

CHAPTER

1

INTRODUCTION TO R.C.C. AND PRINCIPLES OF WORKING STRESS METHOD

Chapter Outline

- 1.0 Introduction
- 1.1 Concrete
- 1.2 Ingredients of Concrete
- 1.3 Important Properties of Concrete
- 1.4 Advantages of Concrete
- 1.5 Concrete Mix Proportions
- 1.6 Reinforced Concrete
- 1.7 Steel Reinforcement
- 1.8 Loads
- 1.9 Methods of Design of Reinforced Concrete
- 1.10 Code of Practice for RCC Design
- 1.11 Principles of Working Stress Method
- 1.12 Analysis of Singly Reinforced Sections

1.0 INTRODUCTION

Concrete structures have become very common in Civil engineering construction. Concrete has established as a universal building material because of its high compressive strength, its adoptability to take any form and shape and resistant to fire and corrosion with negligible maintenance cost. Concrete is very strong in compression but very weak in tension. Its low tensile strength is compensated by introducing steel reinforcement in the tension zone. Thus, the concrete is strengthened by steel and the resultant composite mass is known as Reinforced Cement Concrete. Thus RCC is used extensively in construction of buildings, bridges, tanks, dams etc. The design of these modern reinforced concrete structures may appear to be complex. However, most of these structures are the assembly of several basic structural elements such as beams, columns, slabs, walls and foundations. Accordingly, the designer has to learn the design of these basic reinforced concrete elements. The joints and connections are then carefully developed.

1.1 CONCRETE

Concrete is a composite material consisting of cement, aggregate and water in suitable proportions. The chemical interaction between cement and water binds the aggregates in to a solid mass. Fresh concrete will be plastic, so that it can be moulded to any desired shape in the moulds and compacted to form a dense mass. Water has to be applied for few days over the concrete surface soon after its setting because the hydration reactions between cement and water continue for a longer period due to which hardening of concrete takes place. This period when concrete is kept moist during which concrete gains strength is called curing period. Hence, the strength of concrete increases with age. The process of solidification of concrete from plastic stage is called setting while gaining of strength after setting is called hardening. Usually, setting completes within a maximum duration of 10 hours, while about 90% of hardening is completed by 28 days.

The properties and quality of cement concrete are influenced by the properties of its ingredients and quality control maintained during its making and curing.

1.2 INGREDIENTS OF CONCRETE

The main ingredient materials in concrete are:

- (a) Cement
- (b) Aggregates
- (c) Water

1.2.1 CEMENT

Cement is the binding material which is obtained by burning calcareous, siliceous and argillaceous materials together in definite proportions at high temperature and grinding the resultant clinker in to a fine powder. Various types of cements have been developed for the use in different types of structures under different situations. According to IS : 456-2000, the types of cements and their suitability for a specific situations are given in Table 1.1.

TABLE 1.1: Types of Cements and their Suitability

S.No.	Type of Cement	IS Code	Where Used
1.	Ordinary Portland cement		
	33 Grade	IS: 269	All general concreting works
	43 Grade	IS: 8112	Multi story structures
	53 Grade	IS: 12269	Bridges, Tall structures, Prestressed concrete work (Compressive strength of cement at 28 days in N/mm ² is called as grade of cement)
2.	Rapid hardening cement	IS: 8041	Road works and repairs
3.	Low heat Portland cement	IS: 12600	Mass concreting-Dams
4.	Port land slag cement	IS: 455	Marine structures
5.	Portland pozzolana cement	IS: 1489	General building works, mass concrete, Marine structures
6.	Sulphate resisting Portland cement	IS: 12330	Marine structure foundations in sulphate bearing soils
7.	Hydrophobic cement	IS: 8043	Swimming pools, floors of food processing plants
8.	High alumina cement	IS: 6452	Marine structures
9.	Supersulphated cement	IS: 6909	Marine structures, construction of sewers

1.2.2 AGGREGATES

Around 75% volume of concrete is occupied by the aggregates. Hence, the structural behaviour of concrete is significantly influenced by the type of aggregates used. The aggregates used for the concrete should be durable, strong, hard, chemically inert and well graded.

Aggregates whose particle size varies from 0.075 mm to 4.75 mm are called as fine aggregate. Aggregates with particle sizes more than 4.75 mm are called as coarse aggregates. Usually sand is used as fine aggregate whereas crushed rock and gravel is used as coarse aggregate.

The nominal maximum size of the coarse aggregate shall be as large as possible but it should be limited to $\frac{1}{4}$ th of the minimum thickness of the member.

The various properties of aggregates like specific gravity, strength, toughness, hardness, soundness, particle size distribution and grading should comply with the IS code IS:383-1979.

1.2.3 WATER

Water plays an active role in the chemical process of hydration of cement and in curing of concrete. Hence, the water used for mixing and curing of concrete should be clean and free from injurious amount of oils, acids, alkalies, salts, organic matter etc. that may be deleterious to concrete and steel. Drinking water is generally considered satisfactory for mixing of concrete. Sea water should not be used for mixing and curing because of presence of harmful salts in it. The PH value of water should not be less than 6. The physical and chemical tests for water should be done as per IS: 3025.

1.2.4 ADMIXTURES

Admixtures are added to the concrete before or during mixing, to modify one or more of the specific properties of concrete in the fresh or hardened states. IS:9103-1979 lays down the procedures for evolution of admixtures for concrete. The different types of admixtures used are given below.

1. **Accelerating Admixtures:** These are added to concrete to increase the rate of early strength development, which in turn facilitates earlier removal of form work. Common accelerators are calcium chloride, fluosilicates and trietanolamine.
2. **Retarding Admixtures:** These are added to slow down the rate of setting of cement. They are useful in hot weather concreting. Common types of retarders are starches and cellulose products, sugar and hydroxyl-carboxylic acids.
3. **Water Reducing or Plasticizing Admixtures:** The addition of plasticizer allows greater workability for given water cement ratio or alternatively retains the workability while reducing the water content. The basic ingredients of water reducing agents are either lignosulphonate salts or polyhydroxy compounds.
4. **Air-Entraining Admixtures:** These are used to incorporate air in the form of minute bubbles in concrete usually to increase workability and resistance to freezing and thawing. Commonly used air-entraining agents are animal and

vegetable oils, natural wood resins and their sodium salts, alkali salts of sulphated and sulphonated organic compounds.

1.3 IMPORTANT PROPERTIES OF CONCRETE

1.3.1 COMPRESSIVE STRENGTH/GRADE OF CONCRETE

The compressive strength of concrete is the characteristic compressive strength (f_{ck}) of 150 mm concrete cubes at the age of 28 days in N/mm^2 . The characteristic strength is defined as the strength of material below which not more than 5% of the test results are expected to fall.

IS 456-2000 specifies the characteristic compressive strength of 150 mm cube at the age of 28 days in N/mm^2 , as the Grades of concrete varying from 15 to 80 N/mm^2 designated as M 15 to M 80. Based on the characteristic compressive strength (f_{ck}), concrete is graded as given below :

TABLE 1.2 : Grades of Concrete (Table 2 of IS 456-2000)

Group	Grade Designation	Characteristic Compressive Strength of 150 mm Cubes at 28 Days in N/mm^2 , f_{ck}
Ordinary concrete	M 10	10
	M 15	15
	M 20	20
Standard concrete	M 25	25
	M 30	30
	M 35	35
	M 40	40
	M 45	45
	M 50	50
	M 55	55
High strength concrete	M 60	60
	M 65	65
	M 70	70
	M 75	75
	M 80	80

In the designation of concrete mix, M refers to the mix and the number to the characteristic compressive strength of 150 mm cubes at 28 days in N/mm^2 .

For reinforced concrete, the minimum grade of concrete to be used is M 20. For various weather conditions, the minimum grade of concrete to be used for reinforced concrete is given in Table 1.3.

TABLE 1.3 : Minimum Grade of Concrete for Different Exposure Conditions

Exposure	Minimum Grade of Concrete for RCC
Mild	M 20
Moderate	M 25
Severe	M 30
Very severe	M 35
Extreme	M 40

1.3.2 TENSILE STRENGTH

The tensile strength of concrete is very low and hence it is not taken into account in the design of reinforced concrete. But it is an important property which affects the extent and width of cracks in the structure. According to IS 456-2000, the tensile strength of concrete can be calculated from the compressive strength using the following relation.

$$f_{cr} = 0.7 \sqrt{f_{ck}} \text{ N/mm}^2$$

Where f_{ck} is the characteristic cube compressive strength of concrete.

1.3.3 MODULUS OF ELASTICITY

Modulus of elasticity of concrete is an important property required for computation of deflections of structural concrete members. In the absence of test data, the modulus of elasticity of concrete is related to compressive strength by the following relation as per IS 456 - 2000.

$$E_c = 5000 \sqrt{f_{ck}} \text{ N/mm}^2$$

Where E_c is the short term static modulus of elasticity in N/mm^2

1.3.4 POISSON RATIO

Poisson ratio varies between 0.1 for high strength concrete and 0.2 for weak concrete. Usually it is taken as 0.15 for strength design and 0.2 for serviceability calculations.

1.3.5 SHRINKAGE

The property of decreasing in volume during the process of drying and hardening of concrete is called shrinkage. Shrinkage in concrete may result in surface cracks. Shrinkage of concrete also influences the deflections of reinforced concrete members.

It depends up on the ingredients of concrete and environmental conditions like temperature and humidity. The IS : 456-2000 recommends the total shrinkage strain of 0.0003 for design purpose.

1.3.6 CREEP

Creep is defined as plastic deformation under constant load or stress. The creep coefficient which is defined as the ratio of ultimate creep strain to the elastic strain at various ages of loadings as recommended by IS 456-2000 is given below.

Age at Loading	Creep Coefficient
7 days	2.2
28 days	1.8
1 year	1.1

Creep of concrete considerably increases the deflections of reinforced concrete flexural members. Higher the creep coefficient more will be the deflections. The value of creep coefficient is useful in the calculation of time dependent deflections in reinforced concrete members.

1.3.7 WORKABILITY

The workability of freshly mixed concrete is that property which determines the ease and homogeneity with which it can be mixed, placed, compacted and finished. A workable concrete should not show any segregation (separation of constituents of concrete in its plastic stage) or bleeding (rise of water in the mix to the surface when it is compacted).

A workable concrete possesses required lubrication for handling concrete without segregation, can be placed in the forms without loss of homogeneity, compacted with minimum effort and shall be finished easily.

The factors which influence the workability are

- (i) Water-Cement ratio
- (ii) Type and grading of aggregate.
- (iii) Ratio of fine and coarse aggregate.
- (iv) Use of admixtures.
- (v) Efficiency of mixing.

Recommended tests to measure workability are

- (i) Slump test
- (ii) Compacting factor test
- (iii) Vee Bee consistometer test
- (iv) Flow test

A Suggested ranges of workability of concrete in accordance of IS : 1199 are given below.

TABLE 1.4 : Workability for Different Placing Conditions (Clause 7.1 of IS 456-2000)

Placing Conditions	Degree of Workability	Slump mm
Blinding concrete, shallow sections, pavements using pavers.	Very low	(0.75-0.8 compaction factor)
Mass concrete, lightly reinforced sections in slabs, beams walls, columns, floors, Hand placed pavements, canal linings, strip footing	Low	25-75
Heavily reinforced sections in slabs, beams, walls, columns.	Medium	50-100
Slip form work, pumped concrete	Medium	75-100
Trench fill, Insitu piling	High	100-150
Tremie concrete	Very high	Based on flow test

1.3.8 UNIT WEIGHT

The unit weight of concrete depends up on the type of aggregates and amount of voids. The unit weight as specified by the IS : 456-2000 for plain concrete and reinforced concrete are 24 kN/m^3 and 25 kN/m^3 respectively.

1.4 ADVANTAGES OF CONCRETE

The following are the advantages of concrete due to which concrete is extensively used in construction industry.

1. Compressive strength of concrete is very high
2. Concrete can be moulded to any desired shape.
3. The materials for concrete are easily available.
4. It is easy to make.
5. It is durable
6. By proper proportioning of mix, concrete can be made watertight.

7. Its monolithic character gives it more rigidity.
8. It is fire resisting.
9. Its maintenance cost is practically nil.
10. Strength of concrete increases with age.

Disadvantages of Concrete:

1. Tensile strength of concrete is very low and hence plain concrete cannot be used in situations where tensile stresses are developed.
2. Strict quality control has to be maintained during production, placing and compaction.
3. Curing has to be done for at least 14 days and hence time of construction increases.
4. Once the members caste with concrete, it is very difficult to dismantle it.

1.5 CONCRETE MIX PROPORTIONS

The mix proportions shall be selected to ensure the workability of fresh concrete and when concrete is hardened, it shall have the required strength, durability and surface finish.

The main objective of concrete mix proportioning is to select the optimum proportion of the various ingredients of concrete (cement, aggregate, and water), to attain the required workability (when it is fresh), strength and durability (when it is hardened) with minimum cost. It can be done in the following two ways.

- (a) Nominal mix concrete
- (b) Design mix concrete

1.5.1 NOMINAL MIX CONCRETE

In nominal mix, the proportions of cement, aggregate and water are nominally adopted. Nominal mix concrete is used on small works for routine concrete construction and for concrete M 20 and lower. However it requires high cement content. Nominal proportions such as 1:2:3, 1:3:6 etc. have no significance for strength and durability. In these, the quantity of fine aggregate is fixed irrespective of the cement content and the maximum size of aggregate. Hence, the variations in the quality of concrete produced are inevitable. The proportions of materials for nominal mix concrete prescribed by IS : 456 - 2000 are given in Table. 1.3.

TABLE 1.5 : Proportions for Nominal Mix Concrete (Table 9 of IS 456-2000)

Grade Concrete	Total Quantity of Dry Aggregates by Mass Per 50 kg of Cement, kg	Proportion of Fine Aggregate to Coarse Aggregate (by Mass)	Quantity of Water Per 50 kg of Cement Max
M 5	800		60
M 7.5	625	Generally 1:2 but subjected to an upper limit of 1:1 1/2 and a lower limit of 1:2 1/2	45
M 10	480		34
M 15	330		32
M 20	250		30

Note: The proportion of the fine to coarse aggregates should be adjusted from upper limit to lower limit progressively as the grading of fine aggregates becomes finer and the maximum size of coarse aggregate becomes larger.

Example : For an average grading of fine aggregate (Zone-II), the proportions shall be 1:1 1/2, 1:2, 1:2 1/2 for maximum size of aggregate 10 mm, 20 mm and 40 mm respectively.

The proportions for nominal mix concrete generally adopted in practice are given below.

Grade of Concrete	Nominal Mix Proportions (Cement : Fine Aggregate : Coarse Aggregate)
M 5	1 : 5 : 10
M 7.5	1 : 4 : 8
M 10	1 : 3 : 6
M 15	1 : 2 : 4
M 20	1 : 1 1/2 : 3

1.5.2 DESIGN MIX CONCRETE

In design mix, the proportions of cement, aggregate, water and mineral admixtures if any are actually designed. For all important works involving large quantities of concrete, it is preferable to use design mix, which results in considerable economy ensuring the required strength.

The process of mix design consists of selecting optimum proportions of cement, aggregate and water to produce the grade of concrete with the required workability, strength and durability with minimum cost.

Over the years, several mix design methods have been developed, which are listed below.

- (i) Minimum voids method
- (ii) Maximum density method
- (iii) Fineness modulus method
- (iv) Road research laboratory (RRL) method
- (v) American concrete Institute (ACI) method
- (vi) IS Code method

1.6 REINFORCED CONCRETE

Reinforced concrete (RCC) is concrete that contains embedded steel bars, plates, or fibres that strengthen the material.

Concrete has a high compressive strength and a low tensile strength. This led to the invention of reinforced concrete, which is a combination of concrete and steel in which steel provides the tensile strength lacking in concrete. In addition, steel can also carry the compressive load as found in column structures.

The combination of concrete and steel in reinforced concrete produces a structural element that takes the advantage of each material. The high tensile capacity of steel is used to overcome the low strength of concrete. On the other hand, the high resistance of concrete against corrosive environment is used to protect steel, which is prone to corrosion. Also both concrete and steel are bonded together very well, thus able to resist the load as a unit. This excellent bond is achieved by chemical adhesion between two materials, the surface roughness and closely spaced ribs along steel.

To produce good reinforced concrete elements, concrete and steel must be of good quality. Mix design for concrete must be prepared such that the produced concrete satisfies the workability, strength, and exposure condition requirements. The steel is produced in factory and normally in good quality. However, make sure that the steels are clean and not corroded.

1.7 STEEL REINFORCEMENT

Steel bars are essentially used in the tension zone of flexural members of concrete to resist the tensile stresses as concrete is weak in tension and in compression members to increase the load carrying capacity.

Steel is used as reinforcement to take up the tensile stresses in RCC construction because of the following reasons.

- (a) Its tensile strength is high
- (b) It can develop good bond with concrete.

- (c) Its coefficient of expansion is nearly same as for concrete.
- (d) It is easily available.

1.7.1 SIGNIFICANCE/FUNCTIONS OF REINFORCEMENT IN R.C.C

The reinforcement in RCC serves the following different types of functions.

- (i) To resist the bending tension in flexural members like slabs, beams and walls of water tanks etc.
- (ii) To increase the load carrying capacity of compression members like columns.
- (iii) To resist diagonal tension due to shear.
- (iv) To resist the effects of secondary stresses like temperature etc.
- (v) To reduce the shrinkage of concrete.
- (vi) To resist spiral cracking due to torsion.
- (vii) To prevent the development of wide cracks in concrete due to tensile strains.

1.7.2 TYPES OF REINFORCEMENT

Reinforcing steel consists of bars usually circular in cross section. The following four types of steel reinforcement are generally used in reinforced concrete construction.

- (a) Mild steel and medium tensile steel bars conforming to IS : 432 (Part- I)
- (b) High yield strength deformed steel bars (HYSD bars) conforming to IS : 1786
- (c) Steel wire fabric conforming to IS : 1566.
- (d) Structural steel conforming to Grade-A of IS : 2062

All reinforcement shall be free from loose mill scale, loose rust, oil, mud, and any other substances which reduces bond between steel and concrete. The grades of steel normally used for reinforcement are Fe 250, Fe 415 and Fe 500. In mentioning the grade of steel, Fe refers to ferrous metal and the number following it refers to specified guaranteed yield strength in N/mm^2 . The stress strain curves for different grades of steel are shown in Fig. 1.1.

Mild Steel Bars (Fe 250) : These plain bars of mild steel are of grade Fe 250 conforming to IS : 432 - 1982. It has well defined yield point giving yield stress of $250 N/mm^2$ and excellent ductility with high percentage of elongation ($> 23\%$). The actual and idealized stress strain curve for mild steel is shown in Fig 1.1. The properties of mild steel are given in Table 1.4.

TABLE 1.5 : Properties of Mild Steel (IS 432-1982, Part-I)

Type of Steel	IS Code	Bar Diameter ≤ 20 mm	Minimum Yield Stress N/mm ² (f _y)	Minimum Elongation %	Min Ultimate Stress N/mm ²
Mild Steel Grade - I	IS 432-1982 Part - I	≤ 20 mm	250	23	410
		20 to 50 mm	240	23	410
Mild Steel Grade - II	IS 432-1982 Part - I	≤ 20 mm	225	23	370
		20 to 50 mm	215	23	370
Medium tensile steel	IS 432-1982 Part - I	≤ 16 mm	350	20	530
		16 to 32 mm	340	20	540
		> 32 to 50 mm	330	20	510

High Yield Strength Deformed Bars (Fe 415 & Fe 500) : As the name indicates, these HYSD bars have high yield strength but the yield point is not well defined. The yield stress or characteristic stress is given by 0.2 % proof stress. These bars possess ribs, lugs or deformations on their surface due to which the bond strength is improved. The properties of HYSD bars are given in Table 1.6.

TABLE 1.6 : Properties of HYSD Bars (IS 1786-1985)

Property	Fe 415	Fe 500
0.2% proof stress, min N/mm ²	415	500
Elongation %, min	14.5	12.0
Ultimate stress, min N/mm ²	15 % more than actual 0.2% proof stress	10 % more than actual 0.2 % proof stress

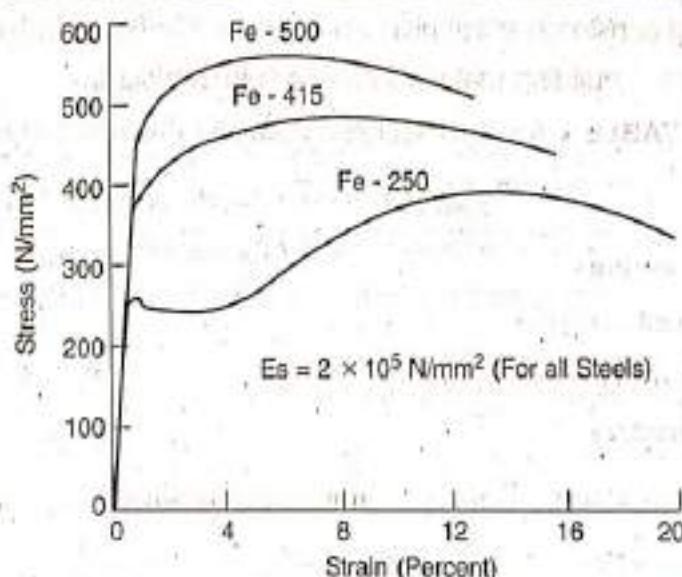


FIG 1.1 : Typical Stress-Strain Curves for Reinforcing Steels

1.7.3 MODULES OF ELASTICITY OF STEEL

The module of elasticity of steel of all grades is taken as $2 \times 10^5 \text{ N/mm}^2$ (200 kN/mm^2).

1.7.4 UNIT WEIGHT OF STEEL

The unit weight of steel is 78.5 kN/m^3 (7850 kg/m^3). A quicker method to find the weight of bar of circular section is given by the following equation.

$$\text{Weight of bar in kg/m} = \frac{\phi^2}{162.2}$$

Where ϕ = Diameter of the bar in mm

1.8 LOADS

The reinforced concrete structures are designed to resist the following types of loads.

- (i) Dead loads
- (ii) Live loads or Imposed loads
- (iii) Wind loads
- (iv) Earthquake forces
- (v) Snow loads
- (vi) Shrinkage, creep and Temperature effects.
- (vii) Other forces and effects

1.8.1 DEAD LOADS

Dead loads are permanent or stationary loads which are transferred to the structure throughout their life span. Dead loads mainly due to self weight of structural members, permanent partitions, fixed equipments and fittings. These loads shall be calculated by estimating the quantity of each material and then multiplying it with the unit weight. The unit weights of various materials used in building construction are given in the code IS : 875 (Part-1) - 1987. The unit weight of commonly used building materials are given in Table 1.8.

TABLE 1.8 : Unit Weights Common Building Materials

S.No.	Material	Unit Weight KN/m ³
1.	Plain concrete	24
2.	Reinforced concrete	25
3.	Brick masonry , cement plaster	20
4.	Stone masonry	24
5.	Wood	8
6.	Steel	78.5
7.	Floor finish	0.6 - 1.2

1.8.2 LIVE LOADS (OR) IMPOSED LOADS

These are the loads that changes with respect to time. Live loads or imposed loads include loads due to the people occupying the floor, weight of movable partitions, weight furniture and materials. The live loads to be taken in design of buildings have been given in IS : 875 (Part - 2) - 1987. Some of the common live loads used in the design of buildings are given in Table 1.8.

TABLE 1.8 : Live loads on floors (IS : 875, Part-2)

S.No.	Type of Floors	Minimum Live Load kN/m ²
1.	Floors in dwelling houses, tenements, hospital wards, hostels and dormitories	2.0
2.	Office floors other than entrance halls, floors of light	2.5 - 4.0 (2.5, when separate storage work rooms facility is provided, otherwise 4.0)
3.	Floors of banking halls, office entrance halls and reading rooms	3.0
4	Shops, educational buildings, assembly buildings, restaurants	4.0
5	Office floors for storage, assembly floor space without fixed seating, public rooms in hotels, dance halls and waiting halls	5.0
6.	Ware houses, work shops and factories (a) Light weight loads (b) Medium weight loads (c) Heavy weight loads	5.0 7.5 10.0
7.	Garages (light-handling vehicles of weight < 25 kN) Garages (heavy- vehicles of weight > 25 kN)	4.0 7.5
8.	Stairs, landings, Balconies and corridors for floors mentioned in 1, but not liable to over crowding Stairs, landings and corridors for floors mentioned in 1, but liable to overcrowding and for all other floors	3.0 5.0
9	Flat slabs, sloped roofs (a) Access provided (b) Access not provided	1.5 0.75

1.8.3 WIND LOADS

The horizontal load caused by the wind is called as wind loads. It depends up on the velocity of wind and shape and size of the building. Complete details of calculating wind loads on structures are given in IS : 875 (Part - 3) - 1987. Brief idea of these provisions is given below.

