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Incremental mining of sequential patterns in large databases

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Abstract

In this paper, we consider the problem of the incremental mining of sequential patterns when new transactions or new customers are added to an original database. We present a new algorithm for mining frequent sequences that uses information collected during an earlier mining process to cut down the cost of finding new sequential patterns in the updated database. Our test shows that the algorithm performs significantly faster than the naive approach of mining the whole updated database from scratch. The difference is so pronounced that this algorithm could also be useful for mining sequential patterns, since in many cases it is faster to apply our algorithm than to mine sequential patterns using a standard algorithm, by breaking down the database into an original database plus an increment.

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1. Introduction

Most research into data mining has concentrated on the problem of mining association rules [1–8]. Although sequential patterns are of great practical importance (e.g. alarms in telecommunications networks, identifying plan failures, analysis of Web access databases, etc.) they have

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received relatively little attention [9–11]. First introduced in [9], where an efficient algorithm called AprioriAll was proposed, the problem of mining sequential patterns is to discover temporal relationships between facts embedded in the database. The facts under consideration are simply the characteristics of individuals, or observations of individual behavior. For example, in a video database, a sequential pattern could be “95% of customers bought ‘Star Wars and The Empire Strikes Back’, then ‘Return of the Jedi’, and then ‘The Phantom Menace’”. In [10], the definition of the problem was extended to handle time constraints and taxonomies (*is-a* hierarchies) and a new algorithm, called generalized sequential patterns (GSP), which outperformed AprioriAll by up to 20 times, was proposed.

As databases evolve the problem of maintaining sequential patterns over a significantly long period of time becomes crucial, since a large number of new records may be added to a database. To reflect the current state of the database where previous sequential patterns would become irrelevant and new sequential patterns might appear, there is a need for efficient algorithms to update, maintain and manage the information discovered [12]. Several efficient algorithms for maintaining association rules have been developed [12–15]. Nevertheless, the problem of maintaining sequential patterns is much more complicated than maintaining association rules, since timestamps and sequence permutation have to be taken into account [16]. In order to illustrate the problem, let us consider an original and an incremental database. In order to compute the set of sequential patterns embedded in the updated database, we need to discover all sequential patterns which were not frequent in the original database but have become frequent with the increment. We also need to examine all transactions in the original database that can be extended to become frequent. Furthermore, old frequent sequences may become invalid when a new customer is added. The challenge is thus to discover all the frequent patterns in the updated database with far greater efficiency than the naive method of mining sequential patterns from scratch.

In this paper, we propose an efficient algorithm, called incremental sequence extraction (ISE), for computing the frequent sequences in the updated database when new transactions and new customers are added to the original database. ISE minimizes computational costs by re-using the minimal information from the old frequent sequences, i.e. the support of frequent sequences. The main new feature of ISE is that the set of candidate sequences to be tested is substantially reduced. Furthermore, some optimization techniques for improving the approach are also provided.

Empirical evaluations were carried out to analyze the performance of ISE and compare it against cases where GSP is applied to the updated database from scratch. Experiments showed that ISE outperforms the GSP algorithm by a factor of 4–6. Indeed the difference is so pronounced that our algorithm may be useful for mining sequential patterns as well as incremental mining, since in many cases, instead of mining the database with the GSP algorithm, it is faster to extract an increment from the database then apply our approach, considering that the database is broken down into an original database plus an increment. Our comparative experimental results show an improvement in performance by a factor of 2–5.

The rest of this paper is organized as follows. Section 2 states the problem and describes related research. The algorithm ISE is described in Section 3. Section 4 describes the experiments in detail and interprets the performance results obtained. Finally, Section 5 concludes the paper with future avenues for research.

2. Statement of the problem

In this section, we give the formal definition of the problem of incremental sequential pattern mining. First, we formulate the concept of sequence mining summarizing the formal description of the problem introduced in [9] and extended in [10]. A brief overview of the GSP algorithm is also provided. Second, we examine the incremental update problem in detail.

2.1. Mining of sequential patterns

Let DB be a set of customer transactions where each transaction T consists of a customer-id, a transaction time and a set of items involved in the transaction. Let $I = \{i_1, i_2, \dots, i_m\}$ be a set of literals called *items*. An *itemset* is a non-empty set of items. A sequence s is a set of itemsets ordered according to their time stamp. It is denoted by $\langle s_1, s_2, \dots, s_n \rangle$, where s_j , $j \in 1, \dots, n$, is an itemset. A k -sequence is a sequence of k items (or of length k). For example, let us consider that a given customer purchased items 1, 2, 3, 4, 5, according to the following sequence: $s = \langle (1)(2, 3)(4)(5) \rangle$. That is, items in the sequence were bought separately, apart from 2 and 3 that were purchased together, i.e. during the same transaction, s is a 5-sequence. A sequence $\langle s_1, s_2, \dots, s_n \rangle$ is a sub-sequence of another sequence $\langle s'_1, s'_2, \dots, s'_m \rangle$ if there exist integers $i_1 < i_2 < \dots < i_j < \dots < i_n$ such that $s_1 \subseteq s'_{i_1}, s_2 \subseteq s'_{i_2}, \dots, s_n \subseteq s'_{i_n}$. For example, the sequence $s' = \langle (2)(5) \rangle$ is a sub-sequence of s because $(2) \subseteq (2, 3)$ and $(5) \subseteq (5)$. However, $\langle (2)(3) \rangle$ is not a sub-sequence of s since items were not bought during the same transaction.

Property 1. If $A \subseteq B$ for sequences A, B then $\text{supp}(A) \geq \text{supp}(B)$ because all transactions in DB that supports necessarily also support A .

All transactions from the same customer are grouped together and sorted in increasing order and are called a *data sequence*. A support value ($\text{supp}(s)$) for a sequence gives its number of actual occurrences in DB . Nevertheless, a sequence in a data sequence is taken into account only once to compute the support even if several occurrences are discovered. In other words, the support of a sequence is defined as the fraction of all the distinct data sequences that contain s . A data sequence contains a sequence s if s is a sub-sequence of the data sequence. In order to decide whether a sequence is frequent or not, a minimum support value (minSupp) is specified by the user, and the sequence is said to be *frequent* if the condition $\text{supp}(s) \geq \text{minSupp}$ holds.

Given a database of customer transactions, the problem of sequential pattern mining is to find all the sequences whose support is greater than a specified threshold (minimum support). Each of these represents a *sequential pattern*, also called a *frequent sequence*.

The task of discovering all the frequent sequences in large databases is quite challenging since the search space is extremely large (e.g. with m attributes there are $O(m^k)$ potentially frequent sequences of length k) [11]. To the best of our knowledge, the problem of mining sequential patterns according to the previous definitions has received relatively little attention.

We shall now briefly review the GSP algorithm. For building up candidate and frequent sequences, the GSP algorithm makes multiple passes over the database. The first step aims at computing the support of each item in the database. When this step has been completed, the frequent items (i.e. those that satisfy the minimum support) have been discovered. They are

considered as frequent 1-sequences (sequences having a single itemset, itself a singleton). The set of candidate 2-sequences is built up according to the following assumption: candidate 2-sequences could be any pair of frequent items, whether embedded in the same transaction or not. Frequent 2-sequences are determined by counting the support. From this point, candidate k -sequences are generated from frequent $(k - 1)$ -sequences obtained in pass- $(k - 1)$. The main idea of candidate generation is to retrieve, from among $(k - 1)$ -sequences, pairs of sequences (s, s') such that discarding the first element of the former and the last element of the latter results in two fully matching sequences. When such a condition holds for a pair (s, s') , a new candidate sequence is built by appending the last item of s' to s . The supports for these candidates are then computed and those with minimum support become frequent sequences. The process iterates until no more candidate sequences are formed.

2.2. Incremental mining on discovered sequential patterns

Let DB be the original database and minSupp be the minimum support. Let db be the increment database where new transactions or new customers are added to DB . We assume that each transaction on db has been sorted by customer-id and transaction time. $U = DB \cup db$ is the updated database containing all sequences from DB and db .

Let L^{DB} be the set of frequent sequences in DB . The problem of incremental mining of sequential patterns is to find frequent sequences in U , noted L^U , with respect to the same minimum support. Furthermore, the incremental approach has to take advantage of previously discovered patterns in order to avoid re-running all the mining algorithms when the data is updated.

