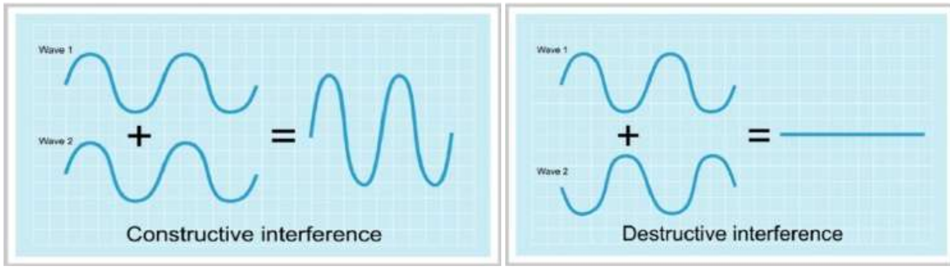
INTERFERENCE

Interference:

When two or more waves are superimposed then there is a modification of intensity or amplitude in the region of superposition. This modification of intensity or amplitude in the region of super position is called **Interference.**

When the resultant amplitude is the sum of the amplitudes due to two waves, the interference is known as **Constructive interference** and when the resultant amplitude is equal to the difference of two amplitudes, the interference is known as **destructive interference**.



PRINCIPLE OF SUPERPOSITION:

This principle states that the resultant displacement of particle in a medium acted upon by two or more waves simultaneously is the algebraic sum of displacements of the same particle due to individual waves in the absence of the others.

Consider two waves traveling simultaneously in a medium. At any point let y_1 be the displacement due to one wave and y_2 be the displacement of the other wave at the same instant.

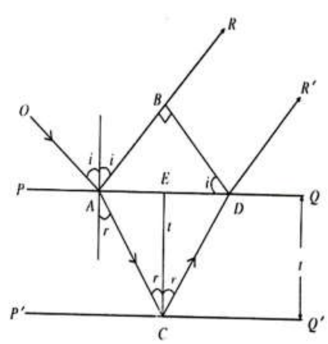
Then the resultant displacement due to the presence of both the waves is given by

$$\boxed{y = y_1 \pm y_2}$$

$+ve$ Sign has to be taken when both the displacements y_1 & y_2 are in the same direction

$-ve$ Sign' has to be taken when both the displacements y_1 & y_2 are in the opposite direction.

INTERFERENCE IN THIN FILMS



Consider a thin film of thickness t and refractive index μ. A ray of light OA incident on the surface at an angle i is partly reflected along AB and partly refracted into medium along AC, making an angle of refraction r .at C it is again partly reflected along CD. Similar refractions occur at E.

To find the path difference between the rays, draw DB perpendicular to AB

Then the path difference $= \mu(AC + CD) - AB$ …………….(1)

From triangle ACE

$$\cos r = \frac{CE}{AC}$$

$$AC = \frac{CE}{\cos r} = \frac{t}{\cos r} \text{ ………………..(2)}$$

From triangle CDE

$$\cos r = \frac{CE}{CD}$$

$$CD = \frac{CE}{\cos r} = \frac{t}{\cos r} \text{ ……………….(3)}$$

From triangle ABD

$$\cos(90 - i) = \frac{AB}{AD}$$

$$AB = AD\cos(90 - i) = 2AE\sin i \text{ ………..(4)} \qquad (\because AD = 2AE)$$

FROM triangle ACE

$$\sin r = \frac{AE}{AC} \Rightarrow AE = AC\sin r$$

$$AE = \frac{t\sin r}{\cos r} \qquad (\because AC = \frac{t}{\cos r})$$

From Eq (4)

$$AB = \frac{2t\sin r}{\cos r} \times \sin i$$

$$AB = \frac{2t\sin r\sin i}{\cos r} \times \frac{\sin r}{\sin r}$$

$$AB = \frac{2\mu t\sin^2 r}{\cos r} \text{ ………………………(5)} \qquad (\because \mu = \frac{\sin i}{\sin r})$$

On substituting the values of $AC, CD \,\&\, AB$ from Eq(2),(3)&(5) in Eq(1) ,we get

The path difference $= \mu(\frac{t}{\cos r} + \frac{t}{\cos r}) - \frac{2\mu t\sin^2 r}{\cos r}$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r) = \frac{2\mu t\cos^2 r}{\cos r} = 2\mu t\cos r$$

$\therefore$ The path difference $= 2\mu t\cos r$

According to the theory of reversibility, when the light ray reflected at rarer-denser interface, it introduces an extra phase difference π (or) path difference of $\frac{\lambda}{2}$

$\therefore$ The actual path difference $= 2\mu t\cos r - \frac{\lambda}{2}$

Case.1: **<u>condition for maximum intensity</u>**

We know that the intensity is maximum when path difference= $n\lambda$

$\therefore$ From Eq.(6) $2\mu t\cos r - \frac{\lambda}{2} = n\lambda$

$$\boxed{2\mu t\cos r = (2n+1)\frac{\lambda}{2}}$$

Case.2: **<u>condition for minimum intensity</u>**

We know that the intensity is minimum when path difference $=(2n+1)\frac{\lambda}{2}$