- (i) Basic wind pressure V_b has to be picked up depending upon the locality of the building.
- (ii) To get the design wind velocity V_z the following expression shall be used,

$$V_z = k_1 k_2 k_3 V_b$$

Where

k_1 = Risk coefficient

k_2 = Coefficient based on terrain, height and structure size.

k_3 = Topography factor

- (iii) The design wind pressure is given by

$$P_z = 0.6 V_z^2$$

Where P_z is in N/m^2 at height Z and V_z is in m/sec . Up to a height of 30 m, the wind pressure is considered to act uniformly. Above 30 m height, the wind pressure increases.

1.8.4 EARTH QUAKE FORCES

Earthquake shocks cause movement of foundation of structures. Due to inertia additional forces develop on the super structure. The impact of earthquake on structures depends on the stiffness of the structure, stiffness of the soil media, height and location of the structure etc. Accordingly, the country has been divided into several zones depending on the magnitude of the earthquake. Depending on the problem, one of the following two methods may be used for computing the seismic forces.

- (i) Seismic coefficient method.
- (ii) Response spectrum method.

The details of these methods are prescribed in IS : 1893 code and also in National Building Code of India.

1.8.5 SNOW LOADS

These are important loads for structures located in areas having snow fall, which gets accumulated in different parts of the structure depending on projections, height, slope etc. of the structure. The standard values of snow loads are specified in Part 4 of IS : 875.

1.8.6 SHRINKAGE, CREEP AND TEMPERATURE EFFECTS

Shrinkage, creep and temperature (high or low) may produce stresses and cause deformations like other loads and forces. Hence, these are also considered as

loads which are time dependent. The safety and serviceability of structures are to be checked following the stipulations of clauses 6.2.4, 5 and 6 of IS : 456 : 2000 and Part 5 of IS : 875.

1.8.7 OTHER FORCES AND EFFECTS

IS : 456 : 2000 stipulates the following forces and effects to be taken into account in case they are liable to affect materially the safety and serviceability of the structures. The relevant codes as mentioned therein are also indicated below:

- Foundation movement (IS : 1904)
- Elastic axial shortening
- Soil and fluid pressures (IS : 875 - Part - 5)
- Vibration
- Fatigue
- Impact (IS : 875 - Part-5)
- Erection loads (IS : 875 - Part - 2)
- Stress concentration effect due to point of application of load and the like.

1.8.8 COMBINATION OF LOADS

Load combinations have to be taken in accordance to the IS : 875 (Part - 5) - 1987.

1.9 METHODS OF DESIGN OF REINFORCED CONCRETE

The aim of design is to decide the size of the member and amount of reinforcement required, so that the structure will perform satisfactorily during its life period with minimum cost. With an appropriate degree of safety the structure should

- (i) Sustain all loads
- (ii) Sustain the deformations during and after construction
- (iii) Should have adequate durability and
- (iv) Should have adequate resistance to misuse and fire.

In any method of design of reinforced concrete structures, the following are the common steps to be followed:

- (i) To assess the dead loads and other external loads and forces likely to be applied on the structure,
- (ii) To determine the design loads from different combinations of loads,
- (iii) To estimate structural responses (bending moment, shear force, etc.) due to the design loads,
- (iv) To determine the cross-sectional areas of concrete sections and amounts of reinforcement needed.

Many of the above steps have lot of uncertainties. Estimation of loads and evaluation of material properties are to name a few. Hence, some suitable factors of safety should be taken into consideration depending on the degrees of such uncertainties.

The following three methods have been developed for the design of reinforced concrete structures.

- (a) Working stress method
- (b) Ultimate load method
- (c) Limit state method

1.9.1 WORKING STRESS METHOD

Working stress method is based on elastic theory assuming reinforced concrete as elastic material. The stress strain curve of concrete is assumed as linear from zero at the neutral axis to a maximum value at the extreme fiber.

This method adopts permissible stresses which are obtained by dividing ultimate stress by a factor known as factor of safety. For concrete a factor of safety of 3 is used and for steel it is 1.78. This factor of safety accounts for any uncertainties in ~~working loads and moments~~.

CHAPTER

2

PHILOSOPHY OF LIMIT STATE DESIGN

Chapter Outline

- 2.1 Introduction
- 2.2 Philosophy of Limit State Method
- 2.3 Limit States
- 2.4 Characteristic Strength of Materials
- 2.5 Characteristic Load
- 2.6 Partial Safety Factors
- 2.7 Design Strength of Materials
- 2.8 Design Loads
- 2.9 Differences between Working Stress Method & Limit State Method

2.1 INTRODUCTION

As discussed in the previous chapter, working stress method gives satisfactory performance of the structure at working loads; it is unrealistic at ultimate state of collapse. Similarly, while the Ultimate load method provides realistic assessment of safety, it does not guarantee the serviceability requirements at working loads. An ideal method is the one which takes into account not only the ultimate strength of the structure but also the serviceability requirements. Limit state method oriented towards the simultaneous satisfaction of all these requirements.

2.2 PHILOSOPHY OF LIMIT STATE METHOD

A structure may become unfit for use not only when it collapses but when it violates the serviceability requirements such as deflections, cracking etc. A structure is said to have reached its limit state, when the structure as a whole or a part becomes unfit for use, during its expected life. The limit state of a structure is the condition of its being not fit for its intended use.

The philosophy of limit state method of design is to see that the structure remains fit for use throughout its life period by assuring safety against strength and serviceability requirements i.e. the structure will not reach the limit state in its life time. All the relevant limit states have to be considered in the design. The loads and strength of materials are to be estimated by probabilistic approach (characteristic values). The design loads and strengths are derived from the characteristic values through the use of partial safety factors.

2.3 LIMIT STATES

Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs. Many types of limit states or failure conditions can be specified. The two major limit states which are usually considered are the following.

- (i) Limit state of collapse
- (ii) Limit state of serviceability

(i) Limit State of Collapse :

It is the limit state at which the structure is likely to collapse. The structure may collapse due to rupture of one or more critical sections or loss of overall stability due to buckling or overturning. This limit state may correspond to

- (a) Flexure
- (b) Compression

- (c) Shear
- (d) Torsion

(II) Limit State of Serviceability :

Limit state of serviceability relate to the performance of the structure at working loads. It is the limit state at which the structure undergo excessive deflection, which adversely affect the finishes causing discomfort to the users and excessive cracking which effects the efficiency or appearance of the structure. This limit state may correspond to

- (a) Deflection
- (b) Cracking
- (c) Other limit states (Vibrations, Fire resistance, Durability)

All relevant limit states have to be considered in the design to ensure adequate degree of safety and serviceability. The structure shall be designed on the basis of the most critical limit state and shall be checked for other limit states.

2.4 CHARACTERISTIC STRENGTH OF MATERIALS

In Limit state design, design should be based on characteristic values for material strengths and applied loads, which take into account the variations in the material strengths and in the loads to be supported. The characteristic values should be based on statistical data if available; where such data are not available they should be based on experience.

The characteristic strength of material is that value of the strength of the material below which not more than 5% of the test results are expected to fall.

For concrete, characteristic strength is compressive strength of 150mm cubes of 28 days in N/mm², below which not more than 5% of the test results are expected to fall. Concrete grades are specified based on this strength.

For steel, the minimum yield strength or 0.2% proof strength is taken as the Characteristic strength of steel.

2.5 CHARACTERISTIC LOAD

The maximum working load that the structure has to withstand and for which it has to be designed is called Characteristic load. The characteristic loads are calculated based on statistical analysis.

The characteristic load is defined as that value of load which has 95% probability of not being exceeded during the life of the structure.

The various loads acting on structures consist of dead loads, live loads, wind or earthquake loads etc. These are discussed in sec. 1.7. However, the researches made so far fail to estimate the actual loads on the structure. Accordingly, the loads are predicted based on statistical approach, where it is assumed that the variation of the loads acting on structures follows the normal distribution (Fig. 2.1).

Characteristic load should be more than the average or mean load. Accordingly,

Characteristic load = Average or mean load + K (standard deviation for load)

The value of K is assumed such that the actual load does not exceed the characteristic load during the life of the structure in 95 per cent of the cases.

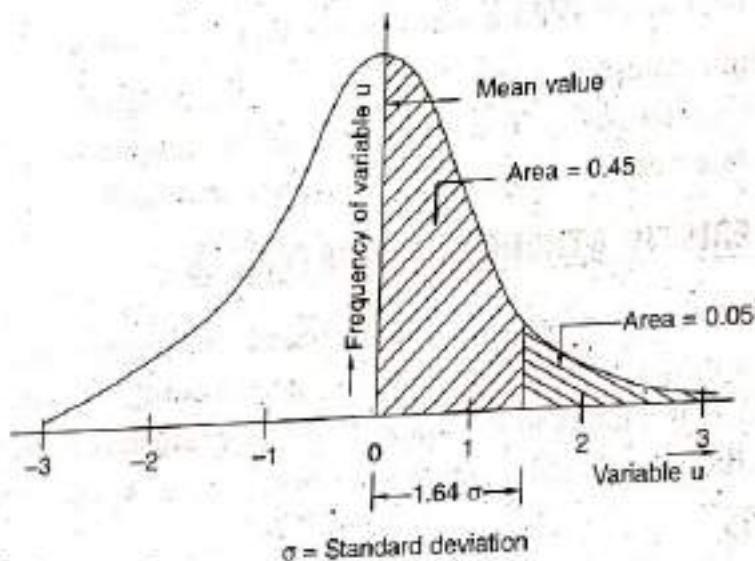


FIG 2.1: Normal Distribution Curve

2.6 PARTIAL SAFETY FACTORS

The design values are derived from the characteristic values through the use of partial safety factors, one for material strengths and the other for loads.

The characteristic values of loads as discussed in sec. 2.5 are based on statistical data. It is assumed that in ninety-five per cent cases the characteristic loads will not be exceeded during the life of the structures (Fig. 2.1). However, structures are subjected to overloading also. Hence, structures should be designed with loads obtained by multiplying the characteristic loads with suitable factors of safety depending on the nature of loads or their combinations, and the limit state being considered. These factors of safety for loads are termed as partial safety factors (γ_f) for loads.

Similarly, the characteristic strength of a material as obtained from the statistical approach is the strength of that material below which not more than five per cent of the test

results are expected to fall. However, such characteristic strengths may differ from sample to sample also. Accordingly, the design strength is calculated dividing the characteristic strength further by the partial safety factor for the material (γ_m), where γ_m depends on the material and the limit state being considered.

2.7 DESIGN STRENGTH OF MATERIALS

The design strength of materials is obtained by dividing the characteristic strength by a factor known as partial safety factor. The partial safety factor takes in to account variation of material strength, local weakness etc.

The design strength of the materials, f_d is given by

$$\text{Design strength} = \frac{\text{characteristic strength}}{\text{partial safety factor}}$$

$$f_d = \frac{f}{\gamma_m}$$

Where

f = characteristic strength of the material

γ_m = Partial safety factor appropriate to the material and limit state being considered.

The value of partial safety factors materials as recommended by IS 456-2000 is given in Table 2.1.

Table 2.1 Partial safety factor for material strengths, γ_m as per IS 456-2000

Material	Limit State of Collapse	Limit State of Serviceability
Steel	1.15	1.0
Concrete	1.5	1.0

A higher value of partial safety factor for concrete (1.5) has been adopted because there are greater chances of variation of strength of concrete due to improper compaction, inadequate curing, improper batching & mixing and variations in properties of ingredients. The partial safety factor for steel (1.15) is comparatively lower because the steel for reinforcement is produced in steel plants and commercially available in specific diameters with expected better quality control than that of concrete.

Further, in case of concrete the characteristic strength is calculated on the basis of test results on 150 mm standard cubes. But the concrete in the structure has different sizes. To take the size effect into account, it is assumed that the concrete in the structure

develops a strength of 0.67 times the characteristic strength of cubes. Accordingly, in the calculation of strength employing the limit state of collapse, the characteristic strength (f_{ck}) is first multiplied with 0.67 (size effect) and then divided by 1.5 (γ_m for concrete) to have 0.446 f_{ck} as the maximum strength of concrete in the stress block.

2.8 DESIGN LOADS

Design loads or factored loads, for which the structure is to be designed, are obtained by multiplying the characteristic load with a factor known as partial safety factor. The partial safety factor takes into account inaccurate assessment of load, unexpected stress redistribution and variation in dimensional accuracy.

The design load, F_d is given by

$$\text{Design load} = \text{characteristic load} \times \text{Partial safety factor}$$

$$F_d = F \cdot \gamma_f$$

Where

F = characteristic load

γ_f = Partial safety factor appropriate to the nature of loading and limit state being considered.

The value of partial safety factors for loads as recommended by IS 456-2000 is given in Table 2.2.

Table 2.2 Partial safety factors for loads, γ_f (Table 18 of IS 456-2000)

Load Combination	Limit State of Collapse			Limit State of Serviceability		
	DL	LL	WL	DL	LL	WL
DL + LL	1.5	1.5	—	1.0	1.0	—
DL + WL	1.5 or 0.9*	—	1.5	1.0	—	1.0
DL+LL+WL	1.2	1.2	1.2	1.0	0.8	0.8

* This value is to be used when stability against overturning or stress reversal is critical

2.9 DIFFERENCES BETWEEN WORKING STRESS METHOD & LIMIT STATE METHOD

The differences between Working stress method (WSM) and Limit state method (LSM) are mentioned in Table 2.3.

TABLE 2.3 : Differences between WSM and LSM

S.No.	Working Stress Method	Limit State Method
1.	The Stresses in an element is obtained load and compared with design strength. permissible stresses.	The stresses are obtained from design from the working loads and compared with
2.	The method follows linear stress-strain behaviour of both the materials (Elastic approach).	In this method, it follows non linear stress relationship (plastic design) but linear strain relationship.
3.	Material strengths are under estimated to large extent. Factor of safety are used in working stress method.	The material capabilities are not underestimated as much compared WSM. Partial safety factors are used in limit state method.
4.	WSM is stress based method.	LSM is strain based method.
5.	WSM substantially reduces the calculation efforts due to linear stress-strain behaviour.	LSM involves more calculation efforts due to nonlinear stress behaviour.
6.	Average or statistic values are used for loads and material strengths in WSM.	Characteristic values (derived from probabilistic approach which is more realistic and reasonably accurate) are used in case of LSM.
7.	This method yields to uneconomical design.	This method yields to economical design.

REVIEW QUESTIONS

- Define characteristic strength of concrete. Explain what is meant by M20 mix. (Oct. 2005)
- State the methods of design of reinforced concrete structures. (March/April. 2019)
- Differentiate between Nominal mix concrete and design mix concrete.
- State the loads to be considered in the design of reinforced concrete elements. (March. 2006, 2010)
- Define the term Limit state. Mention the different types of limit states. (March/April. 2017 ; April/May. 2011)
- Name the three methods recommended by IS 456 for the design of R.C.C members. (Oct. 2003)

Unit-1

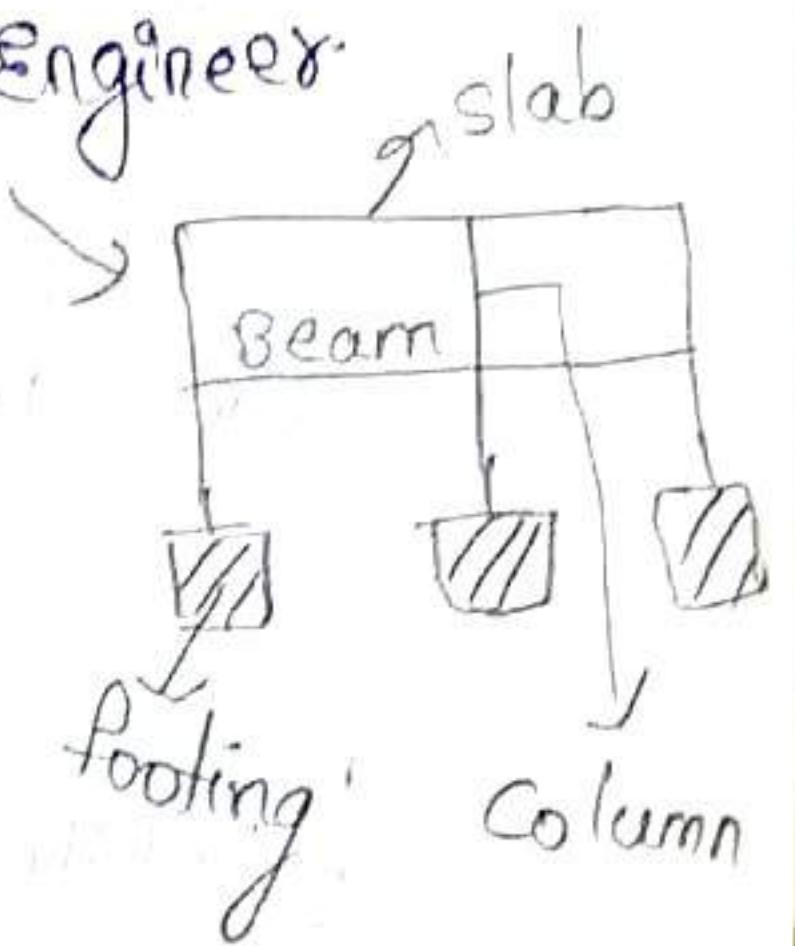
Introduction

Limit state Design

Design → 1) Functional design - role of Architecture ex:- No of rooms

2) Structural design - structural engineer

ex:- Beam, slab.



Design means proportioning structural elements.

Beam undergoes flexure (Bending).

Concrete :- Concrete weak in tension and strong in compression.

1) Plain Concrete. → Without steel

2) Reinforced Concrete. → with steel in Concrete.

which gives strength to particular material.

Design Codes and hand books :-

Basic Code for design :-

IS:456:2000 → Code of practice for plain and reinforced concrete (Fourth revision)

Loading standards :-

IS 875 - (Part 1-5) Code of practice for design

loads (Other than earth quakes)

Part-1 Dead load, Part-2 Live loads, Earthquake & wind loads, Snow Part-IV
Part-3 High floors.

Design hand book :-

SP 16:1980 - Design Aids for Reinforced Concrete to

IS 456:1978

Sp 34:1987 — Hand Book on Concrete reinforcement and detailing.

The following are the basal codes and hand books for designing a Concrete Structure.

Design Consideration:-

Flexure, shear, torsion, axial force (tensile (σ) Compressive (ε)) etc.

- 1) Safety — Strength, Stability, integrity
- 2) Serviceability — implies adequate performance of structure.

in terms of deformation, cracks, vibrations,

Concrete is only fire resistance.

Concrete is used to bare the compression load because in foundation more earthquake. (Sudden) collapses.

Loads:-

Horizontal loads — wind, floods

Vertical loads — live load, wind

longitudinal loads — sudden stopping of moving loads.

Loading Standards:- Dead load + Live load will comes.

Dead load (DL): - Static load of buildings (Columns, Beams, walls)

Live load (LL): - Cal. By unit weight \times quantity (volume or $b \times h$)

Not static, moving loads, persons, furniture, cars.

Imposed load (IL): - Marching, cranes; cal. Percentage of live loads.

Wind load (WL): - Due to wind.

Earthquake load (EL): - 0.891×0.192

Longitudinal load (LL): -

1) Working stress method :- (WSM)

2) Limit state Method (LSM)

1.) Working Stress method :- This method simply uses within elastic limit theory.

The both Steel and Concrete behaves as elastic.

Stress in the material < Permissible Stress.

$$\text{Permissible stress} = \frac{\text{Characteristic strength of Concrete } f_{ck}}{\text{(or)} \quad \text{yield strength of steel } f_y}$$

(or) Ultimate load

Permissible stress < Strength of material.

Factor of Safety.

$$\frac{M}{I} = \frac{E}{R} = \frac{F}{y} + \text{Flexure formula.}$$

Factor of Safety

For Concrete - 3 : For IS 456

Steel - 1.8 ≈

~~Stress in Steel is equal to the stress in concrete~~ surrounding concrete is called

~~Stress~~ Stress Compatibility.

Draw back:- → WSM fails due to different loads combinations.

→ It does not consider creep and shrinkage.

→ It is uneconomical design.

Theory of simple Bending

$$\frac{M}{I} = \frac{E}{R} = \frac{F}{y}$$

The design of a structural member is carried out by two methods.

1) Working Stress method (WSM): (or) [elastic theory]
(or)

2) Limit State method (LSM) ^{present}

modular ratio method

$$M = \frac{E_s S}{E_c C}$$

i) Working Stress Method (WSM): - → shape is linear.

→ It is based on elastic theory and assumes

both Steel and Concrete are elastic and obeys Hooke's law
possible

→ The bases of this method is at worst combination of working states the stresses in the materials are not exceeded beyond permissible values.

i.e. permissible stress < strength of material

Permissible stress are calculate by using factor of safety. f_{ck} (or) f_y

Permissible Stress = characteristic strength of material
(or) f_{ck}

Yield Strength of Steel (f_y)

Factor of Safety.

AS per IS: 456:2000 a factor of safety 3 is used for Concrete and in Bending Compression and 1.8 is used for ~~Steel~~ yield strength of Steel.

Drawbacks:- (less life time) (In this only bear particular load, when extra load apply it does not bear failure will occur)

1) The assumption Concrete is elastic is not true. Since Concrete is heterogeneous material.

2) It uses factor of safety for stresses not for loads.

i.e It fails to explain load Combinations and to access failure load.

3) It doesn't account for Shrinkage and Creep which are time dependent and plastic in nature.

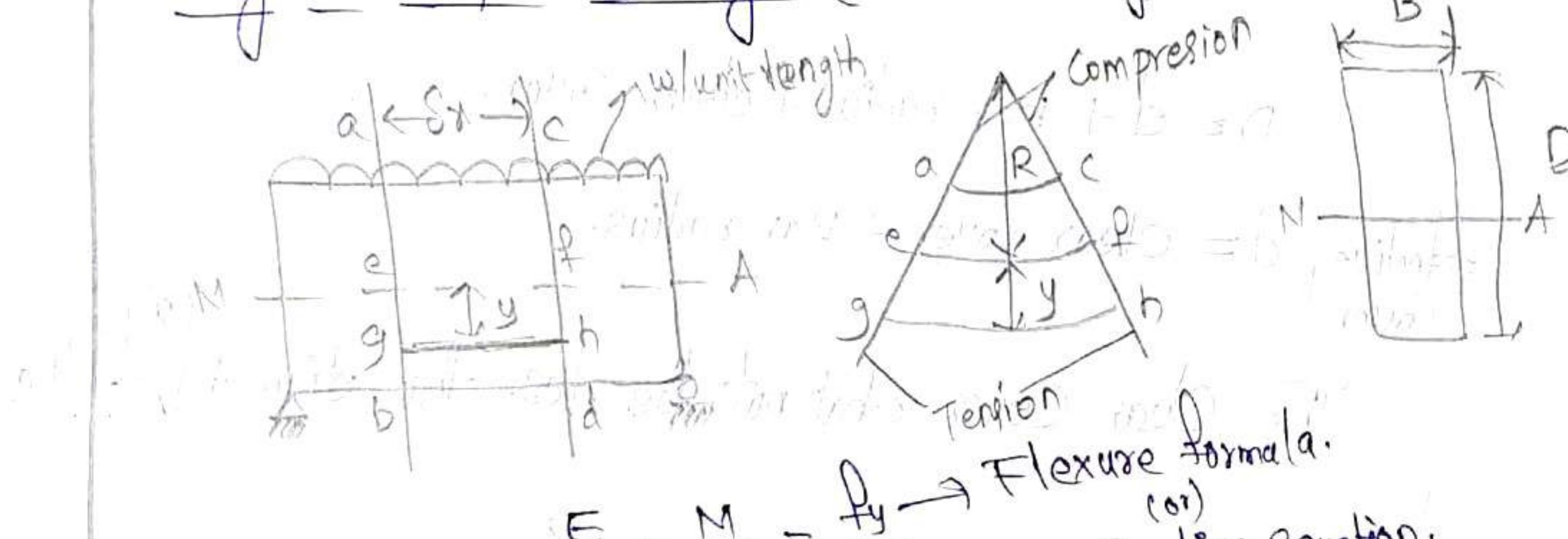
4) This method gives uneconomical (or) larger Sections.

Assume there is no change in R.C = Steel + Concrete length

Strain Steel \neq Strain Concrete

= Strain in R.C.