First, we consider the problem of when new transactions are appended to customers already existing in the database. In order to illustrate this problem, let us consider the base DB given in Fig. 1, giving facts about a population reduced to just four customers. Transactions are ordered according to their time-stamp. For instance, the data sequence of customer $C3$ is $\langle(10\ 20)(40)(30)\rangle$. Let us assume that the minimum support value is 50%, which means that in order to be considered as frequent a sequence must be observed for at least two customers. The set of all maximum frequent sequences embedded in the database is the following: $L^{DB} = \{\langle(10\ 20)(30)\rangle, \langle(10\ 20)(40)\rangle\}$. After some update activities, let us consider the increment database db (described in Fig. 1) where new transactions are appended to customers $C2$ and $C3$. Assuming that the support value is the same, the following two sequences $\langle(60)(90)\rangle$ and $\langle(10\ 20)(50\ 70)\rangle$

Cust-Id	Itemsets			Itemsets	
$C1$	10 20	20	50 70	50 60 70	80 100
$C2$	10 20	30	40	50 60	80 90
$C3$	10 20	40	30		
$C4$	60	90			

(DB)
(db)

Fig. 1. An original database (DB) and an increment database with new transactions (db) and db .

Cust-Id	Itemsets				Itemsets	
C1	10	20	20	50	70	
C2	10	20	30	40	50	60
C3	10	20	40	30	50	60
C4	60	90				
C5					10	40

(DB) (db)

Fig. 2. An original database (DB) and an increment database with new transactions and new customers (db).

become frequent after the database update since they have sufficient support. Let us consider the first of these. The sequence is not frequent in *DB* since the minimum support does not hold (it only occurs for the last customer). With the increment database, this sequence becomes frequent since it appears in the data sequences of customers *C3* and *C4*. The sequence $\langle(1020)\rangle$ could be detected for customers *C1*, *C2* and *C3* in the original database. By introducing the increment database the new frequent sequence $\langle(1020)(5070)\rangle$ is discovered because it matches with transactions of *C1* and *C2*. Furthermore, new frequent sequences are discovered: $\langle(1020)(30)(5060)(80)\rangle$ and $\langle(1020)(40)(5060)(80)\rangle$. $\langle(5060)(80)\rangle$ is a frequent sequence in *db* and on scanning *DB* we find that the frequent sequences in L^{DB} are its predecessor.

Let us now consider the problem when new customers and new transactions are appended to the original database (Fig. 2). Let us consider that the minimum support value is still 50%, which means that in order to be considered as frequent a sequence must now be observed for at least three customers since a new customer *C5* has been added. According to this constraint the set of frequent sequences embedded in the original database becomes $L^{DB} = \{\langle(1020)\rangle\}$ since the sequences $\langle(1020)(30)\rangle$ and $\langle(1020)(40)\rangle$ occur only for customers *C2* and *C3*. Nevertheless, the sequence $\langle(1020)\rangle$ is still frequent since it appears in the data sequences of customer *C1*, *C2* and *C3*. By introducing the increment database, the set of frequent sequences in the updated database is $L^U = \{\langle(1020)(50)\rangle, \langle(10)(70)\rangle, \langle(10)(80)\rangle, \langle(40)(80)\rangle, \langle(60)\rangle\}$. Let us now take a closer look at the sequence $\langle(1020)(50)\rangle$. This sequence could be detected for customer *C1* in the original database but it is not a frequent sequence. Nevertheless, as the item 50 becomes frequent with the increment database, this sequence also matches with transactions of *C2* and *C3*. In the same way, the sequence $\langle(10)(70)\rangle$ becomes frequent since, with the increment, it appears in the data sequences of *C1*, *C2* and the new customer *C5*.

2.3. Related work

A considerable body of work has been carried out on the problem of incremental association rule mining [12,13,17–21], but incremental sequential pattern mining has received very little attention. Furthermore, among the available work in the field, no research has dealt with time constraints or is ready to do so. This section is intended to give two points of view: FASTUP [22] and a SuffixTree approach [23] on the one hand, and ISM [16] on the other.

2.3.1. SuffixTree and FASTUP approaches

In [23], the authors proposed a solution based on suffix tree techniques. The structure used in that context acquires the data and builds up the frequent sequences in one scan, by means of a suffix tree. This method is thus very appropriate to incremental sequence extraction, because it only has to continue the data reading after the update. Though the effectiveness of such a method cannot be denied, it has the drawback that the complexity in space of the proposed algorithm (as well as that of ISM, described below) depends on the size of the database.

FASTUP, proposed by [22], is an example of the first work done on incremental sequential pattern mining, where complexity in space depends on the size of the result. Indeed, FASTUP is in effect an enhanced GSP, taking into account the previous mining result, before generating and validating candidates using the generating-pruning method.

The main idea is that FASTUP, by means of the previous result, takes advantage of information about sequence thresholds to generate candidates. It can therefore avoid generating some sequences, depending on their support.

2.3.2. ISM

The ISM algorithm, proposed by [16], is actually an extension of SPADE [24], which aims at considering the update by means of the negative border and a rewriting of the database.

Fig. 3 is an example of a database and its update (items in bold characters). We observe that three clients have been updated.

The first iterations of SPADE on *DBspade*, ended in the lattice given in Fig. 4 (without the gray section). The main idea of ISM is to keep the negative border (in gray, Fig. 4) *NB*, which is made of *j*-candidates, at the bottom of the hierarchy in the lattice. In other words, let *s* be a sequence in *NB*, then $\overline{A}s'/s'$ is child of *s* and $s' \in NB$, and more precisely *NB* is made of sequences which are not frequent but which are generated by frequent subsequences. We can observe, in Fig. 4 the lattice and negative border for *DBspade*. Note that hash lines stand for a hierarchy that does not end in a frequent sequence.

The first step of ISM aims at pruning the sequences that become infrequent from the set of frequent sequences after the update. One scan of the database is enough to update the lattice as well as the negative border. The second step aims at taking into account the new frequent sequences one by one, in order to make the information browse the lattice using the SPADE generating process. The field of observation considered by ISM is thus limited to the new items. For further information you can refer to [16,25].

Example 1. Let us consider item “C” in *DBspade*. This item only has a threshold of 1 sequence according to SPADE. After the update given in Fig. 3, ISM will consider that support, which is now of four sequences. “C” is now going from NB to the set of frequent sequences. In the same way, the sequences $\langle(A)(A)(B)\rangle$ and $\langle(A)(B)(B)\rangle$ become frequent after the update and go from *NB* to the set of frequent sequences. This is the goal of the first step.

The second step is intended to consider the generation of candidates, but is limited to the sequences added to the set of frequent sequences during the first step. For instance, sequences $\langle(A)(A)(B)\rangle$ and $\langle(A)(B)(B)\rangle$ can generate the candidate $\langle(A)(A)(B)(B)\rangle$ which will have a support of 0 sequences and will be added to the negative border. After the update, the set of frequent

Client	Itemset	Items
1	10	A B
	20	B
	30	A B
	100	A C
2	20	A C
	30	A B C
	50	B
3	10	A
	30	B
	40	A
	110	C
	120	B
4	30	A B
	40	A
	50	B
	140	C

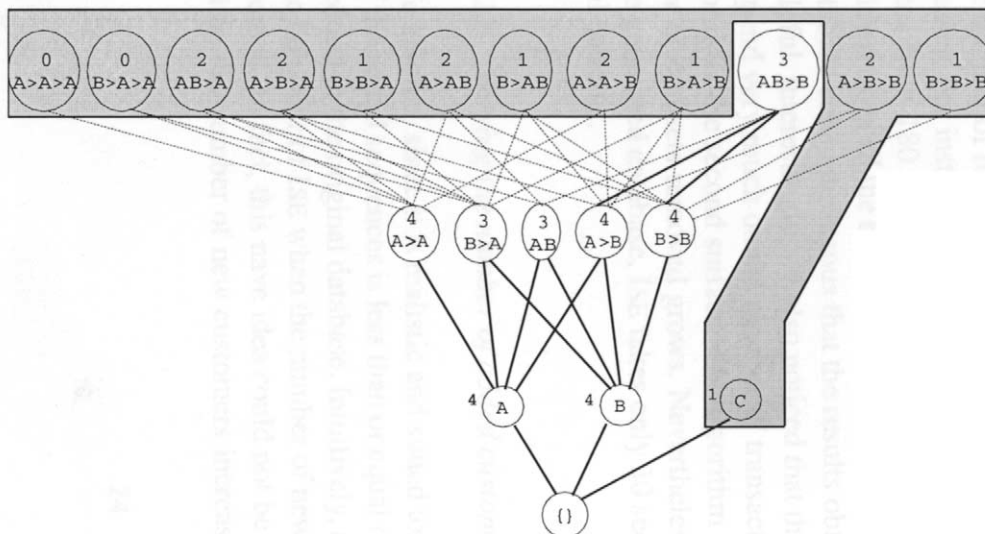
Fig. 3. *DBspade*, a database and its update.

