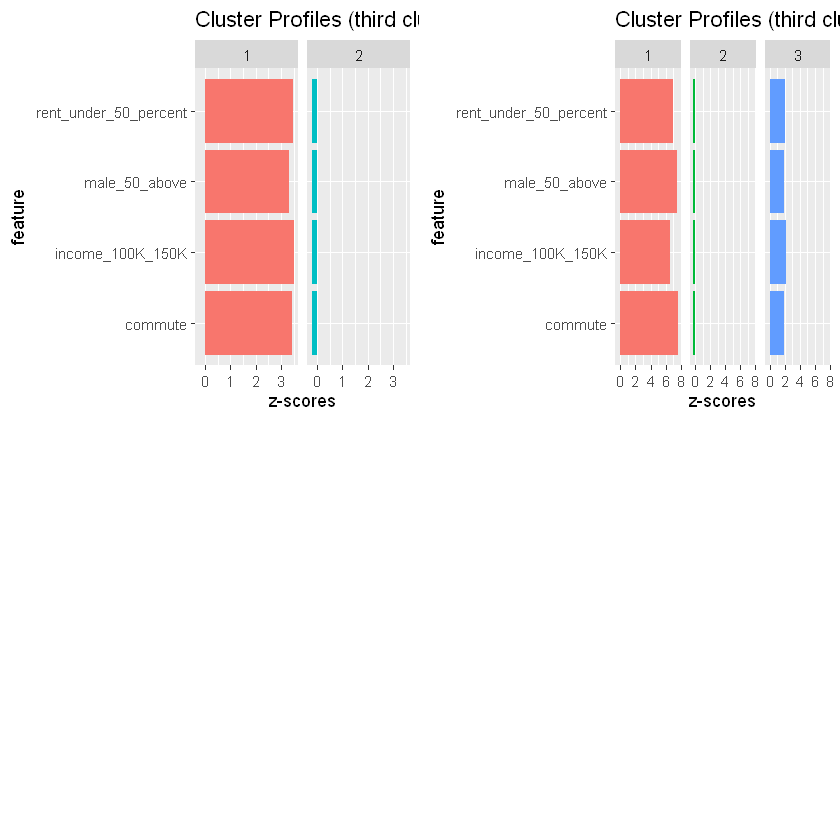
 

Fig 18. Visualization for third k-means clustering

To further validate the above conclusion, we used visualization of the clustering as shown above. Looking at the clusters on the left, we can see that we only need two clusters. When an additional cluster is added, the clusters don’t properly represent the datapoints. The third cluster seems to cluster outliers. Similarly, looking at the plot on the right, we can see that the second cluster has almost same distribution for all the features indicating that it does identify any significant variance of the dataset. It also supports that there are only 2 clusters needed for this dataset. Therefore, based on all of the assessments, we can say that 2 is the optimum number of clusters for the dataset.

### Fourth k-means Clustering

Before performing k-means clustering with certain number of clusters, we calculated the Hopkins statistic. The value was 0.99 which is close to 1 indicating that the dataset had a strong clustering tendency. Additionally, we also created following plots to visualize distance matrix of the dataset. Based on the factoextra plot, we can say that we might need 3 clusters in the k-means clustering for this dataset.

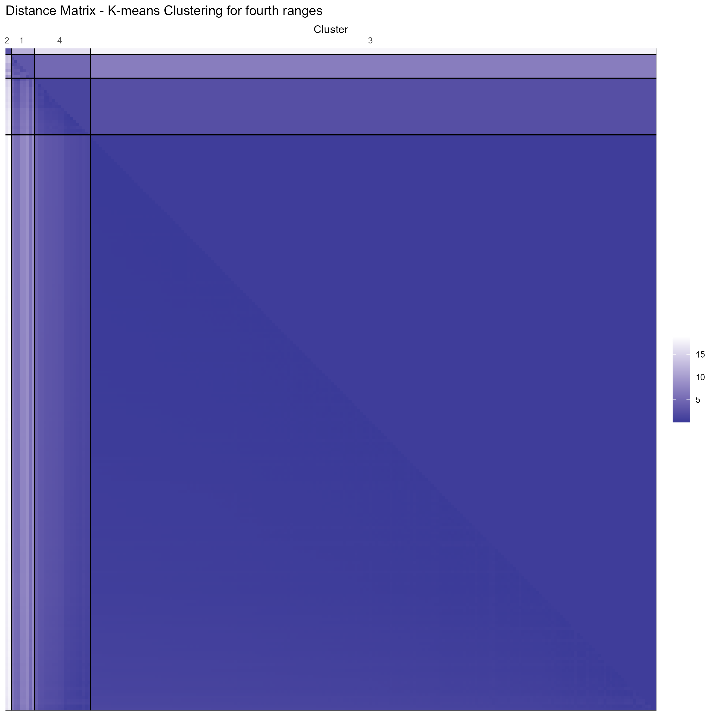
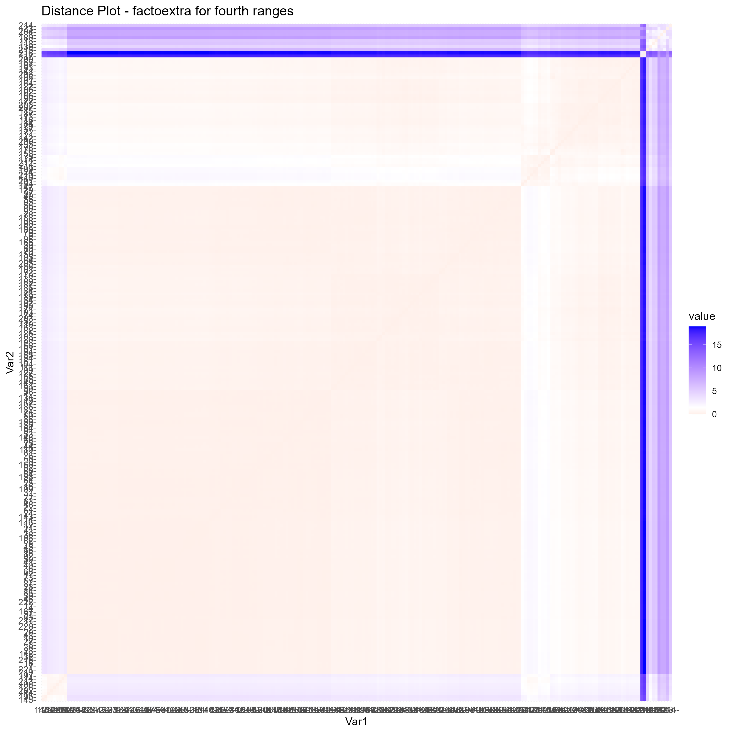
 

Fig 15. Distance matrix (left) and factoextra plot (right) for fourth k-means clustering

**Determine suitable number of clusters**

**Average Silhouette Width**

To determine suitable number of clusters for the dataset, we started with analyzing the average silhouette width for various number of clusters. Based on the plot below, we can see that the highest value is at 2 clusters, and the value starts to decrease rapidly with a greater number of clusters. Thus, based on this assessment, we just need 2 clusters for the dataset.

