

ASSIGNMENT - 1

2. 1. $x(n) = \{6, 3, 9, 4\}$ ($0 \leq n \leq 3, n: \text{integer}$)

$$N = 4$$

DFT equation:

$$X(\omega(m)) = X\left(\frac{2\pi \cdot m}{N}\right)$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\left(\frac{2\pi m}{N}\right)n} \quad (0 \leq m \leq N-1, m: \text{integer})$$

$$\Rightarrow X(\omega(m)) = X\left(\frac{2\pi \cdot m}{4}\right) = X\left(\frac{\pi m}{2}\right)$$

$$= \sum_{n=0}^3 x(n) e^{-j\left(\frac{\pi m}{2}\right)n}$$

$$m = 0$$

$$\begin{aligned} \Rightarrow X(\omega(0)) &= X\left(\frac{\pi \cdot 0}{2}\right) = x(0) + x(1) + x(2) + x(3) \\ &= 6 + 3 + 9 + 4 \end{aligned}$$

$$\boxed{\therefore X(\omega(0)) = \underline{\underline{22}}}$$

$$m = 1$$

$$\begin{aligned} \Rightarrow X(\omega(1)) &= X\left(\frac{\pi}{2}\right) = x(0) + x(1) \cdot e^{-j\pi/2} + x(2) \cdot e^{-j\pi} \\ &\quad + x(3) \cdot e^{-j3\pi/2} \end{aligned}$$

$$= 6 + 3(-i) + 9(-1) + 4(i)$$

$$\boxed{\therefore X(\omega(1)) = \underline{\underline{-3 + i}}}$$

$$m=2$$

$$\Rightarrow X(\omega(2)) = X(\pi) = x(0) + x(1)e^{-i\pi} + x(2)e^{-i2\pi} + x(3)e^{-i3\pi}$$

$$= 6 + 3(-1) + 9(1) + 4(-1)$$

$$\therefore X(\omega(2)) = \underline{\underline{8}}$$

$$m=3$$

$$\Rightarrow X(\omega(3)) = X\left(\frac{3\pi}{2}\right) = x(0) + x(1)e^{-i3\pi/2} + x(2)e^{-i6\pi/2} + x(3)e^{-i9\pi/2}$$

$$= 6 + 3(i) + 9(-1) + 4(-i)$$

$$\therefore X(\omega(3)) = \underline{\underline{-3 - i}}$$

~~These are~~ In matrix form:

$$\begin{bmatrix} X(\omega(0)) \\ X(\omega(1)) \\ X(\omega(2)) \\ X(\omega(3)) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 9 \\ 4 \end{bmatrix} = \begin{bmatrix} 22 \\ -3 + i \\ 8 \\ -3 - i \end{bmatrix}$$

To calculate magnitude:

$$|X(\omega(0))| = 22$$

$$|X(\omega(1))| = \sqrt{(-3)^2 + 1^2} = \sqrt{10} \approx 3.16$$

$$|X(\omega(2))| = 8$$

$$|X(\omega(3))| = \sqrt{(-3)^2 + (-1)^2} = \sqrt{10} \approx 3.16$$