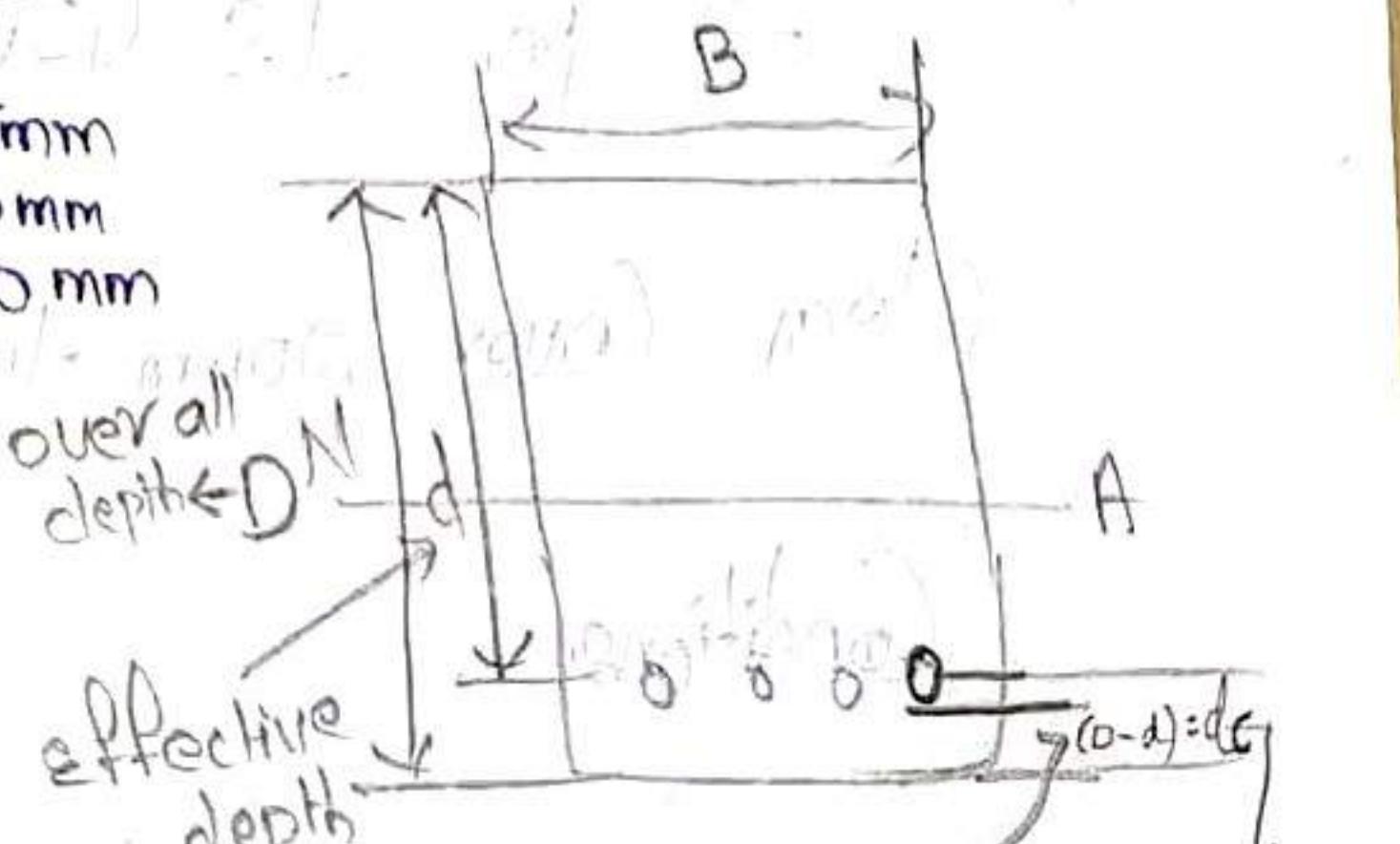
Theory of Simple Bending:- (elastic theory).



Size of Beam :-

Beam - 205mm
Column - 40mm
Footings - 50 mm

1) Size of Beam in general



depends on Architectural

requirements.

2) The Breadth of the

$$D = d + \text{Effective cover}$$

Beam is often restricted $D = d + \frac{\pi}{2} + \text{clear cover}$

by wall thickness and depth is restricted by clear space between beam and floor level.

3) In general B and D are provided as follows.

B = 115, 230, 250, 300, 350mm etc.

D = 230, 250, 300, 350, 375, 400, 450, 500, 600mm etc.

Cover to reinforcement:-

1) Cover is provided to ensure the grip of concrete over reinforcement. So, that they act as one to resist loads.

(i) To prevent reinforcement from corrosion.

$$D = d + \text{Bar radius} + \text{clear cover}$$

effective, $d = \text{clear cover} + \text{bar radius}$.

The clear cover shall not be less than diameter of the bar.

As per IS 456:2000 Table No. 16 provide a minimum

clear cover 20mm to the stirrups from the mild exposure

conditions.

∴ Clear cover to the Main Reinforcement = 20mm

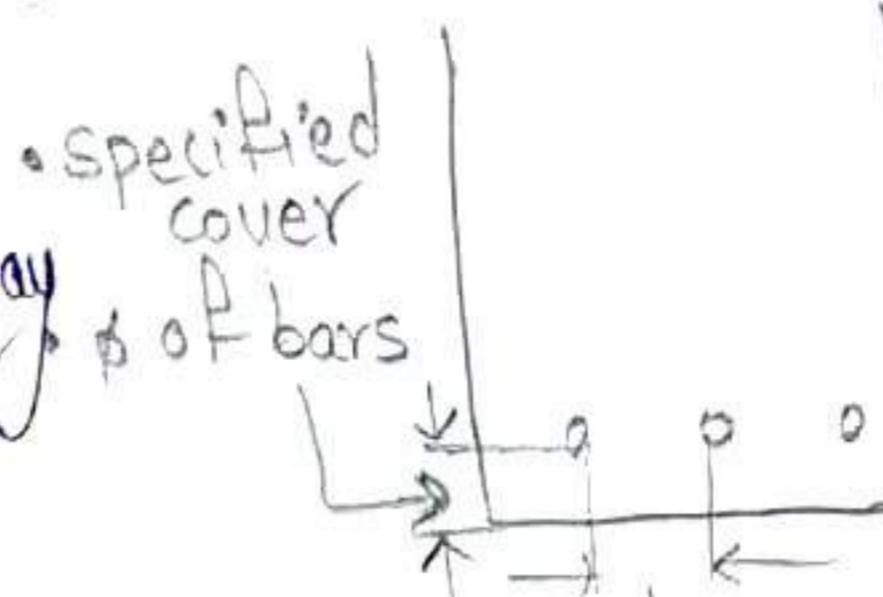
+ Stirrup diameter
= 8 (or) 10mm.
= 30mm.

Spacing of bars:-

Bars shall be placed in such a way that they allowed concrete to enter when

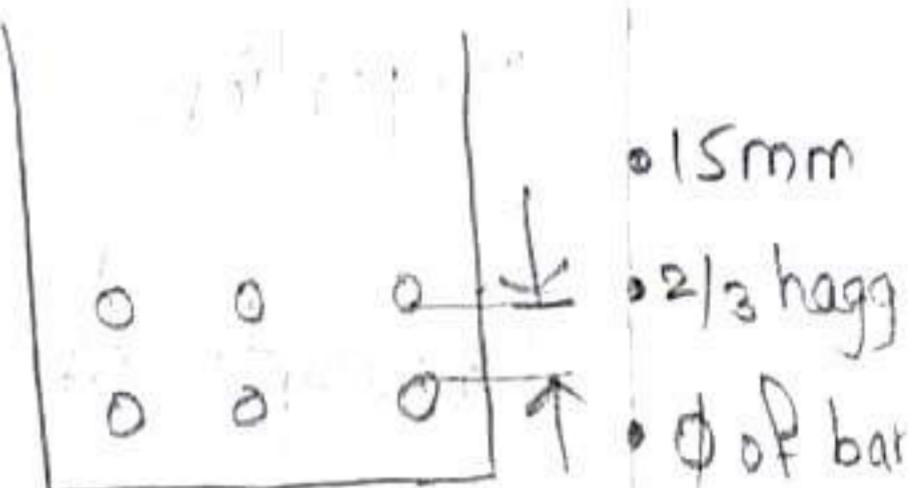
placing; (or) A vibrator can be immersed.

The minimum Spacing of bars as follows shown in fig.



Not less than (k)

$\cdot h_{agg} + 5\text{mm}$
 $\cdot d \text{ of bar}$



h_{agg} = Nominal maximum size of aggregate.

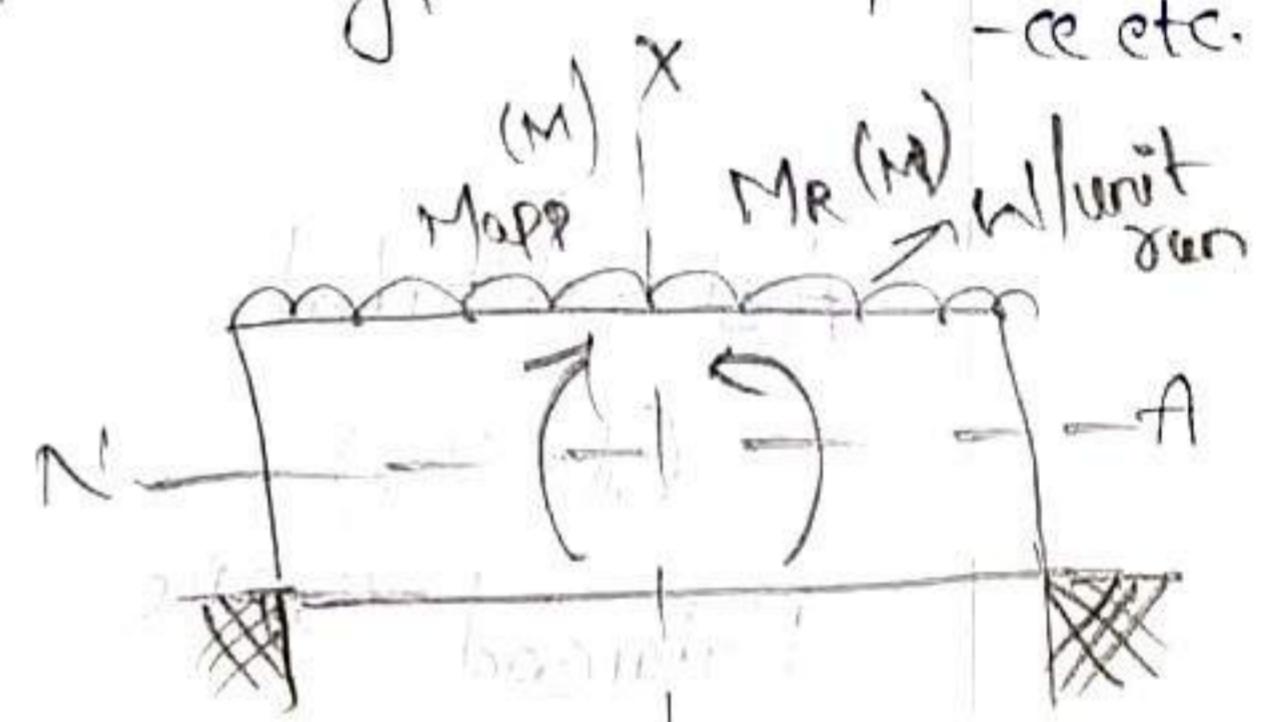
Design requirements of a Beam :-

Mainly the beam is designed for

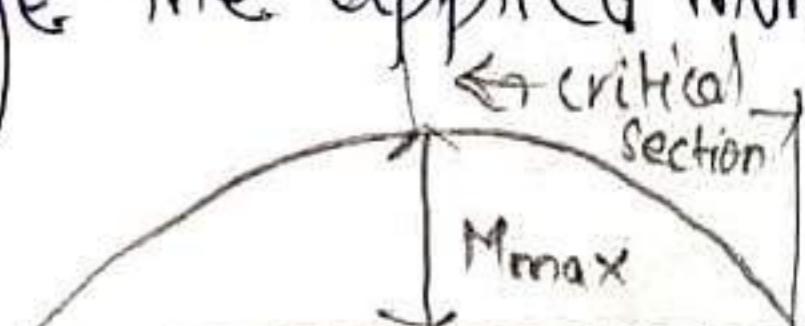
- 1) Strength \rightarrow Includes Flexure, Shear, Tension, axial force (tensile or compressive) etc.
- 2) Serviceability \rightarrow Includes Deflection, cracking, Vibrations, fire resistance etc.

Fundamental of design:-

$$M_{app} \leq M_R(M)$$



Let Consider a Simply Supported beam subjected to UDL unit run as shown in fig. As the load gradually increases the applied moment also increases. At one particular stage the applied moment becomes maximum. The



The Section which carries maximum bending

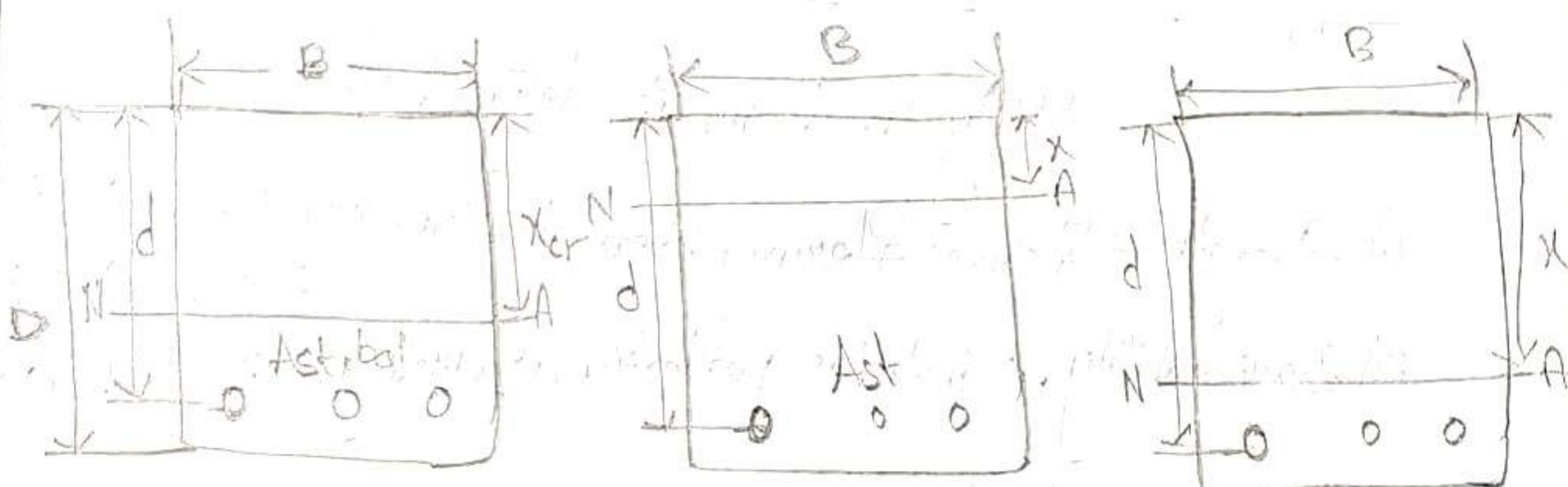
Moment is called "critical Section."

At critical section, $M_{app} \leq M_R (N)$
 ($M_{applied}$) Moment of resistance
 (or)
 Strength of section
 (or)
 Capacity of section

If the above condition is satisfied the
 Critical section remains in its position. The design
 proportions are also adequate. This is the fundamental
 concept of the design.

Types of Beams: - 3 types. Based on sections.

Balanced, Under-reinforced and Over-reinforced designs (or) Section.



$A_{st} = A_{st, bal}$
 $(x = x_{cr})$
 Balanced $n_a = n_c$

$A_{st} < A_{st, bal}$
 $(x < x_{cr})$
 under reinforced

$A_{st} > A_{st, bal}$
 $(x > x_{cr}) n_a > n_c$
 over-reinforced

Balanced design:

Tension failure.

Brittle failure
Sudden failure

In this design both Concrete and Steel reaches maximum permissible stresses. (design stress)

at the same time and the failure is called

Balanced Failure:

In this the steel placed in the section.

$$A_{st} = A_{st, bal}$$

→ gradually failure.

Under reinforced design (or) section:- Mostly used this only.

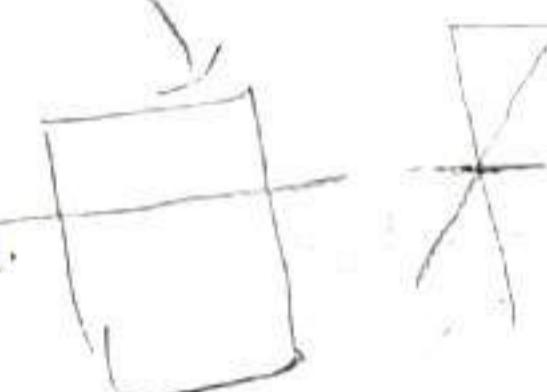
In this design the stresses in steel reaches maximum permissible stress earlier than concrete.

Compression Zone.

As $A_{st} < A_{st, bal}$ and the failure is called Tension failure.

→ Under reinforced design suffers from large deflections

and more cracks. Cracks widths.



↑ sudden failure.

Over-Reinforced design (or) section :-

In this design the concrete in compression zone reaches maximum permissible stress earlier than

Steel as $A_{st} > A_{st, bal}$.

This type of failure is called brittle failure. (or) sudden failure.

→ Normally, structures are designed as under reinforced design because it gives warning to the occupant (or) user by giving larger deflections

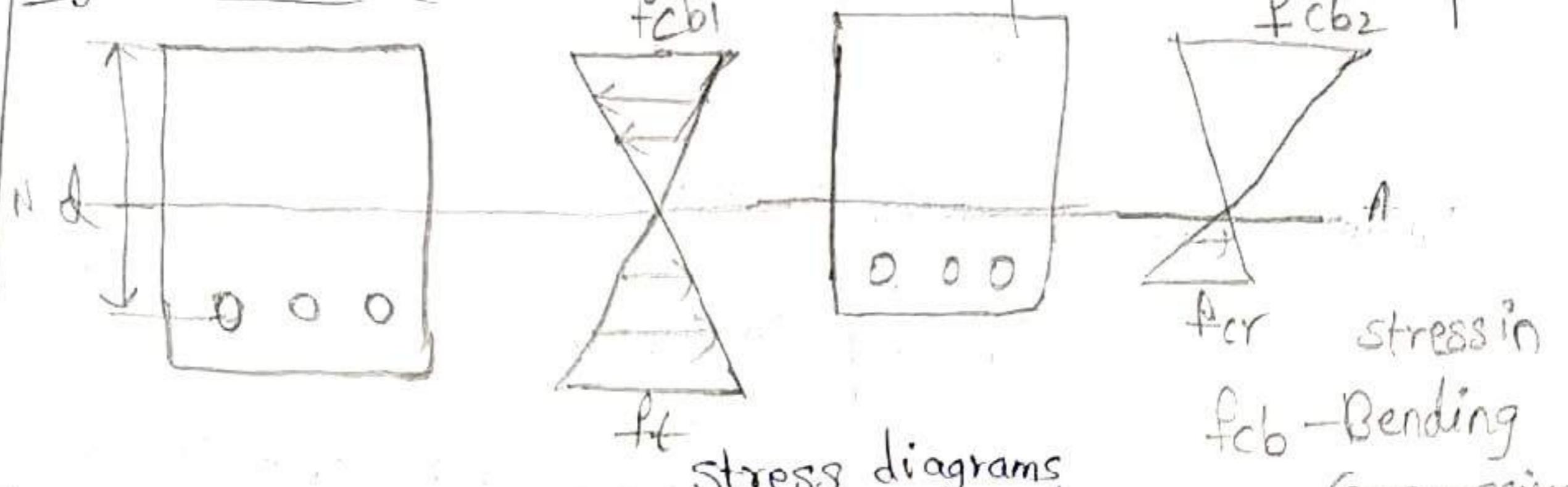
and crack widths. Unlike brittle failure of

Over reinforced design.

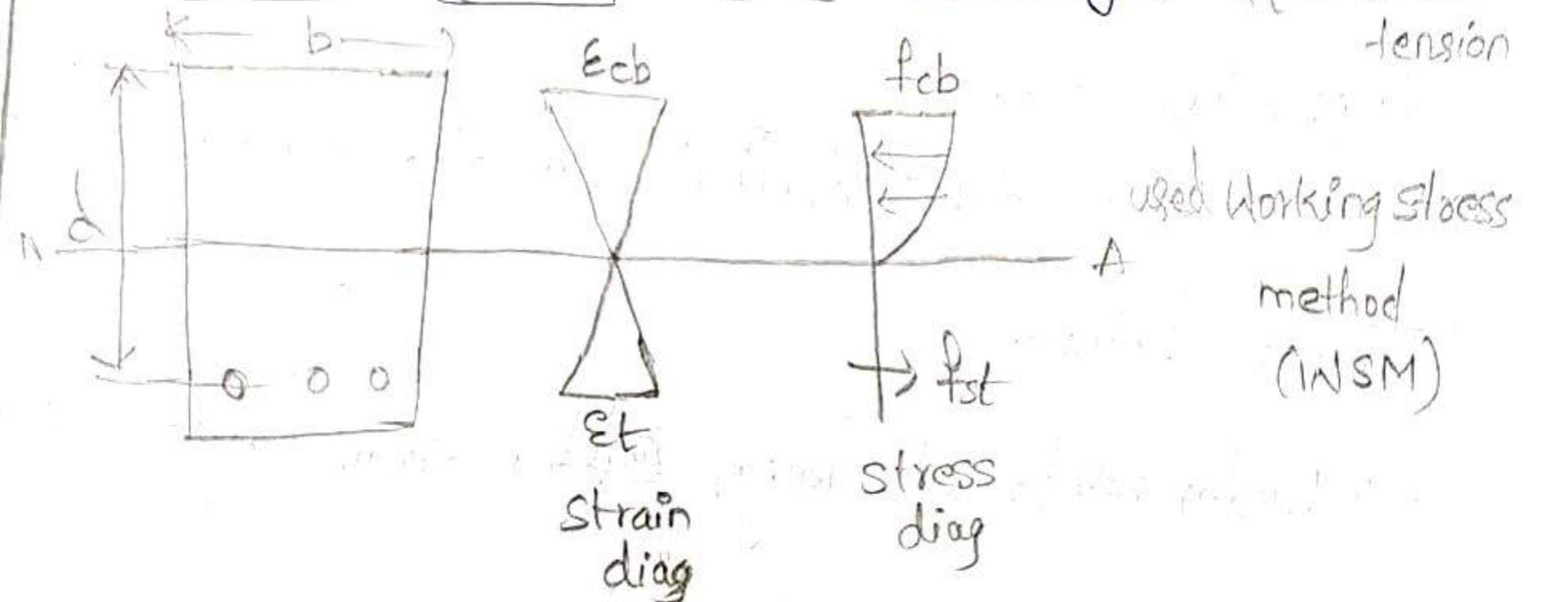
(M21) bottom

Bending of R.C.C Beam:-

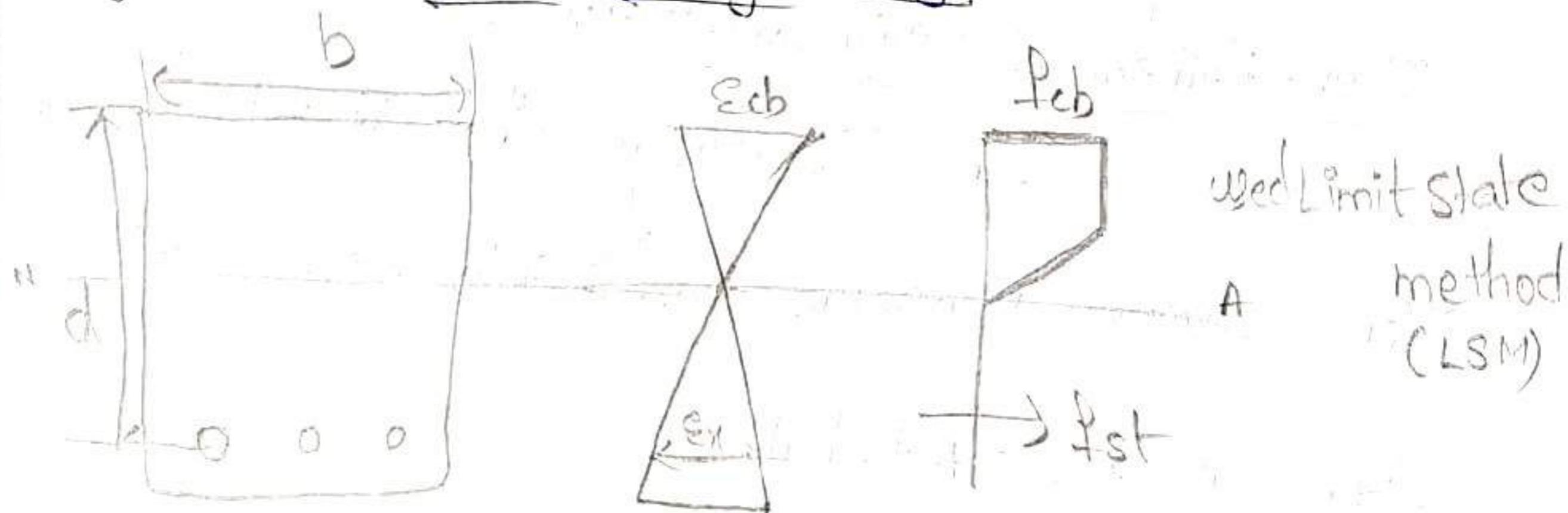
Stages Uncracked Concrete Stage:-



Stage 2 Concrete Cracked elastic stress stage



Stage 3 ultimate strength Stage



A Beam under flexure passes through above stages before failure.