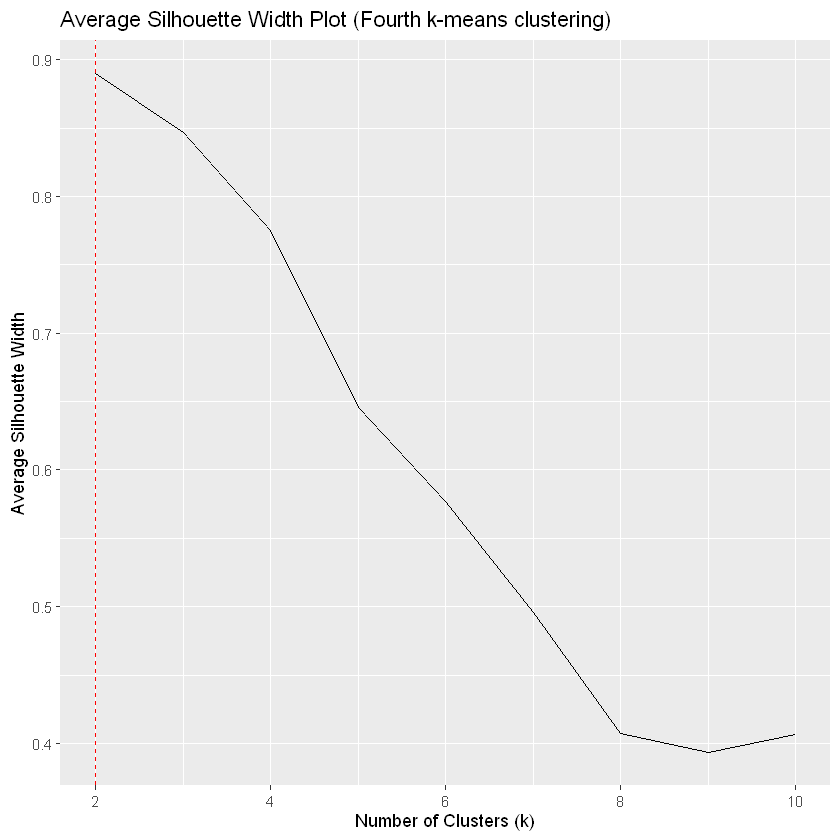


Fig 16. Average silhouette width plot for fourth k-means clustering

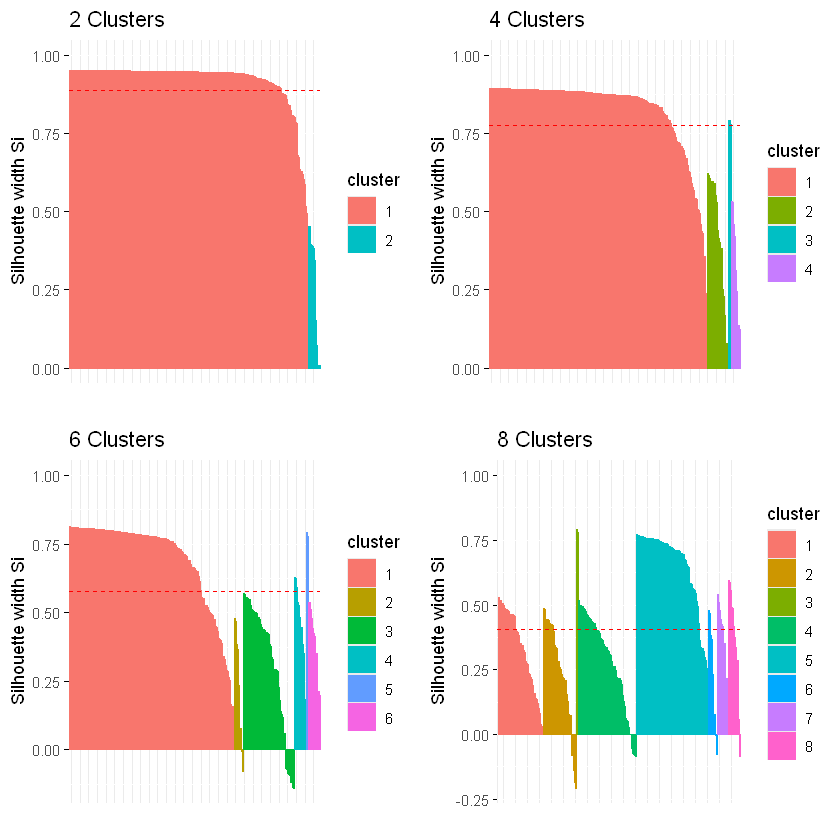
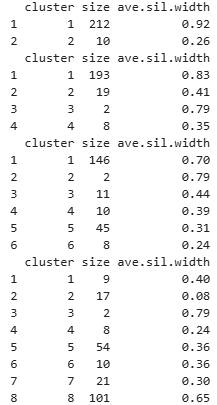
 

Fig 17. Silhouette plots for various number of clusters for fourth k-means clustering

Next, we look at analyzing the silhouette widths and plots for various number of clusters. For the clustering with 2

clusters, 2 clusters show a very high silhouette width for one cluster but a very low value for the other, suggesting that one cluster is well-defined while the other is not. 4 clusters present a more balanced solution with reasonably high silhouette widths for three clusters (0.83, 0.41, 0.79) and a slightly lower but still reasonable value for one cluster (0.35). However, clusters 3 and 4 of this clustering have very few datapoints. 6 clusters and 8 clusters show more mixed results, with some clusters having low silhouette widths, indicating less well-defined clusters. Therefore, the

optimum number of clusters could not be identified with high confidence using this assessment.

**Elbow Method: Within-Cluster Sum of Square**

Next, we used the Elbow method to visually identify the optimum number of clusters. Based on the elbow plot below, we can see that the within sum of squares value starts to decrease not so significantly after about 4 clusters. Since the elbow point (or knee) indicates a point where adding more clusters does not significantly improve the compactness of the clusters, suggesting that the optimal number of clusters has been reached, we can say that the optimum number of clusters is 4.

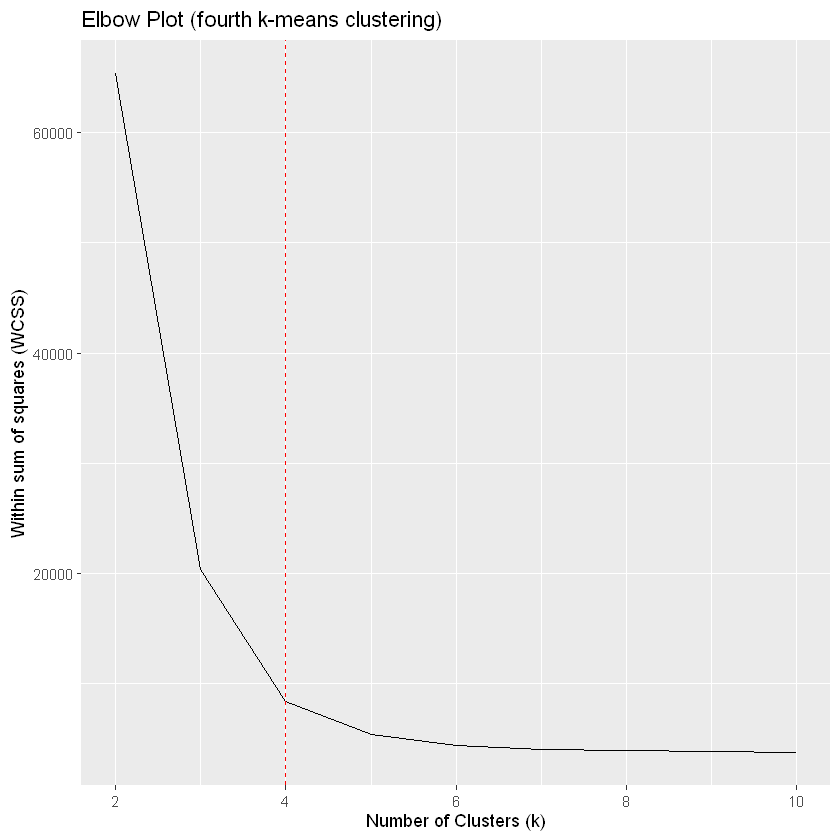


Fig 18. Elbow plot for fourth k-means clustering

**Gap Statistic**

Next, we used gap statistic method to find optimum number of clusters. Based on the plot below, we can see that the max gap value is at k = 8. So the optimum value of k using the 1-SE rule is 8 indicating that the optimum number of clusters is 11.

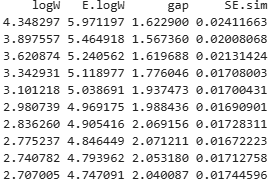
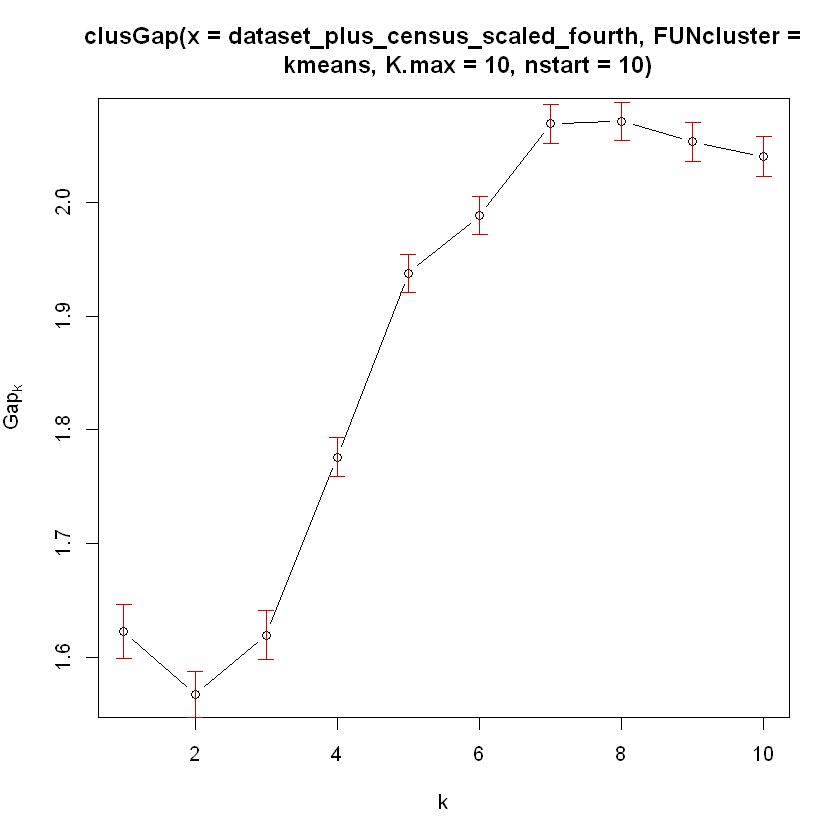


Fig 19. Gap statistic for fourth k-means clustering

**Unsupervised Evaluation**

Next, we computed various metrics for different number of clusters as shown below. Based on the values of various metrics, we can clearly see that all of those values are largest for 2 clusters. Therefore, the optimum number of clusters is 2.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Metric** | **2 clusters** | **4 clusters** | **8 clusters** | **10 clusters** |
| within.cluster.ss | 380.2993 | 76.5326 | 50.10439 | 49.19994 |
| avg.silwidth | 0.8896608 | 0.7753996 | 0.4071046 | 0.363984 |
| pearsongamma | 0.8246153 | 0.655894 | 0.2225344 | 0.1924984 |
| dunn | 0.08043983 | 0.05822636 | 0.004683524 | 0.00467794 |

Table 9. Unsupervised evaluation of fourth k-means clustering with different number of clusters