Fig. 4. The negative border, considered by ISM after using SPADE on the database from Fig. 3, before the update.

sequences will thus be: $A, B, C, \langle(A)(A)\rangle, \langle(B)(A)\rangle, \langle(AB)\rangle, \langle(A)(B)\rangle, \langle(B)(B)\rangle, \langle(A)(C)\rangle, \langle(B)(C)\rangle, \langle(A)(A)(B)\rangle, \langle(AB)(B)\rangle, \langle(A)(B)(B)\rangle, \langle(A)(A)(C)\rangle, \langle(A)(B)(C)\rangle$.

At the end of the second and last step, the lattice is updated and ISM can give the new set of frequent sequences, as well as a new negative border, allowing the algorithm to take a new update into account. As we observe in Fig. 4, the lattice storing the frequent itemsets and the negative border can be very large and memory intensive. Our proposal aims to provide better memory management and study candidate generation in order to reduce the number of sequences to be evaluated at each scan of the database.

3. ISE algorithm

In this section, we introduce the ISE algorithm for computing frequent sequences in the updated database. After a brief description of our proposal, we explain, step by step, our method for efficiently mining new frequent sequences using information collected during an earlier mining process. Then we present the associated algorithm and the optimization techniques.

3.1. An overview

How can we solve the problem of the incremental mining of frequent sequences by using previously discovered information? To find all new frequent sequences, three kinds of frequent sequences need to be considered. First, sequences embedded in DB could become frequent since they have sufficient support with the incremental database, i.e. sequences similar to sequences embedded in the original database appear in the increment. Next, there may be new frequent sequences embedded in db but not appearing in the original database. Finally, sequences of DB might become frequent when items from db are added.

To discover frequent sequences, the ISE algorithm executes iteratively. In Table 1, we summarize the notation used in the algorithm. Since the main consequence of adding new customers is to verify the support of the frequent sequences in L^{DB} , in the next section we first illustrate iterations through examples mainly concerning added transactions by existing customers. Finally, Example 5 illustrates the behavior of ISE when new transactions and new customers are added to the original database.

Table 1
Notation for algorithm

L_{DB}	Frequent sequences in the original database
L_1^{db}	Frequent 1-sequences embedded in db and validated on U
$candExt$	Candidate sequences generated from db
$freqExt$	Frequent sequences obtained from $candExt$ and validated on U
$freqSeed$	Frequent sub-sequences of L^{DB} extended with an item from L_1^{db}
$candInc$	Candidate sequences generated by appending sequences of $freqExt$ to sequences of $freqSeed$
$freqInc$	Frequent sequences obtained from $candInc$ and validated on U
L^U	Frequent sequences in the updated database

3.1.1. First iteration

During the first pass on db , we count the support of individual items and are provided with 1-candExt standing for the set of items occurring at least once in db . Considering the set of items embedded in DB we determine which items of db are frequent in U . This set is called L_1^{db} .

At the end of this pass, if there are additional customers, we prune out frequent sequences in L^{DB} that no longer verify the minimum support.

Example 1. Consider the increment database in Fig. 1. When db is scanned we find the support of each individual item during the pass over the data: $\{(\langle(50)\rangle, 2), (\langle(60)\rangle, 2), (\langle(70)\rangle, 1), (\langle(80)\rangle, 2), (\langle(90)\rangle, 1), (\langle(100)\rangle, 1)\}$. Let us consider that a previous mining of DB provided us with the items embedded in DB with their support:

Item	10	20	30	40	50	60	70	90
Support	3	3	2	2	1	1	1	1

Combining these items with the result of the scan db , we obtain the set of frequent 1-sequences which are embedded in db and frequent in U : $L_1^{db} = \{(\langle(50)\rangle), (\langle(60)\rangle), (\langle(70)\rangle), (\langle(80)\rangle), (\langle(90)\rangle)\}$.

We use the frequent 1-sequences in db to generate new candidates. This candidate generation works by joining L_1^{db} with L_1^{db} and yields the set of candidate 2-sequences. We scan db and obtain the 2-sequences embedded in db . Such a set is called 2-candExt . This phase is quite different from the GSP approach since we do not consider the support constraint. We assume, according to Lemma 2 (cf. Section 3.2), that a candidate 2-sequence is in 2-candExt if and only if it occurs at least once in db . The main reason is that we do not want to provide the set of all 2-sequences, but rather to obtain the set of potential extensions of items embedded in db . In other words, if a candidate 2-sequence does not occur in db it cannot possibly be an extension of an original frequent sequence of DB , and thus cannot give a frequent sequence for U . In the same way, if a candidate 2-sequence occurs in db , this sequence might be an extension of previous sequences in DB .

Next, we scan U to find out frequent 2-sequences from 2-candExt . This set is called $freqExt$ and it is achieved by discarding the 2-sequences that do not verify the minimum support from 2-candExt .

Example 2. Consider L_1^{db} in the previous example. From this set, we can generate the following sequences $\langle(50\ 60)\rangle, \langle(50)(60)\rangle, \langle(50\ 70)\rangle, \langle(50)(70)\rangle, \dots, \langle(80)(90)\rangle$. To discover 2-candExt in the updated database, we only have to consider if an item occurs at least once in db . For instance, since the candidate $\langle(50)(60)\rangle$ does not appear in db , it is no longer considered when U is scanned. After the scan of U with remaining candidates, we are thus provided with the following set of frequent 2-sequences, $2\text{-freqExt} = \{(\langle(50\ 60)\rangle), (\langle(50)(80)\rangle), (\langle(50\ 70)\rangle), (\langle(60)(80)\rangle), (\langle(60)(90)\rangle)\}$.

An additional operation is performed on the frequent items discovered in db . Based on Property 1 and Lemma 2 (cf. Section 3.2) the main idea is to retrieve in DB the frequent sub-sequences of L^{DB} preceding items of db , according to their order in time.

In order efficiently to find the frequent sub-sequences preceding an item, we create for each frequent sub-sequence an array that has as many elements as the number of frequent items in db .

When scanning U , for each data sequence and for each frequent sub-sequence, we check whether it is contained in the data sequence. In such a case, the support of each item following the sub-sequence is incremented.

During the scan to find 2-freqExt , we also obtain the set of frequent sub-sequences preceding items of db . From this set, by appending the items of db to the frequent sub-sequences we obtain a new set of frequent sequences. This set is called $freqSeed$. In order to illustrate how this new set of frequent sequences is obtained, let us consider the following example.

Example 3. Consider item 50 in L_1^{db} . For customer C_1 , 50 is preceded by the following frequent sub-sequences: $\langle(10)\rangle$, $\langle(20)\rangle$ and $\langle(1020)\rangle$. If we now consider customer C_2 with the updated transaction, we are provided with the following set of frequent sub-sequences preceding 50: $\langle(10)\rangle$, $\langle(20)\rangle$, $\langle(30)\rangle$, $\langle(40)\rangle$, $\langle(1020)\rangle$, $\langle(10)(30)\rangle$, $\langle(10)(40)\rangle$, $\langle(20)(30)\rangle$, $\langle(20)(40)\rangle$, $\langle(1020)(30)\rangle$ and $\langle(1020)(40)\rangle$. The process is repeated until all transactions are examined. In Fig. 5, we show the frequent sub-sequences as well as their support in U .

Let us now examine item 90. Even if the sequence $\langle(60)(90)\rangle$ is detected for C_3 , and C_4 , it is not considered since 60 was not frequent in the original database, i.e. $60 \notin L^{DB}$. Actually, this sequence is discovered as frequent in 2-freqExt .