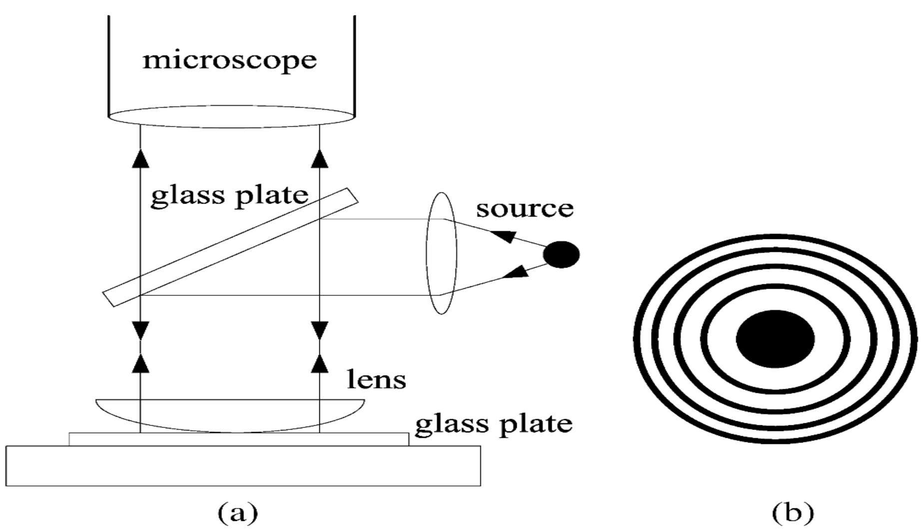
$\therefore$ from Eq.(6) $2\mu t\cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$$\boxed{2\mu t\cos r = (n+1)\lambda}$$

<u>NEWTON'S RING EXPERIMENT</u>

A Plano convex lens(L) having large focal length is placed with its convex surface on the glass plate(G_2).a gradually increasing air film will be formed between the plane glass plate and convex surface of Plano convex lens. The thickness of the air film will be zero at the point of contact and symmetrically increases as we go radially from the point of contact.

A monochromatic light of wavelength 'λ' is allowed to fall normally on the lens with the help of glass plate (G_1) kept at 45^0 to the incident monochromatic beam. A part of the incident light rays are reflected up at the convex surface of the lens and the remaining light is transmitted through the air film. Again a part of this transmitted light is reflected at on the top surface of the glass plate (G_1).both the reflected rays combine to produce an interference pattern in the form of alternate bright and dark concentric circular rings, known as Newton rings. The rings are circular because the air film has circular symmetry. These rings can be seen through the travelling microscope.



THEORY

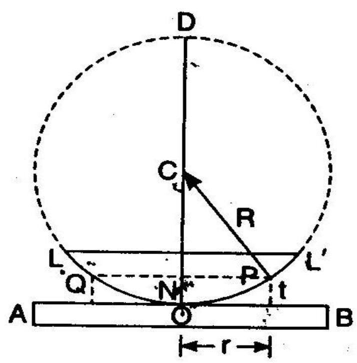
Consider a Plano convex lens is placed on a glass plate. Let R be the radius of curvature and r be the radius of NEWTON ring, corresponding to constant film thickness.

As one of the rays suffers reflection at denser medium, so a further phase changes of π or path difference of $\frac{\lambda}{2}$ takes place.

The path difference between the rays =2μt cos r + $\frac{\lambda}{2}$ ---------------------------- **(i)**

For air $\mu = 1$, and normal incidence $r = 0$

$\therefore$ **Path difference** $= 2t + \frac{\lambda}{2}$



AT THE POINT OF CONTACT

The thickness of the air film t=0, μ=1 & for normal incidence r = 0.

Then the path difference = $\frac{\lambda}{2}$.

If the Then the path difference = $\frac{\lambda}{2}$ **then the corresponding phase difference is π.so that gives a dark spot is formed at the centre.**

For bright ring

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1)\frac{\lambda}{2} \quad \text{----------------------------------- (ii)}$$

For Dark ring

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2t = n\lambda \quad \text{--- (iii)}$$

In the above fig, from the property of the circle

$$NP \times NQ = NO \times ND$$

$$r \times r = 2t \times (2R - t)$$

$$r^2 = 2Rt - t^2$$

As t is small, t^2 is very small. So t^2 is neglected.

$$\therefore \; r^2 = 2Rt$$

$$t = \frac{r^2}{2R} \quad \Rightarrow \quad t = \frac{D^2}{8R} \text{ ------------------------ (iv)}$$

Thus for bright ring

From Eq (ii) & (iv)

$$\frac{2D^2}{8R} = (2n-1)\frac{\lambda}{2}$$

$$\boxed{D_n^2 = 2(2n-1)\lambda R} \text{ -- (v)}$$

Thus for dark ring

From Eq. (iii) & (iv)

$$\frac{2D^2}{8R} = n\lambda$$

$$D_n^2 = 4Rn\lambda \text{ .... (vi)}$$

$$\boxed{D_n^2 = 4Rn\lambda}$$

Determination of wave length of monochromatic light

From Eq(vi) $\quad D_n^2 = 4Rn\lambda$

For $n = m$, $\quad D_m^2 = 4Rm\lambda$

$$\therefore \; D_m^2 - D_n^2 = 4Rm\lambda - 4Rn\lambda \quad = 4R\lambda(m-n)$$

$$\therefore \quad \lambda = \frac{D_m^2 - D_n^2}{4R(m-n)} \text{ ------------------------------------(vii)}$$

This is the expression for wave length of monochromatic light.

Determination of refractive index of a liquid

The experimental set up as shown in fig. is used to find the refractive index of a liquid.