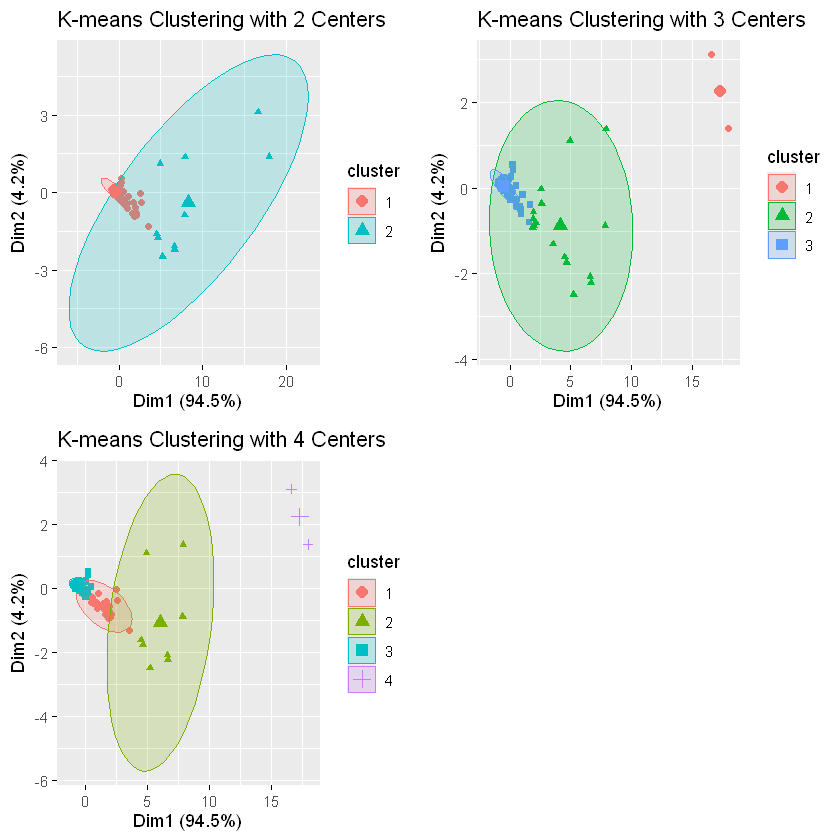
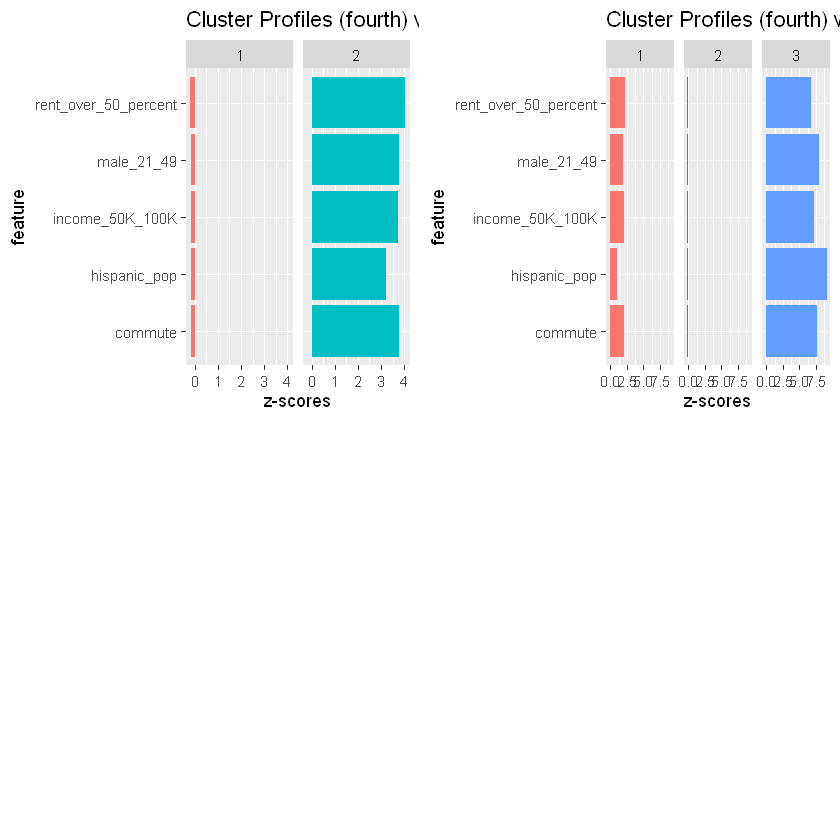
 

Fig 20. Visualization for fourth k-means clustering

To further validate the above conclusion, we used visualization of the clustering as shown above. Looking at the clusters on the left, we can see that we only need two clusters. When an additional cluster is added, the clusters don’t properly represent the datapoints. The third cluster seems to cluster outliers. Similarly, looking at the plot on the right, we can see that the second cluster has almost same distribution for all the features indicating that it does identify any significant variance of the dataset. It also supports that there are only 2 clusters needed for this dataset. Therefore, based on all of the assessments, we can say that 2 is the optimum number of clusters for the dataset.

**Supervised Evaluation (Compare all four k-means clusterings)**

To perform supervised evaluation, we chose “median\_income” variable as the ground truth. Since the minimum and maximum incomes are 24794 and 80938, we created three labels: low (0-50K), medium (51-75K), and high (76K above). We used purity score and entropy to evaluate how well different clustering align with the predefined income labels (i.e. ground truth). For each cluster of a clustering, the purity score is the proportion of counties in that cluster that are correctly classified into their majority ground truth label. So a higher purity score means that the clustering is more accurate in terms of assigning similar income counties to the same cluster. Similarly, the entropy function is used to measure the disorder or uncertainty in the clustering results with respect to the ground truth labels. It returns a value that quantifies how mixed or pure the clusters are with respect to the known labels. So low entropy score suggests that the clusters are mostly composed of counties with the same income group, which means that the k-means algorithm has effectively separated the counties based on income. Similarly, The Corrected Rand Index (corrected.rand) is a measure of the similarity between two clusterings. So, score of 1 means perfect agreement between the two clusterings (i.e. identical). Likewise, The Variation of Information (vi) is a measure of the difference between two clusterings. So, a smaller vi score indicates more similarity between the two clusterings.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Metric** | **corrected.rand** | **vi** | **purity** | **entropy** |
| truth | 1 | 0 | 1 | 0 |
| k\_means\_first | 0.010545 | 0.901685 | 0.657658 | 0.460574 |
| k\_means\_second | 0.002179 | 0.819546 | 0.657658 | 0.073128 |
| k\_means\_third | 0.006321 | 0.914865 | 0.657658 | 0.472569 |
| k\_means\_fourth | -0.01869 | 0.891209 | 0.657658 | 0.127927 |

Comparison of four k-means clusterings

Based on the above table, the first clustering has the highest corrected Rand index, indicating it is the closest to the ground truth. The second clustering has the lowest VI score indicating that it is the closest to the ground truth. All four clusterings have the same purity, so this metric does not help distinguish between them. The second clustering has the lowest entropy indicating it is the closest to the ground truth. Based on the Variation of Information (VI) and Entropy, the second clustering appears to be the closest to the ground truth and thus performed the best clustering.

Hierarchical Clustering

## Hierarchical Clustering

Hierarchical clustering is an alternative approach to for identifying groups in the dataset. It does not require us to pre-specify the number of clusters to be generated. Furthermore, hierarchical clustering has an added advantage that it results in an attractive tree-based representation of the observations, called a dendrogram.

### Identifying the Linkage method

The agglomerative coefficient of the methods below was calculated and the best method was chosen. The table below shows these methods and the corresponding coefficient for this dataset

| **Linkage Method** | **Description** | **Agglomerative coefficient for this dataset** |
| --- | --- | --- |
| Complete linkage clustering: | It computes all pairwise dissimilarities between the elements in cluster 1 and the elements in cluster 2, and considers the largest value (i.e., maximum value) of these dissimilarities as the distance between the two clusters. It tends to produce more compact clusters. | 0.9877028 |
| Single linkage clustering | computes all pairwise dissimilarities between the elements in cluster 1 and the elements in cluster 2, and considers the smallest of these dissimilarities as a linkage criterion. It tends to produce long, “loose” clusters. | 0.9679563 |
| Mean or average linkage clustering: | It computes all pairwise dissimilarities between the elements in cluster 1 and the elements in cluster 2, and considers the average of these dissimilarities as the distance between the two clusters. | 0.9859381 |
| Ward’s minimum variance method: | It minimizes the total within-cluster variance. At each step the pair of clusters with minimum between-cluster distance are merged. | 0.9940167 |

Based on the Agglomerative Coefficient results, the maximum coefficient was for the Wards minimum variance method, which is also used for clustering.

### First Hierarchical Clustering

#### Dendogram

A graph of data overlay

Description automatically generated

We will be evaluating the 4 different clusters with different number of clusters. We have selected clusterings with number of cluster 2,4,6,8

#### Determine the suitable number of clusters

**Cluster Gap Analysis**

A graph with red lines and numbers

Description automatically generated

Gap Analysis suggests 2 as the optimal number of clusters

**Average Silhouette Width**

A graph of a number of numbers

Description automatically generated with medium confidence

A graph of a plot of numbers

Description automatically generated with medium confidence

Fig : Avg Silhouette Width

The highest Silhouette coefficient is for the Cluster with number of clusters = 2.

Hence, we conclude that number of cluster = 2 is the most optimal clustering for the features that were selected.

#### Cluster Visualization

The cluster visualization is performed for the best cluster identified by the un-supervised cluster evaluation method.

A grid of red dots

Description automatically generated

Fig : Pair correlation plot

From this Diagram, We can see that that the clusters can be easily distinguished based on the feature that were selected.

A screenshot of a graph

Description automatically generated

The above figure shows that the Clusters can be fairly separated.

Also, below are the counties' look based on how that is plotted in a Texas Map. The grey counties are the counties that were excluded from the clustering as they were the counties that fall under the major metropolitan cities (Houston, Dallas and SanAntonio), The green and the Orange show the counties separated based on the clustering’s.

A map of texas with blue and red squares

Description automatically generated

Fig: Heat map of clusters

### Second Hierarchical Clustering

#### Dendogram

A graph of data

Description automatically generated

We will be evaluating the 4 different clusters with different number of clusters. We have selected clusterings with number of cluster 2,4,6,8

#### Determine the suitable number of clusters

**Cluster Gap Analysis**

A graph of a graph

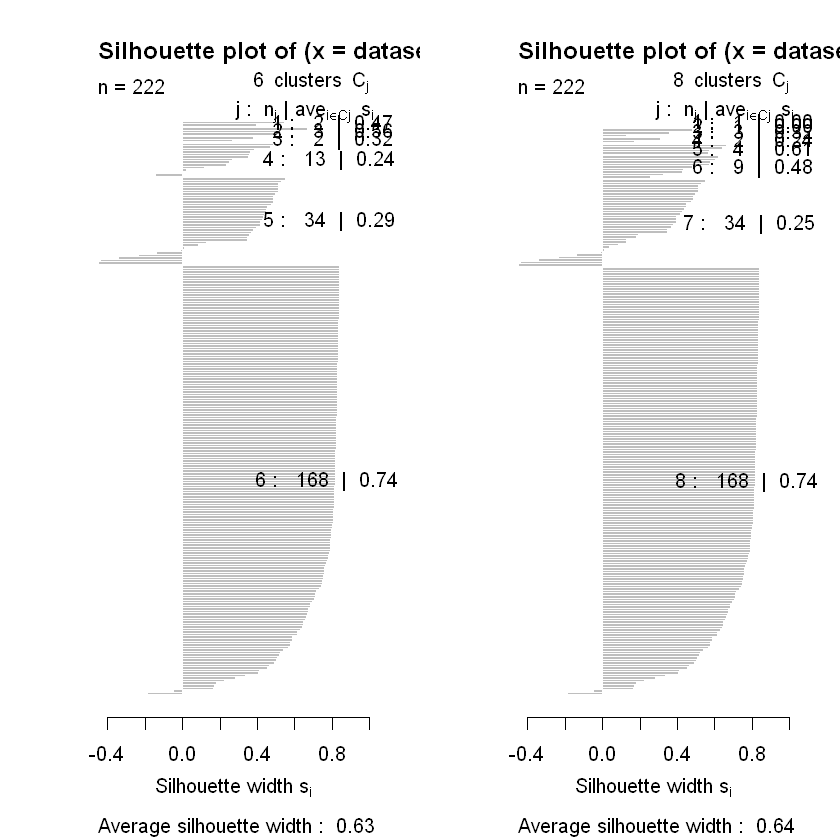
Description automatically generated

Gap Analysis suggests 2 as the optimal number of clusters

**Average Silhouette Width**

A graph of a plot of a number

Description automatically generated with medium confidence



The highest Silhouette coefficient is for the Cluster with number of clusters = 2.

