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High-Frequency Transistor Primer

Part II

Noise and S-parameter Characterization

This is the second part of the Agilent Technologies High Frequency Transistor Primer series. It is an introduction to the noise and S-parameter characterization of GaAs FET and silicon bipolar transistors for the microwave engineer. The contents are based on questions often received by Agilent application engineers.

The other parts of the High Frequency Transistor Primer series currently available are: Part I, *Electrical Characteristics* (of bipolar microwave transistors); Part III, *Thermal Properties* (of silicon bipolar and GaAs FET transistors); Part III-A, *Thermal Resistance* (of power FETs) and Part IV, *GaAs FET Characteristics.*

Copies of the Agilent Technologies High Frequency Transistor Primer volumes are located on the world wide web at http://www.semiconductor.agilent.com

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Introduction

This Primer is a short summary of the S-parameter and noise parameters commonly used on Agilent Technologies transistor data sheets and their functional relationships to noise figure, gain, stability, impedance matching and other parameters necessary for high frequency circuit design. Much of this information has been published in various journals over the years. The intent of this primer is to provide a short, concise booklet containing the key functional relationships necessary for circuit design.

I. S-parameters

By far the most accurate and conveniently measured microwave two-port parameters are the scattering parameters. These parameters completely and uniquely define the small signal gain and the input/ output emittance properties of any linear two-port network. Simply interpreted, the scattering parameters are merely insertion gains, forward and reverse, and reflection coefficients, input and output, with the driven and non-driven ports both terminated in equal impedances; usually 50 ohms, real. This type of measurement system is particularly attractive because of the relative ease in obtaining highly accurate 50 ohm measurement hardware at microwave frequencies.

Proceeding more specifically, S-parameters are defined analytically by:

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

or, in matrix form,

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

where (referring to Figure 1):

a_1 = (Incoming power at Port 1)$^{1/2}$

b_1 = (Outgoing power at Port 1)$^{1/2}$

a_2 = (Incoming power at Port 2)$^{1/2}$

b_2 = (Outgoing power at Port 2)$^{1/2}$

E_1, E_2 = Electrical Stimuli at Port 1, Port 2

z_o = Characteristic Impedance = (50 + j0) Ohms

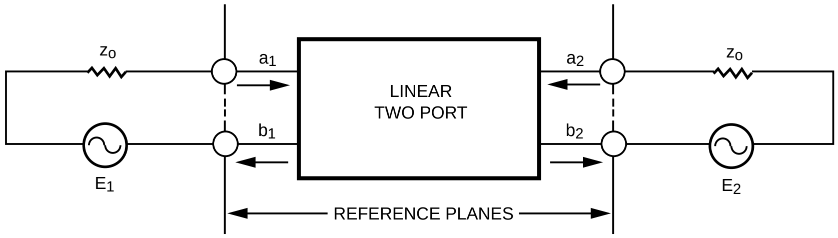

Figure 1. S-Parameters Definition Schematic

From Figure 1 and defining linear equations for $E_2 = 0$, then $a_2 = 0$, and:

$$S_{11} = \frac{b_1}{a_1} = \left[\frac{\text{Outgoing Input Power}}{\text{Incoming Input Power}}\right]^{1/2} \qquad (1)$$

$$= \frac{\text{Reflected Voltage}}{\text{Incident Voltage}}$$

$$= \text{Input Reflection Coefficient}$$

$$S_{21} = \frac{b_2}{a_1} = \left[\frac{\text{Outgoing Input Power}}{\text{Incoming Input Power}}\right]^{1/2} \qquad (2)$$

$$= [\text{Forward Transducer Gain}]^{1/2}$$

or in the case of S_{21}:

$$\text{Forward Transducer Gain} = |S_{21}{}^2| \qquad (3)$$

Similarly at Port 2 for $E_1 = 0$, $a_1 = 0$:

$$S_{12} = \left[\frac{\text{Outgoing Input Power}}{\text{Incoming Output Power}}\right]^{1/2} \qquad (4)$$