To find the refractive index of a liquid, the plane glass plate and Plano convex lens set up is placed in a small metal container. The diameter of n^{th} and m^{th} dark rings are determined, when there is air between Plano convex lens and plane glass plate.

Then we have,

$$D_m^2 - D_n^2 = 4Rm\lambda - 4Rn\lambda$$
$$= 4R\lambda(m-n).$$

Now the given liquid whose refractive index (μ) is to be introduced in to the space between Plano convex lens and plane glass plate without disturbing the experimental set up.

Then the diameters of Newton's rings are changed. Now the diameter of n^{th} and m^{th} dark rings are measured.

Then $D_m^2 - D_n^2$ **= 4Rλ (m-n)/μ--------------------------- (viii)**

Therefore from (vii) & (viii)μ= $\underline{D_m^2 - D_n^2}$

CONDITIONS TO GET STATIONARY INTERFERENCE FRINGES

1. The two sources should be coherent.
2. The two sources must emit continuous waves of the same wavelength and same frequency.
3. The distance between the two sources (d) should be small.
4. The distance between the sources and the screen (D) should be large.
5. To view interference fringes, the back ground should be dark.
6. The amplitude of interfering waves should be equal.
7. The sources must be narrow, i.e., they must be extremely small.
8. The source must be monochromatic source.

Production of Colors in thin films:

With monochromatic light alternate dark and bright interference fringes are obtained. With white light, the fringes obtained are colored. it is because the path difference $2\mu t\cos r - \frac{\lambda}{2}$ depends upon $\mu, t \,\&\, r$

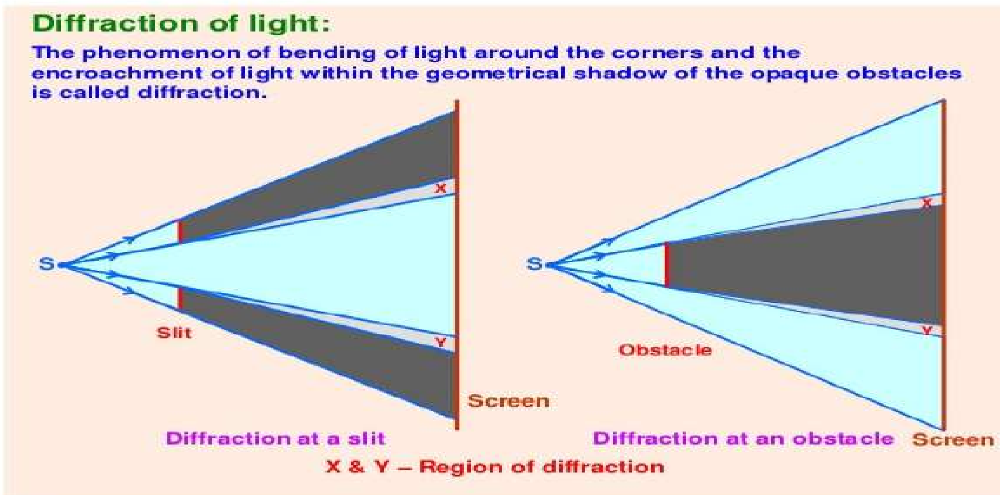
(i) Even if t and r kept constant, the path difference will change with $\mu \,\&\, \lambda$ of light used. White light composed of various colors from violet to red. The path difference also changes due to reflection at denser medium by $\frac{\lambda}{2}$ as $\lambda_V \langle \lambda_R$.

(ii) If the thickness of the film varies with uniformly, if at beginning it is thin, which will appear black. as path difference varies with thickness of the film, it appears different colors with white light.

(iii) If the angle of incidence changes, the angle of refraction is also changes, so that with white light, the film appears various colors when viewed from different directions.

DIFFRACTION

"When light is incident on the obstacles or small apertures whose size is comparable to wavelength of light, then there is a departure from straight line propagation, the light bends round the corners of the obstacles and enters into geometrical shadow. This bending of light is called diffraction."

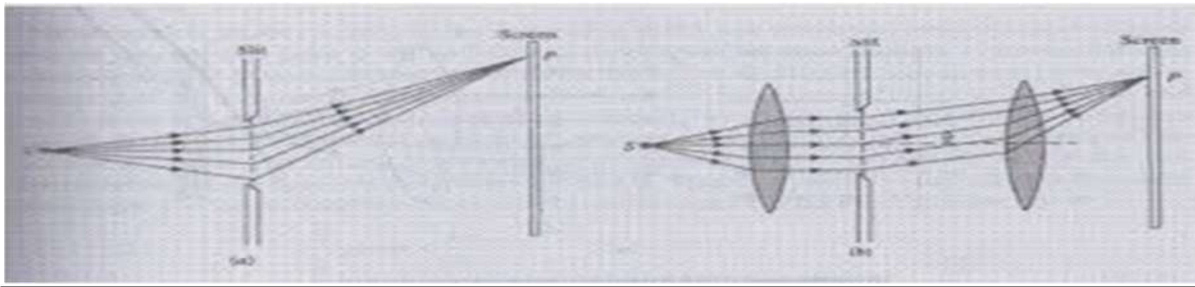


Differences between Interference and diffraction

INTERFERENCE	DIFFRACTION
1. Superposition is due to two separate wave fronts originating from two coherent sources.	1. Superposition is due to secondary wavelets originating from different parts of same wave front.
2. Interference fringes may or may not be of same width.	2. Diffraction fringes are not of the same width
3. Points of minimum intensity are perfectly dark	3. Points of minimum intensity are not perfectly dark.
4. All bright bands are of uniform intensity	4. All bright bands are not of same intensity.