Hence, we conclude that number of cluster = 2 is the most optimal clustering for the features that were selected.

#### Cluster Visualization

A screenshot of a graph

Description automatically generated

Fig 3.2.4 Pair correlation plot

From this Diagram, We can see that that the clusters can be easily distinguished based on the feature that were selected.

A diagram of a clustering scale

Description automatically generated

The above figure shows that the Clusters can be fairly separated.

Also, below are the counties' look based on how that is plotted in a Texas Map. The grey counties are the counties that were excluded from the clustering as they were the counties that fall under the major metropolitan cities (Houston, Dallas and SanAntonio), The green and the Orange show the counties separated based on the clustering’s.

A map of texas with red and blue squares

Description automatically generated

### Third Hierarchical Clustering

#### Dendogram

A graph of data

Description automatically generated

We will be evaluating the 4 different clusters with different number of clusters. We have selected clusterings with number of cluster 2,4,6,8

#### Determine the suitable number of clusters

**Cluster Gap Analysis**

A graph with red lines and numbers

Description automatically generated

Gap Analysis suggests 2 as the optimal number of clusters

**Average Silhouette Width**

A graph of a number of numbers

Description automatically generated with medium confidence

A graph of a number of numbers

Description automatically generated with medium confidence

The highest Silhouette coefficient is for the Cluster with number of clusters = 2.

Hence, we conclude that number of cluster = 2 is the most optimal clustering for the features that were selected.

#### Cluster Visualization

A grid of black and red dots

Description automatically generated

Fig 3.2.4 Pair correlation plot

From this Diagram, We can see that that the clusters can be easily distinguished based on the feature that were selected.

A diagram of a diagram

Description automatically generated

The above figure shows that the Clusters can be fairly separated.

Also, below are the counties' look based on how that is plotted in a Texas Map. The grey counties are the counties that were excluded from the clustering as they were the counties that fall under the major metropolitan cities (Houston, Dallas and SanAntonio), The green and the Orange show the counties separated based on the clustering’s.

### Fourth Hierarchical Clustering

#### Dendogram

A graph of data

Description automatically generated

We will be evaluating the 4 different clusters with different number of clusters. We have selected clusterings with number of cluster 2,4,6,8

#### Determine the suitable number of clusters

**Cluster Gap Analysis**

A graph with numbers and lines

Description automatically generated

Gap Analysis suggests 2 as the optimal number of clusters

**Average Silhouette Width**

A graph of a number of numbers

Description automatically generated with medium confidence

A graph of a plot of numbers

Description automatically generated with medium confidence

The highest Silhouette coefficient is for the Cluster with number of clusters = 2.

Hence, we conclude that number of cluster = 2 is the most optimal clustering for the features that were selected.

#### Cluster Visualization

A grid of black dots

Description automatically generated

Fig 3.2.4 Pair correlation plot

From this Diagram, We can see that that the clusters can be easily distinguished based on the feature that were selected.

A diagram of a clustering diagram

Description automatically generated with medium confidence

The above figure shows that the Clusters can be fairly separated.

Also, below are the counties' look based on how that is plotted in a Texas Map. The grey counties are the counties that were excluded from the clustering as they were the counties that fall under the major metropolitan cities (Houston, Dallas and SanAntonio), The green and the Orange show the counties separated based on the clustering’s.

A map of texas with different colored squares

Description automatically generated

### Supervised Cluster evaluation

We compared 4 clusterings and identified the best cluster using un-supervised evaluation for each of the clusters. We are using that best cluster for supervised cluster evaluation. The first step in supervised evaluation is to create a ground truth. For this we are using the death per case to calculate the ground truth.

The ground truth that was selected to evaluate the cluster was 3 groups created by using the death\_per\_case feature in the dataset

Purity measure was used to compare the Cluster with the ground truth. Below snippet shows the result of this comparison.

|  |  |
| --- | --- |
| **Cluster** | **Purity Measure** |
| **1** | 0.747747747747748 |
| **2** | 0.774774774774775 |
| **3** | 0.801801801801802 |
| **4** | **0.815315315315315** |

The higher purity means that the best hierarchical cluster that we evaluated against a supervised cluster. In this case the best clustering using the 4th Hierarchical clustering is considered as the best cluster using the supervised cluster evaluation.

## Density Based (DBSCAN) Clustering

In Density based clustering algorithm, the data is partitioned into groups with similar characteristics or clusters, but it does not require specifying the number of those groups in advance. It discovers clusters of arbitrary shapes in spatial databases with noise.

In DBSCAN clustering, dependence on distance-curve of dimensionality is more. The algorithm is as follows:

1. Randomly select a point “p”.
2. Retrieve all the points that are density reachable from p with regard to Maximum radius of the neighborhood(EPS) and minimum number of points within eps neighborhood(Min Pts).
3. If the number of points in the neighborhood is more than Min Pts, then p is a core point.
4. For p core points, a cluster is formed. If p is not a core point, then mark it as a noise/outlier and move to the next point.
5. Continue the process until all the points have been processed.

### First Density Based Clustering

#### Selecting the min and eps

We are plotting the distance between the points in the dataset in the below graph. The EPS is decided by visually looking for the knee in the plot. These are identified by the lines shown in the graph. We are using 4 eps values near the knee to create 4 different clustering

A screen shot of a black screen

Description automatically generated

Fig Distance Plot

#### Identified Clusters.

The below output snippet shows the 4 Density based clusters that are created.

**Clustering 1:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.65, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 2 cluster(s) and 17 noise points.

0 1 2

17 4 201

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 2:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.7, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 2 cluster(s) and 16 noise points.

0 1 2

16 5 201

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 3:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.8, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 2 cluster(s) and 14 noise points.

0 1 2

14 6 202

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 4:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.9, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 14 noise points.

0 1

14 208

Available fields: cluster, eps, minPts, metric, borderPoints

#### Unsupervised Cluster evaluation

We are using Silhouette Coefficient to evaluate the 4 clusters that are created. Below figure shows the Silhouette coefficients of all the Clusters that we are evaluating.

A screenshot of a computer screen

Description automatically generated

Fig 3.3.2 Average Silhouette Width

As we can identify from the above figure, the Silhouette width is similar for all three cluster. The for the one had only one cluster and noise clusters, Hence we are trying a different method to evaluate the clusters. We are using the Dunn Index to evaluate the clusters

**Dunn Index**

Like all other such indices, the aim of this Dunn index to identify sets of clusters that are compact, with a small variance between members of the cluster, and well separated, where the means of different clusters are sufficiently far apart, as compared to the within cluster variance. Higher the Dunn index value, better is the clustering. The below table shows the Dunn Index of all the 4 Density based clustering

|  |  |
| --- | --- |
| **Db Clustering Number** | **Dunn Index** |
| Cluster 1 ( EPS=0.5, Min Pts = 4) | **0.200692885450029** |
| Cluster 2 ( EPS=0.5, Min Pts = 4) | 0.195220905956125 |
| Cluster 3 ( EPS=0.5, Min Pts = 4) | 0.192685680236223 |
| Cluster 4 ( EPS=0.5, Min Pts = 4) | 0.18189007202032 |

From the above table, **Cluster number 1 has the highest Dunn Index and hence the better clustering** among the 4 Density Based Clustering

#### Cluster visualizations

The cluster visualizations are performed for the cluster which is identified as the best cluster among the ones created by cluster evaluation methods used above. From the below cluster hulls, We can see that the clusters are fairly separated out, We can also see the noise points that are present in the clustering.

A screenshot of a video game

Description automatically generated

Fig DB Scan Clustering Hull Plot

The below cluster plot also show a similar story. The cluster are identified by the Pink and Blue shaded area and the orange shaded area is the Cluster 0 which is noise, Also after ignoring the noise, We can see that the Cluster 2 consists of mostly higher income counties with lesser death per case

A screen shot of a computer screen

Description automatically generated

Fig 3.3.4 Db Scan Clustering Cluster Plot

A graph with different colored dots

Description automatically generated

Fig 3.3.5 Death\_per\_case vs Income

**Plotting the clusters over a Texas map**

A map of the state of texas

Description automatically generated

|  |  |  |
| --- | --- | --- |
| **Cluster Number** | 1 | 2 |
| **avg\_death\_per\_case** | 0.01687944 | 0.02727379 |
| **avg\_income\_100K\_150K** | 9162.2500 | 872.9574 |
| **avg\_commute\_1000** | 848.0232 | 741.1578 |
| **avg\_worked\_at\_home\_1000** | 14.30697 | 13.69180 |
| **no\_counties\_in\_cluster** | 4 | 188 |

From the average features for each cluster, We can again see that the Cluster 1 with higher avegare\_income\_100K\_150K, the death percentage is far lesser than other counties, Also you can see that this cluster has comparatively higher worked from home too. This may also imply that the higher income brackets tend to work from home more.

### Second Density Based Clustering

#### Selecting the min and eps

We are plotting the distance between the points in the dataset in the below graph. The EPS is decided by visually looking for the knee in the plot. These are identified by the lines shown in the graph. We are using 4 eps values near the knee to create 4 different clustering

A screen shot of a black screen

Description automatically generated

Fig Distance Plot

#### Identified Clusters.

The below output snippet shows the 4 Density based clusters that are created.

**Clustering 1:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.65, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 13 noise points.