$$= \frac{b_1}{a_2}$$

$$= \text{Reverse Transducer Gain}$$

$$S_{22} = \left[\frac{\text{Outgoing Output Power}}{\text{Incoming Output Power}}\right]^{1/2} \qquad (5)$$

$$= \frac{b_2}{a_2}$$

$$= \text{Output Reflection Coefficient}$$

Since many measurement systems actually “read out” the magnitude of S-parameters in decibels, the following relationships are particularly useful:

$$| S_{11} |_{dB} = 10 \log | S_{11} |^2 = 20 \log | S_{11} | \qquad (6)$$

$$| S_{22} |_{dB} = 20 \log | S_{22} | \qquad (7)$$

$$| S_{21} |_{dB} = 20 \log | S_{21} | \qquad (8)$$

$$| S_{12} |_{dB} = 20 \log | S_{12} | \qquad (9)$$

Using scattering parameters, it is possible to calculate the reflection coefficients and transducer gains for arbitrary load and source impedance where the load and source impedances are described by their reflection coefficients Γ_L and Γ_s respectively:

$$S'_{11} = \frac{b_1}{a_1} = \frac{S_{11}(1-S_{22}\Gamma_L) + S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L}$$

$$= S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L} \qquad (10)$$

$$S'_{22} = \frac{b_2}{a_2} = \frac{S_{22}(1-S_{11}\Gamma_S) + S_{21}S_{12}\Gamma_S}{1-S_{11}\Gamma_S}$$

$$= S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1-S_{11}\Gamma_S} \qquad (11)$$

$$\text{Transducer Power Gain} = \frac{\text{Power Delivered to Load}}{\text{Power Available from Source}}$$

$$= \left|\frac{b_2}{b_S}\right|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)$$

$$= \frac{|S_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|(1 - S_{11}\Gamma_S)(1 - S_{22}\Gamma_L) - S_{12}S_{21}\Gamma_L\Gamma_S|^2} \qquad (12)$$

II. Functional Relationships

With this information, the functional relationships to gain, stability, input and output matching impedance can be readily derived from S-parameters. Since much of the literature[1, 2, 3] gives the complete derivation of these relationships, the mathematics of their derivation is omitted. Table 1 lists the most useful relationships required for circuit design.

Table 1.

1. Available Power Gain = $\dfrac{\text{Power Available from Network}}{\text{Power Available from Generator}}$

$$G_A = \frac{|S_{21}|^2 (1 - |\Gamma_S|^2)}{(1-|S_{22}|^2) + |\Gamma_S|^2(|S_{11}|^2 - |D|^2) - 2\,\mathrm{Re}\,(\Gamma_S C_1)}$$

2. Stability

$$K = \frac{1 + |D|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|}$$

3. Maximum Stable Gain

$$G_{msg} = \left| \frac{S_{21}}{S_{12}} \right|$$

4. Maximum Available Gain (for K>1)

$$G_{max} = \left| \frac{S_{21}}{S_{12}} \left(K \pm \sqrt{K^2 - 1}\right) \right|$$

5. Maximum Unilateral Power Gain

$$U = \frac{|S_{21}|^2}{(1 - |S_{11}|)^2 \; (1 - |S_{22}|)^2}$$

6. Source and Load Match for Maximum Available Power Gain

$$\Gamma_{ms} = C_1^* \left[\frac{B_1 \pm \sqrt{B_1^2 - 4\,|C_1|^2}}{2\,|C_1|^2} \right]$$

$$\Gamma_{mL} = C_2^* \left[\frac{B_2 \pm \sqrt{B_2^2 - 4\,|C_2|^2}}{2\,|C_2|^2} \right]$$

Use minus sign when B_1 or B_2 is positive, plus sign when B_1 or B_2 is negative.

where:

B_1	=	$1 + \lvert S_{11}\rvert^2 - \lvert S_{22}\rvert^2 - \lvert D\rvert^2$
B_2	=	$1 + \lvert S_{22}\rvert^2 - \lvert S_{11}\rvert^2 - \lvert D\rvert^2$
C_1	=	$S_{11} - D(S_{22}^*)$
C_2	=	$S_{22} - D(S_{11}^*)$
D	=	$\det [s] = S_{11} S_{22} - S_{12} S_{21}$

