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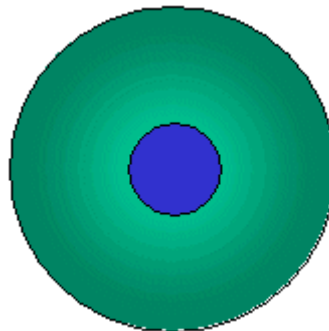
The Fastest Software Tool  
for  
System-Level Thermal Design

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## Thermal Spreading Resistance

This month's topic takes the theme of thermal calculations into the realm of conduction within a solid. Up to now, we've covered calculations for the pressure drop and thermal behavior of flow in rectangular channels. Those calculations can be used in lots of different ways, including for parallel flow through heat sink fins. Now, drawing on some of my work experience in heat sink design, I'd like to include some other essential components of analytically looking at heat sink performance.

Besides the more obvious fin-side thermal performance, there is the question of how heat gets from the contact area of the heat source out into the fin structure. Unless you are very lucky, or you have a very small heat sink, the heat source area is typically smaller than the heat sink base area that is covered with fins. Here's the picture for a circular shape, where the solid circle in the middle represents the heat source:



If the heat were evenly spread over the entire large slab area, the source temperature would be lower than what you'll find in this situation. Not only that, the thickness of the slab will affect the temperature. So how do we estimate the resulting thermal penalty?

Luckily, there is a handy reference paper by Song et al that gives an approximation that is good for design work. Yes, it's got some Greek-letter variables, but take heart - no integrals! (Believe me, this is a major step forward.) What follows is a summary of the method presented in the paper. Keep in mind some general caveats: this assumes that the heating is uniform within the source area. It also assumes that the heat source is reasonably centered. The exact shape isn't as important as the fact that the whole heat source perimeter has roughly equal access to heat dissipation area.

Let's start with an outline of the general calculation procedure.

1. First gather the inputs. You will need the thermal resistance of the fin-side of the heat sink  $\theta$  as well as all these variables: the source area  $A_{source}$ , base area  $A_{base}$ , base thermal conductivity  $k$ , and plate thickness  $t$ .
2. Calculate the dimensionless parameters for the problem: Biot number  $Bi = 1/(\pi k b \theta)$ , equivalent source radius  $a = (A_{source} / \pi)^{1/2}$ , equivalent base radius  $b = (A_{base} / \pi)^{1/2}$ , dimensionless heat source radius  $\varepsilon = a / b$ , and dimensionless plate thickness  $\tau = t / b$ . Then crunch through the intermediate variables:
3.  $\lambda = \pi + 1/(\pi^{1/2} \varepsilon)$
4.  $\Phi = (\tanh(\lambda \tau) + \lambda / Bi) / (1 + (\lambda / Bi) \tanh(\lambda \tau))$
5.  $\Psi = (\varepsilon \tau + (1 - \varepsilon) \Phi) \pi^{1/2}$
6. The base spreading resistance, including the resistance due to the bulk material thickness, is  $R = \Psi / (\pi^{1/2} k a)$ .

This is the resistance based on the maximum temperature in the heat source area. This can be taken in series with the fin-side resistance for a quick estimate of heat sink performance with reduced source size.

If you, like me, tend to put calculations in spreadsheets (or similar tools) and then make graphs, you might find that there is a minimum resistance available when the x-axis is slab thickness. (This is because the resistance of the bulk thickness is taken into account in the approximation.) My experience with this for heat sink base thickness has been that it's not worth making the heat sink base thicker than 10-15 mm; the extra material doesn't buy you performance, and just costs you weight and therefore money. But you do need to keep the sources centered.

What to do if the sources aren't centered? Your best bet is to set up a conduction model with a commercial modeling tool, using the equivalent fin-side thermal performance for the heat sink that includes air temperature rise (which you did remember to include, didn't you? See the March column if in doubt...).

Calculator

**Reference:**

Song, S., Lee, SI, and Au, V., 1994, "Closed-Form Equation for Thermal Constriction/Spreading Resistances with Variable Resistance Boundary Condition," Proceedings of the 1994 International Electronics Packaging Conference, Atlanta, Georgia, pp. 111-121.