

Part 1:

$A = \{a, b, c, d\}$, $B = \{v, w, x, y, z\}$, $f = \{(a, z), (b, y), (c, x), (d, w)\}$

The relation is a function.

The function is injective.

$A = \{a, b, c, d\}$, $B = \{x, y, z\}$, $f = \{(a, z), (b, y), (c, x), (d, z)\}$

The relation is a function.

The function is surjective.

$A = \{a, b, c, d\}$, $B = \{w, x, y, z\}$, $f = \{(a, z), (b, y), (c, x), (d, w)\}$

The relation is a function.

The function is bijective.

The inverse function is: $\{(x, c), (w, d), (y, b), (z, a)\}$

$A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4, 5\}$, $f = \{(a, 4), (b, 5), (c, 1), (d, 3)\}$

The relation is a function.

The function is injective.

$A = \{a, b, c\}$, $B = \{1, 2, 3, 4\}$, $f = \{(a, 3), (b, 4), (c, 1)\}$

The relation is a function.

The function is injective.

$A = \{a, b, c, d\}$, $B = \{1, 2, 3\}$, $f = \{(a, 2), (b, 1), (c, 3), (d, 2)\}$

The relation is a function.

The function is surjective.

$A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4\}$, $f = \{(a, 4), (b, 1), (c, 3), (d, 2)\}$

The relation is a function.

The function is bijective.

The inverse function is: $\{(3, c), (4, a), (1, b), (2, d)\}$

$A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4\}$, $f = \{(a, 2), (b, 1), (c, 2), (d, 3)\}$

The relation is a function.

The function is not injective nor surjective nor bijective.

$A = \{a, b, c\}$, $B = \{1, 2, 3, 4\}$, $f = \{(a, 2), (a, 4), (b, 1), (c, 3)\}$

The relation is not a function

Part 2:

$$662/414 = 1 \text{ R } 248$$

$$414/248 = 1 \text{ R } 166$$

$$248/166 = 1 \text{ R } 82$$

$$166/82 = 2 \text{ R } 2$$

$$82/2 = 41 \text{ R } 0$$

$$\gcd(414, 662) = 2$$

$$14/6 = 2 \text{ R } 2$$

$$6/2 = 3 \text{ R } 0$$

$$\gcd(6, 14) = 2$$

$36/24 = 1 \text{ R } 12$
 $24/12 = 2 \text{ R } 0$
 $\gcd(24, 36) = 12$

$42/12 = 3 \text{ R } 6$
 $12/6 = 2 \text{ R } 0$
 $\gcd(12, 42) = 6$

$252/198 = 1 \text{ R } 54$
 $198/54 = 3 \text{ R } 36$
 $54/36 = 1 \text{ R } 18$
 $36/18 = 2 \text{ R } 0$
 $\gcd(252, 198) = 18$

Part 3:

First Pass:

$$662 = 414 * 1 + 248$$

$$414 = 248 * 1 + 166$$

$$248 = 166 * 1 + 82$$

$$166 = 82 * 2 + 2$$

$$82 = 2 * 41 + 0$$

Back Pass:

$$2 = 1 * 166 - 2 * 82$$

$$2 = 1 * 166 - 2 * (248 - 1 * 166)$$

$$2 = 3 * 166 - 2 * 248$$

$$2 = 3 * (414 - 1 * 248) - 2 * 248$$

$$2 = 3 * 414 - 5 * 248$$

$$2 = 3 * 414 - 5 * (662 - 1 * 414)$$

$$2 = 8 * 414 - 5 * 662$$

First Pass:

$$14 = 6 * 2 + 2$$

$$6 = 2 * 3 + 0$$

Back Pass:

$$2 = 1 * 14 + -2 * 6$$

First Pass:

$$36 = 24 * 1 + 12$$

$$24 = 12 * 2 + 0$$

Back Pass:

$$12 = 1 * 36 + -1 * 24$$

First Pass:

$$42 = 12 * 3 + 6$$

$$12 = 6 * 2 + 0$$

Back Pass:

$$6 = 1 * 42 + -3 * 12$$

First Pass:

$$252 = 198 * 1 + 54$$

$$198 = 54 * 3 + 36$$

$$54 = 36 * 1 + 18$$

$$36 = 18 * 2 + 0$$

Back Pass:

$$18 = 1 * 54 - 1 * 36$$

$$18 = 1 * 54 - 1 * (198 - 3 * 54)$$

$$18 = 4 * 54 - 1 * 198$$

$$18 = 4 * (252 - 1 * 198) - 1 * 198$$

$$18 = 4 \cdot 252 + -5 \cdot 198$$

Part 4

$$\begin{aligned} q_1 &= 1, q_2 = 1, q_3 = 1, q_4 = 2, q_5 = 41, \\ s_0 &= 1, s_1 = 0, s_2 = s_0 - s_1 \cdot q_1 = 1 - 0 \cdot 1 = 1, s_3 = s_1 - s_2 \cdot q_2 = 0 - 1 \cdot 1 = -1, \\ s_4 &= s_2 - s_3 \cdot q_3 = 1 - -1 \cdot 1 = 2, s_5 = s_3 - s_4 \cdot q_4 = -1 - 2 \cdot 2 = -5, \\ t_0 &= 0, t_1 = 1, t_2 = t_0 - t_1 \cdot q_1 = 0 - 1 \cdot 1 = -1, t_3 = t_1 - t_2 \cdot q_2 = 1 - -1 \cdot 1 = 2, \\ t_4 &= t_2 - t_3 \cdot q_3 = -1 - 2 \cdot 1 = -3, t_5 = t_3 - t_4 \cdot q_4 = 2 - -3 \cdot 2 = 8, \\ \gcd(414, 662) &= 8 \cdot 414 + -5 \cdot 662 \end{aligned}$$

$$\begin{aligned} q_1 &= 2, q_2 = 3, \\ s_0 &= 1, s_1 = 0, s_2 = s_0 - s_1 \cdot q_1 = 1 - 0 \cdot 2 = 1, \\ t_0 &= 0, t_1 = 1, t_2 = t_0 - t_1 \cdot q_1 = 0 - 1 \cdot 2 = -2, \\ \gcd(6, 14) &= -2 \cdot 6 + 1 \cdot 14 \end{aligned}$$

$$\begin{aligned} q_1 &= 1, q_2 = 2, \\ s_0 &= 1, s_1 = 0, s_2 = s_0 - s_1 \cdot q_1 = 1 - 0 \cdot 1 = 1, \\ t_0 &= 0, t_1 = 1, t_2 = t_0 - t_1 \cdot q_1 = 0 - 1 \cdot 1 = -1, \\ \gcd(24, 36) &= -1 \cdot 24 + 1 \cdot 36 \end{aligned}$$

$$\begin{aligned} q_1 &= 3, q_2 = 2, \\ s_0 &= 1, s_1 = 0, s_2 = s_0 - s_1 \cdot q_1 = 1 - 0 \cdot 3 = 1, \\ t_0 &= 0, t_1 = 1, t_2 = t_0 - t_1 \cdot q_1 = 0 - 1 \cdot 3 = -3, \\ \gcd(12, 42) &= -3 \cdot 12 + 1 \cdot 42 \end{aligned}$$

$$\begin{aligned} q_1 &= 1, q_2 = 3, q_3 = 1, q_4 = 2, \\ s_0 &= 1, s_1 = 0, s_2 = s_0 - s_1 \cdot q_1 = 1 - 0 \cdot 1 = 1, s_3 = s_1 - s_2 \cdot q_2 = 0 - 1 \cdot 3 = -3, \\ s_4 &= s_2 - s_3 \cdot q_3 = 1 - -3 \cdot 1 = 4, \\ t_0 &= 0, t_1 = 1, t_2 = t_0 - t_1 \cdot q_1 = 0 - 1 \cdot 1 = -1, t_3 = t_1 - t_2 \cdot q_2 = 1 - -1 \cdot 3 = 4, \\ t_4 &= t_2 - t_3 \cdot q_3 = -1 - 4 \cdot 1 = -5, \\ \gcd(252, 198) &= 4 \cdot 252 + -5 \cdot 198 \end{aligned}$$