Table 2. y and h Parameters in Terms of S-Parameters

$$y_{11} = \frac{S_{12}S_{21} + (1 - S_{11})(1 + S_{22})}{(1 + S_{11})(1 + S_{22}) - S_{21}S_{12}} Z_o^{-1}$$

$$y_{21} = \frac{-2S_{21}}{(1 + S_{11})(1 + S_{22}) - S_{21}S_{12}} Z_o^{-1}$$

$$y_{12} = \frac{-2S_{12}}{(1 + S_{11})(1 + S_{22}) - S_{21}S_{12}} Z_o^{-1}$$

$$y_{22} = \frac{S_{21}S_{12} + (1 + S_{11})(1 + S_{22})}{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}} Z_o^{-1}$$

$$h_{11} = \frac{(1 + S_{11})(1 + S_{22}) - S_{21}S_{12}}{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}} Z_o$$

$$h_{21} = \frac{-2S_{21}}{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}$$

$$h_{12} = \frac{+2S_{12}}{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}$$

$$h_{22} = \frac{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}} Z_o^{-1}$$

III. Stability

A two port network is unconditionally stable if there exists no combination of passive load or source impedances which will allow the circuit to oscillate. In terms of S-parameters, unconditional stability is assured if the following equations are simultaneously satisfied:

$$| S_{11} | < 1 \quad (13)$$

$$| S_{22} | < 1 \quad (14)$$

$$\left| \frac{|S_{12}S_{21}| - |C_1^*|}{|S_{11}|^2 - |D|^2} \right| > 1 \quad (15)$$

$$\left| \frac{|S_{12}S_{21}| - |C_2^*|}{|S_{22}|^2 - |D|^2} \right| > 1 \quad (16)$$

Under these conditions, Rollett's Stability Factor, K > 1 and Maximum Available Gain is real and defined (Equation 4, Table 1):

When K < 1, the 2 port network is potentially unstable, but there may exist areas of the Γ_s and Γ_L plan in which the real part of the total impedance in the input (or output) loop is positive and the network is conditionally stable. The regions of instability occur within the stability circles, the centers and radii of which are defined by,

rS_1 = Center of the stability circle on the input plane

$$= \frac{C_1^*}{|S_{11}|^2 - |D|^2} \tag{17}$$

RS_1 = Radius of stability circle on the input plane

$$= \left[\frac{|S_{12}S_{21}|}{|S_{11}|^2 - |D|^2}\right] \tag{18}$$

rS_2 = Center of the stability circle on the output plane

$$= \frac{C_2^*}{|S_{22}|^2 - |D|^2} \tag{19}$$

RS_2 = Radius of the stability circle on the output plane

$$= \left[\frac{|S_{12}S_{21}|}{|S_{22}|^2 - |D|^2}\right] \tag{20}$$

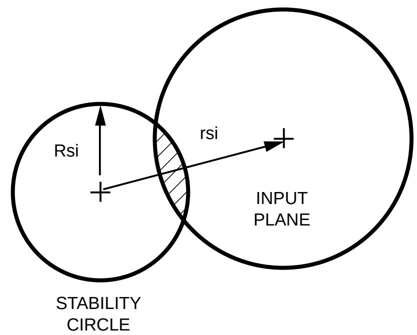

Figure 2.

Figure 2 is a typical example of the input plane of a conditionally stable network and the location of the stability circle. The shaded area represents the area of the input plane in which instability (or oscillation) occurs.

IV. Gain Contours

By manipulating Equation 1, Table 1, circles of constant power gain can be generated in the Γ_s plane.