There are two types of Diffractions are there, they are
1. Fresnel Diffraction
2. Fraunhofer Diffraction

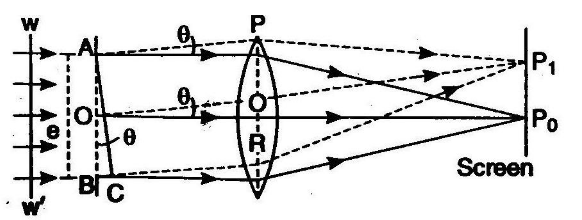
Fresnel diffraction **Fraunhofer diffraction**



Differences between Fresnel Diffraction and Fraunhofer Diffraction

Fresnel Diffraction	Fraunhofer Diffraction
1. Eighter a point source or an illuminated narrow slit is used.	1. Extended source at infinite distance is used.
2. The wave front undergoing diffraction is either spherical or cylindrical.	2. The wave front undergoing diffraction is plane wave front.
3. The source and screen are at finite distances from the obstacle.	3. The source and screen are at infinite distances from the obstacle.
4. No lens is used to focus the rays.	4. Converging lens is used to focus the rays.

FRAUNHOFER DIFFRACTION AT SINGLE SLIT:



Consider a slit AB of width "e" and a plane wave front WW^1 of monochromatic light of wavelength "λ" is incident normally on the slit. The diffracted light through the slit is focused with the help of a convex lens on a screen. The screen is placed at the focal plane of the lens. Here the secondary wave lets spared out to the right in all directions.

The waves travelling along OP_o are brought out to focus at P_o by the lens. Hens P_O is the bright central image.

The secondary wavelets at angle "θ" with normal are focused at P_1 on the screen. Depending upon path difference, P_1onmay be of maximum (or) minimum intensity point.

To find intensity at P_1 we drawn a normal AC from A to the light ray at B the path difference between the wave lets from A and B in the direction "θ" is given by

From $TraingleABC,\ \sin\theta = \frac{BC}{AB} \Rightarrow BC = AB\sin\theta = e\sin\theta$

$\therefore$Phase difference = $\frac{2\pi}{\lambda} e \sin\theta$

Let us consider the width of the slit is divided into 'n' equal parts. Then the phase difference between any two consecutive waves from these parts would be.

$$\frac{1}{n}(total.phase) = \frac{1}{n}\left(\frac{2\pi}{\lambda}.e\sin\theta\right) = d \quad \text{(say)}$$

$\because$ **Resultant amplitude** $R = \dfrac{a\sin\frac{nd}{2}}{\sin\frac{nd}{2}}$

$$\therefore R = \frac{a\sin\left(\frac{n}{2}\times\frac{2\pi}{n\lambda}.e\sin\theta\right)}{\sin\left(\frac{2\pi}{2n\lambda}.e\sin\theta\right)}$$

$$= \frac{a\sin\left(\frac{\pi e\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e\sin\theta}{n\lambda}\right)}$$

Let $\alpha = \frac{\pi e\sin\theta}{\lambda}$. Then

$$R = \frac{a\sin\alpha}{\sin\frac{\alpha}{n}}$$

As $\frac{\alpha}{n}$ is small, $\sin\frac{\alpha}{n} \approx \frac{\alpha}{n}$

$$\therefore R = \frac{a\sin\alpha}{\frac{\alpha}{n}} = na\frac{\sin\alpha}{\alpha}$$

Now the intensity $$\boxed{\mathrm{I} = \mathrm{R}^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \text{.......(1)}}$$

<u>Principal Maximum:</u>

$$R = A\frac{\sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right]$$

The value of R will be maximum, when α=0, i.e $\frac{\pi e\sin\theta}{\lambda} = 0$ or $\sin\theta = 0$

Or $\theta = 0$

$\therefore$ Maximum intensity $I=R^2=A^2$, this is occurred at $\theta = 0$, this maximum is known as principal maximum.

Minimum intensity Positions:

The intensity will be minimum, when sin α=0.

$$\therefore \alpha = \pm\pi, \pm 2\pi, \pm 3\pi, ...\pm m\pi$$

$$\alpha = \pm m\pi$$

$$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{e\sin\theta = \pm m\lambda}$$

In this way, we obtain the points of min. inf. on either side of the principle maxima.

Secondary maxima:

In addition to principle maxima at α=0. There are weak secondary maxima between equally spaced minima. The points of secondary maxima obtained as follows.