0 1

13 209

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 2:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.7, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 12 noise points.

0 1

12 210

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 3:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.8, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 11 noise points.

0 1

11 211

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 4:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.9, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 11 noise points.

0 1

11 211

Available fields: cluster, eps, minPts, metric, borderPoints

#### Unsupervised Cluster evaluation

We are using Silhouette Coefficient to evaluate the 4 clusters that are created. Below figure shows the Silhouette coefficients of all the Clusters that we are evaluating.

A screenshot of a computer

Description automatically generated

Fig 3.3.2 Average Silhouette Width

As we can identify from the above figure, the Silhouette width is similar for all three cluster. The for the one had only one cluster and noise clusters, Hence we are trying a different method to evaluate the clusters. We are using the Dunn Index to evaluate the clusters

**Dunn Index**

Like all other such indices, the aim of this Dunn index to identify sets of clusters that are compact, with a small variance between members of the cluster, and well separated, where the means of different clusters are sufficiently far apart, as compared to the within cluster variance. Higher the Dunn index value, better is the clustering. The below table shows the Dunn Index of all the 4 Density based clustering

|  |  |
| --- | --- |
| **Db Clustering Number** | **Dunn Index** |
| Cluster 1 ( EPS=0.5, Min Pts = 4) | **inf** |
| Cluster 2 ( EPS=0.5, Min Pts = 4) | inf |
| Cluster 3 ( EPS=0.5, Min Pts = 4) | inf |
| Cluster 4 ( EPS=0.5, Min Pts = 4) | inf |

From the above table, Dunn Index is Inf for all the clusterings as the selected features created only one cluster and noise. However, based on the Silhoette width, We can shoose Cluster 4 as the best cluster from these.

#### Cluster visualizations

The cluster visualizations are performed for the cluster which is identified as the best cluster among the ones created by cluster evaluation methods used above.

The below cluster plot also show a similar story. The cluster are identified by the Pink and Blue shaded area and the orange shaded area is the Cluster 0 which is noise, Also after ignoring the noise, We can see that the Cluster 2 consists of mostly higher income counties with lesser death per case

A screen shot of a computer

Description automatically generated

Fig 3.3.4 Db Scan Clustering Cluster Plot

A screen shot of a computer

Description automatically generated

Fig 3.3.5 Death\_per\_case vs Income

**Plotting the clusters over a Texas map**

A map of texas with different colored areas

Description automatically generated

| A tibble: 2 × 5 | | | | |
| --- | --- | --- | --- | --- |
| **cluster** | **avg\_death\_per\_case** | **avg\_income\_50K\_100K** | **avg\_worked\_at\_home** | **no\_counties\_in\_cluster** |
| 0 | 0.01888781 | 12770.6923 | 4776.0000 | 13 |
| 1 | 0.02674057 | 928.1579 | 338.6077 | 209 |

From the average features for each cluster, We can again see that the Cluster 1 with higher avegare\_income\_50K\_100K, the death percentage is far lesser than other counties, Also you can see that this cluster has comparatively higher worked from home too. This may also imply that the higher income brackets tend to work from home more.

### Third Density Based Clustering

#### Selecting the min and eps

We are plotting the distance between the points in the dataset in the below graph. The EPS is decided by visually looking for the knee in the plot. These are identified by the lines shown in the graph. We are using 4 eps values near the knee to create 4 different clustering

A screen shot of a black screen

Description automatically generated

Fig Distance Plot

#### Identified Clusters.

The below output snippet shows the 4 Density based clusters that are created.

**Clustering 1:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.65, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 14 noise points.

0 1

14 208

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 2:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.7, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 11 noise points.

0 1

11 211

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 3:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.8, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 11 noise points.

0 1

11 211

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 4:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.9, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 11 noise points.

0 1

11 211

Available fields: cluster, eps, minPts, metric, borderPoints

#### Unsupervised Cluster evaluation

We are using Silhouette Coefficient to evaluate the 4 clusters that are created. Below figure shows the Silhouette coefficients of all the Clusters that we are evaluating.

A screenshot of a computer

Description automatically generated

Fig 3.3.2 Average Silhouette Width

As we can identify from the above figure, the Silhouette width is similar for all three cluster. The for the one had only one cluster and noise clusters, Hence we are trying a different method to evaluate the clusters. We are using the Dunn Index to evaluate the clusters

**Dunn Index**

Like all other such indices, the aim of this Dunn index to identify sets of clusters that are compact, with a small variance between members of the cluster, and well separated, where the means of different clusters are sufficiently far apart, as compared to the within cluster variance. Higher the Dunn index value, better is the clustering. The below table shows the Dunn Index of all the 4 Density based clustering

|  |  |
| --- | --- |
| **Db Clustering Number** | **Dunn Index** |
| Cluster 1 ( EPS=0.5, Min Pts = 4) | **inf** |
| Cluster 2 ( EPS=0.5, Min Pts = 4) | inf |
| Cluster 3 ( EPS=0.5, Min Pts = 4) | inf |
| Cluster 4 ( EPS=0.5, Min Pts = 4) | inf |

Similar to the previous case, this is also producing only one cluster. Hence trying out other next clustering.

### Fourth Density Based Clustering

#### Selecting the min and eps

We are plotting the distance between the points in the dataset in the below graph. The EPS is decided by visually looking for the knee in the plot. These are identified by the lines shown in the graph. We are using 4 eps values near the knee to create 4 different clustering

A screen shot of a black screen

Description automatically generated

Fig Distance Plot

#### Identified Clusters.

The below output snippet shows the 4 Density based clusters that are created.

**Clustering 1:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.65, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 2 cluster(s) and 13 noise points.

0 1 2

13 7 202

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 2:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.7, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 13 noise points.

0 1

13 209

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 3:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.8, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 13 noise points.

0 1

13 209

Available fields: cluster, eps, minPts, metric, borderPoints

**Clustering 4:-**

DBSCAN clustering for 222 objects.

Parameters: eps = 0.9, minPts = 4

Using euclidean distances and borderpoints = TRUE

The clustering contains 1 cluster(s) and 11 noise points.

0 1

11 211

Available fields: cluster, eps, minPts, metric, borderPoints

#### Unsupervised Cluster evaluation

We are using Silhouette Coefficient to evaluate the 4 clusters that are created. Below figure shows the Silhouette coefficients of all the Clusters that we are evaluating.

A screenshot of a computer

Description automatically generated

Fig 3.3.2 Average Silhouette Width

As we can identify from the above figure, the Silhouette width is similar for all three cluster. The for the one had only one cluster and noise clusters, Hence we are trying a different method to evaluate the clusters. We are using the Dunn Index to evaluate the clusters

**Dunn Index**

Like all other such indices, the aim of this Dunn index to identify sets of clusters that are compact, with a small variance between members of the cluster, and well separated, where the means of different clusters are sufficiently far apart, as compared to the within cluster variance. Higher the Dunn index value, better is the clustering. The below table shows the Dunn Index of all the 4 Density based clustering

|  |  |
| --- | --- |
| **Db Clustering Number** | **Dunn Index** |
| Cluster 1 ( EPS=0.5, Min Pts = 4) | **0.331449051368392** |
| Cluster 2 ( EPS=0.5, Min Pts = 4) | inf |
| Cluster 3 ( EPS=0.5, Min Pts = 4) | inf |
| Cluster 4 ( EPS=0.5, Min Pts = 4) | inf |

From the above table, **Cluster number 1 has the highest Dunn Index and hence the better clustering** among the 4 Density Based Clustering. The inf Dunn Indexes are ignored as it has only one cluster and a noise cluster

#### Cluster visualizations

The cluster visualizations are performed for the cluster which is identified as the best cluster among the ones created by cluster evaluation methods used above. From the below cluster hulls, We can see that the clusters are fairly separated out, We can also see the noise points that are present in the clustering.

A screen shot of a computer

Description automatically generated

Fig DB Scan Clustering Hull Plot

The below cluster plot also show a similar story. The cluster are identified by the Pink and Blue shaded area and the orange shaded area is the Cluster 0 which is noise, Also after ignoring the noise, We can see that the Cluster 2 consists of mostly higher income counties with lesser death per case

A screenshot of a computer screen

Description automatically generated

Fig 3.3.4 Db Scan Clustering Cluster Plot

A graph with different colored dots

Description automatically generated

Fig 3.3.5 Death\_per\_case vs Income

**Plotting the clusters over a Texas map**

A map of the state of texas

Description automatically generated

| A tibble: 3 × 5 | | | | |
| --- | --- | --- | --- | --- |
| **cluster** | **avg\_death\_per\_case** | **avg\_income\_50K\_100K** | **avg\_commute\_1000** | **no\_counties\_in\_cluster** |
| 0 | 0.01888781 | 32393.846 | 811.6873 | 13 |
| 1 | 0.02012825 | 14330.143 | 853.9955 | 7 |
| 2 | 0.02696971 | 2008.678 | 739.4994 | 202 |

From the average features for each cluster, We can again see that the Cluster 1 with higher avegare\_income\_50K\_100K, the death percentage is far lesser than other counties, Also you can see that this cluster has comparatively higher worked from home too. This may also imply that the higher income brackets tend to work from home more.

### Supervised Cluster evaluation

We compared 4 clusterings and identified the best cluster using un-supervised evaluation for each of the clusters. We are using that best cluster for supervised cluster evaluation. The first step in supervised evaluation is to create a ground truth. For this we are using the death per case to calculate the ground truth.

The ground truth that was selected to evaluate the cluster was 3 groups created by using the death\_per\_case feature in the dataset

Purity measure was used to compare the Cluster with the ground truth. Below snippet shows the result of this comparison.

|  |  |
| --- | --- |
| **Cluster** | **Purity Measure** |
| **1** | **0.391891891891892** |
| **2** | 0.364864864864865 |
| **3** | 0.369369369369369 |
| **4** | 0.378378378378378 |

The higher purity means that the best hierarchical cluster that we evaluated against a supervised cluster. In this case the best clustering using the first DB Scan feature is considered as the best cluster using the supervised cluster evaluation.