Equation 1, Table 1, may be expressed as:

$$G_A = |S_{21}|^2 G_1 \tag{21}$$

where

$$G_1 = \frac{|1 - |\Gamma_S|^2|}{(1 - |S_{22}|^2) + |\Gamma_S|^2(|S_{11}|^2 - |D|^2) - 2\text{Re}\Gamma_S C_1} \tag{22}$$

The radius and location of a constant G_1 gain circle is given by:

$$r_g = \frac{(1 - 2K|S_{12}S_{21}| G_1 + |S_{12}S_{21}|^2 G_1^2)^{1/2}}{1 + M_1G_1} \tag{23}$$

$$R_g = \left(\frac{G_1}{1 + M_1G_1}\right) C_1^* \tag{24}$$

where

$$M_1 = |S_{11}|^2 - |D|^2 \tag{25}$$

Figures 3 and 4 are typical examples of gain contour plots on a Smith chart. In this case, the contours are of a typical AT-41435 transistor measured at 2 GHz and 4 GHz; since K >1 and the transistor is unconditionally stable, the maximum available gain is uniquely defined at a single point.

To realize the specified gain for any arbitrary Γ_S, the output matching impedance is obtained by conjugately matching S'_{22} (Equation 11) or

$$\Gamma_L = \left[S_{22} + \frac{S_{21} S_{12}\, \Gamma_S}{1 - S_{11}\, \Gamma_S} \right]^* \qquad (26)$$

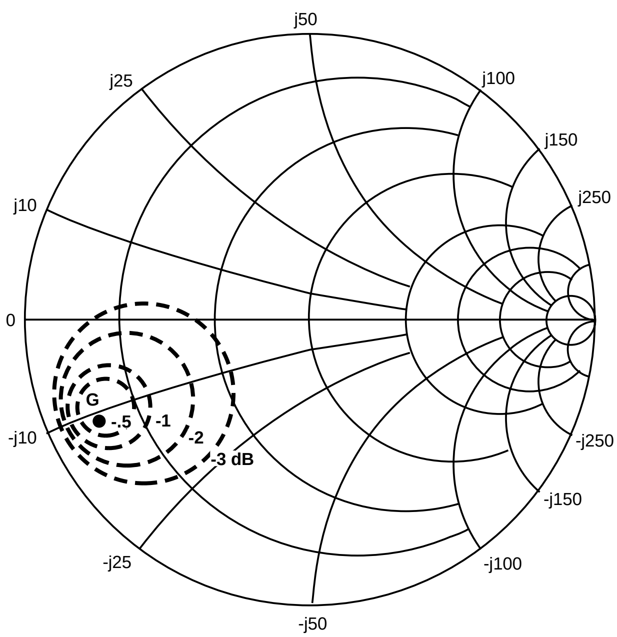

Figure 3. Constant Gain vs. Source Impedance – AT-41435
Frequency = 2 GHz, V_{CE} = 8 V, I_C = 10 mA, MAG = 16.3 dB, Γ_{ms} = 0.84, -160

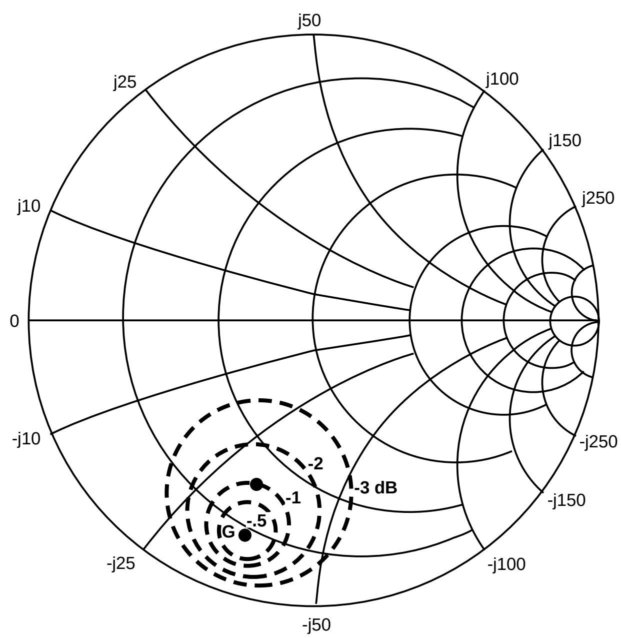

Figure 4. Constant Gain vs. Source Impedance – AT-41435
Frequency = 4 GHz, V_{CE} = 8 V, I_C = 10 mA, MAG = 10.6 dB, Γ_{ms} = 0.84, -111

V. Noise Characterization

While S-parameters completely define the stability, gain and power matching conditions of a linear two port network, they are not sufficient to describe the noise behavior of a noisy, linear, two port network such as a small signal transistor. Another set of parameters, namely noise parameters, are required in addition to S-parameters to describe the noisy linear two port.