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\frac{dI}{d\alpha} = A^2.2\frac{\sin\alpha}{\alpha}.\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

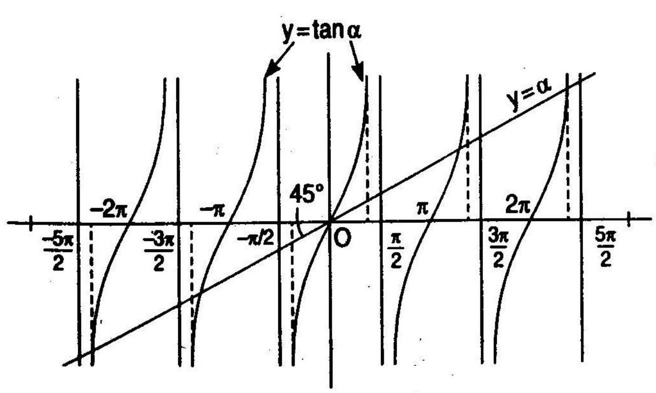
From above either sin α=0, or α cosα -sin α =0 if sin α=0, it is min. intensity position. Hence positions of maximum are obtained by

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha\cos\alpha = \sin\alpha$$

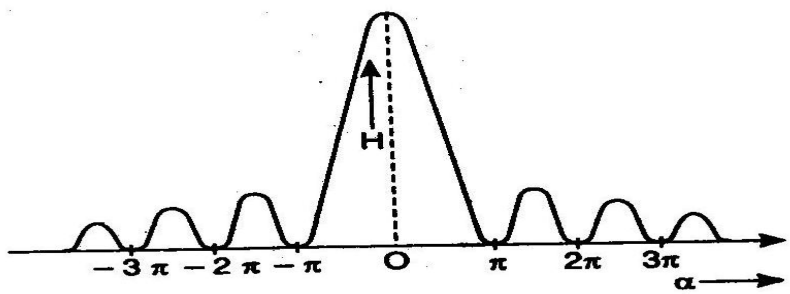
$$\boxed{\alpha = \tan\alpha} \quad \text{----------------} \ (2)$$

The values of α satisfying the above equation are obtained graphically by plotting curves $y = \alpha$, $y = \tan\alpha$ on the same graph. The points of intersection of two curves give the values of α which satisfy the equation (2)

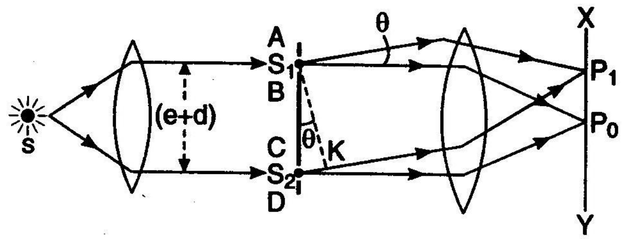


From fig The points of intersections are $\alpha = 0, \pm 3\frac{\pi}{2}, \pm 5\frac{\pi}{2}, ...,$ at these points we get secondary maxima

Intensity distribution graph



FRAUN HOFFER DIFFRACTION AT DOUBLE SLIT:



Let A B and CD be two parallel slits of equal width 'e' separated by an opaque distance d . The distance between the corresponding middle points of the two slits is $(e+d)$. Let a parallel beam of monochromatic beam of wave length λ be incident normally upon to the two slits. .

When a wave front is incident normally on both slits all the points with in the slits become the sources of secondary wavelets. The secondary waves traveling in the direction of incident light come to focus at P_o while the secondary waves traveling in the direction making an angle with θ the incident light come to focus at P_1.

According to the theory of diffraction at a single slit. The amplitude R due to all the wavelets diffracted from each slit in a dissection θ is given by.

$$R = A\frac{\sin\alpha}{\alpha} \text{ where } \alpha = \frac{\pi e \sin\theta}{\lambda}$$

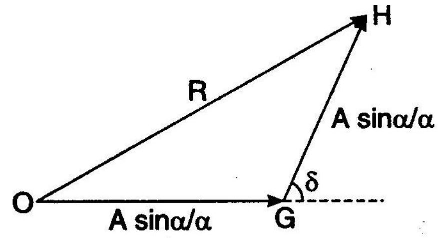
Thus for simplicity we can take two slits as equivalent to two sources S_1 and S_2 placed at mid points of the slits and each slit sending a wavelet of amplitude $\frac{A\sin\alpha}{\alpha}$ in the direction θ.

$\therefore$ Resultant amplitude at a point P_1 on the screen will be a result of interference between two waves of amplitude $\frac{A\sin\alpha}{\alpha}$ and having a phase difference.

The path difference between the wavelets from S_1 and S_2 in the dissection θ = S_2k.

$$path.difference = (e+d)\sin\theta$$

$$\therefore \; phase.difference(\delta) = \frac{2\pi}{\lambda}(e+d)\sin\theta$$



From figure $R\cos\theta = \frac{A\sin\alpha}{\alpha} + \frac{A\sin\alpha}{\alpha}\cos\delta$ ....(1)

$R\sin\theta = \frac{A\sin\alpha}{\alpha}\sin\delta$ ……………………..(2)

Squaring & adding eq(1)&(2)

$$I = R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2\cos\delta + 2\left(\frac{A\sin\alpha}{\alpha}\right)^2\cos\delta + \left(\frac{A\sin\alpha}{\alpha}\right)^2\sin\delta$$

$$= \left(\frac{A\sin\alpha}{\alpha}\right)^2 [2 + 2\cos\delta]$$

$$= 2\left(\frac{A\sin\alpha}{\alpha}\right)^2 [1 + \cos\delta]$$

$$= 2\left(\frac{A\sin\alpha}{\alpha}\right)^2 2\cos^2\frac{\delta}{2}$$

$$I = 4\left(\frac{A\sin\alpha}{\alpha}\right)^2 \cos^2\beta \text{........(3)} \text{, Where } \beta = \frac{\delta}{2} = \frac{\pi}{\lambda}(e+d)\sin\theta$$

Discussion of Intensity:

From equation (3) the resultant intensity depending upon the following two factors.