## Fuzzy Clustering

Fuzzy clustering addresses the limitation of other clustering algorithms by allowing data points to belong to multiple clusters simultaneously. It has the following advantages over normal clustering. In fuzzy clustering each point is assigned a percentage on which cluster the point has a tendency towards, it is considered by data scientists to handle noise better than other traditional algorithms

### First Fuzzy Clustering

#### Fuzzy Clusters

We created 4 Fuzzy clusertings with different number of clusters 3,5,7 and 9. The below table shows a sample membership matrix of one of such clusters

| A data.frame: 6 × 10 | | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **white\_pop** | **income\_100K\_150K** | **rent\_under\_50\_percent** | **male\_21\_49** | **commute** | **1** | **2** | **3** | **4** | **5** |
| **1** | 2.9612278 | 7.379360 | 8.585387 | 8.310508 | 8.296710 | 0.01368586 | 0.01229809 | 0.01842358 | 0.92295329 | 0.03263917 |
| **2** | 1.2764403 | 5.693880 | 5.438001 | 7.719448 | 6.974905 | 0.02458624 | 0.02170197 | 0.03481091 | 0.85349230 | 0.06540859 |
| **3** | 4.9940727 | 3.343135 | 3.431992 | 2.789336 | 3.211183 | 0.02541254 | 0.01935258 | 0.05703270 | 0.01979302 | 0.87840916 |
| **4** | -0.3186604 | 1.795505 | 1.536212 | 2.235001 | 2.128970 | 0.22888514 | 0.16485510 | 0.42188055 | 0.02788566 | 0.15649356 |
| **5** | 0.7176674 | 2.581921 | 2.661243 | 3.414305 | 3.431921 | 0.13293897 | 0.10212284 | 0.28116375 | 0.06329871 | 0.42047572 |
| **6** | 3.1286271 | 4.846130 | 4.414474 | 3.190818 | 3.717860 | 0.06194688 | 0.04939587 | 0.12197340 | 0.09153978 | 0.67514407 |

From the above sample table, Point 1 could possibly in Cluster 1 with the highest value in the membership matrix and the second point has the highest value for cluster 5. This Membership matrix is evaluated for all points to identify the cluster to which the observation is tagged to.

#### Un Supervised Cluster evaluation

We are using **Davies-Bouldin Index** for unsupervised cluster evaluation. Davies Bouldin Index is calculated as the average similarity measure for each cluster with the cluster most similar to it. It can be also defined as the ratio between inter cluster and intra cluster distances.

Higher Db Index values correspond to poor clustering solutions and lower Db Index value corresponds to a better clustering solution. The below table shows the DB Index for all the Fuzzy cluster that we created for this project

|  |  |
| --- | --- |
| **Fuzzy Cluster** | **DB Index** |
| Cluster 1 – Number of Center = 5 | 0.659402825860543 |
| Cluster 2 – Number of Center = 3 | 0.654372790609573 |
| **Cluster 3 – Number of Center = 7** | **0.860569002796028** |
| Cluster 4 – Number of Center = 9 | 0.781519523190392 |

Clustering with 7 clusters has the lowest DBI Index, hence considered as the best cluster among the four clusters that we created.

#### Cluster Visualizations

Below cust plot shows that the clusters are fairly separated out.

A screenshot of a computer

Description automatically generated

The visualizations are performed for the best cluster identified by the cluster evaluation methods. The below Hull plot shows that the Fuzzy clusters are separated nicely.

A screenshot of a computer screen

Description automatically generated

A group of colorful dots

Description automatically generated

The above correlation chart also shows that the clusters are fairly separated out based on the features that we selected for the Fuzzy Clustering

A graph of a clustering chart

Description automatically generated with medium confidence

The below tale shows the median values of the feature selected for the fuzzy clustering, We can infer from this table that the commute and median income is linked, the clusters with higher median income are the clusters that have less commute to work and less affected by COVID cases.

| A tibble: 7 × 4 | | | |
| --- | --- | --- | --- |
| **cluster** | **avg\_death\_per\_case** | **avg\_male\_21\_49** | **no\_counties\_in\_cluster** |
| 1 | 0.02107392 | 11769.400 | 25 |
| 2 | 0.01928942 | 52798.800 | 5 |
| 3 | 0.02546976 | 4951.679 | 56 |
| 4 | 0.02888205 | 1362.843 | 121 |
| 5 | 0.01928446 | 28949.800 | 10 |
| 6 | 0.01445483 | 68155.667 | 3 |
| 7 | 0.02689154 | 161828.000 | 2 |

### Second Fuzzy Clustering

#### Fuzzy Clusters

We created 4 Fuzzy clusertings with different number of clusters 3,5,7 and 9. The below table shows a sample membership matrix of one of such clusters

| A data.frame: 6 × 9 | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **hispanic\_pop** | **income\_50K\_100K** | **female\_21\_49** | **worked\_at\_home** | **1** | **2** | **3** | **4** | **5** |
| **1** | 8.660063 | 8.073754 | 8.175117 | 7.768368 | 0.009383979 | 0.006608704 | 0.01597332 | 0.960806179 | 0.007227819 |
| **2** | 9.759021 | 6.338627 | 8.065484 | 9.673992 | 0.008272996 | 0.006027075 | 0.01307199 | 0.966088498 | 0.006539440 |
| **3** | 1.083512 | 3.277058 | 2.725398 | 2.030037 | 0.185618434 | 0.040044983 | 0.71164855 | 0.007630546 | 0.055057482 |
| **4** | 3.092729 | 1.636767 | 2.346927 | 1.402428 | 0.282612986 | 0.098742591 | 0.47285150 | 0.015812209 | 0.129980719 |
| **5** | 4.622239 | 3.096164 | 3.648852 | 2.048811 | 0.145281225 | 0.069430720 | 0.66340592 | 0.038334881 | 0.083547256 |
| **6** | 2.681424 | 4.056543 | 3.209328 | 2.103794 | 0.029206558 | 0.010370069 | 0.94329365 | 0.003968798 | 0.013160927 |

From the above sample table, Point 1 could possibly in Cluster 4 with the highest value in the membership matrix and the second point has the highest value for cluster 3. This Membership matrix is evaluated for all points to identify the cluster to which the observation is tagged to.

#### Un Supervised Cluster evaluation

We are using **Davies-Bouldin Index** for unsupervised cluster evaluation. Davies Bouldin Index is calculated as the average similarity measure for each cluster with the cluster most similar to it. It can be also defined as the ratio between inter cluster and intra cluster distances.

Higher Db Index values correspond to poor clustering solutions and lower Db Index value corresponds to a better clustering solution. The below table shows the DB Index for all the Fuzzy cluster that we created for this project

|  |  |
| --- | --- |
| **Fuzzy Cluster** | **DB Index** |
| Cluster 1 – Number of Center = 5 | 0.698357819262781 |
| Cluster 2 – Number of Center = 3 | **0.601713255833001** |
| **Cluster 3 – Number of Center = 7** | 0.778706202869979 |
| Cluster 4 – Number of Center = 9 | 1.17427221861615 |

Clustering with 7 clusters has the lowest DBI Index, hence considered as the best cluster among the four clusters that we created.

#### Cluster Visualizations

Below cust plot shows that the clusters are fairly separated out.

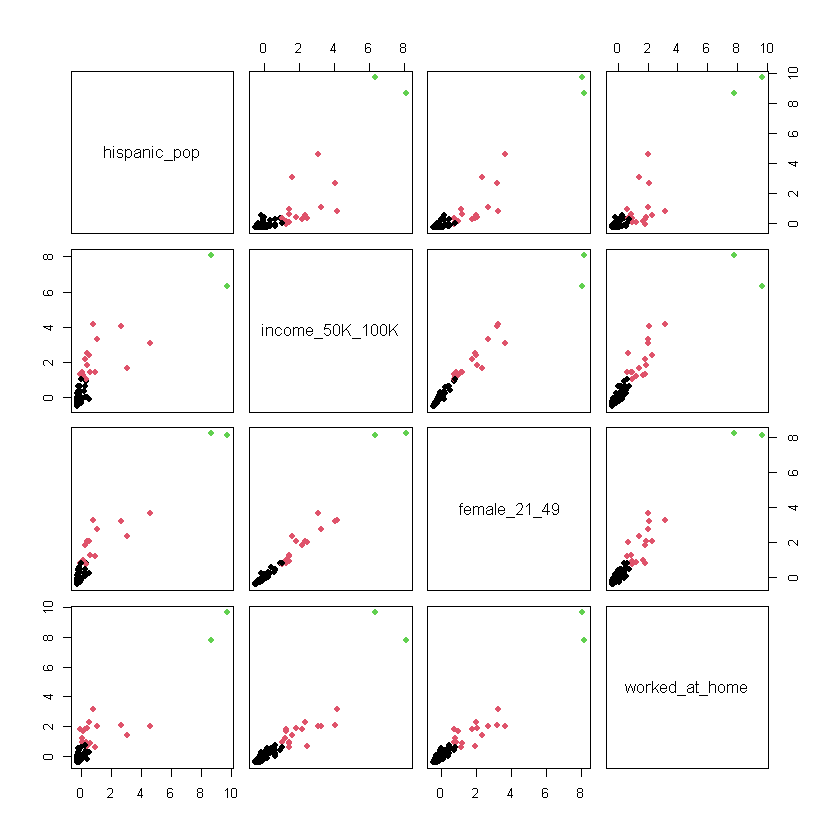
A computer screen shot of a computer screen

Description automatically generated

The visualizations are performed for the best cluster identified by the cluster evaluation methods. The below Hull plot shows that the Fuzzy clusters are separated nicely.