The noise figure of a linear two port network as a function of source admittance may be represented by: [3]

$$F = F_{OPT} + \frac{R_n}{G_S} \left[(G_{OPT} - G_S)^2 + (B_{OPT} - B_S)^2 \right] \qquad (27)$$

where:

$G_s + jB_s$ = the source admittance presented to the input of the two port

$G_{opt} + jb_{opt}$ = the source admittance at which optimum noise figure occurs

R_n = an empirical constant relating the sensitivity of the noise figure to source admittance, with dimensions of resistance

It may be noted that for an arbitrary noise figure measurement with a known source admittance, Equation (27) has four unknowns, F_{opt}, R_n, G_{opt}, and B_{opt}. By choosing four known values of source admittance, a set of four linear equations is formed and the solution of the four unknowns can be found.

Equation (27) may be transformed to:

$$F = F_{OPT} + \frac{R_n |Y_{OPT}|^2}{G_S} - 2R_n G_{OPT} + \frac{R_n |Y_S|^2}{G_S} - 2R_n B_{OPT} \left(\frac{B_S}{G_S} \right) \tag{28}$$

or,

$$F = F_{OPT} + \frac{R_n}{G_S} \bullet |Y_S - Y_{OPT}|^2$$

Let,

$$X_1 = F_{OPT} - 2R_n G_{OPT} \tag{29}$$

$$X_2 = R_n - |Y_{OPT}|^2 \tag{30}$$

$$X_3 = R_n \tag{31}$$

$$X_4 = R_n B_{OPT} \tag{32}$$

Then the generalized equation may be written as:

$$F_i = X_1 + \frac{1}{G_{Si}} X_2 + \frac{|Y_{Si}|^2}{G_{Si}} X_3 - 2\left(\frac{G_{Si}}{B_{Si}} \right) X_4 \tag{33}$$

Or, in matrix form:

$$[F] = [A]\,[X] \tag{34}$$

and the solution becomes:

$$[X] = [A]^{-1}\,[F] \tag{35}$$

These parameters completely characterize the noise behavior of the two port network. Direct measurement of these noise parameters by this method would be possible only if the receiver on the output of the two port were noiseless and insensitive to its input admittance. In actual practice, the receiver itself behaves as a noisy two port network and can be characterized in the same manner. What is actually being measured is the system noise figure of the two port and the receiver.

The two port noise figure, however, can be calculated using the system formula:

$$F_{1i} = F_{(Sys)i} - \frac{F_2 - 1}{G_{1i}} \tag{36}$$

Where:

F_{1i}	=	Two port noise figure when driven from the ith source admittance
F_2	=	Second stage noise figure (or receiver noise figure)
$F_{(sys)i}$	=	System noise figure when driven from the ith source
G_{1i}	=	Available gain of the two port when driven from the ith source.

It is important to note that F_2 is assumed to be independent of the impedance of the first stage two port, which means that an isolator must be inserted between the first stage two port and the receiver. Thus, it becomes apparent that to do a complete two port noise characterization, the system noise characterization, the receiver noise characterization, and the gain of the two port must be measured. In addition, any losses in the input matching networks must be carefully accounted for, because they add directly to the measured noise figure reading.

Figure 5 shows a generalized block diagram of a typical noise figure setup used to obtain noise parameters.