1. $A^2\frac{\sin^2\alpha}{\alpha^2}$ Which is same as the intensity in the case of single slit diffraction thus it gives intensity distribution in the diffraction pattern.
2. $\cos^2\beta$ Which gives the intensity pattern due to two waves interfere.

The resultant intensity at any point on the screen is given by the product of these two factors.

$\therefore$ Diffraction term $\frac{\sin^2\alpha}{\alpha^2}$ gives the

(i)Central maximum at $\theta = 0$

(ii)Minimum intensity positions $\alpha = \pm m\pi$

$$\frac{\pi e\sin\theta}{\lambda} = \pm m\pi$$

$$e\sin\theta = \pm m\lambda$$

.

(iii) .Secondary maxima obtained at $\alpha = \pm 3\frac{\pi}{2}, \pm 5\frac{\pi}{2}, \ldots\ldots\ldots\ldots\ldots$

On taking these three points plotted as graph as shown in the fig(a).

The interference term $\cos^2\beta$ gives the maximum

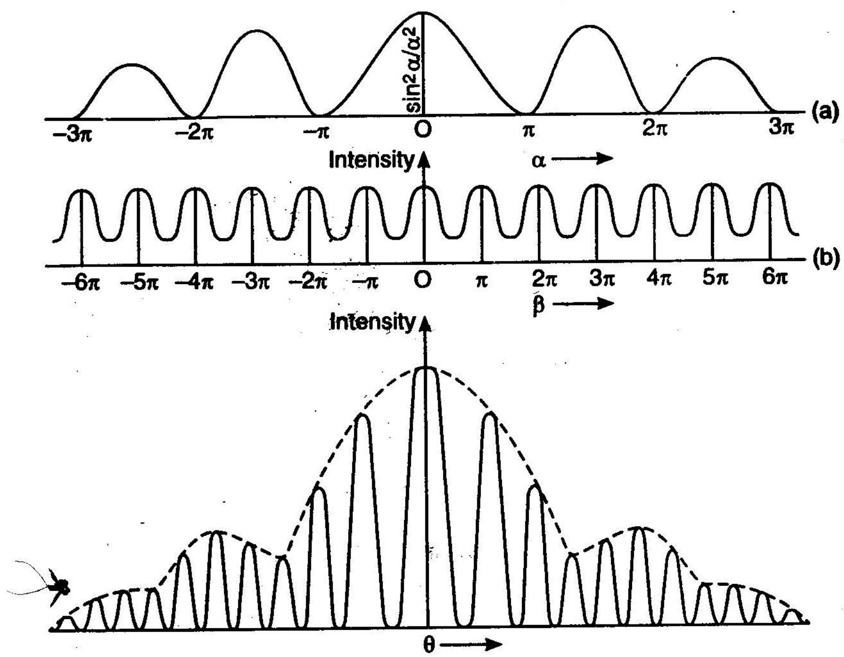
$$\cos^2\beta = 1 \Rightarrow \beta = \pm m\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

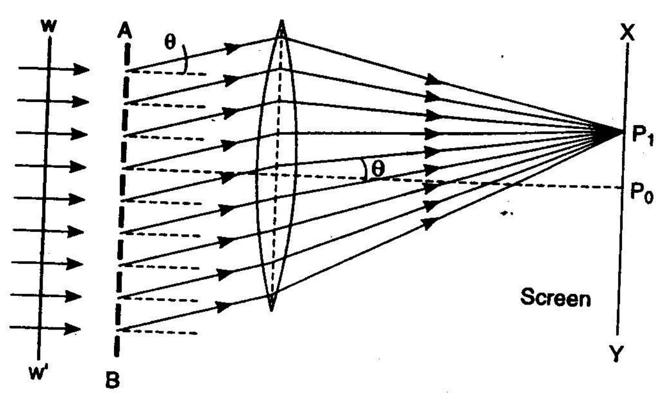
$$(e+d)\sin\theta = \pm m\lambda$$

This is plot as shown in fig.(b)

The resultant intensity graph is as shown in fig. (c)



Diffraction at N-Parallel slits [Diffraction grating]



An arrangement consists of large no. of parallel slits of same width and separated by equal opaque spaces is known as diffraction grating.

If there are N slits.

The path difference between any two consecutive slits is $=(e+d)\sin\theta$

$\therefore$ Phase difference = $\frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta$

By the method of vector addition of amplitudes

$$R = \frac{a \sin \frac{nd}{2}}{\sin \frac{d}{2}}$$

In this case $a = \frac{A \sin \alpha}{\alpha}$, $n = N$ and $d = 2\beta$

$$\therefore\ R = \frac{A \sin \alpha}{\alpha} . \frac{\sin N\beta}{\sin \beta}$$

$$I = R^2 = \left(\frac{A \sin \alpha}{\alpha} \right)^2 \frac{\sin \beta}{\beta} \left(\frac{\sin N\beta}{\sin \beta} \right)^2$$

The factor $\left(\frac{A \sin \alpha}{\alpha} \right)^2$ gives the distribution of intensity due to single slit. While the factor $\left(\frac{\sin N\beta}{\sin \beta} \right)^2$ gives the distribution of intensity as a combined effect of all the slits.