Stage 2 :- Concrete Cracked elastic stage is used for designing structures in working stress Method (W.S.M).

Stage 3:- Ultimate strength stage is used to design
Limit state method (LSM)

Permissible stress:-

$$\text{Permissible stress} = \frac{\text{Strength of the material } f_{ck} \text{ (or) } f_y}{\text{Yield strength } f_{sy}} \times \frac{f_{sy}}{\text{Factor of Safety.}}$$

Permissible stress are obtained from Table No. 21 and 22 of
IS 456:2000 Code book.

Modular Ratio:- (M)

$$M = \frac{E_s}{E_c}$$

$$E_s = 2 \times 10^5 \text{ N/mm}^2$$

$$E_c = 5000 \sqrt{f_{ck}}$$

It is the ratio of Modulus of elasticity of steel to
Modulus of elasticity of concrete.

→ Static Modulus, where $E_c = 5000 \sqrt{f_{ck}}$.

Also E_c is a static Modulus of elasticity.
(Short-term modulus).

→ Long term modulus, $E_{ce} = \frac{E_c}{1+\alpha}$, where α = creep coefficient.
of elasticity has to be used to determine modular ratio and

To consider long term effects like creep.

→ IS 456 gave a value $m = \frac{280}{36 C_{bc}}$,
Code book

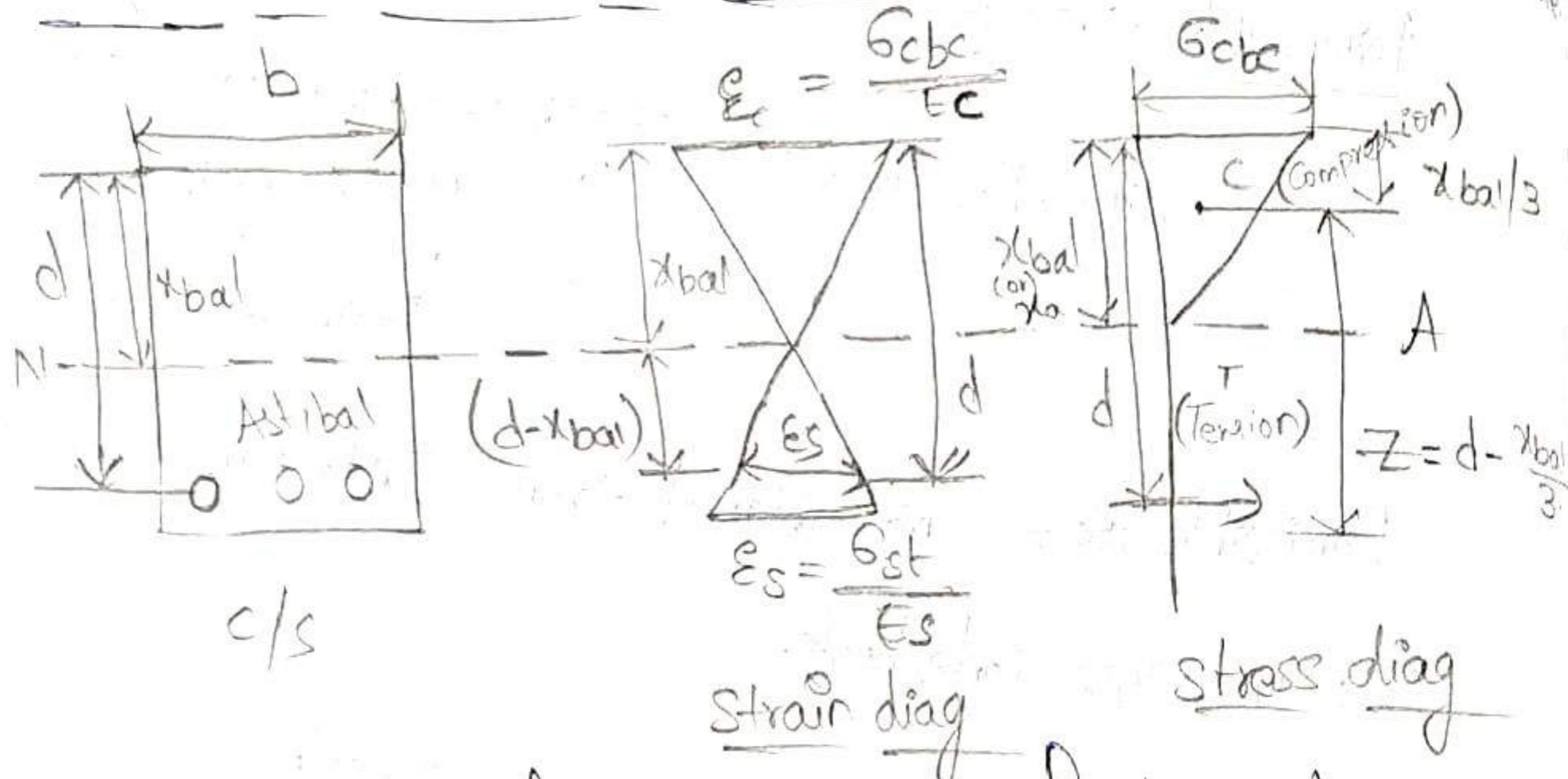
As modular ratio are considering into long term
effects.

$$\therefore M = \frac{E.S}{E.C} = \frac{280}{6c_{bc}}$$

Neutral axis:-
In a Concrete beam through axis, where stress is equal (or) zero

Neutral Axis:- An axis which is divided into Compression Zone and Tension Zone.

Derivation of formulae for balanced Section:-



Let A_{st} = Reinforcement provided for balanced section.

σ_{st} = Max. permissible stress in steel.

σ_{cb} = Max. permissible stress in Concrete.

E_s = Modulus of elasticity of steel.

E_c = Modulus of elasticity of concrete.

ϵ_c = Strain in Concrete in extreme Compression fiber = $\frac{\sigma_{cb}}{E_c}$.

ϵ_s = Strain in Steel.

$$= \frac{\sigma_{st}}{E_s} = \frac{\sigma_{st}}{M \cdot E_c}$$

d = effective depth. (The distance between the extreme Compression fiber to centroid of steel.)

x_{bal} = depth of balanced Neutral axis (distance between extreme Compression fiber to Neutral axis)

$Z = \text{Lever arm}$. [It is the distance between centroids of (T) tensile and Compressive forces]

Assumptions in WSM :- (Working stress method)

1) At any cross section plane sections remain before bending remain plane after bending.

2) All tensile stresses are taken up by reinforcement and none by Concrete.

3) The stress-strain relationship of steel and Concrete under working load is a straight line.

4) There exist a perfect bond between Steel and Concrete.

5) The molecular ratio (M) as the value $280/3\sigma_{cbc}$.

σ_{cbc} = permissible compressive stress in building compression.

$$M = \frac{280}{3\sigma_{cbc}}$$

To find NA (x_{bal}).

For linear relationship of strain diagram.

$$\frac{x_{bal}}{d-x_{bal}} = \frac{\sigma_{cbc}}{\frac{E_c}{\frac{G_{st}}{E_s}}}$$

$$= \frac{\sigma_{cbc}}{E_c} \cdot \frac{E_s}{G_{st}}$$

$$= \frac{\sigma_{cbc}}{G_{st}} \cdot \frac{E_s}{E_c}$$

$$\frac{x_{bal}}{d-x_{bal}} = \frac{\sigma_{cbc}}{G_{st}} \cdot M$$

Solving for x_{bal}

$$x_{bal} \cdot G_{st} = (d - x_{bal}) \cdot m \cdot G_{cbc}$$

$$x_{bal} \cdot G_{st} = d \cdot m G_{cbc} - m \cdot G_{cbc} \cdot x_{bal}$$

$$x_{bal} [G_{st} + m \cdot G_{cbc}] = m \cdot d \cdot G_{cbc}$$

$$x_{bal} = \frac{m \cdot d \cdot G_{cbc}}{G_{st} + m \cdot G_{cbc}}$$

$$x_{bal} = \frac{m \cdot d \cdot G_{cbc}}{m \cdot G_{cbc} \left(\frac{G_{st}}{m \cdot G_{cbc}} + 1 \right)}$$

$$x_{bal} = \frac{d}{\frac{G_{st}}{m \cdot G_{cbc}} + 1}$$

$$x_{bal} = \left(\frac{1}{1 + \frac{G_{st}}{m \cdot G_{cbc}}} \right) \cdot d$$

$$\therefore x_{bal} = k \cdot d$$

(or)

Where.

$$\therefore k = \frac{1}{1 + \frac{G_{st}}{m \cdot G_{cbc}}} = \text{NA Constant.}$$

$$x_{bal} = \frac{d}{1 + \frac{G_{st}}{m \cdot G_{cbc}}} \quad m = \frac{280}{3 G_{cbc}}$$

$$x_{bal} = \frac{d}{1 + \frac{G_{st}}{\frac{G_{cbc} \times 280}{3 G_{cbc}}}}$$

$$x_{bal} = \frac{d}{1 + \frac{G_{st} \times 3}{280}}$$

$$x_{bal} = \frac{d}{1 + 0.0107 G_{st}}$$

To find lever arm (Z):-

$$Z = d - \frac{2b_{bal}}{3}$$

Substituting $2b_{bal} = k \cdot d$.

$$Z = d - \frac{k \cdot d}{3}$$

$$Z = d \left(1 - \frac{k}{3}\right)$$

$$Z = d \cdot j$$

Where $j = \left(1 - \frac{k}{3}\right)$ = Lever arm constant.

Total Compressive force (C) = $\frac{1}{2} \times (G_{cbc} \times Z_{bal}) \times b$.

Total strain Tensile force (T) = $G_{st} \times A_{st} \times b_{bal}$.

To find (MR) Moment of resistance:-

$$MR = T \times Z \text{ or } C \times Z$$

MR = $M_{bal} = C \times Z \rightarrow$ with respect to Compression.

$$= \frac{1}{2} \times G_{cbc} \times Z_{bal} \times b \times j \cdot d$$

$$= \frac{1}{2} \times G_{cbc} \times k \cdot d \cdot b \cdot j \cdot d$$

$$= \frac{1}{2} G_{cbc} \times k \times j \cdot b \cdot d^2$$

$$= Q_{bal} \times b \cdot d^2$$

where $Q_{bal} = \frac{1}{2} G_{cbc} \times k \cdot j$

MR = $M_{bal} = T \times Z \rightarrow$ with respect to Tension

$$= G_{st} \times A_{st} \times j \cdot d$$

Types of Beams:-

- 1.) Balanced or critical section.
- 2.) Under reinforced section.
- 3.) Over reinforced section.

1.) Balanced (or) Critical Section:-

The characteristic property of a reinforced Concrete member is that its Components namely Concrete & steel act together as a single unit.

Elastic theory:- Components act together as a single unit as long as they remain in the elastic condition i.e. the two components are bound together.

$$R.C = \text{Concrete} + \text{Steel} \rightarrow \text{so that can be no relative displacement}$$

When subjected to loads, b/w them.

Assume that there is no change in length.

Strain in steel = strain in concrete = strain in R.C member.

$$\frac{w_{se}}{A_{se} E_s} \rightarrow ① \quad \frac{w_c}{A_c E_c} = \frac{w_c + w_{se}}{A_{se} E_s + A_c E_c} \rightarrow ② \quad \frac{w_c + w_{se}}{A_{se} E_s + A_c E_c} \rightarrow ③$$

By equating ① & ②.

m = modular ratio.

$$\frac{w_s}{A_{se} E_s} = \frac{w_c}{E_c \cdot A_c}$$

$$\frac{f_{sc}}{E_s} = \frac{f_c}{E_c}$$

$$f_{sc} = \frac{E_s}{E_c} \cdot f_c$$

$$f_{sc} = m \cdot f_c$$

$$\frac{w_c}{E_c} = f_c \cdot \text{stress}$$

$$\frac{E_s}{E_c} = m$$

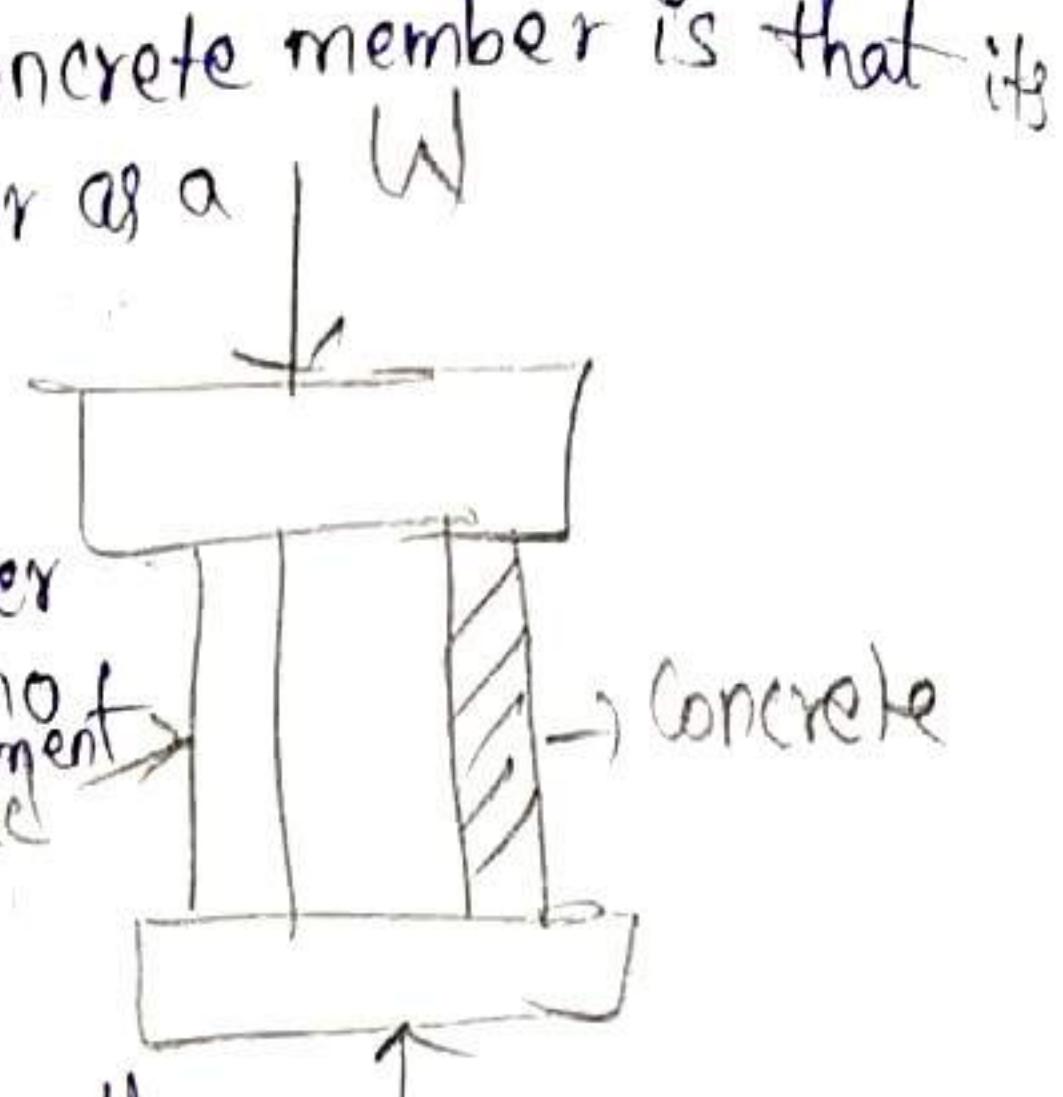
Calculate the load

By comparing ① & ③.

$$\frac{w_s}{A_{se} E_s} = \frac{w_c + w_{se}}{A_{se} E_s + A_c E_c}$$

w_s - load in steel.
 w_c - load in concrete.
 E_c - young's modulus of concrete
 E_s - young's modulus of steel

A_c - Area of concrete.
 A_{se} - Area of steel in concrete.
 f_{sc} - stress in steel concrete.
 f_c - stress in concrete.



$$\frac{W_s}{A_{sc} \cdot E_s} = \frac{W}{A_c E_c + A_{sc} E_s}$$

$$W_s = \left[\frac{W}{A_c E_c + A_{sc} E_s} \right] A_{sc} \cdot E_s$$

By dividing it with E_c

$$W_s = \frac{W}{A_c \frac{E_c}{E_c} + A_{sc} \frac{E_s}{E_c}} \cdot \frac{A_{sc} \cdot E_s}{E_c}$$

$$\frac{E_s}{E_c} = m$$

$$W_s = \frac{m w A_{sc}}{A_c + m A_{sc}}$$

steel will bear more load when

Compare to concrete.

eg ② & ③.

$$W_c = \frac{w \cdot A_c}{m \cdot A_{sc} + A_c}$$

$$W = W_c + W_s$$

$$\frac{W_c}{A_c \cdot E_c} = \frac{W_c + W_s}{A_{sc} \cdot E_s + A_c \cdot E_c}$$

Ans-

$$\frac{W_c}{A_c \cdot E_c} = \frac{W}{A_{sc} \cdot E_s + A_c \cdot E_c}$$

$$W_c = \left[\frac{W}{A_{sc} \cdot E_s + A_c \cdot E_c} \right] A_c \cdot E_c$$

divide with E_c

$$W_c = \frac{W}{A_{sc} \frac{E_s}{E_c} + A_c \frac{E_c}{E_c}} \cdot \frac{A_c \cdot E_c}{E_c}$$

$$W_c = \frac{W \cdot A_c}{A_{sc} \cdot m + A_c}$$

$$W_c = \frac{W \cdot A_c}{m \cdot A_{sc} + A_c}$$

Equivalent Concrete area (or) transformed Concrete area:-

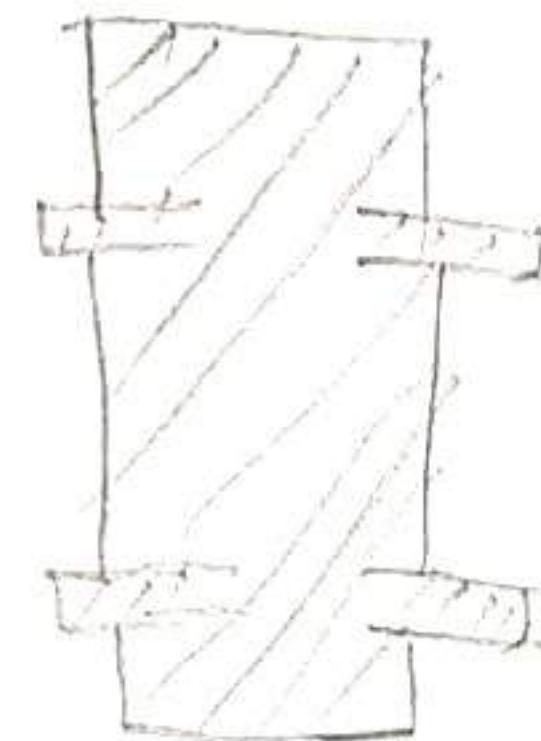
* The strains are equal $[f_{sc} = m f_c]$.

Load on Steel + Load on Concrete = Total load.

$$f_{sc} \cdot A_{sc} + f_c \cdot A_c = W.$$

$$m f_c \cdot A_{sc} + f_c \cdot A_c = W.$$

$$f_c [m \cdot A_{sc} + A_c] = W.$$



$$f_c = \frac{W}{(m A_{sc} + A_c)}$$

, Equivalent Concrete area. (A_e)

- 1.) A Concrete Beam of size 300x300mm Subjected to an a Compressive load of 440kN and it is reinforced with 125 t mm^2 of Steel Supports . Calculate the stress in Concrete and steel take $m = 13.33$.

Given that :-

Ques:

Size of Concrete Beam, $A_c = 300 \times 300 \text{ mm}^2$.

Compressive load, $W = 440 \text{ kN}$

reinforced , $A_{sc} = 125 \text{ t mm}^2$

$$m = 13.33.$$

$$f_s, f_{sc} = ?$$

$$A_e = m A_{sc} + A_c$$

$$= 13.33 \times 125 + 300 \times 300$$

$$A_e = 106.75 \times 10^3 \text{ mm}^2$$

$$f_c = \frac{440 \times 10^3}{106.75 \times 10^3}$$

$$f_c = 4.12 \text{ N/mm}^2$$

$$\therefore f_{st} = \frac{100 \text{ Ast}}{\text{bd.}}$$

$$f_{sc} = m \cdot f_c$$

$$f_{sc} = m \cdot f_c = 13.33 \times 4.12$$

$$f_{sc} = 54.9 \text{ N/mm}^2$$

Neutral axis:- To calculate depth of N.A

Case i:- when we know the stress in the R.C. Beam.

n = Neutral axis depth.

σ_{st} = permissible tension stress.

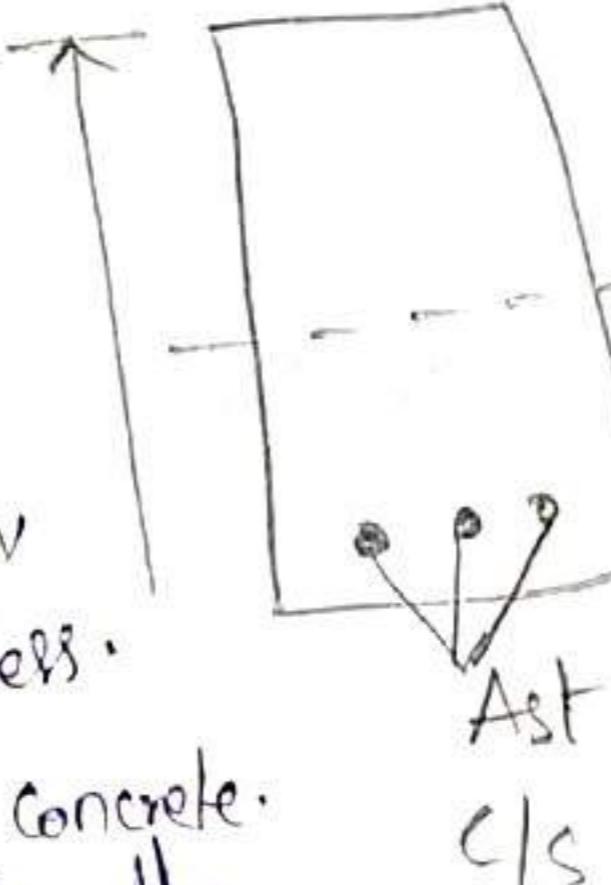
σ_{cbc} = permissible compression stress.

E_{sc} = Modulus of elasticity of steel.