A screenshot of a computer

Description automatically generated



The above correlation chart also shows that the clusters are fairly separated out based on the features that we selected for the Fuzzy Clustering

A graph with red and green dots

Description automatically generated

The below tale shows the median values of the feature selected for the fuzzy clustering, We can infer from this table that the commute and median income is linked, the clusters with higher median income are the clusters that have less commute to work and less affected by COVID cases.

| A tibble: 3 × 5 | | | | |
| --- | --- | --- | --- | --- |
| **cluster** | **avg\_death\_per\_case** | **avg\_hispanic\_pop** | **avg\_female\_21\_49** | **no\_counties\_in\_cluster** |
| 1 | 0.02692633 | 6276.77 | 3425.74 | 204 |
| 2 | 0.01797299 | 97230.31 | 43345.25 | 16 |
| 3 | 0.02689154 | 728483.50 | 163360.50 | 2 |

### Third Fuzzy Clustering

#### Fuzzy Clusters

We created 4 Fuzzy clusertings with different number of clusters 3,5,7 and 9. The below table shows a sample membership matrix of one of such clusters

| A data.frame: 6 × 9 | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **income\_100K\_150K** | **rent\_under\_50\_percent** | **male\_50\_above** | **commute** | **1** | **2** | **3** | **4** | **5** |
| **1** | 7.379360 | 8.585387 | 7.969696 | 8.296710 | 0.011367254 | 0.01024474 | 0.01568615 | 0.932918652 | 0.02978320 |
| **2** | 5.693880 | 5.438001 | 7.063784 | 6.974905 | 0.024335804 | 0.02129370 | 0.03682869 | 0.828685051 | 0.08885675 |
| **3** | 3.343135 | 3.431992 | 2.661676 | 3.211183 | 0.003803802 | 0.00294918 | 0.01008608 | 0.002151353 | 0.98100959 |
| **4** | 1.795505 | 1.536212 | 1.710948 | 2.128970 | 0.081509287 | 0.05252204 | 0.72843789 | 0.008220008 | 0.12931078 |
| **5** | 2.581921 | 2.661243 | 3.865434 | 3.431921 | 0.039120082 | 0.03015021 | 0.09686898 | 0.022124126 | 0.81173661 |
| **6** | 4.846130 | 4.414474 | 3.954835 | 3.717860 | 0.052679122 | 0.04336076 | 0.10375155 | 0.093911156 | 0.70629741 |

From the above sample table, Point 1 could possibly in Cluster 4 with the highest value in the membership matrix and the second point has the highest value for cluster 3. This Membership matrix is evaluated for all points to identify the cluster to which the observation is tagged to.

#### Un Supervised Cluster evaluation

We are using **Davies-Bouldin Index** for unsupervised cluster evaluation. Davies Bouldin Index is calculated as the average similarity measure for each cluster with the cluster most similar to it. It can be also defined as the ratio between inter cluster and intra cluster distances.

Higher Db Index values correspond to poor clustering solutions and lower Db Index value corresponds to a better clustering solution. The below table shows the DB Index for all the Fuzzy cluster that we created for this project

|  |  |
| --- | --- |
| **Fuzzy Cluster** | **DB Index** |
| Cluster 1 – Number of Center = 5 | 0.739791885493779 |
| Cluster 2 – Number of Center = 3 | **0.689610898672008** |
| **Cluster 3 – Number of Center = 7** | 0.917229768702394 |
| Cluster 4 – Number of Center = 9 | 0.861722305390436 |

Clustering with 7 clusters has the lowest DBI Index, hence considered as the best cluster among the four clusters that we created.

#### Cluster Visualizations

Below cust plot shows that the clusters are fairly separated out.

A screen shot of a computer

Description automatically generated

The visualizations are performed for the best cluster identified by the cluster evaluation methods. The below Hull plot shows that the Fuzzy clusters are separated nicely.

A screenshot of a computer

Description automatically generated

A group of green and red dots

Description automatically generated

The above correlation chart also shows that the clusters are fairly separated out based on the features that we selected for the Fuzzy Clustering

A graph with red and blue dots

Description automatically generated

The below table shows the median values of the feature selected for the fuzzy clustering, We can infer from this table that the commute and median income is linked, the clusters with higher median income are the clusters that have less commute to work and less affected by COVID cases.

| A tibble: 3 × 4 | | | |
| --- | --- | --- | --- |
| **cluster** | **avg\_death\_per\_case** | **avg\_male\_50\_above** | **no\_counties\_in\_cluster** |
| 1 | 0.02696971 | 3688.322 | 202 |
| 2 | 0.02689154 | 103378.500 | 2 |
| 3 | 0.01848090 | 30637.056 | 18 |

### Fourth Fuzzy Clustering

#### Fuzzy Clusters

We created 4 Fuzzy clusterings with different number of clusters 3,5,7 and 9. The below table shows a sample membership matrix of one of such clusters

| A data.frame: 6 × 10 | | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **hispanic\_pop** | **income\_50K\_100K** | **male\_21\_49** | **commute** | **rent\_over\_50\_percent** | **1** | **2** | **3** | **4** | **5** |
| **1** | 8.660063 | 8.073754 | 8.310508 | 8.296710 | 6.891311 | 0.004956708 | 0.004561457 | 0.006306465 | 0.972809266 | 0.01136610 |
| **2** | 9.759021 | 6.338627 | 7.719448 | 6.974905 | 6.514913 | 0.005405325 | 0.004970625 | 0.006883826 | 0.970259258 | 0.01248097 |
| **3** | 1.083512 | 3.277058 | 2.789336 | 3.211183 | 4.478906 | 0.020097942 | 0.016286282 | 0.041450469 | 0.007413267 | 0.91475204 |
| **4** | 3.092729 | 1.636767 | 2.235001 | 2.128970 | 2.028639 | 0.138041448 | 0.106334464 | 0.313666841 | 0.022968602 | 0.41898865 |
| **5** | 4.622239 | 3.096164 | 3.414305 | 3.431921 | 3.223631 | 0.091145538 | 0.076625982 | 0.155663326 | 0.066084242 | 0.61048091 |
| **6** | 2.681424 | 4.056543 | 3.190818 | 3.717860 | 3.902913 | 0.033690754 | 0.027823478 | 0.063468415 | 0.021108875 | 0.85390848 |

From the above sample table, Point 1 could possibly in Cluster 4 with the highest value in the membership matrix and the second point has the highest value for cluster 3. This Membership matrix is evaluated for all points to identify the cluster to which the observation is tagged to.

#### Un Supervised Cluster evaluation

We are using **Davies-Bouldin Index** for unsupervised cluster evaluation. Davies Bouldin Index is calculated as the average similarity measure for each cluster with the cluster most similar to it. It can be also defined as the ratio between inter cluster and intra cluster distances.

Higher Db Index values correspond to poor clustering solutions and lower Db Index value corresponds to a better clustering solution. The below table shows the DB Index for all the Fuzzy cluster that we created for this project

|  |  |
| --- | --- |
| **Fuzzy Cluster** | **DB Index** |
| Cluster 1 – Number of Center = 5 | 0.0.670131737347825" |
| Cluster 2 – Number of Center = 3 | **0.0.645257336991538** |
| **Cluster 3 – Number of Center = 7** | 0.0.78578029993446 |
| Cluster 4 – Number of Center = 9 | 1.24486487889565 |

Clustering with 3 clusters has the lowest DBI Index, hence considered as the best cluster among the four clusters that we created.

#### Cluster Visualizations

Below cust plot shows that the clusters are fairly separated out.

A computer screen shot of a computer screen

Description automatically generated

The visualizations are performed for the best cluster identified by the cluster evaluation methods. The below Hull plot shows that the Fuzzy clusters are separated nicely.

A screenshot of a computer

Description automatically generated

A group of green and red dots

Description automatically generated

The above correlation chart also shows that the clusters are fairly separated out based on the features that we selected for the Fuzzy Clustering

A graph with different colored dots

Description automatically generated

The below table shows the median values of the feature selected for the fuzzy clustering, We can infer from this table that the commute and median income is linked, the clusters with higher median income are the clusters that have less commute to work and less affected by COVID cases.

| A tibble: 3 × 4 | | | |
| --- | --- | --- | --- |
| **cluster** | **avg\_death\_per\_case** | **avg\_male\_50\_above** | **no\_counties\_in\_cluster** |
| 1 | 0.02696971 | 3688.322 | 202 |
| 2 | 0.02689154 | 103378.500 | 2 |
| 3 | 0.01848090 | 30637.056 | 18 |

### Supervised Evaluation

We compared 4 clusterings and identified the best cluster using un-supervised evaluation for each of the clusters. We are using that best cluster for supervised cluster evaluation. The first step in supervised evaluation is to create a ground truth. For this we are using the death per case to calculate the ground truth.

The ground truth that was selected to evaluate the cluster was 3 groups created by using the death\_per\_case feature in the dataset for the 3 clusters and 7 groups for the first cluster

Purity measure was used to compare the Cluster with the ground truth. Below snippet shows the result of this comparison.

|  |  |
| --- | --- |
| **Cluster** | **Purity Measure** |
| **1** | 0.225225225225225 |
| **2** | 0.373873873873874 |
| **3** | **0.378378378378378** |
| **4** | 0.36036036036036 |

The higher purity means that the corresponding clustering is the best hierarchical cluster that we evaluated against a supervised cluster. In this case the best clustering using the 3rd Fuzzy CLuster considered as the best cluster using the supervised cluster evaluation.

# Evaluation

## K-means clustering

**First K-means Clustering**

After establishing optimum number of clusters, we generated a heat map of Texas showing all the counties that belonged to one of the two clusters. There are 16 counties that belong to cluster 1, and the name of those counties are shown below. Most of these counties lie far away from the major cities like DFW, Austin, San Antonio, and Houston. We can also see that these counties had higher average confirmed cases per 1000 than the counties that belong to cluster 2. But surprisingly, these counties had lower average deaths per 1000 than those that belong to cluster 2. This could mean that the counties in cluster 1 more confirmed cases but there were effective interventions or better healthcare that kept the mortality rate lower. On the contrary, the counties in cluster 2 could have been less equipped to manage the severity of the virus, or other factors which contributed to higher death rate.