The set $freqSeed$ is obtained by appending to each item of L_1^{db} its associated frequent sub-sequences. For example, if we consider item 70, then the following sub-sequences are inserted into $freqSeed$: $\langle(10)(70)\rangle$, $\langle(20)(70)\rangle$ and $\langle(1020)(70)\rangle$.

At the end of the first scan on U , we are thus provided with a new set of frequent 2-sequences (in 2-freqExt) as well as a new set of frequent sequences (in $freqSeed$). In subsequent iterations we go on to discover the all frequent sequences not yet embedded in $freqSeed$ and 2-freqExt .

Items	Frequent sub-sequences
50	$\langle(10)\rangle_3$ $\langle(20)\rangle_3$ $\langle(30)\rangle_2$ $\langle(40)\rangle_2$ $\langle(10)(30)\rangle_2$ $\langle(10)(40)\rangle_2$ $\langle(20)(30)\rangle_2$ $\langle(20)(40)\rangle_2$ $\langle(1020)\rangle_3$ $\langle(1020)(30)\rangle_2$ $\langle(1020)(40)\rangle_2$
60	$\langle(10)\rangle_2$ $\langle(20)\rangle_2$ $\langle(30)\rangle_2$ $\langle(40)\rangle_2$ $\langle(10)(30)\rangle_2$ $\langle(10)(40)\rangle_2$ $\langle(20)(30)\rangle_2$ $\langle(20)(40)\rangle_2$ $\langle(1020)\rangle_2$ $\langle(1020)(30)\rangle_2$ $\langle(1020)(40)\rangle_2$
70	$\langle(10)\rangle_2$ $\langle(20)\rangle_2$ $\langle(1020)\rangle_2$
80	$\langle(10)\rangle_2$ $\langle(20)\rangle_2$ $\langle(30)\rangle_2$ $\langle(40)\rangle_2$ $\langle(10)(30)\rangle_2$ $\langle(10)(40)\rangle_2$ $\langle(20)(30)\rangle_2$ $\langle(20)(40)\rangle_2$ $\langle(1020)\rangle_2$ $\langle(1020)(30)\rangle_2$ $\langle(1020)(40)\rangle_2$
90	-

Fig. 5. Frequent sub-sequences occurring before items of db .

3.1.2. j th Iteration

Let us assume that we are at the j th pass. In these subsequent iterations, we start by generating new candidates from the two sets found in the previous pass. The main idea of candidate generation is to retrieve from among sequences of *freqSeed* and j -*freqExt*, two sequences ($s \in \text{freqSeed}, s' \in j\text{-freqExt}$) such that the last item of s is the first item of s' . When such a condition hold items for a pair (s, s') , a new candidate sequence is built by dropping the last of s and appending s' to the remaining sequence. Furthermore, an additional operation is performed on j -*freqExt*: we use the same candidate generation algorithm as in GSP to produce new candidate $(j+1)$ -sequences from j -*freqExt*. Candidates occurring at least once in db , are inserted in the $(j+1)$ -*candExt* set. The supports for all candidates are then obtained by scanning U and those with minimum support become frequent sequences. The two sets become, respectively, *freqInc* and $(j+1)$ -*freqExt*. The last one and *freqSeed* are then used to generate new candidates. The process iterates until all frequent sequences are discovered, i.e. until no more candidates are generated.

For ease of understanding, Fig. 6 illustrates candidate generation at the j th iteration. We observe that, for the sake of efficiency, each scan aims at counting support for extensions and incremental candidates obtained by means of previously discovered extensions.

In the end, L^U , the set of all frequent sequences, is obtained from L^{DB} , and the maximal sequences from $\text{freqSeed} \cup J\text{freqInc} \cup J\text{freqExt}$. At this step, ISE provides all the frequent sequences in the updated database, as shown in Theorem 1.

For ease of understanding, Fig. 7 graphically describes the processes in the first and j th iterations.

Example 4. Considering our example, the third iteration, we can thus generate from 2-*freqExt* a new candidate sequence $\langle(5060)(80)\rangle$. Let us now consider how new candidate sequences are generated from *freqSeed* and 2-*freqExt*. Let us consider the sequence $s = \langle(20)(40)(50)\rangle$ from *freqSeed* and $s' = \langle(5060)\rangle$ from 2-*freqExt*. The new candidate sequence $\langle(20)(40)(5060)\rangle$ is obtained by dropping 50 from s and appending s' to the remaining sequence.

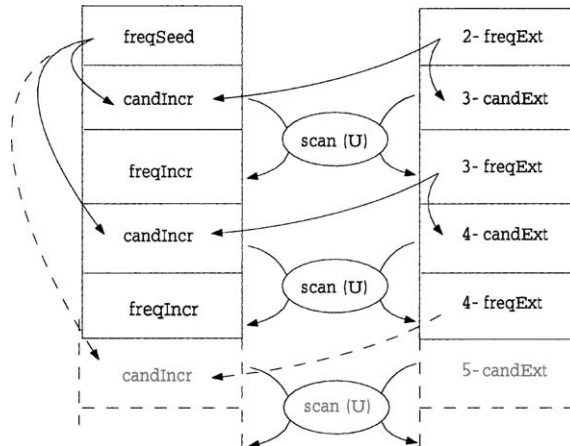
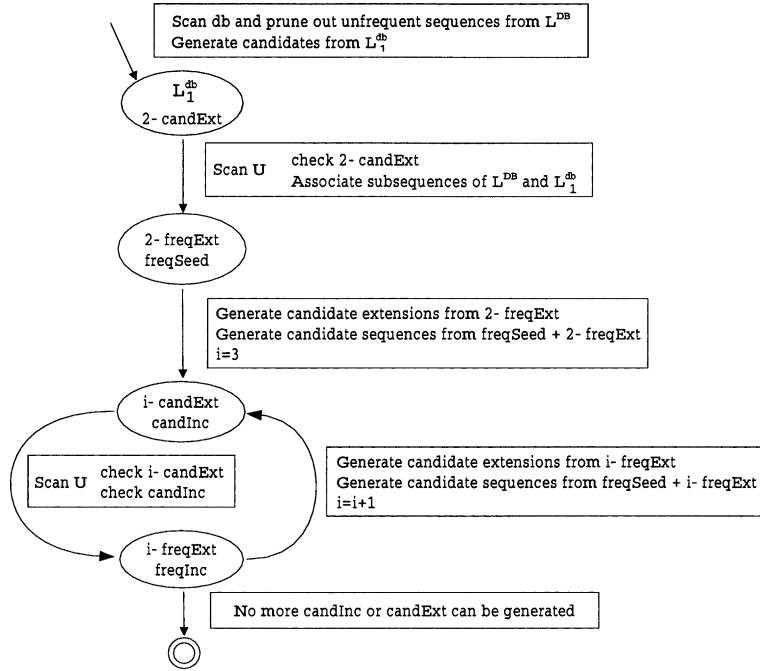


Fig. 6. ISE iterations with $j \geq 2$.

Fig. 7. Processes in the first and j th iterations of ISE.

At the fourth iteration, $\langle(5060)(80)\rangle$ is added to 3-*freqExt* and combined with *freqSeed*, it generates new candidates as example: $\langle(1020)(30)(5060)(80)\rangle$, $\langle(1020)(40)(5060)(80)\rangle$, $\langle(20)(40)(5060)(80)\rangle$ and $\langle(20)(30)(5060)(80)\rangle$. Nevertheless, there are no more candidates generated from 3-*freqExt*, and the process ends by verifying the support of the candidates on U . The final maximal frequent sequence set obtained is $L^U = \{\langle(6090)\rangle, \langle(1020)(5070)\rangle, \langle(1020)(30)(5060)(80)\rangle, \langle(1020)(40)(5060)(80)\rangle\}$.

Now let us examine how new customers are taken into account in the ISE algorithm. As previously described, frequent sequences in the original database may become invalid when adding customer since the support constraint does not hold anymore. The main consequence for the ISE algorithm is to prune out from L^{DB} , the set of sequences that no longer satisfies the support. This is achieved at the beginning of the process. In order to illustrate how such a situation is managed by ISE, consider the following example.