Principle maxima:

The intensity will be maximum when $\sin \beta = 0$

$$\beta = \pm n\pi, n = 0, 1, 2, 3, ...$$

But at the same time Sin N β = 0. So that the factor $\left(\frac{\sin N\beta}{\sin \beta} \right)$ becomes indeterminate.

$$\therefore \lim_{\beta \to n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \to n\pi} \frac{N \cos N\beta}{\cos \beta} = \pm N$$

$$\lim_{\beta \to n\Pi} \left(\frac{\sin N\beta}{\sin \beta} \right)^2 = N^2$$

$\therefore$ The Resultant intensity $I = \left(\frac{A \sin \alpha}{\alpha} \right)^2 N^2$

i.e. The principle maxima obtained for $\beta = \pm n\pi$

$$\frac{\pi (e+d) \sin \theta}{\lambda} = \pm n\pi$$

$$\boxed{(e+d) \sin \theta = \pm n\lambda}$$

Minimum Intensity Positions:

Intensity I is the minimum when $\sin N\beta = 0$, but $\sin \beta \neq 0$

$$\therefore N\beta = \pm \pi, \pm 2\pi, \pm 3\pi, ...$$

$$\frac{N\pi (e+d) \sin \theta}{\lambda} = \pm m\pi$$

$$\boxed{N(e+d) \sin \theta = \pm m\lambda}$$

Where m having all values except

$0, N, 2N, ...nN.$

$i.e, m = 1, 2, ...(N-1), (N+1), ...(2N-1), (2N+1), ...$

Secondary maximum:

I maximum when

$$\frac{dI}{d\beta} = 0$$

$$\frac{d}{d\beta}[(A\frac{\sin\alpha}{\alpha})^2(\frac{\sin N\beta}{\sin\beta})^2] = 0$$

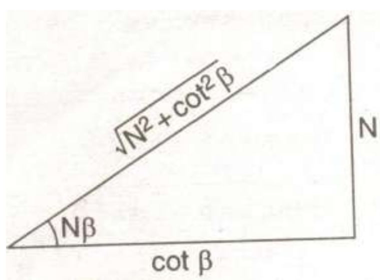
$$(\frac{A\sin\alpha}{\alpha})^2 2[\frac{\sin N\beta}{\beta}][\frac{N\sin\beta\cos N\beta - \sin N\beta\cos\beta}{\sin^2\beta}] = 0$$

$$N\sin\beta\cos N\beta - \sin N\beta\cos\beta = 0$$

$$N\sin\beta\cos N\beta = \sin N\beta\cos\beta$$

$$N\sin\beta = \cos\beta(\frac{\sin N\beta}{\cos N\beta})$$

$$\tan N\beta = \frac{N}{\cot\beta}$$



$$\therefore \sin N\beta = \frac{N}{\sqrt{(N^2 + \cot^2\beta)}}$$

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{(N^2 + \cot^2\beta)\sin^2\beta}$$

$$= \frac{N^2}{N^2\sin^2\beta + \cos^2\beta}$$

$$= \frac{N^2}{N^2\sin^2\beta + 1 - \sin^2\beta}$$

$$\therefore \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{1 + (N^2 - 1)\sin^2\beta}$$

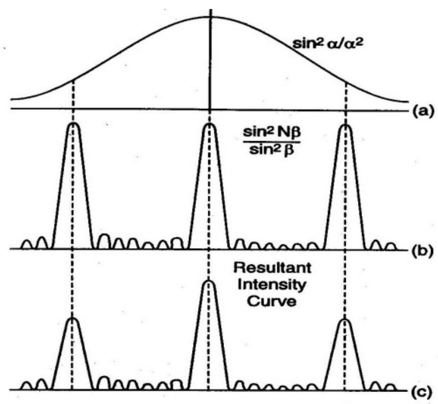
$$I_{\sec} = (A\frac{\sin\alpha}{\alpha})^2[\frac{N^2}{(N^2-1)\sin^2\beta + 1}$$

$$\therefore \frac{\text{intensity.of.secondary maxima}}{\text{intensity.of.principle maxima}}$$

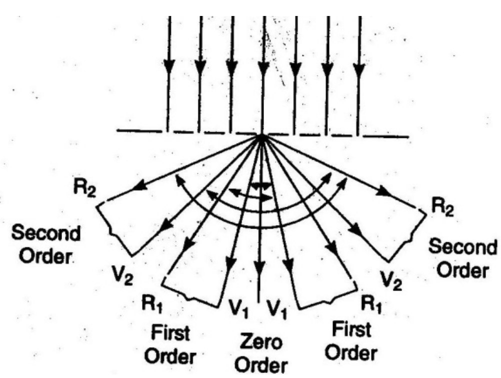
$$= \frac{N^2}{(1+(N^2-1)\sin^2\beta)\times N^2}$$

$$\therefore \frac{\text{intensity.of.secondary maxima}}{\text{intensity.of.principle maxima}} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

From this we conclude that as the value of N increases the intensity of secondary maxima will decreases



GRATING SPECTRA



We know that the principle maxima in a grating are formed in a direction θ is given by

$$(e+d)\sin\theta = \pm n\lambda$$

Where $(e+d)$ grating element is θ is the angle diffraction and λ is Wave length

From the above equation, we conclude that

1. For a particular wave length λ, the angle of diffraction θ is different for different orders.
2. For white light and for an order n the light of different wave lengths will be diffracted in different directions. The longer the wavelength, greater is the angle of diffraction. So violet color being in the innermost position and red color in the outermost position.
3. Most of the intensity goes to zero order and rest is distributed among other orders thus the spectra become fainter as we go to higher orders.