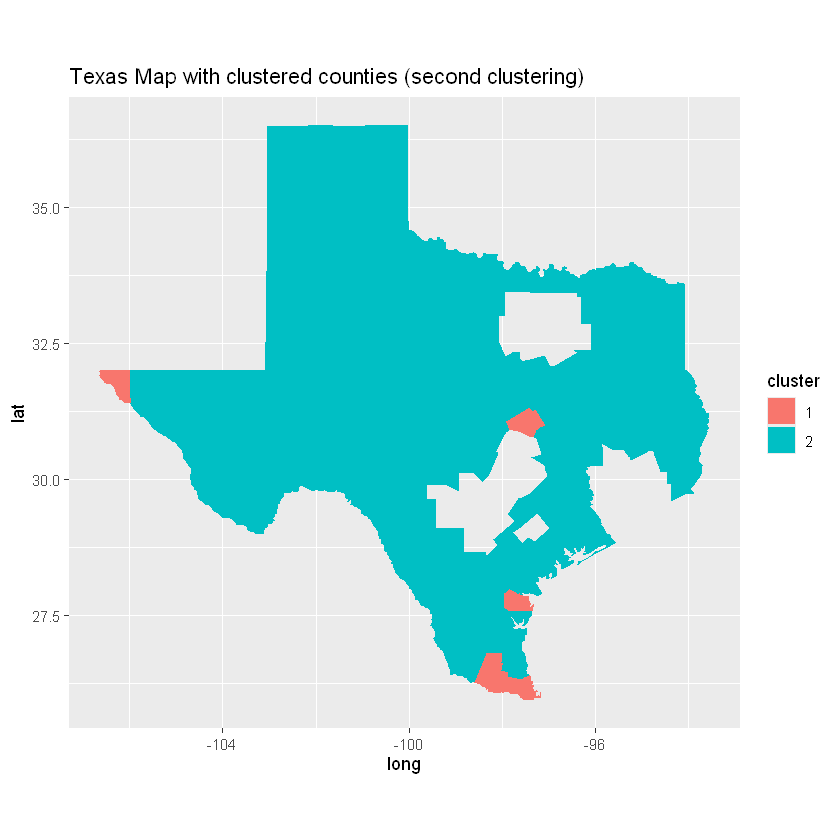
 

|  |  |  |
| --- | --- | --- |
| cluster | avg\_cases\_per\_1000 | avg\_deaths\_per\_1000 |
| 1 | 88.22725 | 1.657642 |
| 2 | 78.07511 | 2.009892 |

Clusters of counties for first k-means clustering

**Second K-means Clustering**

After establishing optimum number of clusters, we generated a heat map of Texas showing all the counties that belonged to one of the two clusters. There are 5 counties that belong to cluster 1, and the name of those counties are shown below. Most of these counties lie far away at the edges of the state. We can also see that these counties had higher average confirmed cases per 1000 than the counties that belong to cluster 2. Additionally, these counties also had higher average deaths per 1000 than those that belong to cluster 2. This could mean that the counties in cluster 2 had less equipped to manage the severity of the virus, or other factors contributing to higher confirmed and death rates.



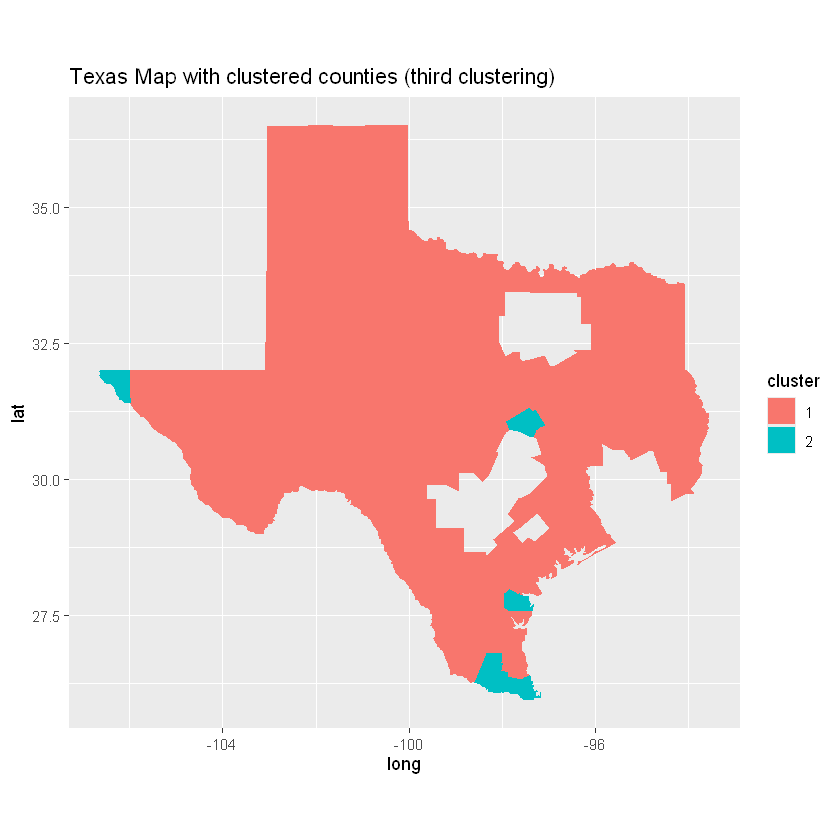


|  |  |  |
| --- | --- | --- |
| cluster | avg\_cases\_per\_1000 | avg\_deaths\_per\_1000 |
| 1 | 98.03851 | 2.363770 |
| 2 | 78.63196 | 1.981056 |

Clusters of counties for second k-means clustering

**Third K-means Clustering**

After establishing optimum number of clusters, we generated a heat map of Texas showing all the counties that belonged to one of the two clusters. There are 5 counties that belong to cluster 1, and the name of those counties are shown below. Most of these counties lie far away at the edges of the state. Interestingly these are the same 5 counties that were in same cluster (i.e. cluster 1) in second k-means clustering. We can also see that these counties had higher average confirmed cases per 1000 than the counties that belong to cluster 1. However, these counties had slightly lower average deaths per 1000 than those that belong to cluster 1. This could mean that the counties in cluster 2 were able to control the deaths better than those counties that belong to cluster 1 despite having higher average confirmed cases.



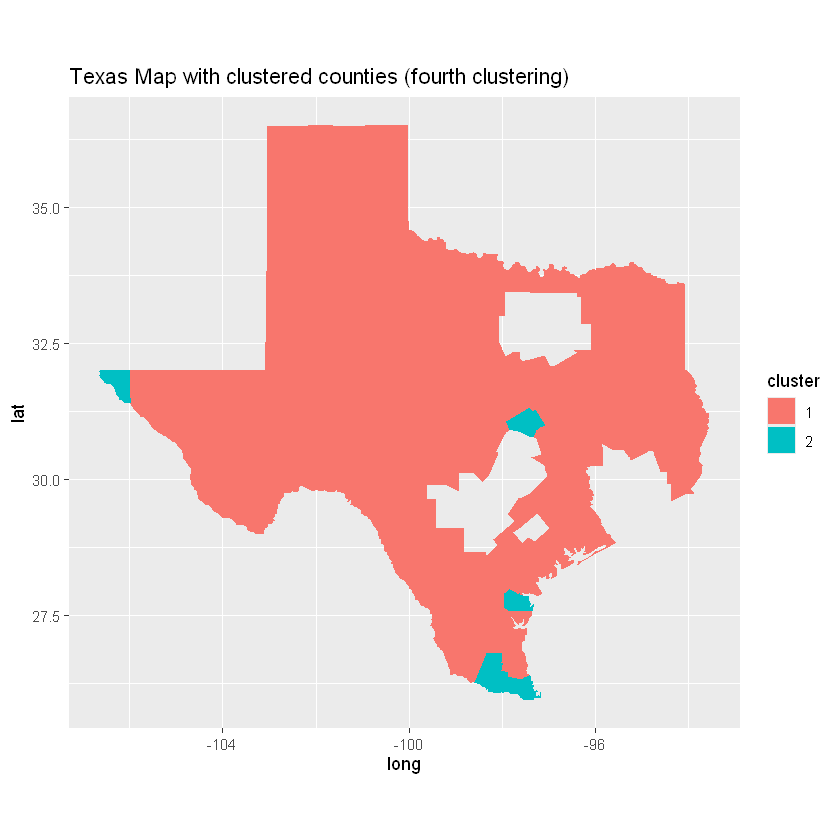


|  |  |  |
| --- | --- | --- |
| cluster | avg\_cases\_per\_1000 | avg\_deaths\_per\_1000 |
| 1 | 78.72391 | 1.986519 |
| 2 | 82.40390 | 1.897068 |

Fig. Clusters of counties for third k-means clustering

**Fourth K-means Clustering**

After establishing optimum number of clusters, we generated a heat map of Texas showing all the counties that belonged to one of the two clusters. There are 5 counties that belong to cluster 2, and the name of those counties are shown below. Most of these counties lie far away at the edges of the state. Interestingly these are the same 5 counties that were in same cluster (i.e. cluster 2) in third k-means clustering. However, it is quite interesting to see that the average cases are higher in counties that belong cluster 1 while this was opposite in third clustering (i.e. higher in counties that belong to cluster 2). Same is true for the average deaths. This indicates that if the features/attributes that were considered for fourth clustering are chosen, the metrics on average confirmed cases and deaths are exactly opposite if the features from third clustering are chosen.





|  |  |  |
| --- | --- | --- |
| cluster | avg\_cases\_per\_1000 | avg\_deaths\_per\_1000 |
| 1 | 82.40390 | 1.897068 |
| 2 | 78.72391 | 1.986519 |

Fig. Clusters of counties for fourth k-means clustering

**Overall Evaluation**

Based on the different clustering that were performed, We were able to identify clusters based on the different features selected and was able to infer that COVID impact based on the income and race features and were able to create clusters of counties which shows these and can be used as a baseline to identify key counties that DHS need to concentrate on……………………………………(to be continued)

## Hierarchical clustering

Based on the different clustering that were performed, We were able to identify clusters based on the different features selected and was able to infer that COVID impact based on the income and race features and were able to create clusters of counties that shows these and can be used as a baseline to identify key counties that DHS need to concentrate on. The cluster points that were included in the noise was those which were slightly different in the features. However these clusters also fall into the clusters that needs DHS concentration

## Density Based clustering

Based on the different clustering that were performed, We were able to identify clusters based on the different features selected and was able to infer that COVID impact based on the income and race features and were able to create clusters of counties that shows these and can be used as a baseline to identify key counties that DHS need to concentrate on

## Fuzzy clustering

The best Fuzzy clustering split the counties in to 3 clusters.

We could identify from the correlation and cluster visualization that the cluster that had most impact due to the COVID were the clusters with the lowest income between 50K and 100K . This also alludes to understanding that DHS need to concentrate on these counties with comparatively lower income

# List of References

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# Appendix

## Graduate level additional work

The below additional work performed for graduate level credits:

* The below additional clustering algorithms are implemented in the project in addition to the K Means and Hierarchical clustering algorithms
  + Density Based Clustering
  + Fuzzy Clustering