Example 5. Now consider Fig. 2, where a new customer as well as new transactions are added to the original database. When db is scanned we find the support of each individual item during the pass over the data: $\{\langle(10)\rangle, 1\}, \langle(40)\rangle, 1\}, \langle(50)\rangle, 2\}, \langle(60)\rangle, 2\}, \langle(70)\rangle, 2\}, \langle(80)\rangle, 3\}, \langle(90)\rangle, 1\}, \langle(100)\rangle, 1\}$. Combining these items with L_1^{DB} , we obtain $L_1^{db} = \{\langle(10)\rangle, \langle(40)\rangle, \langle(50)\rangle, \langle(60)\rangle, \langle(70)\rangle, \langle(80)\rangle\}$. As one customer has been added, in order to be frequent a sequence must appear in at least three transactions. Let us now consider L^{DB} . The set L_1^{DB} becomes: $\{\langle(10), 4\rangle, \langle(20), 3\rangle, \langle(40), 3\rangle$. That is to say that item 30 is pruned out from L_1^{DB} since it is no

longer frequent. According to Property 1, the set L_2^{DB} is reduced to $\{((1020), 3)\}$ and L_3^{DB} is pruned out because the minimum support constraint does not hold anymore. From L_1^{db} , we can now generate new candidates in *2-candExt*: $\{((1040)), ((10)(40)), ((1050)), \dots, ((70)(80))\}$. When *db* is scanned, we prune out candidates not occurring in the increment and are provided with candidate 2-sequences occurring at least once in *db*. Next we scan *U* to verify 2-candidates and sequences of the updated L^{DB} that chronologically precede sequences of L_1^{db} . There are only three candidate sequences that satisfy the support: $2\text{-freqExt} = \{((10)(70)), ((10)(80))((40)(80))\}$. We shall now examine frequent sequences occurring before items of L_1^{db} in more detail:

Items	Frequent sub-sequences
10	$\langle(10)\rangle_0 \langle(20)\rangle_0 \langle(1020)\rangle_0$
40	$\langle(10)\rangle_2 \langle(20)\rangle_2 \langle(1020)\rangle_2$
50	$\langle(10)\rangle_3 \langle(20)\rangle_3 \langle(1020)\rangle_3$
60	$\langle(10)\rangle_2 \langle(20)\rangle_2 \langle(1020)\rangle_2$
70	$\langle(10)\rangle_3 \langle(20)\rangle_2 \langle(1020)\rangle_2$
80	$\langle(10)\rangle_3 \langle(20)\rangle_2 \langle(1020)\rangle_2$

The minimum support constraint holds for the following sequences: $\text{freqSeed} = \{((10)(70)), ((10)(80))((1020)(50))\}$ (The sequences $\langle(10)(70)\rangle$ and $\langle(10)(80)\rangle$ are also in *2-freqExt*, this is a particular case addressed in Section 3.3). Since, we cannot generate new candidates from *freqSeed* and *2-freqExt*, the process completes and all maximal frequent sequences are stored in $L^U = \{((1020)(50)), ((10)(70)), ((10)(80)), ((40)(80)), ((60))\}$.

3.2. The ISE algorithm

Building on the above discussion, we shall now describe the ISE algorithm.

Algorithm ISE

Input: *DB* the original database, L^{DB} the set of frequent sequences in *DB*, the support of each item embedded in *DB*, *db* the increment database, *minSupp* the minimum support threshold and *k* the size of the maximal sequences in L^{DB} .

Output: The set L^U of all frequent sequences in $U = DB \cup db$

Method:

//First Iteration

$L_1^{db} \leftarrow \emptyset$

foreach $i \in db$ **do**

if ($\text{support}_{DB \cup db}(i) \geq \text{minSupp}$) **then** $L_1^{db} \leftarrow L_1^{db} \cup \{i\}$;

enddo

Prune out from L^{DB} the sequences that no longer satisfy the minimum support;

$2\text{-candExt} \leftarrow$ generate candidate 2-sequences by joining L_1^{db} with L_1^{db} ;

//find sequences occurring in *db*

Scan *db* for *2-candExt*;

Generate the set of frequent sub-sequences from L^{DB} ;

Scan *U* to validate candidate *2-candExt* and frequent sub-sequences occurring before items of L_1^{db} ;

```

freqSeed ← frequent sub-sequences occurring before items of  $L_1^{db}$  and appended with the
item;
2-freqExt ← frequent sequences from 2-candExt;
//jth Iteration
j = 2;
While (j-freqExt! =  $\emptyset$ ) do
  candInc ← generate candidates from freqSeed and j-freqExt;
  j++;
  j-candExt ← Generate candidate
  j-sequences from j-freqExt;

  //find sequences occurring in db
  Scan db for j-candExt;
  if (j-candExt! =  $\emptyset$  OR candInc! =  $\emptyset$ ) then
    Scan U for j-candExt and candInc;
  endif
  j-freqExt ← frequent j-sequences;
  freqInc ← freqInc + candidates from candInc verifying the support on U;
enddo
 $L^U \leftarrow L^{DB} \cup \{\text{maximal frequent sequences in } freqSeed \cup freqInc \cup freqExt\}$ ;
end Algorithm ISE

```

To prove that ISE provides the set of frequent sequences embedded in U , we first show in the following two lemmas that every new frequent sequence can be written as the composition of two sub-sequences. The former is a frequent sequence in the original database while the latter occurs at least once in the updated data.

Lemma 1. *Let F be a frequent sequence on U such that F does not appear in L^{DB} . Then F is such that its last itemset occurs at least once in db .*

Proof

- Case $|F| = 1$. Since $F \notin L^{DB}$, F contains an itemset occurring at least once in db , thus F ends with a single itemset occurring at least once in db .
- Case $|F| > 1$. F can be written as $\langle\langle A \rangle\langle B \rangle\rangle$ with A and B sequences such that $0 \leq |A| < |F|$, $0 < |B| \leq |F|$, $|A| + |B| = |F|$ with $B \notin db$. Let M_B be the set of all data sequences containing B . Let M_{AB} be the set of all data sequences containing F . We know that if $|M_B| = n$ and $|M_{AB}| = m$ then $\min Supp \leq m \leq n$ (according to Property 1). Furthermore $M_{AB} \in DB$ (since $B \in db$ and transactions are ordered by time) so $\langle\langle A \rangle\langle B \rangle\rangle$ is frequent in DB . this implies $F \in L^{DB}$ which contradicts the assumption $F \notin L^{DB}$. Thus, if a frequent sequence F does not appear in L^{DB} , F ends with an itemset occurring at least once in db . \square

Lemma 2. *Let F be a frequent sequence on U such that F does not appear in L^{DB} . F can thus be written as $\langle\langle D \rangle\langle S \rangle\rangle$, where D and S are two sequences, $|D| \geq 0$, $|S| \geq 1$, such that S is the maximal sub-sequence occurring at least once in db and D is included in (or is) a frequent sequence from L^{DB} .*

Proof

- Case $|S| = |F|$. Thus $|D| = 0$ and $D \in L^{DB}$.
- Case $1 \leq |S| < |F|$. That is, $D = \langle (i_1)(i_2) \dots (i_{j-1}) \rangle$ and $S = \langle (i_j) \dots (i_t) \rangle$, where S is the maximal sub-sequence ending F and occurring at least once in db (from Lemma 1 we know that $|S| \geq 1$). Let M_D be the set of all data sequences containing D . Let M_F be the set of all data sequences containing F . We know that if $|M_D| = n$ and $|M_F| = m$ then $\minSupp \leq m \leq n$ (according to Property 1). Furthermore, $M_D \in DB$ (since by assumption $i_{j-1} \notin db$ and transactions are ordered chronologically). Thus $D \in L^{DB}$. \square

Considering a new frequent sequence, we show that it can be written as two subsequences such that the latter is generated as a candidate extension by ISE .

Lemma 3. *Let F be a frequent sequence in U such that F does not appear in L^{DB} . F can be written as $\langle \langle D \rangle \langle S \rangle \rangle$, where D and S are two sequences verifying $|D| \geq 0$ and $|S| \geq 1$, S is the maximal sub-sequence occurring at least once in db , D is included in (or is) a frequent sequence from L^{DB} and S is included in $candExt$.*

Proof. Thanks to Lemma 2, we only have to show that S occurs in $candExt$.

- Case S is a one transaction sequence, reduced to a single item. S is thus found at the first scan on db and added to 1- $candExt$.
- Case S contains more than one item. $candExt$ is built up ‘a la GSP’ from all frequent items in db and is thus a superset of all frequent sequences on U occurring in db . \square

The following Theorem guarantees the correctness of the ISE approach.