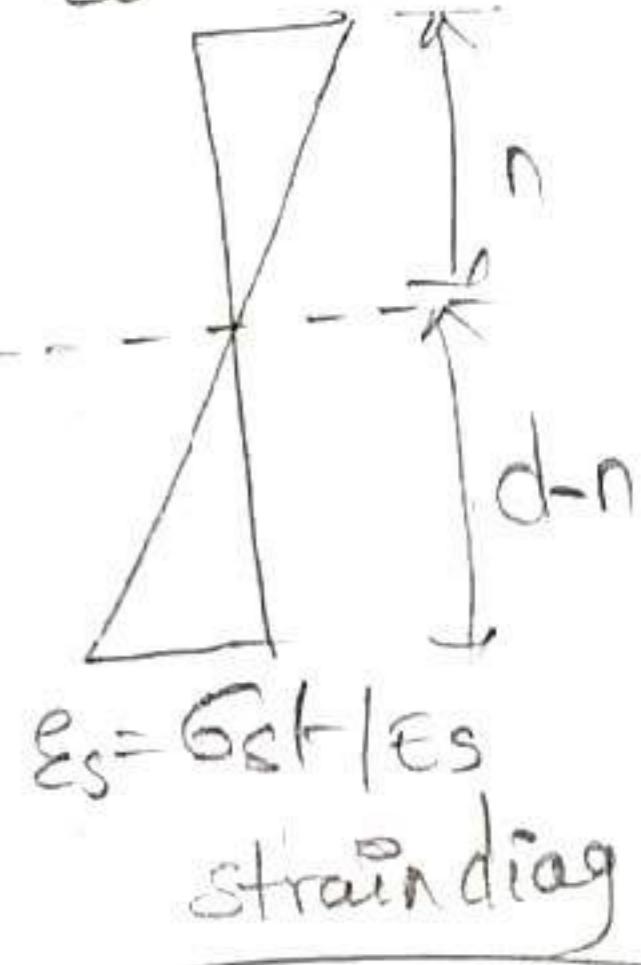
E_c = Modulus of elasticity of concrete.

for cal. of N.A depth, taking the

strains along the depth of N.A.



$$\epsilon_c = \sigma_{cbc}/E_c$$



$$\epsilon_s = \sigma_{st}/E_s$$

$$\frac{\epsilon_c}{\epsilon_{cs}} = \frac{n}{d-n}$$

$$\frac{\sigma_{cbc}}{\sigma_{st}/m} = \frac{n}{d-n}$$

$$\sigma_{sc} = m \cdot f_c$$

$$f_s = m \cdot f_c$$

$$\frac{m \cdot \sigma_{cbc}}{\sigma_{st}} = \frac{n}{d-n}$$

$$\frac{\sigma_{st}}{m \cdot \sigma_{cbc}} = \frac{d-n}{n}$$

$$\frac{\sigma_{st}}{m \cdot \sigma_{cbc}} = \frac{d}{n} - 1$$

$$\frac{d}{n} = 1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}$$

$$n = \left[\frac{1}{1 + \frac{\sigma_{st}}{m \cdot f_{cbc}}} \right] d.$$

Neutral axis depth factor.

Case ii:- When we know the dimensions in the R.C Beam,

→ By taking the internal forces of Concrete and Steel.

$$\text{Force on Compression} = \sigma_{cbc} \times (y_2 \times b \times n)$$

$$= \frac{\sigma_{cbc}}{2} \times b \times n \quad \text{①}$$

$$\text{Force on tension} = \sigma_{st} \times A_{st}$$

$$= (m \cdot f_{st}) \left(\frac{d-n}{n} \right) A_{st} \quad \text{②}$$

By eq. ① & ②.

$$\frac{\sigma_{cbc}}{2} \times b \times n = (m \cdot f_{cbc}) \left(\frac{d-n}{n} \right) A_{st}$$

$$\frac{bn}{2} = m \cdot A_{st} \left(\frac{d-n}{n} \right)$$

$$\boxed{\frac{bn^2}{2} = m \cdot A_{st} (d-n).}$$

$$\frac{bn^2}{2} = m \cdot A_{st} \cdot d - m \cdot A_{st} \cdot n$$

$$\frac{bn^2}{2} + n \cdot m \cdot A_{st} - m \cdot d \cdot A_{st} = 0.$$

$$\frac{b}{8} n^2 + (A_{st} \cdot m) n - m \cdot d \cdot A_{st} = 0.$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$-m \cdot Ast \pm \sqrt{(m \cdot Ast)^2 - 4x(b/2)(-md \cdot Ast)}$$

$$n = \frac{-m \cdot Ast \pm \sqrt{(m \cdot Ast)^2 - 4x(b/2)(-md \cdot Ast)}}{2x(b/2)}$$

$$n = \frac{-m \cdot Ast \pm \sqrt{(m \cdot Ast)^2 + 2bm \cdot d \cdot Ast}}{b}$$

Lever arm :- (for distance)

It is the distance between the centroid of Compressive and tensile stresses.

$$d - n/3$$

$$n = k \cdot d$$

$$= d - \frac{k \cdot d}{3}$$

$$= d \left(1 - \frac{k}{3} \right)$$

J = lever arm constant.

$$Z = d \cdot J$$

Moment of resistance :-

$M \cdot R$ = Compressive load \times Lever arm.

$$= \frac{6cbc}{2} \times b \times n \times dx \cdot j$$

$$= \frac{6cbc}{2} \times b \times k \times dx \times dx \cdot j$$

$$= \frac{6cbc}{2} \times k \times b d^2 \cdot j$$

$$M \cdot R = Q \cdot b d^2$$

$$Q = \frac{6cbc}{2} \times k \cdot j$$

M.O.R Constant.

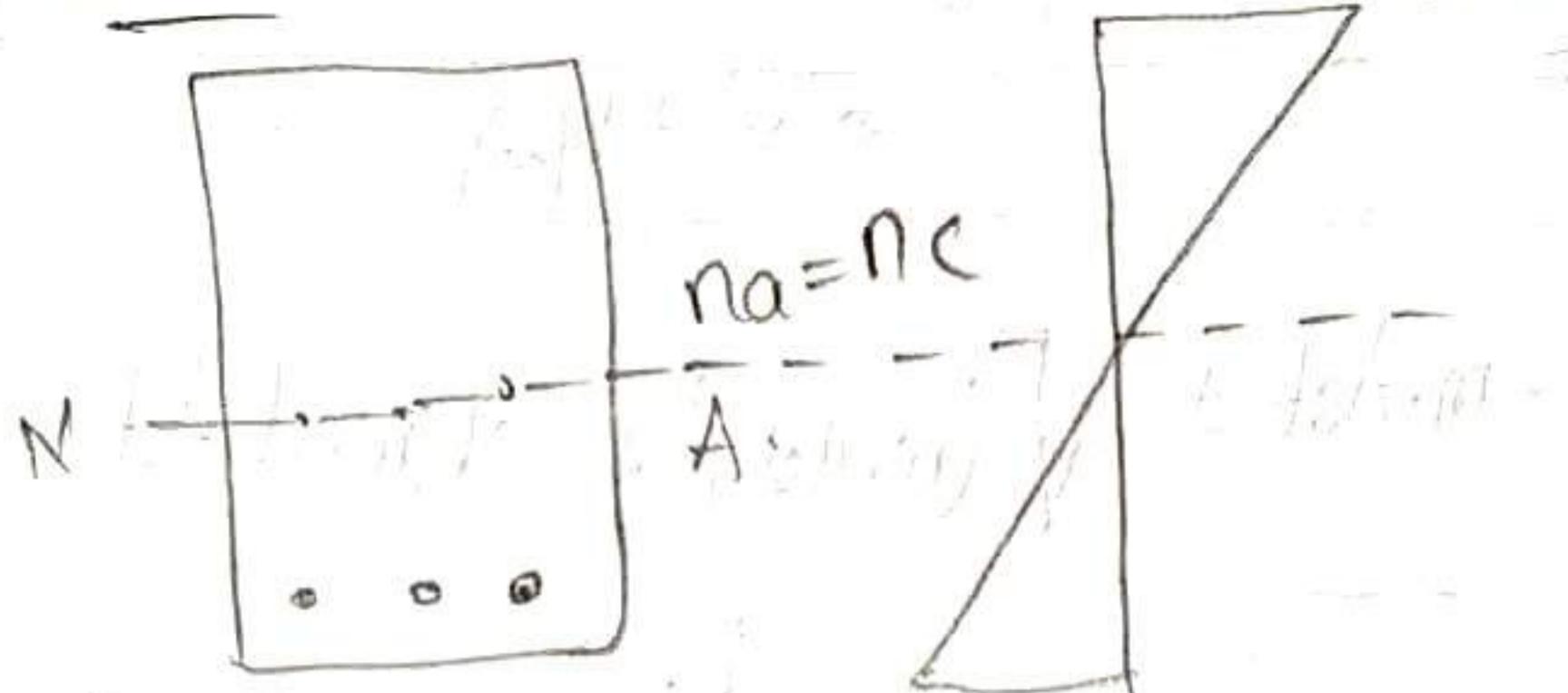
M.O.R = Tension load \times Lever arm

$$= Q_{st} \times Ast \times d \cdot J$$

$$\frac{Q_{st} \times Ast}{Q_{st} \times J \times d} = \frac{M}{J \times d}$$

Types of sections:-

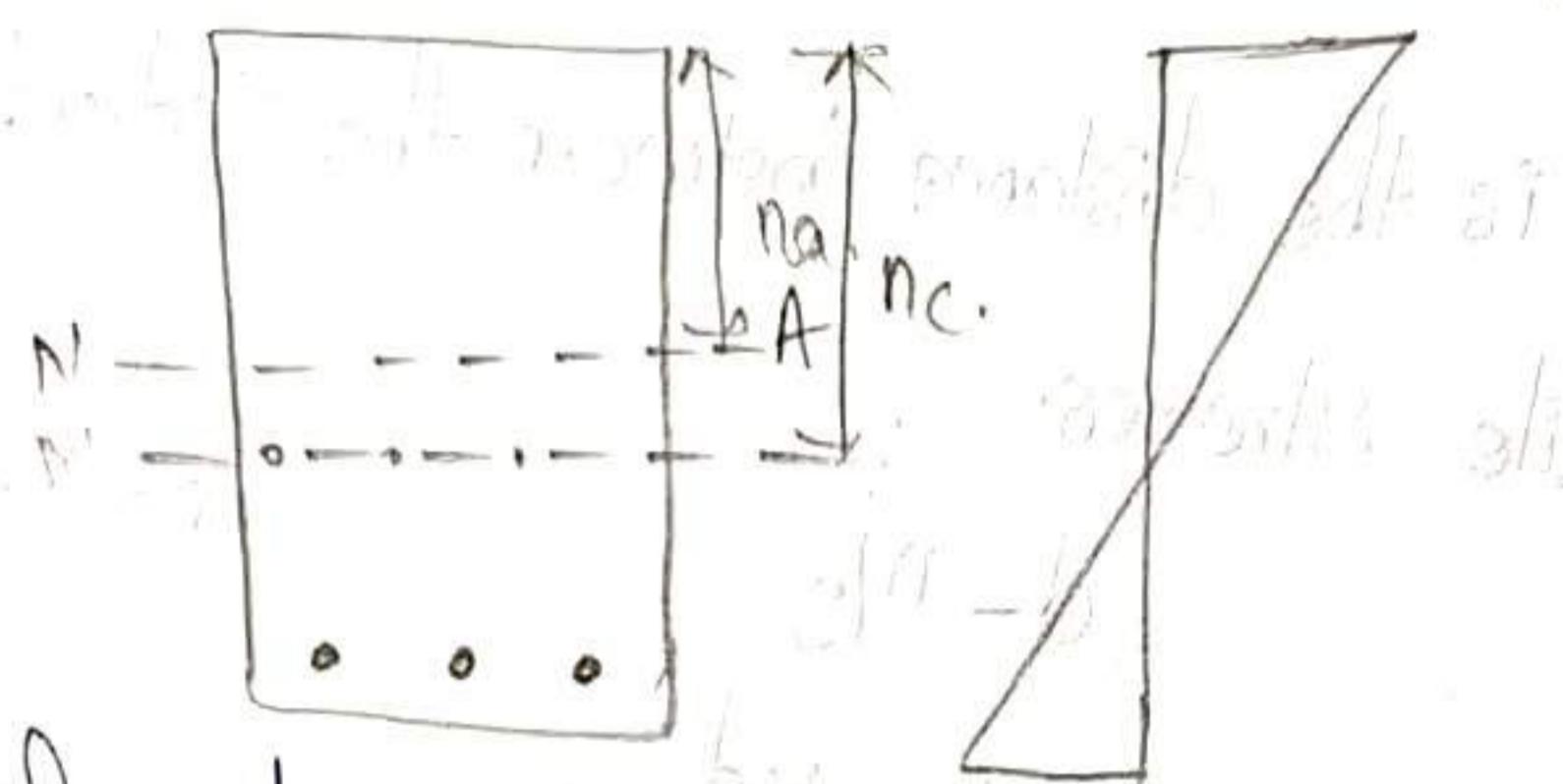
1) Balanced section:



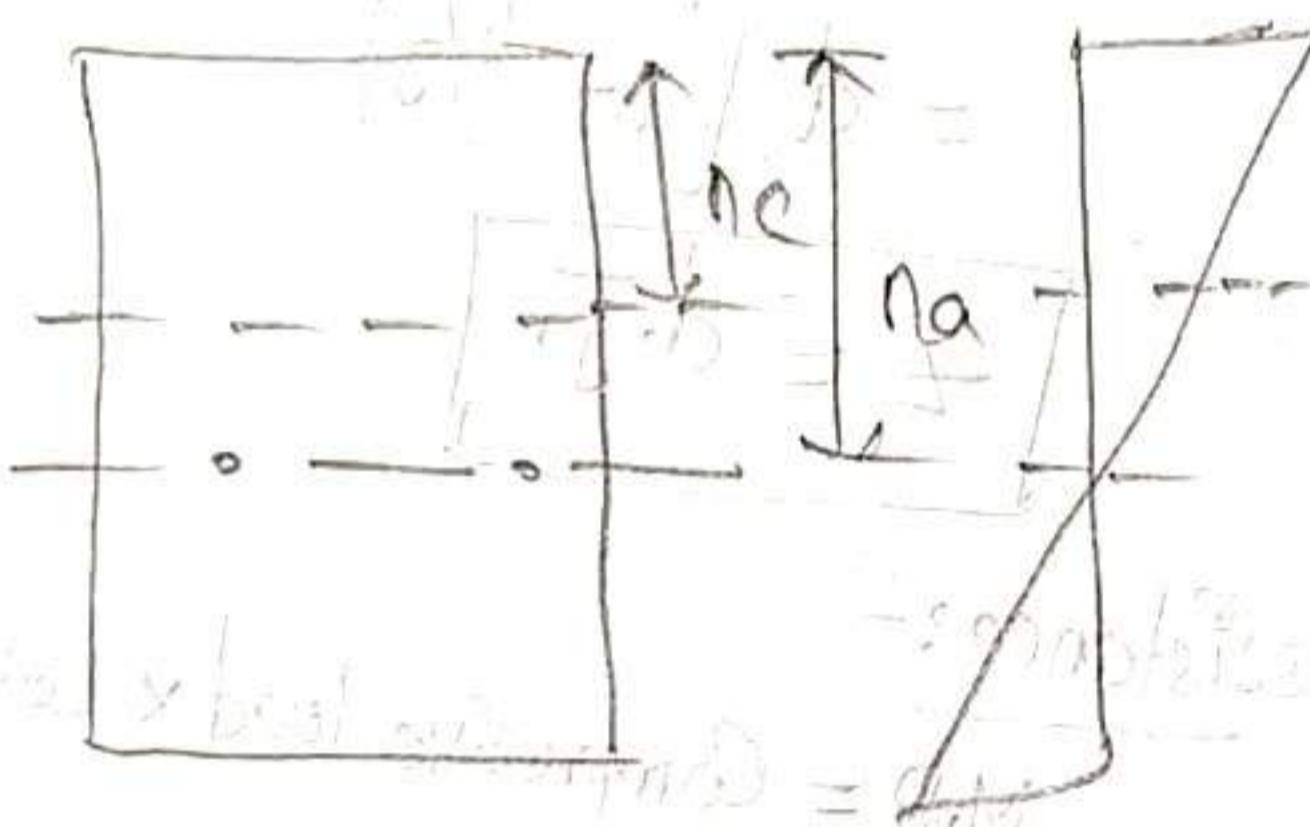
Actual Neutral
axis = C.P.
Neutral axis

2) Under Reinforced section

Mostly used.



3) Over-reinforced section :- $n_a < n_c$.



$n_a > n_c$.

① Consider M_{ao} grade of Concrete and Fe415 Steel find $\alpha_{c1} J_{c1}$, k_c and P_c (Percentage of Concrete).

Q:- Given that ~~M_{ao}~~ grade of Concrete, $f_{ck} = 20 \text{ N/mm}^2$

Fbd Fe415 Steel

Grade of Steel, $f_y = 415 \text{ N/mm}^2$.

Find $\alpha_{c1} J_{c1}$, k_c and P_c .

$$K_c = \frac{1}{1 + \frac{6 G_{cbc}}{m \cdot G_{cbc}}}$$

$$m = \frac{280}{3 G_{cbc}}$$

(Pg No. 80
IS 456 Code)

From table 21, 22 $\sigma_{cbc} \propto f_{st}$.

$$\sigma_{cbc} = \pm N/mm^2 \quad f_{st} = 230 N/mm^2$$

$$m = \frac{280}{3 \times 7}$$

$$m = 13.33$$

$$K_c = \frac{1}{1 + \frac{230}{13.33 \times 7}}$$

$$K_c = 0.289$$

$$K_c = \frac{1}{1 + \frac{f_{st}}{m \cdot \sigma_{cbc}}}$$

$$J_c = \left(1 - \frac{K_c}{3}\right)$$

$$J_c = \left[1 - \frac{0.289}{3}\right]$$

$$J_c = 0.9036$$

$$J_c = 0.904$$

$$\theta_c = \frac{\sigma_{cbc}}{2} \times K_c \times J_c$$

$$= \pm \frac{N}{2} \times 0.289 \times 0.904$$

$$\theta_c = 0.914 \text{ steel} = \text{Concrete}$$

$$G_{st} A_{st} = \frac{\sigma_{cbc}}{2} \times b \times n$$

$$A_{st} = \frac{\sigma_{cbc} \times b \times n}{G_{st}}$$

$$J.P_{st} = \frac{100 A_{st}}{bd}$$

$$P_c = 100 \times \frac{\frac{\sigma_{cbc}}{2} \times b \times n}{bd \cdot G_{st}}$$

$$P_c = \frac{100 \times 7 \times 0.289}{2 \times 230} \times \frac{2 \times G_{st} \times \theta_c}{\theta_c}$$

$$P_c = \frac{100 \times 7 \times 0.289}{2 \times 230} \boxed{P_c = 0.43}$$

$$P_c = 0.48 \times 100$$

$$P_c = 48 \text{ kN}$$

2) For M₂₀ grade of Concrete $G_{cbc} = 7 \text{ N/mm}^2$ and calculate k_c , Q_c and J_c .

$$\text{M}_20 \text{ grade of steel} = 140 \text{ N/mm}^2$$

$$\text{M}_20 \text{ grade of concrete} G_{cbc} = 7 \text{ N/mm}^2$$

Sol:-

$$K_c = \frac{1}{1 + \frac{G_{cbc}}{G_{st}}}$$

$$m = \frac{280}{3 G_{cbc}} = 7$$

$$m = \frac{280}{3 \times 7} = 7$$

$$m = 13.33$$

$$K_c = \frac{1}{1 + \frac{140}{13.33 \times 7}}$$

$$K_c = 0.399$$

$$J_c = \left[1 - \frac{K_c}{3} \right]$$

$$J_c = \left[1 - \frac{0.399}{3} \right] = 0.867$$

$$J_c = 0.867$$

$$Q_c = \frac{G_{cbc}}{2} \times K_c \times J_c$$

$$Q_c = \frac{7}{2} \times 0.399 \times 0.867$$

$$Q_c = 1.910$$

3) For M₂₅ grade of concrete, if $f_{st} = 140 \text{ N/mm}^2$, calculate Ω_c , K_c , J_c .

Sol:-

$$f_{cbc} = 8.5 \text{ N/mm}^2$$

$$f_{st} = 140 \text{ N/mm}^2$$

$$K_c = \frac{1}{1 + \frac{f_{st}}{m \cdot f_{cbc}}}$$

$$m = \frac{280}{3 \cdot f_{cbc}} = \frac{280}{3 \times 8.5}$$

$$m = 10.980$$

$$K_c = \frac{1}{1 + \frac{140}{10.980 \times 8.5}}$$

$$K_c = 0.39$$

$$J_c = \left[1 - \frac{K_c}{3} \right] = \left[1 - \frac{0.399}{3} \right]$$

$$J_c = 0.867$$

$$\Omega_c = \frac{f_{cbc}}{2} \times K_c \times J_c$$

$$\Omega_c = \frac{8.5}{2} \times 0.399 \times 0.867$$

$$\Omega_c = 1.470$$

<u>Grade of concrete :-</u>	<u>and Steel</u>	K_c	J_c	Ω_c
M ₂₀ , $f_c = 15 \text{ N/mm}^2$	$f_{st} = 415 \text{ N/mm}^2$	0.289	0.904	0.914
M ₂₀ , $f_{st} = 140 \text{ N/mm}^2$		0.399	0.867	1.210
M ₂₅ , $f_{st} = 140 \text{ N/mm}^2$		0.39	0.867	1.470

- 4) A reinforced Concrete beam 250mm x 475mm overall depth is reinforced with 3 Bars of 16mm dia at an effective cover of 50mm use M₂₀ grade of Concrete and Fe415 Steel. Find the depth of the Neutral axis.

Given that

Size of R.c Beam = 250mm x 475mm.

dia of bars = 16mm.

No. of Bars = 3

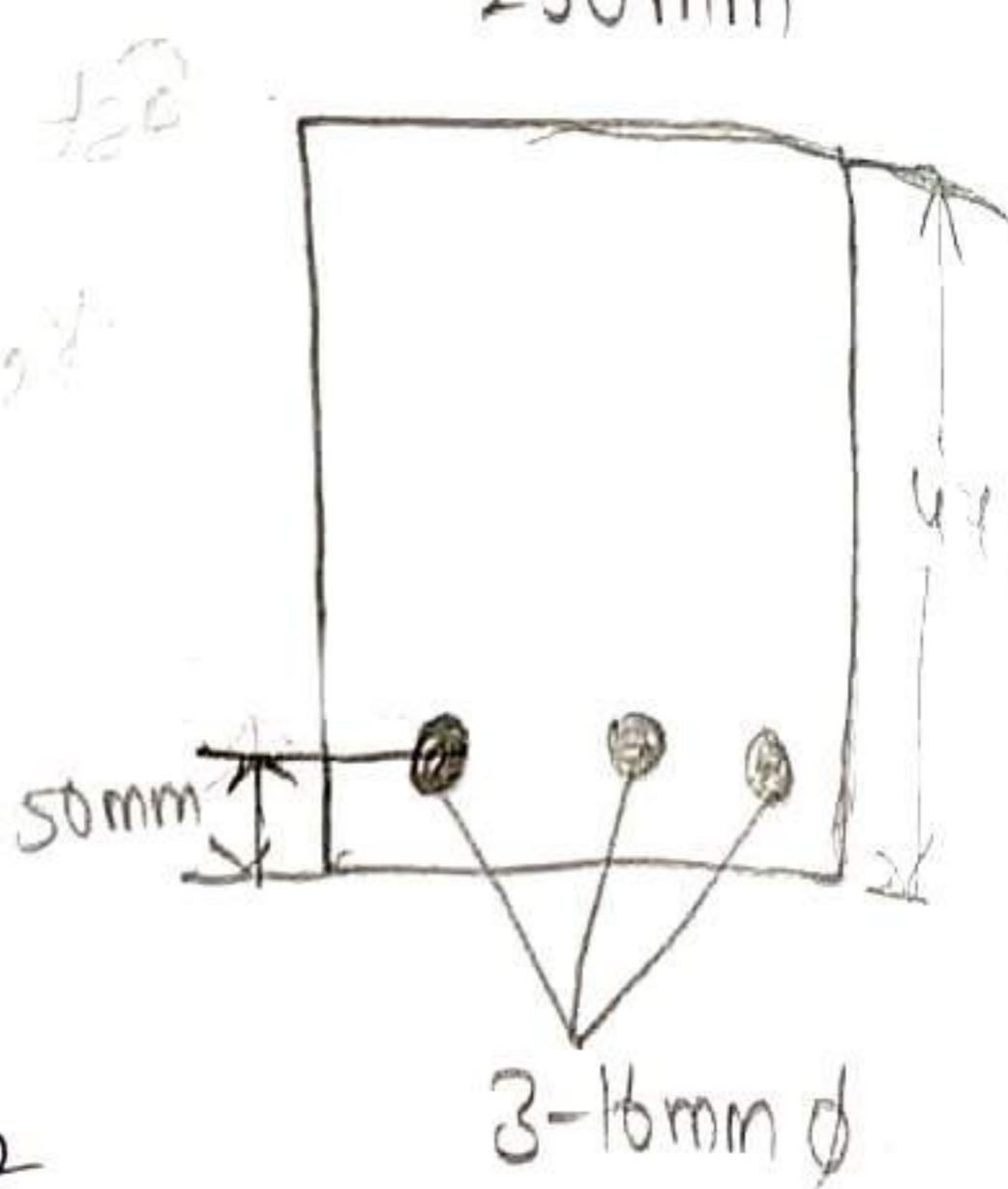
Effective Covere = 50mm

Grade of Concrete, $f_{ck} = 20 \text{ N/mm}^2$.