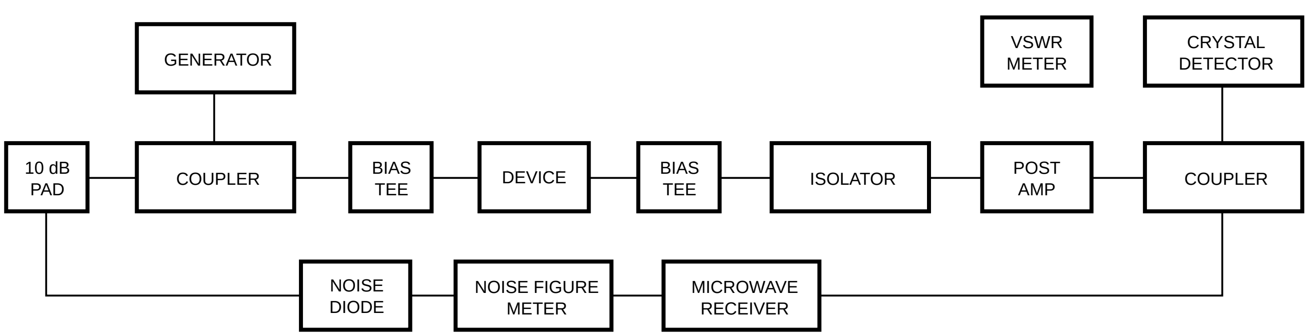

Figure 5.

VI. Noise Contours

Noise figure can be graphically presented on a Smith chart of the input plane much the same as gain. This graphical representation can be presented in the impedance plane (Z plane), admittance plane (Y plane) or reflection coefficient plane (Γ plane), all of which can be functionally related to each other. Since the noise parameters were derived in terms of admittance parameters, the noise contours will be derived in terms of normalized admittance parameters, which may be easily converted into the Z plane by a 180° angular rotation.

If we define the normalized admittances as:

$$y_S = g_S + jb_S = \frac{1}{Y_o}(G_S + jB_S) \tag{37}$$

$$y_{OPT} = g_{OPT} + jb_{OPT} = \frac{1}{Y_o}(G_{OPT} + jB_{OPT}) \tag{38}$$

where: Y_o is the real characteristic admittance of the input transmission line.

From the literature[3] it can be shown that the center of the circle of constant noise figure ($F_i \geq F_{OPT}$) is:

$$R_{Fi} = \frac{\left[(1 - g^2_{OPT} - b^2_{OPT})^2 + 4b^2_{OPT}\right]^{1/2}}{(1 + g_{OPT})^2 + b^2_{OPT} + 2\,\delta_{Fi}} \tag{39}$$

where:

$$\delta_{Fi} = \frac{F_i - F_{OPT}}{2\,R_n\,Y_o} \tag{40}$$

The angle of the vector is:

$$\theta = \tan^{-1}\left[\frac{2b_{OPT}}{1 - g^2_{OPT} - b^2_{OPT}}\right] \tag{41}$$

The radius of the circle of constant noise figure is given by:

$$r_{Fi} = \frac{2N_i}{(1 + g_{OPT})^2 + b^2_{OPT} + 2\,\delta_{Fi}} \tag{42}$$

when:

$$N_i = \frac{1}{Y_o}\left[\frac{G_{OPT}}{R_n}(F_i - F_{OPT}) + \frac{1}{4\,R_n^2}(F_i - F_{OPT})^2\right]^{1/2} \tag{43}$$

Figure 6 shows a typical plot of noise figure of the AT-41435 transistor plotted in the impedance plane.

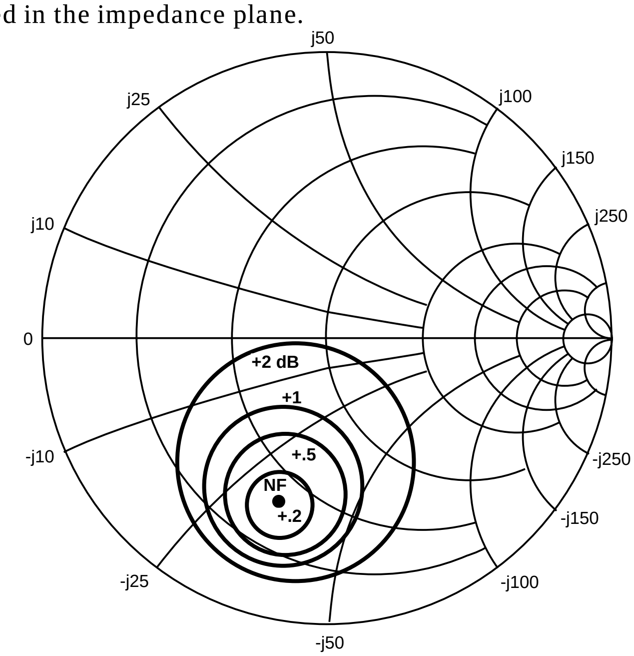

Figure 6. Constant Noise Figure vs. Source Impedance – AT-41435
Frequency = 4 GHz, V_{CE} = 8 V, I_C = 10 mA, NF_O = 3.0 dB, Γ_O = 0.64, -111