Theorem 1. *Let F be a frequent sequence in U such that F does not appear in L^{DB} and $|F| \leq k + 1$. Then F is generated as a candidate by ISE .*

Proof. From Lemma 2, consider the different possibilities for S .

- Case $S = F$. In this case S will be generated in $candExt$ (Lemma 3) and added to $freqExt$.
- Cases $S \neq F$.
 - Case S is a one transaction sequence, reduced to a single item i . Thus $\langle \langle D \rangle \langle (i) \rangle \rangle$ will be considered in the association made by $freqSeed$.
 - Case S contains more than one item. Consider i_{1_1} , the first item from the first itemset of S . i_{1_1} to be frequent in db , in which case $\langle \langle D \rangle \langle (i_{1_1}) \rangle \rangle$ is generated in $freqSeed$. According to Lemma 3, S occurs in $freqExt$ and will be used by ISE to build $\langle \langle D \rangle \langle S \rangle \rangle$ in $candInc$. \square

3.3. Optimization

In order to speed up the performance of the ISE algorithm we consider two optimization techniques for generating candidates.

As the speed of algorithms for mining association rules, as well as sequential patterns, depends very much on the size of the candidate set, we first improve performance by using information on items embedded in L^{db} , i.e. frequent items in db . The optimization is based on the following lemma:

Lemma 4. *Let us consider two sequences ($s \in freqSeed$, $s' \in freqExt$) such that an item $i \in L_1^{db}$ is the last item of s and the first item of s' . If there exists an item $j \in L_1^{db}$ such that j is in s' and j is not associated to s in $freqSeed$, the sequence obtained by appending s' to s is not frequent.*

Proof. If s is not followed by j in $freqSeed$, then $\langle sj \rangle$ is not frequent. Hence $\langle ss' \rangle$ is not frequent since there exists an infrequent sub-sequence of $\langle ss' \rangle$. \square

Using this lemma, at the j th iteration, with $j \geq 2$, we can reduce the number of candidates significantly by avoiding the generation of $\langle ss' \rangle$ as a candidate. In our experiments, the number of candidates was reduced by nearly 40%. The only additional cost is to find out whether there is a frequent sub-sequence matching the first one for each item occurring in the second sequence. As we are provided with an array that stores the items occurring after the sequence for each frequent subsequence, the additional cost of this optimization is relatively low.

In order to illustrate this optimization, consider the following example.

Example 6. Consider the frequent sequence $s \in 2-freqExt$ such that $s = \langle (5070) \rangle$. We have found in it $freqSeed$ the following frequent sequence $\langle (10)(30)(50) \rangle$. According to the previous generation phase, we would generate $\langle (10)(30)(5070) \rangle$. Nevertheless, the sequence $\langle (10)(30) \rangle$ is never followed by 70. So, we can conclude that $\langle (10)(30)(70) \rangle$ is not frequent. This sequence is a sub-sequence of $\langle (10)(30)(5070) \rangle$, so without generating the sequence we know that $\langle (10)(30)(5070) \rangle$ is not frequent. Hence, this last sequence is not generated.

The main concern of the second optimization is to avoid generating candidate sequences that have already been found to be frequent in a previous phase. In fact, when generating a new candidate by appending a sequence of $freqExt$ to a sequence of $freqSeed$ we first test if this candidate was not already discovered frequent. In this case the candidate is no longer considered. To illustrate, consider $\langle (30)(40) \rangle$ to be a frequent sequence in $2-freqExt$. Let us now assume that $\langle (1020)(30)(40) \rangle$ and $\langle (1020)(30) \rangle$ are frequent in $freqSeed$. From the last sequence the generation would provide the following candidate $\langle (1020)(30)(40) \rangle$ which was already found to be frequent. This optimization reduces, at negligible cost, the number of candidates generated before U is scanned.

4. Experiments

In this section, we present the performance results of our ISE algorithm and the GSP algorithm. All experiments were performed on a PC Station with a CPU clock rate of 450 MHz, 64 MB of main memory, a Linux System and a 9GB disk drive (IDE).

4.1. Datasets

We used synthetic datasets to study the algorithm performance. The synthetic datasets were first generated using the same techniques as introduced in [10].² The generation of DB and db was performed as follows. As we wanted to model real life updates very accurately, as in [12], we first generated all the transactions from the same statistical pattern to produce databases of size $|U| = |DB + db|$.

In order to assess the relative performance of $I\mathcal{SE}$ when new transactions were appended to customers already existing in DB , we removed itemsets from the database U using the user defined parameter I^- . The number of transactions which were modified was provided by the parameter $D\%$ standing for the percentage of transactions modified. The transactions embedding removed itemsets were randomly chosen according to $D\%$. Finally, removed transactions were stored in the increment database db while remaining transactions were stored in database DB . In the same way, in order to investigate the behavior of $I\mathcal{SE}$ when new customers were added, the number of customers removed from U was provided by the parameter $C\%$.

Table 2 lists the parameters used in the data generation method and Table 3 shows the databases used and their properties. For experiments we first investigated the behavior of $I\mathcal{SE}$ when new transactions were added. For these experiments, I^- was set to 4 and $D\%$ was set to 90%. Finally, to study the performance of our algorithm with new customers, $C\%$ was set to 10% and 5%.

4.2. Comparison of $I\mathcal{SE}$ with GSP

In this section, we compare the naive approach, i.e. using GSP for mining the updated database from scratch with our incremental algorithm. We also tested how it scaled up as the number of transactions increased. Finally, we carried out experiments to analyze the performance of the $I\mathcal{SE}$ algorithm according to the size of updates.

4.2.1. Naive vs. $I\mathcal{SE}$ algorithm

Fig. 8 shows experiments conducted on the different datasets using different minimum support ranges to get meaningful response times. Note that the minsupport thresholds are adjusted to be as low as possible while retaining reasonable execution times. The label “Incremental Mining” corresponds to the $I\mathcal{SE}$ algorithm while “GSP” stands for GSP used for mining the updated database from scratch. “Mining from scratch” corresponds to $I\mathcal{SE}$ for mining sequential patterns, i.e. assuming that no previous mining has been performed.

Fig. 8 clearly indicates that the performance gap between the two algorithms increases with decreasing minimum support. We can observe that $I\mathcal{SE}$ is 3.5–4 times faster than running GSP from scratch. It can also be noticed that $I\mathcal{SE}$ outperforms GSP for small support as well as large support values: $I\mathcal{SE}$ is still 2.5–3 times faster for large support values. The same results are found even if the number of itemsets is large. For instance, the last graph in Fig. 8 shows an experiment conducted to investigate the effect of the number of itemsets on the performance. When the

² The synthetic data generation program is available at the following URL (<http://www.almaden.ibm.com/cs/quest>).

Table 2
Parameters

$ D $	Number of customers (size of database)
$ C $	Average number of transactions per customer
$ T $	Average number of items per transaction
$ S $	Average length of maximal potentially large sequences
$ I $	Average size of itemsets in maximal potentially large sequences
N_S	Number of maximal potentially large sequences
N_I	Number of maximal potentially large itemsets
N	Number of items
I^-	Average number of itemsets removed from sequences in U to build db
$D^{\%}$	Percentage of updated transactions in U
$C^{\%}$	Percentage of customers removed from U in order to build db

Table 3
Parameter values for synthetic datasets

Name	$-C-$	$-I-$	N	$-D-$	Size (Mo)
C9-I4-N1K-D50K	9	4	1000	50,000	12
C9-I4-N2K-D100K	9	4	2000	100,000	30
C12-I2-N2K-D100K	12	2	2000	100,000	30
C12-I4-N1K-D50K	12	4	1000	50,000	18
C13-I3-N20K-D500K	13	3	20,000	500,000	230
C15-I4-N30K-D600K	15	4	30,000	600,000	320
C20-I4-N2K-D800K	20	4	2000	800,000	460

support is lower the GSP algorithm provides the worst performance. In Section 4.3.1, we shall investigate the correlation between execution times and the number of candidates.

4.2.2. Performance in scaled-up databases

We examined how ISE behaves as the number of total transactions is increased. We would expect the algorithm to have almost linear scale-up. This is confirmed by Fig. 9 which shows that ISE scales up linearly as the number of transactions is increased 10-fold, from 0.1 to 1 million. Experiments were performed on the C12-I4-N1K-D50K dataset with three levels of minimum support (2%, 1.5% and 1%). During our evaluation, the size of the increment database was always proportional ($D^{\%} = 90\%$ and $I^- = 4$) to the number of additional new transactions. The execution times are normalized with respect to the time for the 0.1 million dataset.