Grade of Steel, $f_y = 415 \text{ N/mm}^2$.

$$A_{st} = n \times \pi / 4 d^2$$

$$\text{Area of steel, } A_{st} \quad \text{Area} = 3 \times \pi / 4 (16)^2 = 603.185 \text{ mm}^2$$



$$d = D - e$$

$$d = 475 - 50$$

$$d = 425 \text{ mm}$$

$$m = \frac{280}{3.6 c_{bc}}$$

$$c_{bc} = 7 \text{ N/mm}^2$$

$$= \frac{280}{3 \times 7}$$

$$m = 13.33$$

$$\frac{b n^2}{2} = m \cdot A_{st} (d - n)$$

$$\frac{250 \times n^2}{2} = 13.33 \times 603.185 (425 - n)$$

$$125n^2 + 8040.456n - 341493.8 = 0$$

$$n = 136.27 \text{ mm}$$

$$n_c = K_c \cdot d$$

$$n_c = 0.289 \times 425$$

$$n_c = 129.82$$

$n_a > n_c$. Over reinforced section

Install stiffener bar at 199.2 mm

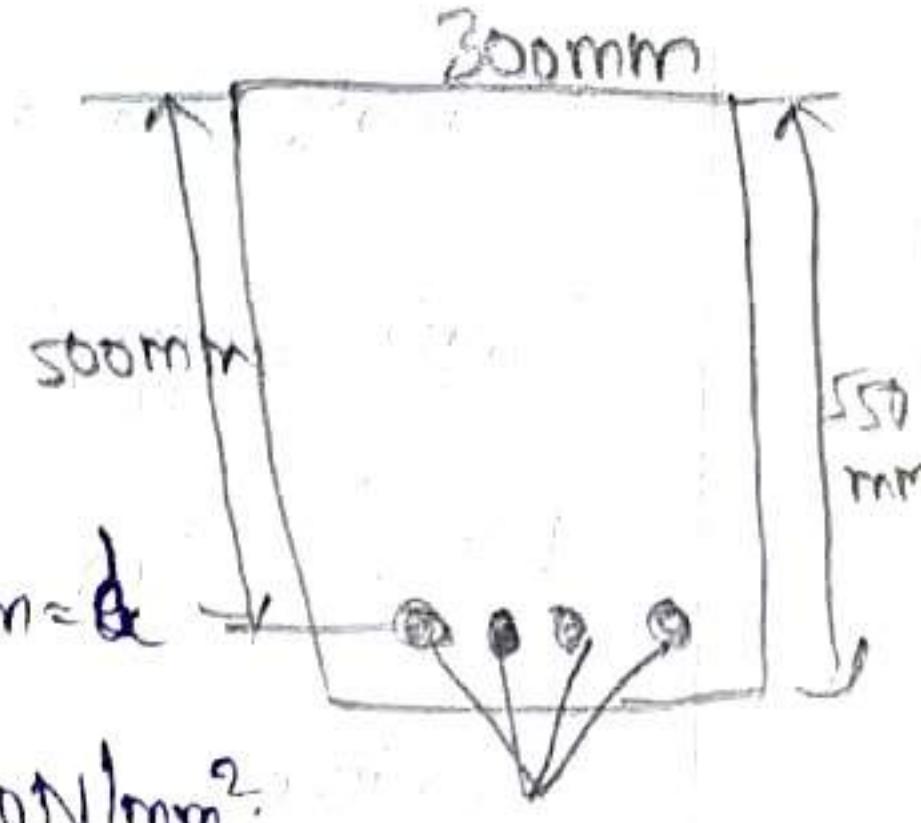
5) A reinforced Concrete Beam 300mm x 550mm overall depth is reinforced with 4 bars of 20mm diameter at an effective depth 500mm, use M20 grade of concrete and Fe415 steel. Calculate the moment of resistance.

Given that, Size of R.C. Beam = 300mm x 550mm.

dia of bars = 20mm

No. of Bars = 4.

Effective depth = 500mm



Grade of Concrete, $f_{ck} = 20 \text{ N/mm}^2$

Grade of Steel, $f_y = 415 \text{ N/mm}^2$.

$$A_{st} = n \times \pi / 4 d^2$$

$$= 4 \times \pi / 4 (20)^2$$

$$A_{st} = 1256.63 \text{ mm}^2$$

$$m = \frac{280}{3 G_{cbc}}$$

$$= \frac{280}{3 \times 7}$$

$$\boxed{m = 13.33}$$

$G_{cbc} = 7 \text{ N/mm}^2$

In Code Book Pg. No

81

$$\frac{n b}{2} = m \cdot A_{st} (d - n)$$

$$\frac{n \times 300}{2} = 13.33 \times 1256.63 (500 - n)$$

$$150n^2 + 16750.8 + n - 8375438.45 = 0$$

$$\boxed{n_a = 186.96 \text{ mm}} \rightarrow \begin{matrix} \text{position of} \\ \text{natural axis.} \end{matrix}$$

$$n_c = k_c \cdot d = 0.289 \times 500$$

$$n_c = 144.5 \text{ mm}$$

$$M.R = \frac{G_{cbc}}{2} \times b \times n_a \times d \times j \quad (\text{or}) \quad \frac{G_{cbc}}{2} \times b \times n \left(d - \frac{n_a}{3} \right)$$

$$k = \frac{1}{1 + \frac{G_{st}}{3n G_{cbc}}}$$

$$= 186.96$$

$$= \frac{7}{2} \times 300 \times 186.96 \times 500 \times 0.903$$

$$j = \left[1 - \frac{k_c}{3} \right]$$

$$j = \left[1 - \frac{0.289}{3} \right]$$

$$\boxed{j = 0.903}$$

$$= \frac{1}{1 + \frac{230}{13.33 \times 7}}$$

$$M.R = 88.63 \times 10^6 \text{ N-mm}$$

$$k = 0.289$$

$$\boxed{M.R = 88.63 \text{ kN-m}}$$

6) A reinforced Concrete Beam $230\text{mm} \times 500\text{mm}$ depth is reinforced with 4 bars of 12mm dia at an effective cover of 50mm use M_{20, f_y} . estimate the moment of resistance of the section. If the beam is Simply Supported over a span of 5m. Find the Max. U.D.L on it. Beam can carry inclusive of its own weight.

Sol:-

Given

size of R.C Beam = $230 \times 500\text{mm}^2$.

No. of bars = $4 = n$

dia of bar = 12mm.

effective Cover, $\bar{c} = 50\text{mm}$.

$$d = D - \bar{c}$$

$$d = 500 - 50$$

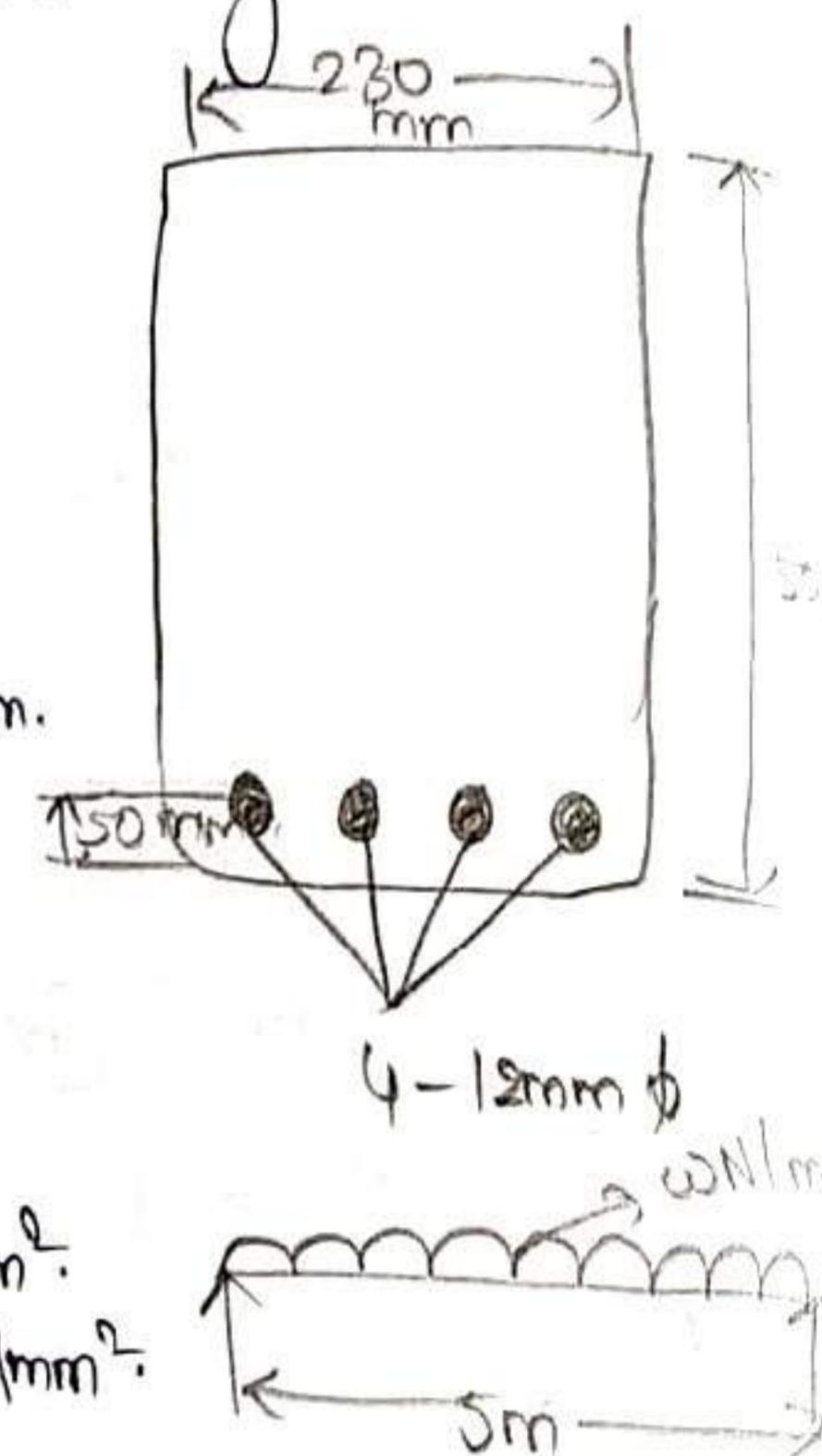
$$\boxed{d = 450\text{mm}.}$$

Grade of Concrete, $f_{ck} = 20\text{N/mm}^2$.

Grade of Steel, $f_y = 415\text{N/mm}^2$.

$$A_{st} = n \times \frac{\pi}{4} d^2 = 4 \times \frac{\pi}{4} (12)^2$$

$$\boxed{A_{st} = 452.38\text{mm}^2.}$$



$$G = 20 \times 250 \times 250 - 250 \times 250 \times 250 \times 6 \times 6 \times 6 \times 6 \rightarrow \text{Pg. No. 80.}$$

$$G_{cbc} = 4\text{N/mm}^2$$

for Codebook,
Pg No. 81.

Table 21

$$m = \frac{280}{3 \times 4}$$

$$\boxed{m = 13.33}$$

$$\frac{bn^2}{2} = m \cdot A_{st} (d - n)$$

$$\frac{230}{2} n^2 = 13.33 \times 452.38 (500 - n)$$

$$115n^2 + 6630.22n - 3015110 = 0.$$

$$\boxed{n_a = 129.81\text{mm}}$$

$$n_c = K_c \cdot d$$

$$n_c = 0.289 \times 500$$

$$\boxed{n_c = 130.05\text{mm}} \quad 144.5\text{mm}$$

~~N_a>N_c~~ over ~~under~~ Reinforced section.

N_a<N_c under Reinforced section.

Tension

$$M.O.R = G_{st} \cdot A_{st} \cdot J$$

$$= 230 \times 452.38 \times 500 \times 0.904 \left[1 - \frac{K}{3} \right].$$

$$M.R = 47.97 \times 10^6 \times 0.904 \times 10^6 \left[1 - \frac{0.289}{3} \right]$$

$$M.R = G_{st} \cdot A_{st} \left(d - \frac{\eta_a}{2} \right) \quad 0.90$$

$$= 230 \times 452.38 \left(500 - \frac{137.81}{3} \right).$$

$$M.R = 47.24 \times 10^6 \text{ N-mm.}$$

$$\boxed{M.R = 47.24 \text{ kN-m.}}$$

B.M for O.D.L

$$\frac{wl^2}{8} = 47.24 \times 10^3$$

$$\frac{w(5)^2}{8} = 47.24 \times 10^3$$

$$w = \frac{47.24 \times 8 \times 10^3}{(5)^2}$$

$$\boxed{O.D.L \quad w = 15.11 \text{ kN/m}}$$

- 7.) Design a reinforced Concrete beam 230mm wide to resist a Bending moment of 30kN-m. Use M40 and Fe415 Steel.

Sol: Given that Max B.M = 30kN-m = $30 \times 10^3 \times 10^3 \text{ N-mm.}$

width of Beam = 230mm

Grade of concrete, f_ck = 20N/mm².

Grade of steel, f_y = 415 N/mm²

$\alpha_c = 0.914$

$J_c = 0.904$

$$M.O.R = \alpha_c b d^2$$

$$30 \times 10^6 = 0.914 \times 230 \times d^2$$

$$\boxed{d = 377.76 \text{ mm.}}$$

$G_{st} = 230$
Code book

Pg No. - 52
Table 22.

$$A_{st} = \frac{M}{G_{st} \cdot J_c \cdot d}$$

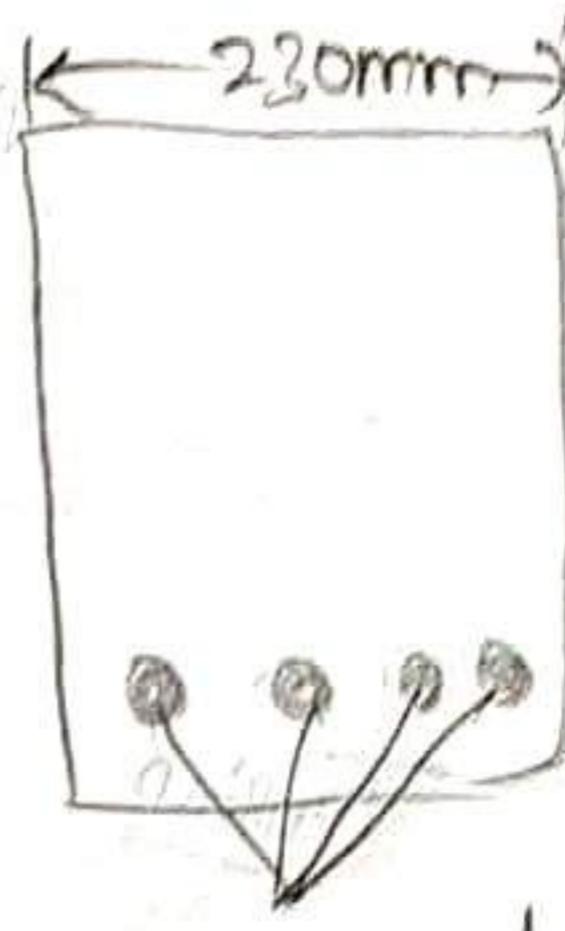
$$\boxed{A_{st} = \frac{30 \times 10^6}{230 \times 0.904 \times 377.76}}$$

$$A_{st} = 381.95 \text{ mm}^2$$

Assuming 12mm dia.

$$A_{st} = n \times \pi / 4 (d^2)$$

$$381.95 = n \pi / 4 (12)^2$$



4-12φ

$$n = 3.37$$

$n = 4 \text{ bars}$

A_{st} should not be less. It will be equal, more than

$$A_{st} = n \times \pi / 4 (d^2)$$

$$A_{st} = 4 \times \pi / 4 (12)^2$$

$$A_{st} = 452.38 \text{ mm}^2$$

- 8.) Design a rectangular Reinforced Concrete Beam to resist a B.M of ~~65~~ KN-m. Use M20 grade of concrete and Fe415 Steel and $b = 0.5d$.

Sol:-

Given that Width of beam $b = 0.5d$.

$$\text{Max B.M} = 65 \text{ KN-m} = 65 \times 10^3 \times 10^3 \text{ N-mm.}$$

Grade of Concrete, $f_{ck} = 20 \text{ N/mm}^2$.

Grade of Steel, $f_y = 415 \text{ N/mm}^2$.

$$M.O.R = Q_c \cdot b \cdot d^2$$

$$65 \times 10^6 = 0.914 \times (0.5)d \cdot d^2$$

$$Q_c = 0.914 \quad J_c = 0.904$$

$$d = 521.99 \text{ mm.}$$

$$A_{st} = \frac{M}{f_{st} \cdot J_c \cdot d}$$

$$A_{st} = \frac{65 \times 10^6}{230 \times 0.904 \times 521.99}$$

$$f_{st} = 230$$

$$A_{st} = 598.90 \text{ mm}^2$$

Assuming 12mm dia. (or) 14, 16 up to 20mm

$$A_{st} = n \times \pi / 4 (d^2)$$

$$598.90 = n \times \pi / 4 (12)^2$$

$$n = 5.29.$$

$$\boxed{n = 6 \text{ bars}}$$

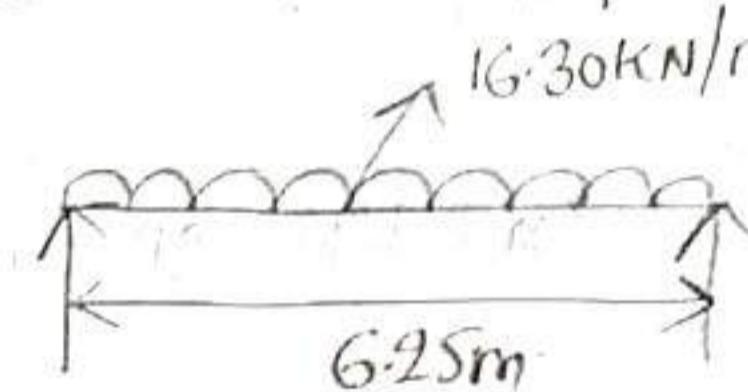
$$A_{st} = n \times \pi / 4 (d^2)$$

provided:

$$A_{st} = 6 \times \pi / 4 (12)^2$$

$$\boxed{A_{st} = 678.58 \text{ mm}^2}$$

- q) A singly reinforced rectangular concrete beam 350 mm wide has a span of 6.25m and carries an inclusive load of 16.30 KN/m. If the stresses in steel and concrete shall not exceed 230 and ± 13.33 N/mm². Find the effective depth and area of tensile reinforcement. Take $m = 13.33$.



Given that:-

width of Beam = 350mm = b

$$\text{Span } (l) = 6.25 \text{ m} \times 10^3 = 6.25 \times 10^3 \text{ mm}$$

$$\text{U.D.L load } (W) = 16.30 \text{ KN/m.} = 16.30 \times 10^6 \text{ N/mm}^2.$$

$$m = 13.33.$$

$$\text{Permissible stress in concrete, } f_{cbc} = \pm 13.33 \text{ N/mm}^2.$$

$$\text{Permissible stress in steel, } f_{st} = 230 \text{ N/mm}^2.$$

$$B.M = \frac{Wl^2}{8} = \frac{16.30 \times 10^6 \times (6.25 \times 10^3)^2}{8}$$

$$B.M = \pm 9.58 \times 10^6$$

$$= \pm 9.58 \times 10^6 \text{ KN-m.}$$

for M₂₀

$$\Omega_c = 0.914$$

$$J_c = 0.904.$$

$$M = \Omega_c b d^2$$

$$\pm 9.58 \times 10^6 = 0.914 \times (350) d^2$$

$$\boxed{d \leq 498.76 \text{ mm.}}$$

$$A_{st} = \frac{M}{f_{st} \cdot J_c \cdot d} = \frac{\pm 9.58 \times 10^6}{230 \times 0.904 \times 498.76}$$

$$\boxed{A_{st} = \pm 67.38 \text{ mm}^2}$$

Assuming 12mm dia.

$$Agt = n \times \pi / 4 (d^2)$$

$$767.38 = n \times \pi / 4 (12)^2$$

$$n = 6.78$$

$$\boxed{n = 7 \text{ bars.}}$$

$$Ast = n \times \pi / 4 (d^2)$$

$$Ast = 7 \times \pi / 4 (12)^2$$

$$\boxed{Ast = 791.68 \text{ mm}^2}$$

- 10) A singly reinforced Concrete beam 300mm wide has an effective depth of 500mm the effective span being 8m. It carries a total load of 160 kN/m on the wholspan. It is reinforced with 804 mm² Steel. Determine the stresses produced in steel and Concrete. Take m = 13.33.

Sol:

Given that

$$\text{width} = 300\text{mm} = b$$

Area of Steel,

$$\text{effective depth } d = 500\text{mm}$$

$$Ast = 804 \text{ mm}^2$$

$$L = 8\text{m.}$$

$$O.D.L.W = 160 \text{ kN/m.}$$

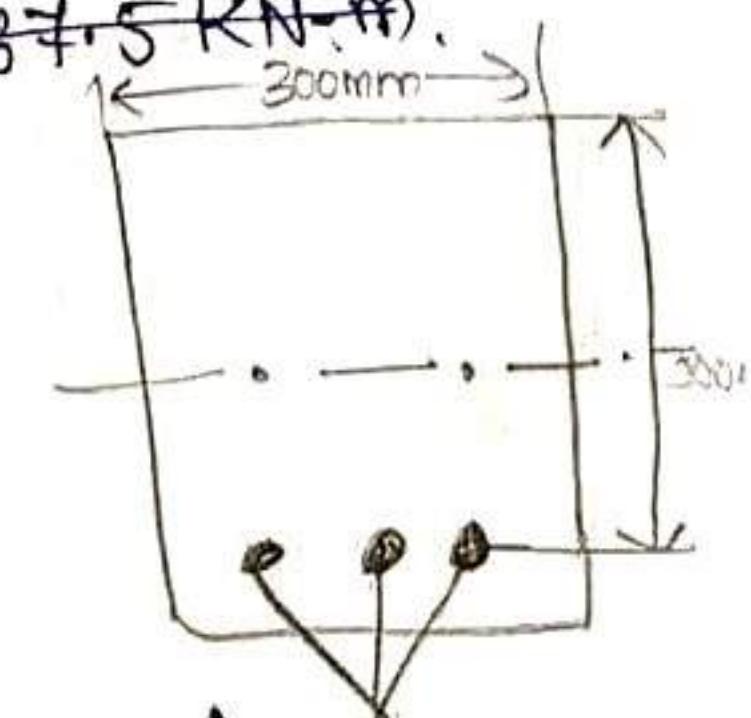
16KN/m
sm

$$\frac{\text{Max. Stress}}{8} = \frac{M \cdot O \cdot R}{8}$$

$$M \cdot O \cdot R = \frac{6cbe}{2} b n_a \left(d - \frac{n_a}{3} \right).$$

$$B.M = \frac{wL^2}{8} = \frac{160 \times 10^3 \times 5^2}{8} = 187.5 \text{ kN-m.}$$

$$= 50 \text{ KN-m.}$$



$$\frac{nb}{2} m.Ast (d-n)$$

$$\frac{n(300)}{2} = 13.33 \times 804 (500-n) \cdot 804$$

$$150n^2 + 10717.32n - 5358660 = 0$$

$$\boxed{n_a = 156.6 \text{ mm}}$$

$$50 \times 10^6 = \frac{6cbc}{2} \times 300 \times \frac{156.63}{3} = 9156.65$$

$$50 \times 10^6 = \frac{6cbc}{2} \times 21.04 \times 10^6 = 403005$$

$$6cbc = \frac{50 \times 10^6 \times 2}{403005 \times 21.04}$$

$$6cbc = 248.13 \text{ N/mm}^2 \text{ or } 4.75 \text{ N/mm}^2$$

$$M.O.R = 6cbc$$

$$M = Gst \times Ast \left(d - \frac{n\alpha}{3} \right)$$

$$Gst = m \cdot 6cbc \left(\frac{d-n}{n} \right)$$

$$Gst = \frac{M}{Ast \left(d - \frac{n\alpha}{3} \right)}$$

$$Gst = 13.33 \times 4.75 \left[\frac{500 - 156.63}{156.63} \right]$$

$$Gst = \frac{50 \times 10^6}{804 \left(500 - \frac{156.63}{3} \right)}$$

$$Gst = 138.80 \text{ N/mm}^2$$

$$Gst = 138.87 \text{ N/mm}^2$$

A singly reinforced beam, 350mm x 550mm deep has an effective span of 6m and carries a load of 20kN/m. The beam is reinforced with 4 bars of 20mm dia. at an effective cover of 35mm. Find the Max. stress provided in Steel and Concrete. Take $m = 13.33$.