VII. Noise and Gain Contours

All practical amplifiers involve more than one internal noise generator, and as a result have an optimum noise source which is not the same as the optimum gain source. From a practical point of view, it becomes desirable to know what the tradeoffs between noise figure and gain involve. This tradeoff is best shown by plotting both the gain and noise circles on the same chart.

By taking the gain contours developed in Section IV and the noise contours developed in Section VI and superimposing them on the same Smith chart, the gain and noise figure tradeoffs become readily apparent.

Figure 7 shows the noise and gain contours of the AT-41435 transistor plotted in the input impedance plane.

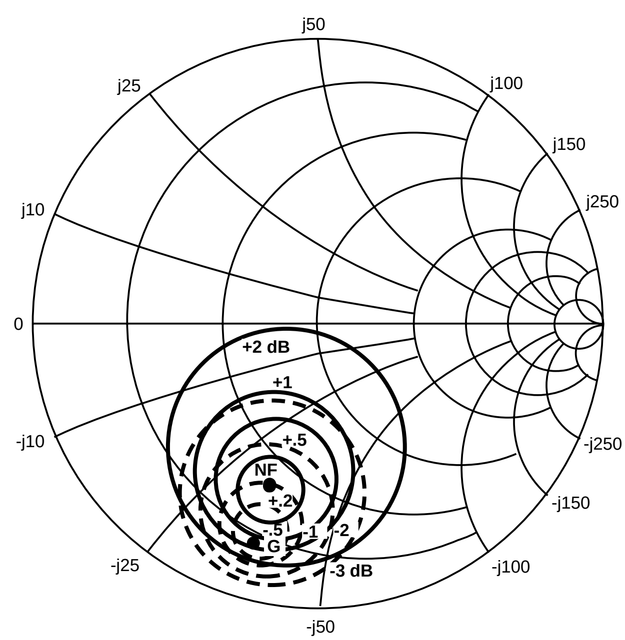

Figure 7. Constant Noise Figure and Gain vs. Source Impedance – AT-41435
Frequency = 4 GHz, V_{CE} = 8 V, I_C = 10 mA, NF_O = 3.0 dB, Γ_O = 0.64, -111, MAG = 10.6 dB, Γ_{ms} = 0.84, -111

With this chart, the circuit designer can easily pick the input matching conditions which will result in the optimum compromise for simultaneously meeting gain, VSWR and noise figure requirements.

Again, to realize the specified gain for any arbitrary point on the input plane, the output matching impedance is obtained by conjugately matching S'_{22} (Equation 11):

$$\Gamma_L = \left[S_{22} + \frac{S_{21} S_{12} \Gamma_S}{1 - S_{11} \Gamma_S} \right]^* \tag{44}$$

Summary

In the previous sections of this booklet, the basic techniques for developing a graphical display of the input plane of a noisy linear two port network have been described, and a number of specific examples were shown. This technique may be used to graphically describe any noisy linear two port network at any microwave frequency, provided that the following parameters are known at the desired frequency and, in the case of a transistor, at the desired bias conditions.

S_{11}, S_{21}, S_{12}, S_{22}, F_{OPT}, Y_{OPT}, R_n

References

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2. Bodway, G.E., Two Port Power Flow Analysis Using Generalized Scattering Parameters, Microwave Journal, Vol. 10, No. 6, May 1967.

3. Fukui, H., Available Power Gain, Noise Figure and Noise Measure of Two Ports and Their Graphical Representation, IEEE Trans. on CT, Vol. CT- 13, No. 2, pp 137-142.

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