4.2.3. Varying the size of added transactions

We carried out some experiments to analyze the performance of the ISE algorithm according to the size of updates. We used the databases C13-I3-N20K-D500K and C12-I2-N2K-D100K for experiments with a threshold of 0.6% and 0.4%, respectively. The number of itemsets removed from the generated database was varied, together with the number of clients. Deleted transactions were stored in the increment database db while the remaining transactions were stored in the DB

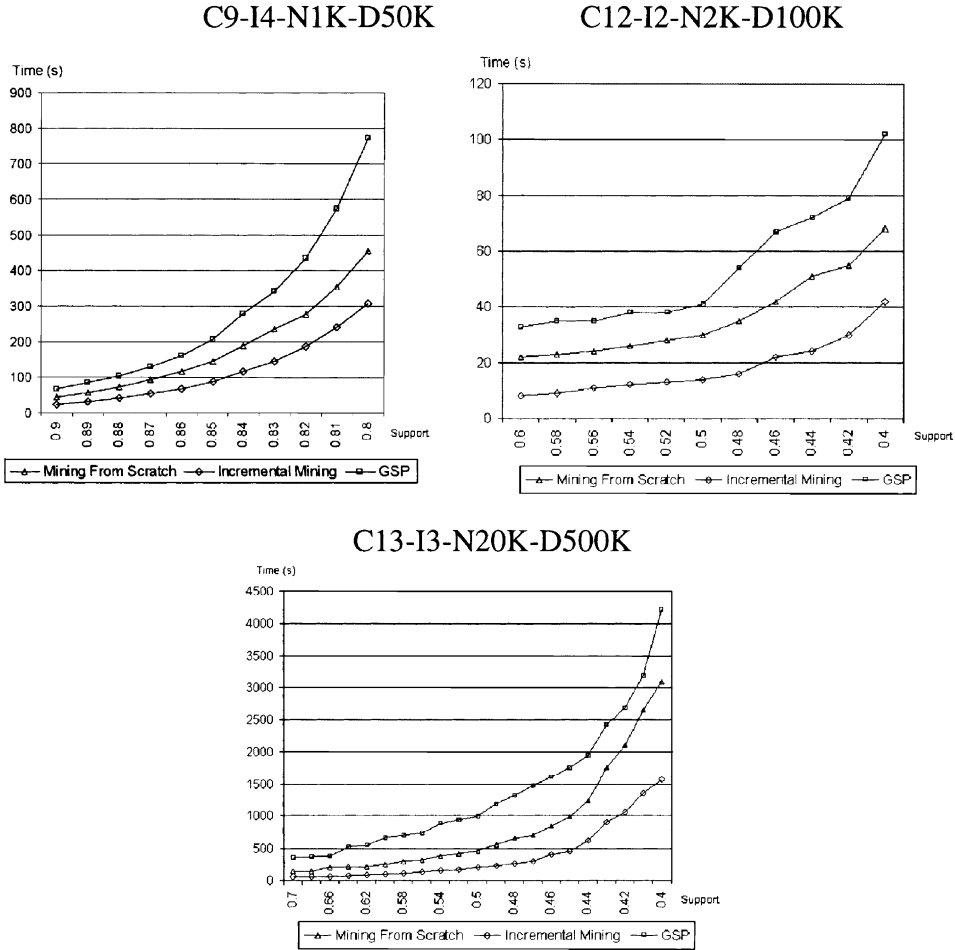


Fig. 8. Execution times.

database. We first ran GSP to mine L^{DB} and then ran ISE on the updated database. Fig. 10 shows the result of this experiment when considering the time for ISE.

For the first one, we can observe that ISE is very efficient from 1 to 6 itemsets removed. The frequent sequences in L^U are obtained in less than 110 s. As the number of removed transactions increases, the amount of time taken by ISE increases. For instance, when 10 itemsets are deleted from the original database, ISE takes from 180 s for 30% of transactions to 215 s if the items were deleted from all the transactions. The main reason is that the changes to the original database are so numerous that the results obtained during an earlier mining are not helpful. Interestingly, we also noticed that the time taken by the algorithm does not depend very much on the number of transactions updated.

Consider the second surface. The algorithm takes more and more time as the number of itemsets removed grows. Nevertheless, when three itemsets are removed from the generated database, ISE takes only 30 s to discover the set of all sequential patterns.

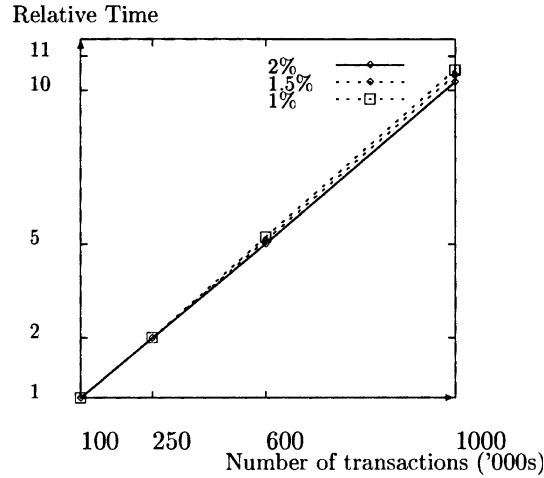


Fig. 9. Scale-up: number of total transactions.

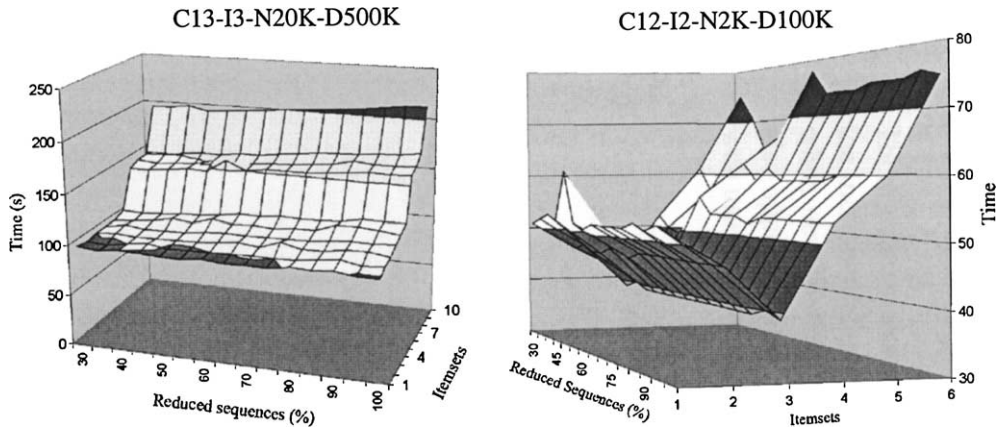


Fig. 10. Size of updates.

4.2.4. Varying the number of added customers

We assume, since it is realistic and suited to real applications, that the average size of the added sequences is less than or equal to the average size of the sequences embedded in the original database. Intuitively, an obvious approach would be to study the behavior of ISE when the number of new customers added to the database is increased. In fact, this naive idea could not be a significant indicator. This is because when the number of new customers increases, then the number of occurrences of a sequence must also be increased to satisfy the minimum support constraint. Obviously, as at the beginning of the ISE algorithm, we prune out from L^{DB} frequent sequences that no longer satisfy the support, so the more customers are added, the more previous frequent sequences are pruned out. The main consequence is that the number of frequent sequences decreases, together with the execution times. A much more interesting approach for