Wide $\Rightarrow b = 350 \text{ mm}$, $d = 550 \text{ mm}$

$d = 6 \text{ m}$

U.D.L, $w = 20 \text{ kN/m}$

$$B.M = \frac{w s^2}{8} = \frac{20 \times 6^2}{8} = 90 \text{ kN-m}$$

$$\frac{n^2 b}{2} = m \cdot Ast \left(d - n \right)$$

$$\frac{n^2 (350)}{2} = 13.33 \cdot Ast (550 - n)$$

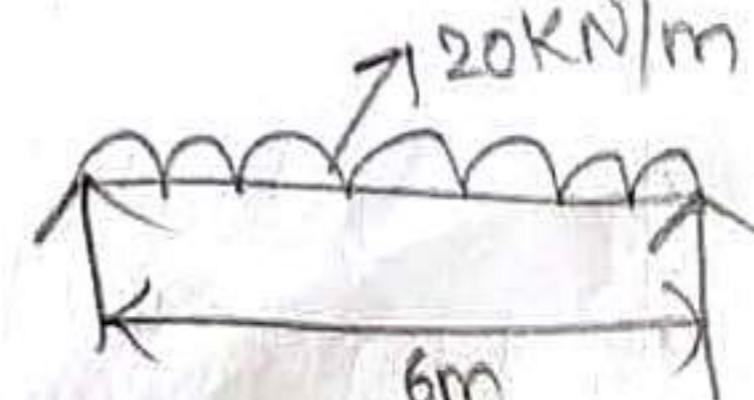
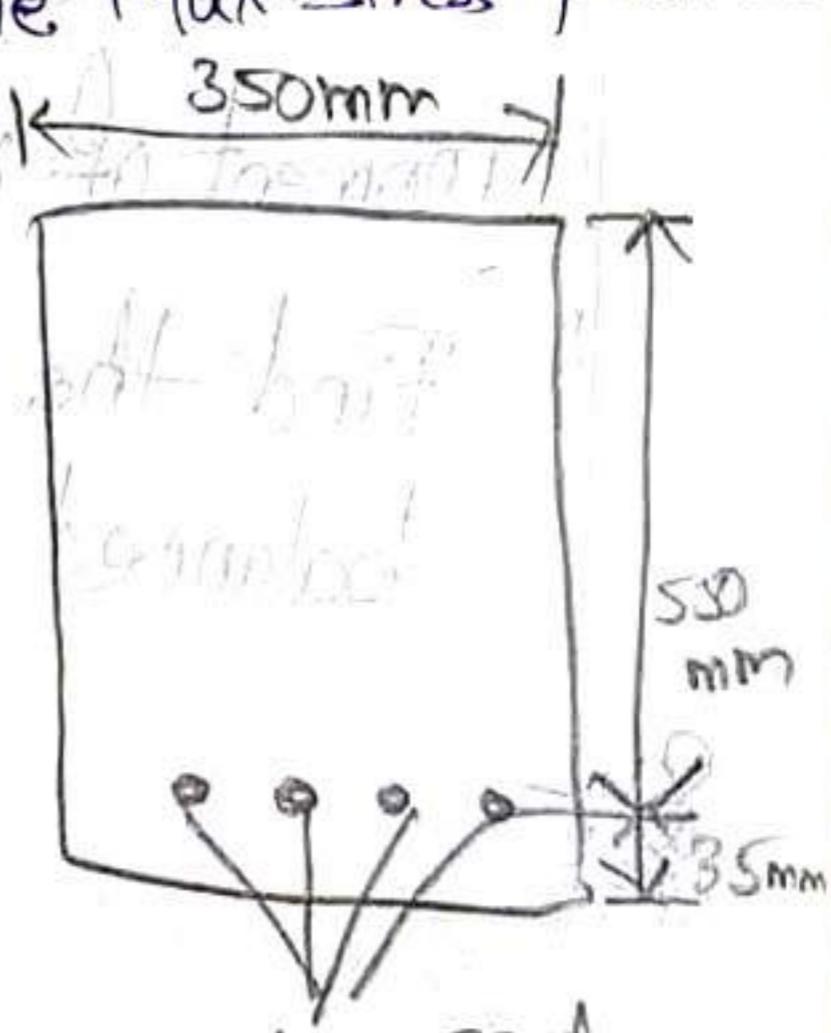
$$Ast = n \left(\frac{\pi d}{4} \right)^2 = Ast = \frac{\pi \times 20^2}{4} \times (20)^2$$

$$Ast = 1256.63 \text{ mm}^2$$

$$175n^2 = 13.33 \times 1256.63 (550 - n)$$

$$175n^2 + 16750.87n - 9113978.5$$

$$n = 185.31 \text{ mm}$$



$$M.O.R = \frac{6cbc}{2} \times b \times n_a \left[d - \frac{n_a}{3} \right]$$

$$90 \times 10^6 = \frac{6cbc}{2} \times 350 \times 185.31 \left[550 - \frac{185.31}{3} \right]$$

$$6cbc = 5.68 \text{ N/mm}^2.$$

$$Gst = 6cbc \cdot m \cdot \cancel{Ast} \left[d - \frac{n_a}{n_a} \right]$$

$$Gst = 5.68 \times 13.33 \left[\frac{550 - 185.31}{185.31} \right]$$

$$Gst = 149.0 \text{ N/mm}^2$$

$$M = Gst \times Ast \left(d - \frac{n_a}{3} \right)$$

$$M = Ast \left(d - \frac{n_a}{3} \right)$$

- Q12) A singly reinforced beam was originally planned to be designed as a balanced section. Later it was desired to increase the moment of resistance of section by double to the reinforcement. Find the ratio of enhanced moment of resistance to the balanced. Use M₂₀ grade of concrete and Fe15 steel.

Given:-

Given that Grade of Concrete = 20N/mm² = f_{ck}

Grade of steel = 415N/mm². = f_y.

Balanced Section

$$Ast = \frac{M}{Gst \cdot J_c \cdot d}$$

$$M = Q_b d^2$$

Table 22
IS 456

J_c = 0.904

Q_c = 0.914

$$0.914 bd$$

$$Gst \times J_c \times d$$

$$Ast = \frac{0.914 bd}{Gst \times J_c \times d}$$

$$(1 - 0.023) \cdot 0.914 \cdot d^2 \cdot 0.904 \times 68.81 = 0.271$$

$$Ast = \frac{0.914 bd}{230 \times 0.904}$$

$$2.8 F P E I / P = 0.78 \cdot 0.08 + Ast = 0.271$$

$$1.128 \times 10^{-3} bd$$

ii) After doubling the reinforcement, the balanced section becomes an over reinforced section.

$$A_{st} = 2 \times 0.0043 \text{ bd.}$$

$$A_{st} = 0.0087 \text{ bd}$$

$$M.O.R = \frac{6cbs}{2} b n_a \left(d - \frac{\eta_a}{3} \right).$$

$$\frac{bn^2}{2} = m \cdot A_{st} (d - n).$$

$$\frac{bn^2}{2} = 13.33 \times 0.0087 b d (d - n).$$

$$0.5n^2 = 13.33 \times 0.0087 \times d^2 - 13.33 \times 0.0087 n d$$

$$0.5n^2 + 0.1159n - 0.1159d = 0.$$

$$\boxed{n = 0.37d}$$

$$0.5n^2 + 0.1159nd - 0.1159d^2 = 0.$$

$$\boxed{n = 0.37d}$$

$$M.O.R = \frac{6cbs}{2} b n_a \left(d - \frac{\eta_a}{3} \right).$$

$$= \frac{7}{2} b (0.37d) \left(d - \frac{0.379d}{3} \right)$$

$$M.O.R = 1.158 b d^2$$

$$\frac{M.O.R}{M.B.S} = \frac{1.158 b d^2}{0.914 b d^2}$$

$$= 1.266$$

It is worthy to note that by increase of amount of steel by 100%, the increase of moment of resistance is only 26.6%.

The maximum value of total F.I. = 1.158 times of 0.914 = 1.266 times of total F.I.

maximum value of total F.I. = 2.000 times of total F.I. has proved.

maximum value of total F.I. = 2.000 times of total F.I. has proved.

2) Limit State Method:-

Limit state collapse

Shear

Torsion

Compression

Flexural

This all
should obeys
the construction
is preferred.

Limit state serviceability

Cracking

Deflections.

Factor of safety, Concrete ≥ 1.5 , Steel $\rightarrow 1.15$. Pg No - 64

Introduction to Limit state method: - The philosophy of L.S.M of

design is to see that the structure remains fit for use throughout its life period by assuring safety against strength and serviceability requirements.

That is the structure will not reach its limit state throughout its life period. A structure is said to have limit state when a structure as a whole (or) a part unfit for use during its expected life time.

Acceptable limits: - These are the acceptable limits for safety and serviceability requirements for the structure before failure occurs.

The two major limit states which are usually considered are:-

1) Limit state of collapse. 2.) Limit state of serviceability.

1) Limit state of collapse: - It is a limit state at which the structure is likely to collapse due to rupture of one (or) more critical sections (or) overturning, buckling, loss of overall stability.

It may lead to Shear, Torsion, Compression and Flexural.

2) Limit state of serviceability: - It relates to the performance of the structure at working loads. It may be due to excessive Cracking and deflection. It may lead to deflection, vibrations, cracking, other limit states (Fire resistance, vibrations, durability).

Generally the crack width on the surface may be taken as 0.3mm.

Characteristic strength of materials:- It is defined as the value of strength of material below which not greater than 5% of test results are expected to fall.

For Concrete: - It is the compressive strength of 150mm cube of 28 days, N/mm². It is denoted by (f_{ck}).

Grade of Concrete	M ₁₅	M ₂₀	M ₂₅	M ₃₀
Characteristic Strength N/mm ²)	15	20	25	30

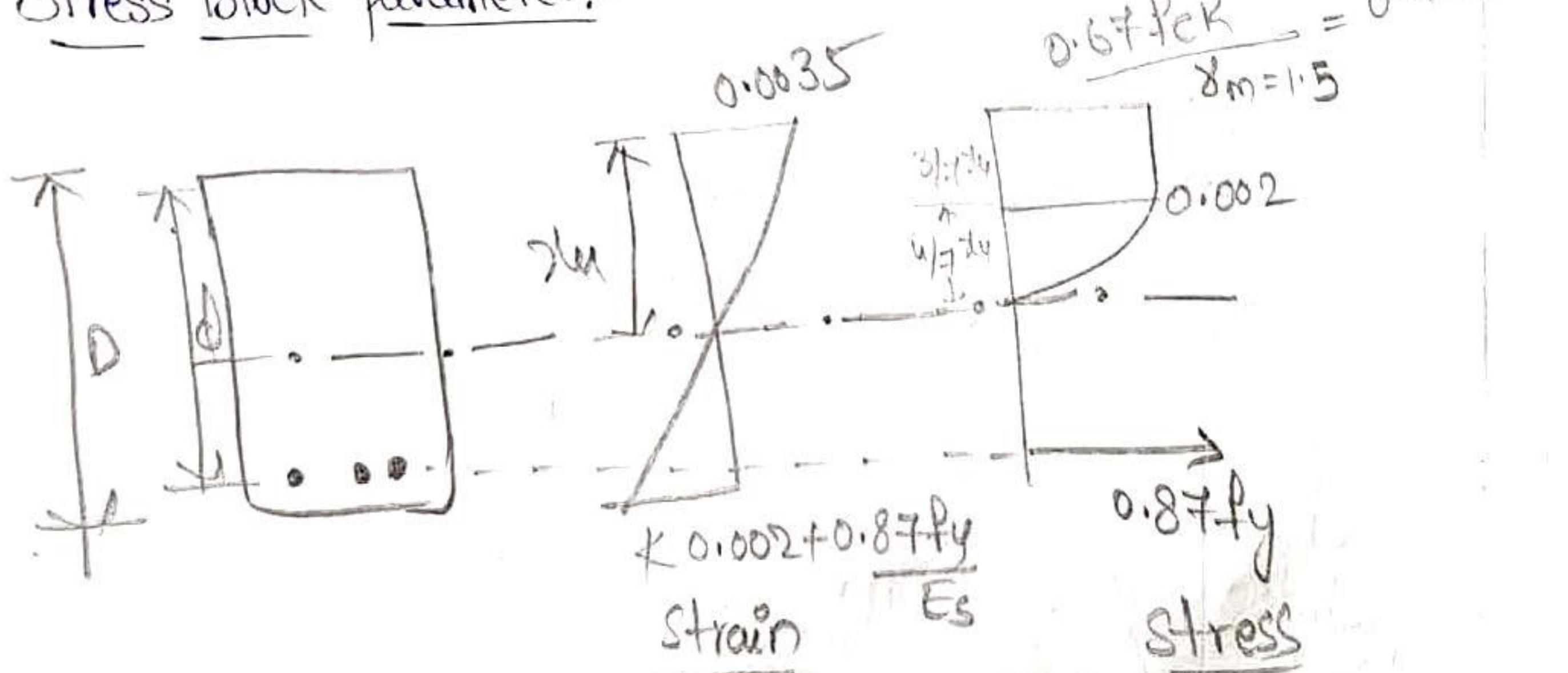
For Steel:- The minimum yield strength (or) 0.2% of proof stress may be taken as characteristic strength.

Grade of Steel	Fe250	Fe415	Fe500
Characteristic Strength N/mm ²	250	415	500

It is denoted by (f_y) .

Characteristic load:- characteristic loads:- Pg No - 67

Stress Block parameter:-



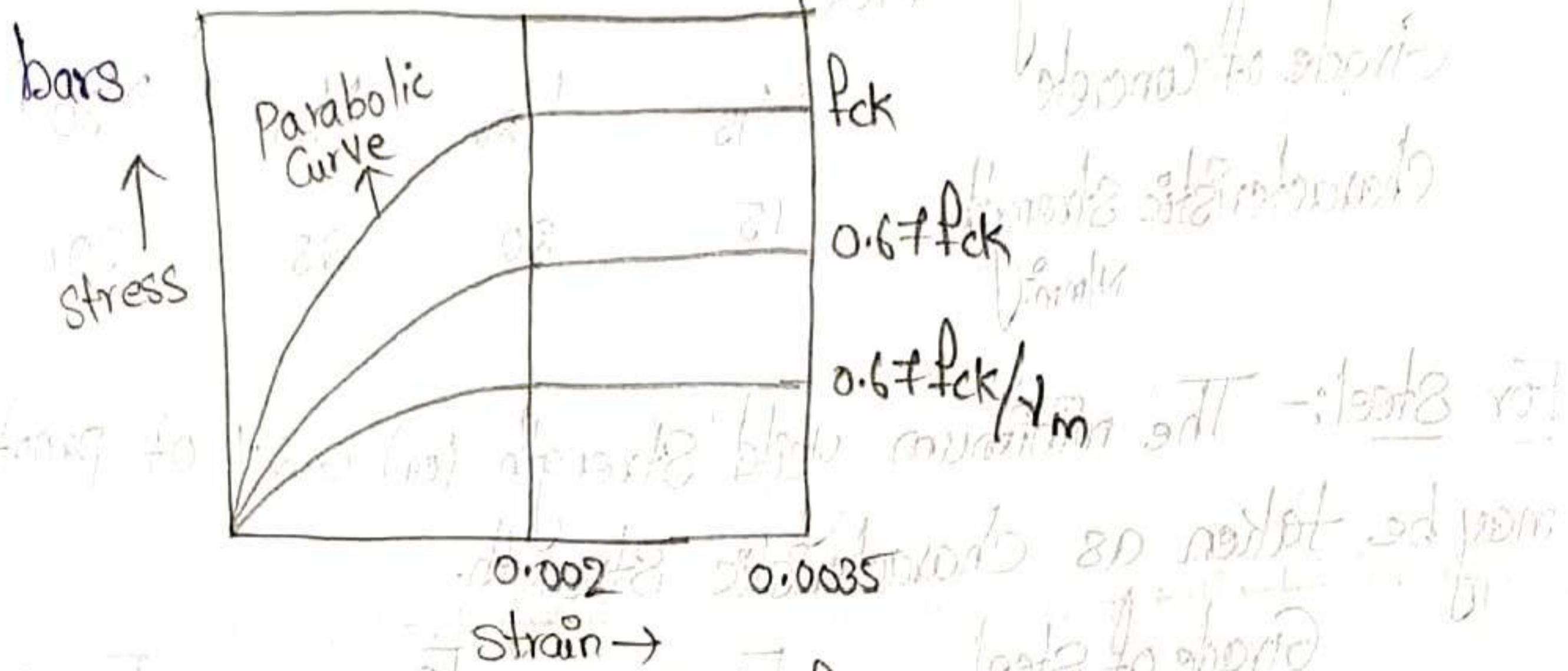
Calculation of Area of Compression:

C_u = Area of rectangle + Area of parabola

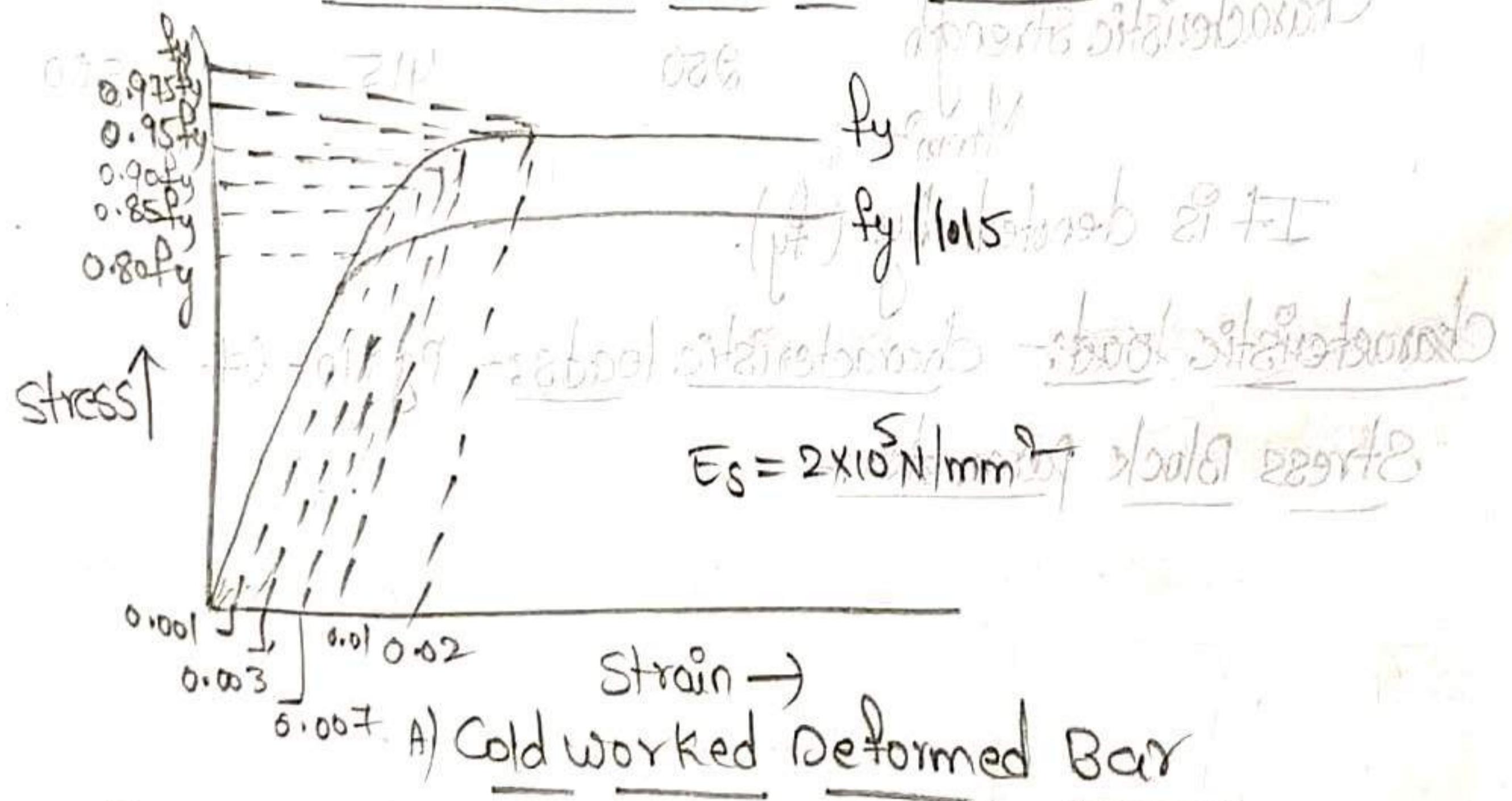
$$= (0.446 f_{ck} \times b) \times 3/4 x_u + 2/3 (0.446 f_{ck} b) (4/7 x_u)$$

$$C_u = 0.366 f_{ck} b x_u$$

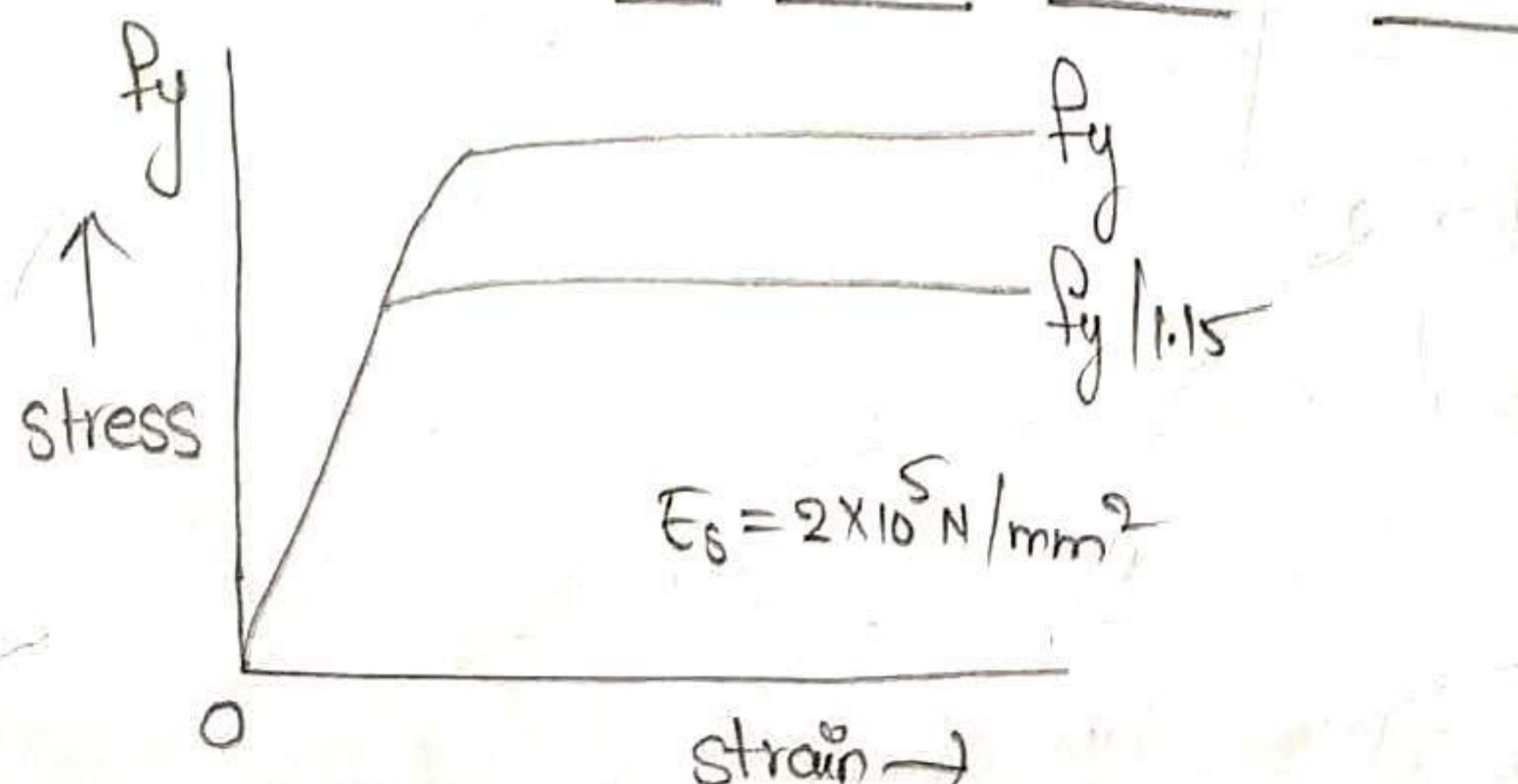
Stress Strain Curves for Cold deformed bars and Mild steel



Stress-strain Curve for Concrete



A) Cold worked Deformed Bar



B.) Steel Bar with definite yield point.