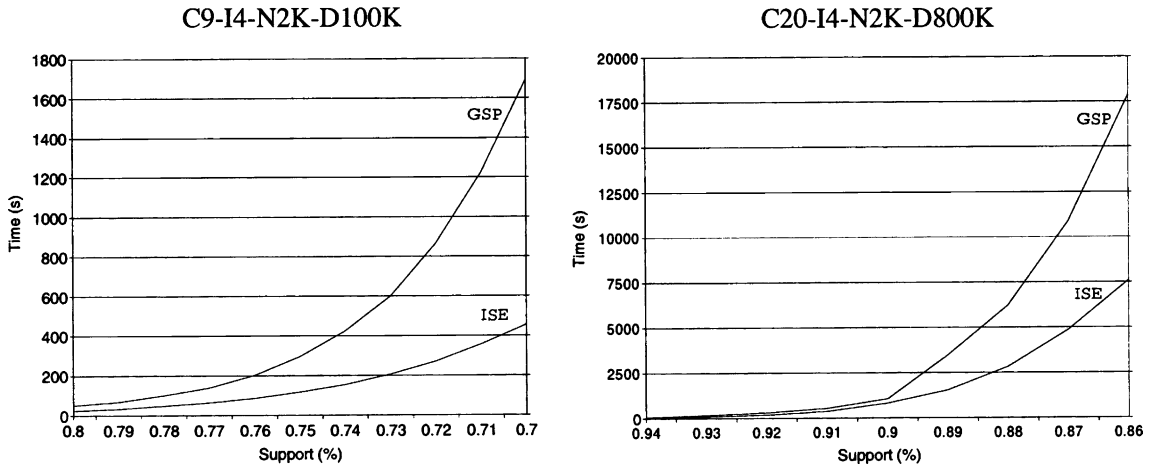


Fig. 11. Execution times when 10% and 5% of customers are added to the original database.

evaluating ISE performance is to carry out experiments comparing the execution times of GSP vs. ISE for different datasets while varying the minimum support. Fig. 11 shows experiments carried out on two datasets C9-I4-N2K-D100K and C20-I4-N2K-D800K to which 10% and 5% of customers have been added respectively. We observe that ISE is very efficient and even when customers are added it is nearly twice as fast as applying GSP from scratch.

4.3. ISE for mining sequential patterns

In this section, we investigate the performance of ISE for mining sequential patterns.

We designed some experiments to analyze the performance of ISE when mining sequential patterns using the same datasets as in Section 4.2.1. However, we performed the following operations on each dataset. First we removed 6 items from a designated 60% of transactions in order to provide the increment database. Second, we ran GSP to mine the k -frequent sequences in DB . Finally, we ran ISE. In other words, the graphs in Fig. 8 show two behaviors. The graph labeled “GSP” indicates the time response of GSP on U , whereas the “ISE” graph shows GSP on DB plus ISE on db . We observe that ISE is from 1.7 to 3 times faster than GSP for mining sequential patterns. The main reason for the gain in performance is the reduced number of candidates. We study this effect in the next section. As expected, we also observe that using ISE for incremental mining instead of mining from scratch is still efficient since the incremental mining is nearly twice as fast as mining from scratch with these data sets.

4.3.1. Candidate sets

In order to explain the correlation between the number of candidates and the execution times, we compared the number of candidate sets generated by GSP and our algorithm. The results are given in Fig. 12. As we can see, the number of candidates for GSP is nearly twice the number for ISE. Let us examine the low support case in more detail. In the first graph, GSP generates more than 7000 candidates while ISE generates only 4000 candidates. The same result is obtained in the

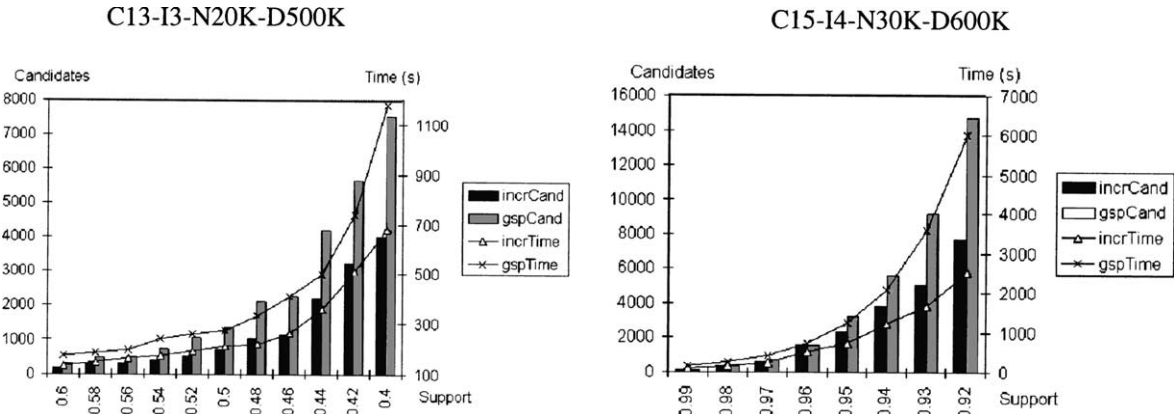


Fig. 12. Candidate sets.

second graph, where GSP generates more than 14,000 candidates while our algorithm generates 8000.

4.3.2. Varying the size of updates

Finally, we carried out some experiments to analyze the performance of the ISE algorithm with respect to the size of updates. Experiments were conducted on datasets C12-I2-N2K-D100K and C13-I3-N20K-D500K with a threshold of 0.4%. Consider the first surface in Fig. 13. The best results are obtained when five itemsets are deleted from the database. All frequent sequences are then obtained in less than 17 s. The algorithm is still very efficient with from 2 to 7 itemsets deleted but when five itemsets are deleted from a designated 10% of customers, ISE is less efficient.

In the second surface of Fig. 13, the performance of ISE is quite similar and best results are obtained when nine itemsets are removed from 80% of customers.

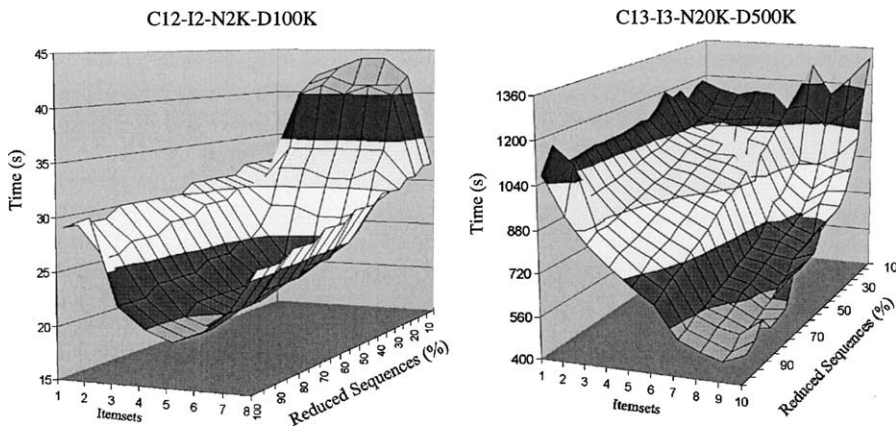


Fig. 13. Size of updates.

5. Conclusion

In this paper, we present the ISE approach for the incremental mining of sequential patterns in large databases. This method is based on the discovery of frequent sequences by only considering frequent sequences obtained by an earlier mining step. By proposing an iterative approach based only on such frequent sequences we are able to handle large databases without having to maintain negative border information, which has been shown to be very memory consuming [16]. Maintaining such a border is highly suited to incremental association mining [19,26], where association rules are only intended to discover intra-transaction patterns (itemsets). Nevertheless, in sequence mining, we also have to discover inter-transaction patterns (sequences) and the set of all frequent sequences is an unbounded superset of the set of frequent itemsets (bounded) [16]. The main consequence is that such approaches are severely limited by the size of the negative border.

Our performance results show that the ISE method is very efficient since it performs much better than re-run discovery algorithms when data is updated. We found by means of empirical evaluations that the proposed approach was so efficient that it was quicker to extract an increment from the original database then apply ISE to mine sequential patterns than to use the GSP algorithm. Experiments on incremental web usage mining were also performed, for further information refer to [27].

There are various avenues for future work on incremental mining. First, while the incremental approach is applicable to databases, which are frequently updated when new transactions or new customers are added to an original database, it is also appropriate to many other fields. For example, both electronic commerce and web usage mining require deletion or modification to be taken into account in order to save storage space or because information is no longer of interest or has become invalid. We are currently investigating how to manage these operations in the ISE algorithm.

Second, we are currently studying how to improve the overall process of incremental mining. By means of experimentation, we would like to discover measures that can suggest to us when ISE should be applied to find out the new frequent sequences in the updated database. Such an approach has been proposed in another context, [28], based on a sampling technique in order to estimate the difference between old and new association rules. We are currently investigating whether other measures could be found by analyzing the data distribution of the original database.

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