Assumptions: Fig No 69, 70

- 1) Plane sections normal to the axis remain plane after bending.
- 2) The tensile strength of concrete is ignored.
- 3) The maximum strain in concrete at the outermost fiber is taken as 0.0035 . In bending regardless of the strength of the concrete.
- 4) The maximum strain in the tension reinforcement in the section at failure shall not be less than $0.002 + \frac{f_y}{1.15\gamma_s}$
(or)
 $0.002 + \frac{0.87f_y}{\gamma_s}$.
- 5) The relationship between stress-strain distribution in concrete is assumed as parabolic.
- 6) Compressive strength = $0.67 \times$ characteristic strength.
 $= 0.67 \times f_{ck}$.
- 7) Design Strength = $0.67f_{ck} / \gamma_m$
 γ_m = Partial safety factor (or) factor of safety.
for Concrete $\gamma_m = 1.5$
 $= \frac{0.67f_{ck}}{1.5} = 0.446f_{ck}$.
- 8) The stress in steel reinforcement is derived from the stress-strain curve for the type of steel used as shown in fig.
- 9) Design Strength = $\frac{f_y}{\gamma_m}$ where $\gamma_m = 1.15$
 $= 0.87f_y$ in Steel

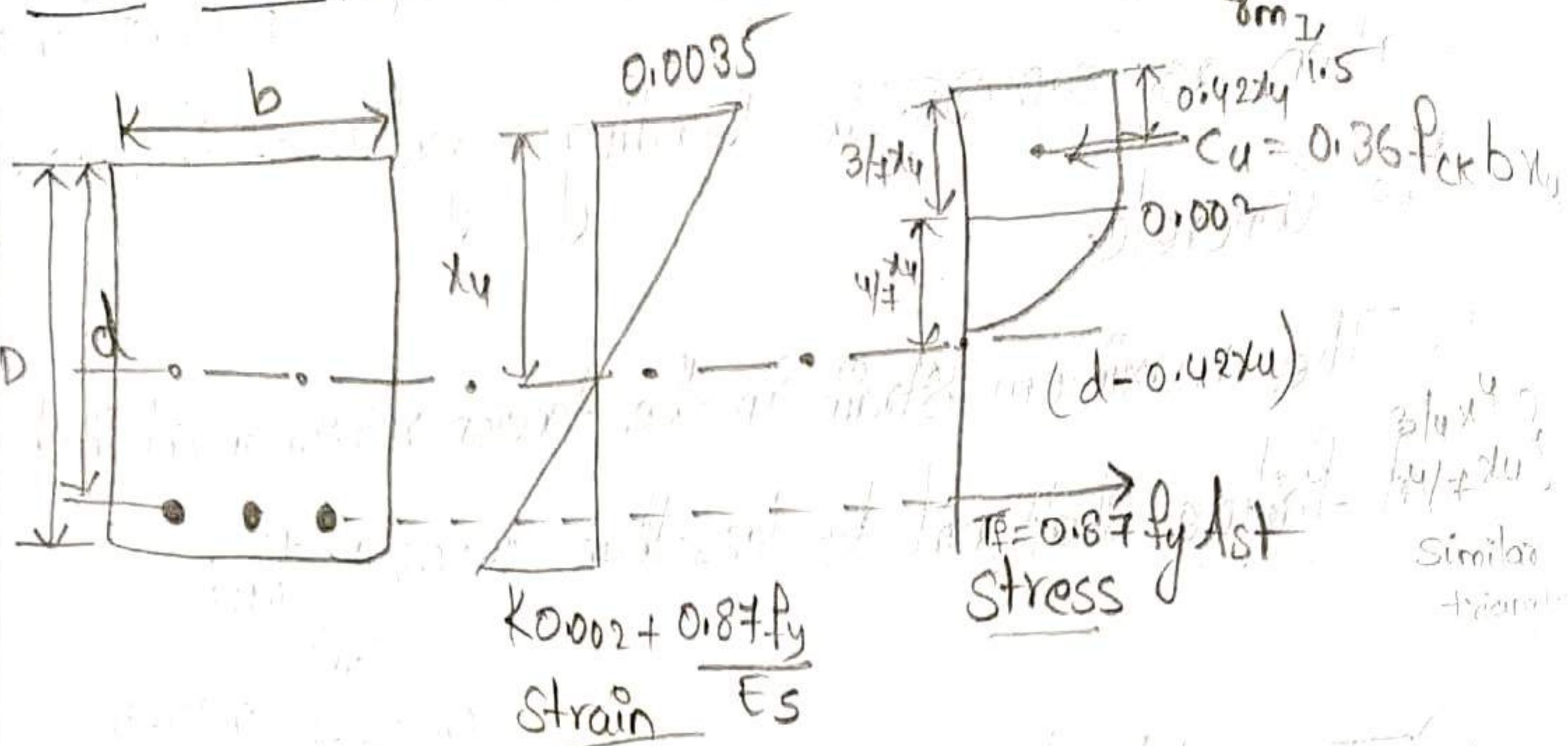
Analysis of a Single Reinforced Beam:

If we provide the reinforcement tension zone of concrete member, then that single member is said to be single reinforced beam.

Effective depth: $D - c = d$

$$d = \text{overall depth} - \text{effective cover}$$

Stress Block Parameters:-



$$\text{By calculation of centroidal distance:}$$

$$= \frac{(3/4 x_u) (0.446 f_{ck} b) (y_2 \times 3/4 x_u) + \left[2/3 (4/7 x_u) (0.446 f_{ck} b) \times (3/4 x_u) (0.446 f_{ck} b) (y_2 \times 3/4 x_u) + 2/3 (4/7 x_u) (0.446 f_{ck} b) \times (3/4 x_u) (0.446 f_{ck} b) (y_2 \times 3/4 x_u) \right]}{0.36 f_{ck} b x_u}$$

$$= \frac{0.1492}{0.366} x_u$$

$$Z = \left(d - 0.42x_4 \right) \text{ mm}$$

The depth of parabolic part of the stress Block.

$$\frac{0.0035}{0.002} \rightarrow x_4$$

Depth of Rectangular portion :-

$$x_u - \frac{4}{7}x_u$$

$$= \frac{4x_u - 4x_u}{7} = \frac{3}{7}x_u$$

Area of the stress Block :-

Area of rectangle + Area of parabola.

$$= 3\frac{1}{4}x_u \times 0.446f_{ck} \cdot b + 4\frac{1}{7}x_u (0.446f_{ck} \cdot b) \times 2/3$$

$$= 0.366f_{ck} \cdot b \cdot x_u$$

The distance of the centroid of stress Block from the top

Fiber: Rectangle + parabola.

$$= 3\frac{1}{4}x_u \times 0.446f_{ck} \cdot b \times y_2 \times 3\frac{1}{4}x_u + 2/3(4\frac{1}{7}x_u)(0.446f_{ck} \cdot b) \times [3\frac{1}{4}x_u + 3/8 \times 4\frac{1}{7}x_u]$$

$$0.36f_{ck} \cdot b \cdot x_u$$

$$= 0.446f_{ck} \cdot b \times (3\frac{1}{4})^2 \cdot x_u^2 \times \frac{1}{2} + \frac{8}{21}x_u \times 0.446f_{ck} \cdot b \cdot x_u \left[3\frac{1}{4} + \frac{12}{56} \right]$$

$$0.36f_{ck} \cdot b \cdot x_u$$

$$= \frac{0.040f_{ck} \cdot b \cdot x_u^2 + 0.1092f_{ck} \cdot b \cdot x_u^2}{0.366f_{ck} \cdot b \cdot x_u} = \frac{(0.1492)f_{ck} \cdot b \cdot x_u^2}{0.366f_{ck} \cdot b \cdot x_u}$$

$$= 0.42x_u$$

The compression force $C_u = 0.36f_{ck} \cdot b \cdot x_u$ acts at a distance of $0.42x_u$ from the top most fiber.

Lever Arm (\bar{z}):- $\bar{z} = d - 0.42x_u$.

$$(d - z) = 0.366f_{ck} \cdot b \cdot x_u$$

$$0.366f_{ck} \cdot b \cdot x_u = x_{out} + x_{inlet}$$

$$\sqrt{b^2 + 0} = x_{out}$$

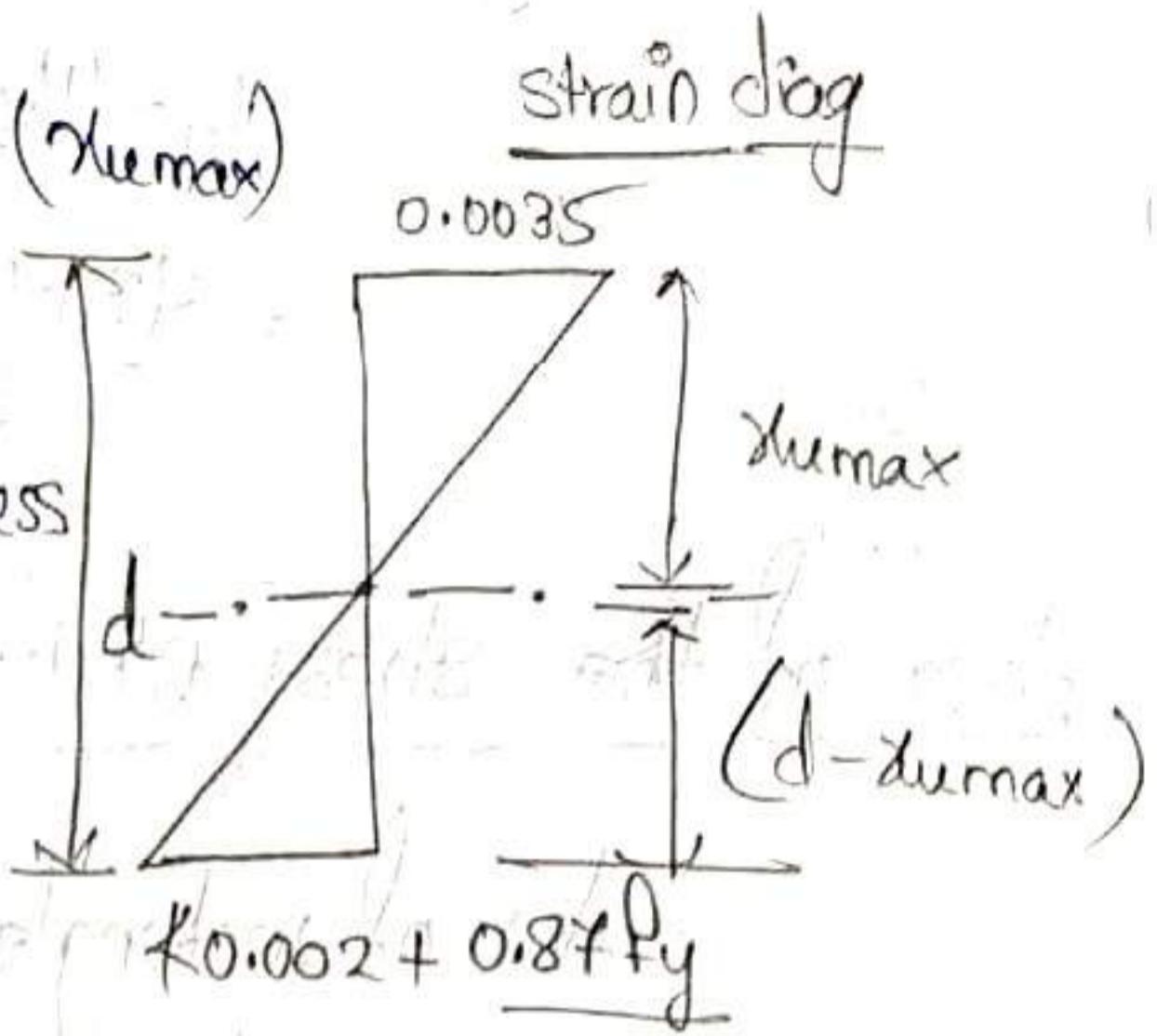
Limiting moment of Resistance:-

Maximum depth of Neutral axis:- (x_{umax})

The max. depth of Neutral axis.

is limited to ensure the tensile stress will reach its maximum yield stress

$$\frac{x_{umax}}{0.0035} = \frac{(d - x_{umax})}{0.002 + 0.87 f_y}$$



$$0.002 + 0.87 f_y = 0.002 + 0.87 f_y$$

before the concrete fails in compression thus brittle failure material

$$x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{0.002 + 0.87 f_y} = \frac{(d - x_{umax}) \times 0.0035}{0.002 + 0.87 \times 415}$$

$$x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{2 \times 10^5}$$

put for $[Fe 415]$, $f_y = 415 \text{ N/mm}^2$ $E_s = 2 \times 10^5 \text{ N/mm}^2$

$$x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{0.002 + 0.87 \times 415}$$

$$x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{2 \times 10^5}$$

$$x_{umax} + x_{umax} \times 0.919 = 0.919 d.$$

$$x_{umax} = 0.478 d.$$

$$x_{umax} = 0.48 d$$

$$[Fe 250] = f_y = 250 \text{ N/mm}^2. \quad x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{0.002 + 0.87 \times 250}$$

$$x_{umax} = \frac{(d - x_{umax}) \times 1.133}{2 \times 10^5}$$

$$x_{umax} + x_{umax} \times 1.133 = 1.433 d$$

$$x_{umax} = 0.531 d$$

$$[Fe 500] = f_y = 500 \text{ N/mm}^2. \quad x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{0.002 + 500 \times 0.87}$$

$$x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{2 \times 10^5}$$

$$x_{umax} = 0.83 (d - x_{umax})$$

$$x_{umax} + 0.83 x_{umax} = 0.83 d$$

$$x_{umax} = 0.45 d$$

f_y	x_{max}/d	Pg No. to of IS 456
250	0.53	
415	0.48	
500	0.45	

Limiting moment of resistance:-

Since the max. depth of Neutral axis is limited to avoid the brittle failure. The maximum moment of resistance is also limited (Mu limit).

$$\text{Mu limit for Concrete} = C_u \times z$$

$$= 0.36 f_{ck} \cdot b \cdot x_{\text{limit}} \times (d - 0.42 x_{\text{limit}})$$

$$\text{Mu limit for steel} = T_f \times z$$

$$= 0.87 A_{st} f_y \times (d - 0.42 x_{\text{limit}})$$

For $f_y = 250$

$$\text{Mu Concrete} = 0.36 f_{ck} \cdot b (0.53/d) (d - 0.42 \times 0.53)$$

$$\text{Mu limit Concrete} = 0.148 f_{ck} \cdot b \cdot d^2$$

$$\text{For } f_y = 415 \quad \text{Mu Steel} = 0.87 A_{st} f_y$$

$$= 0.36 f_{ck} \cdot b (0.48/d) (d - 0.42 \times 0.48)$$

$$\text{Mu limit Concrete} = 0.137 f_{ck} \cdot b d^2$$

For $f_y = 500$

$$= 0.36 f_{ck} \cdot b (0.45/d) (d - 0.42 \times 0.45)$$

$$\text{Mu limit Concrete} = 0.131 f_{ck} \cdot b d^2$$

Grade of Compound Concrete: general Mu limit	Fe 250	Fe 415	Fe 500
General	$0.148 f_{ck} b d^2$	$0.138 f_{ck} b d^2$	$0.133 f_{ck} b d^2$
M20 (f_{ck})	$2.96 b d^2$	$2.76 b d^2$	$2.66 b d^2$
M25	$3.7 b d^2$	$3.45 b d^2$	$3.33 b d^2$

Limiting percentage of Steel :-

The percentage of tensile reinforcement corresponding to the limiting moment of resistance is known as Limiting Percentage of Steel.

If it is obtained by equating the forces of compression and tension.

$$0.36 f_{ck} \cdot b \cdot x_{ulimit} = 0.8 f_y A_{st limit}$$

$$A_{st limit} = \frac{0.36 f_{ck} \cdot b \cdot x_{ulimit}}{0.8 f_y}$$

$$A_{st limit} = \frac{0.412 f_{ck} \cdot b \cdot x_{ulimit}}{f_y}$$

$$\text{Limiting percentage of steel, } P_t \text{ limit} = \frac{100 A_{st limit}}{bd}$$

$$P_t \text{ limit} = \frac{100 \times 0.412 f_{ck} \cdot b \cdot x_{ulimit}}{bd}$$

$$P_t \text{ limit} = \frac{41.2 f_{ck} \cdot b \cdot x_{ulimit}}{f_y d}$$

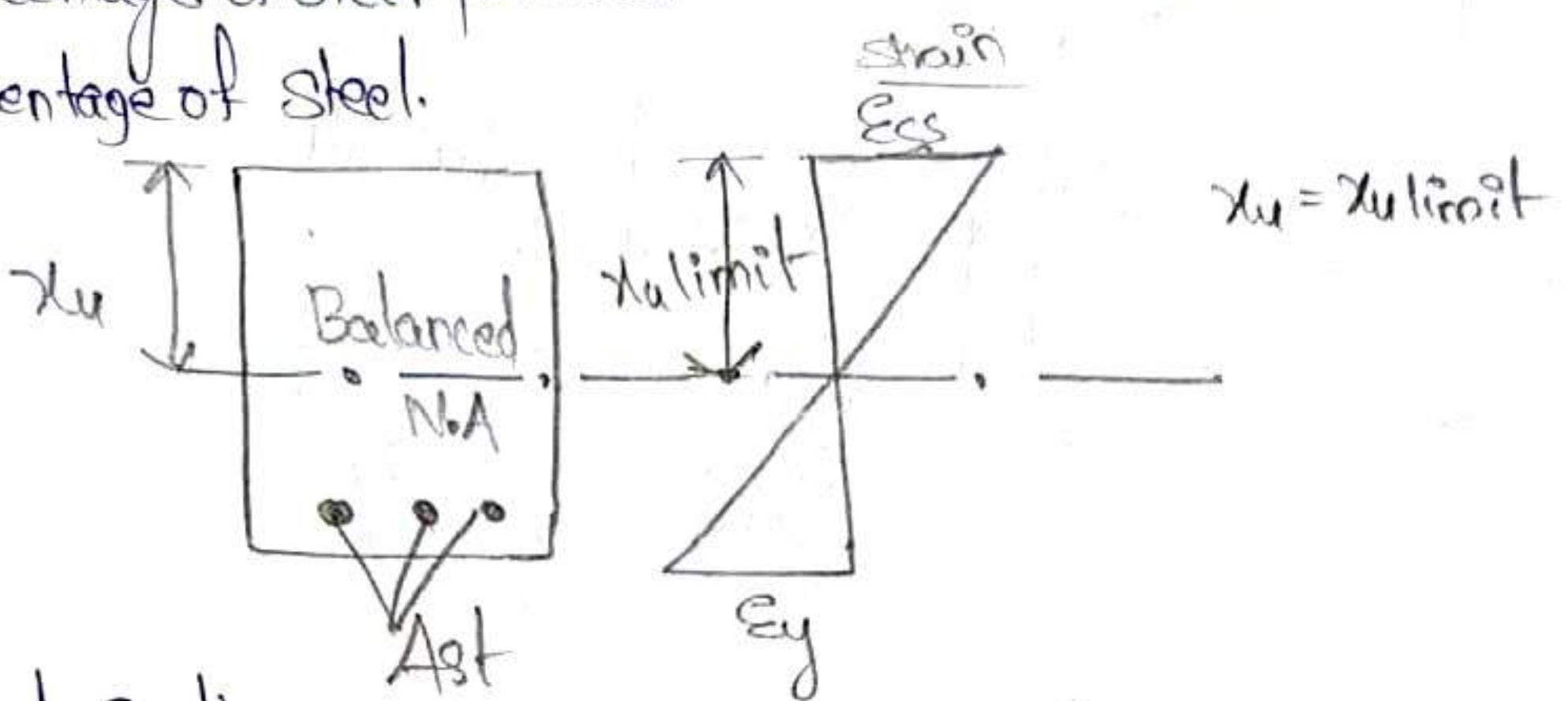
Grade of Concrete	F _c 250	F _c 415	F _c 500
M ₁₅	1.32	0.972	0.57
M ₂₀	1.76	0.96	0.76
M ₂₅	2.02	1.19	0.94

Modes of failures:- (or) Types of Sections:

- A reinforced concrete member considered to have failed when the strain in concrete in extreme fiber reaches its ultimate value i.e., 0.0035.

1) Balanced Section:- When the Max. Strains will reach its maximum values simultaneously then that type of section is called balanced section.

The percentage of steel provided in a balanced section is called limiting se percentage of steel.



2) Under Reinforced Section:-

When the amount of steel in a section is less than the required for a balanced section is called under Reinforced section.

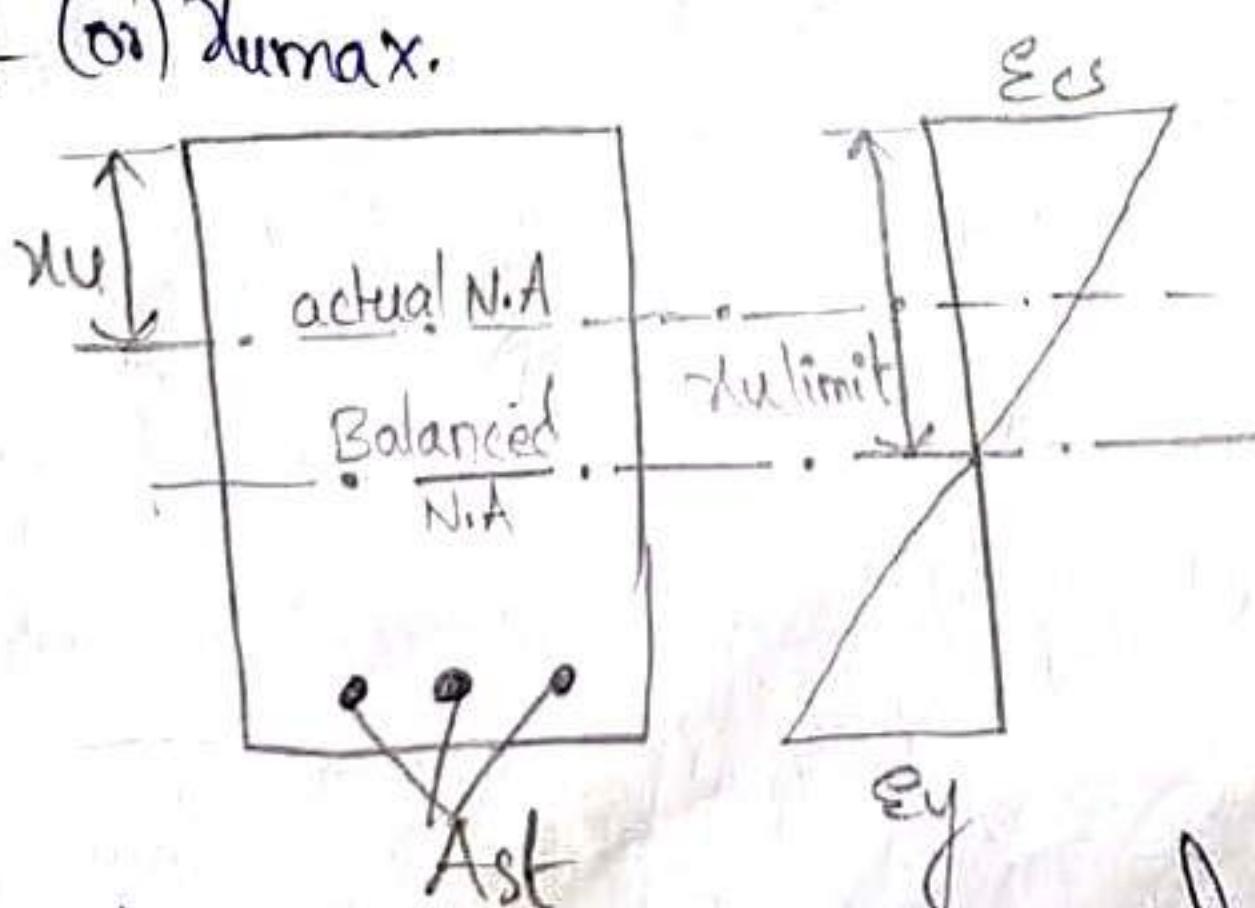
$$x_u < x$$

Since $Ast < Ast_{max}$.

In an under reinforced section the strain in concrete reaches its maximum value while the strain in steel reaches its max. value. does not

The position of Neutral axis will shift upwards to maintain the section in equilibrium b/w the forces of compression and tension.

Since $x_u < x$ (or) x_{umax} .



3) Over-Reinforced Section:- The percentage of steel provided greater than the required for a balanced section is called as over reinforced section. Since $Ast > Ast_{max}$.

In Case of over-reinforced section the

depth of N.A will shift downwards to maintain the structure in equilibrium b/w the forces of compression and tension.

Since $x_u > x_{ulimit}$ (or) $x_u > x_{ulimit}$.