Characteristics of grating spectra

1. Spectrum of different orders are situated symmetrically on both sides of zero order
2. Spectral lines are almost straight and quite sharp.
3. Spectral colors are in the order from violet to red.
4. Most of the intensity goes to zero order and rest is distributed among the other orders.

Maximum no. orders available with a grating

The principle maxima in grating satisfying the condition

$$(e+d)\sin\theta = n\lambda$$

$$n = \frac{(e+d)\sin\theta}{\lambda}$$

$$n_{\max} = \frac{(e+d)\sin 90^0}{\lambda}$$

$$n_{\max} = \frac{(e+d)}{\lambda}$$

DISPERSIVE POWER OF GRATING:

The dispersive power of grating is defined as the rate of variation of angle of diffraction with wavelength i.e., $\frac{d\theta}{d\lambda}$ is known as dispersive power of grating.

The condition for maxima is (e +d) sin θ = nλ

On differentiation we get (e+d) cosθ dθ = n dλ

=

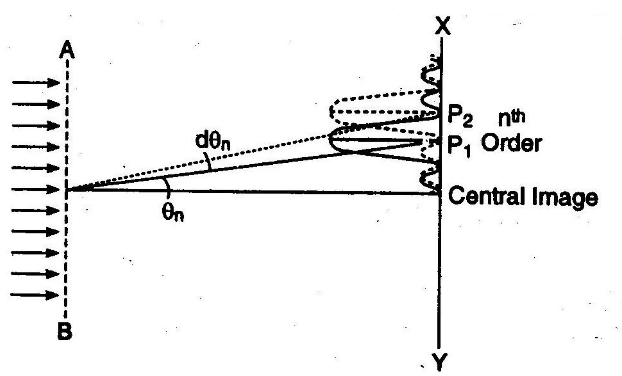
This is the expression for dispersive power of grating.

Conclusions :

- The dispersive power is directly proportional to diffraction order n.
- The dispersive power is inversely proportional to grating element (e+d).
- The dispersive power is inversely proportional to cosθ.

RESOLVING POWER OF GRATING:

The resolving power of a grating is defined as the capacity to form separate diffraction maxima of two wave lengths which are very close to each other



Let A B be a plane grating having grating element $(e+d)$ and N be the total no. of slits. let a beam of wavelengths λ and $\lambda + d\lambda$ is normally incident on the grating in the fig P_1 is the n_{th} primary maximum of wavelength λ at an angle of diffraction θ_n and P_2 is the n_{th} primary maximum of wavelength $\lambda + d\lambda$ at an angle of diffraction $(\theta_n + d\theta_n)$.

According to Rayleigh's criterion, the two wave lengths will be resolved if the principle maximum of one falls on the first minimum of the other.

The principle maximum of λ in the direction θ_n is given by

$$(e+d)\sin\theta_n = \pm n\lambda \text{..........(1)}$$

The wave length $(\lambda + d\lambda)$ form its n_{th} primary maxima in the direction $(\theta_n + d\theta_n)$

$$(e+d)\sin(\theta_n + d\theta_n) = \pm n(\lambda + d\lambda) \text{............(2)}$$

The first minimum of wave length λ from in the direction $(\theta_n + d\theta_n)$

$$N(e+d)\sin(\theta_n + d\theta_n) = (nN+1)\lambda \text{............(3)}$$

Multiplying eq(2) with N

$$N(e+d)\sin(\theta_n + d\theta_n) = \pm nN(\lambda + d\lambda) \text{...........(4)}$$

From (3) & (4)

$$Nn(\lambda + d\lambda) = (nN+1)\lambda$$

$$nN\lambda + nNd\lambda = nN\lambda + \lambda$$

$$nNd\lambda = \lambda$$

$$\frac{\lambda}{d\lambda} = nN$$

But from eq (1) $n = \dfrac{(e+d)\sin\theta_n}{\lambda}$

$\therefore \text{Re}\, solving.power.of.grating$ $\boxed{\dfrac{\lambda}{d\lambda} = \dfrac{N(e+d)\sin\theta_n}{\lambda}}$

PREVIOUS QUESTIONS

1. What is meant by diffraction of light? Explain on the basis of Huygens wave theory.
2. Explain with necessary theory, the Fraunhofer diffraction due to 'n' slits.

   (Or)

   Give the theory of plane diffraction grating. Obtain the condition for the formation of nth order maximum.
3. Distinguish between Interference and Diffraction.

   (Or)

   How is diffraction different from Interference?
4. Calculate the maximum number of orders possible for plane diffraction grating.
5. Write notes on Rayleigh's criterion.
6. Distinguish between Fresnel and Fraunhoffer diffractions.
7. Define Resolving power of grating. Derive the expression for Resolving power of a grating based on Rayleigh's criterion.
8. Describe the action of plane transmission grating in producing diffraction spectrum.
9. Show that grating with 500lines/cm cannot give a spectrum in 4^{th} order for the light of wave length 5890 A^0.

POLARIZATION

Interference and diffraction are the phenomenon which confirmed the wave nature of light. But the phenomenon could not establish whether light waves are longitudinal (or) transverse.
When the phenomenon of polarization was discovered it was established that light waves are transverse in nature.

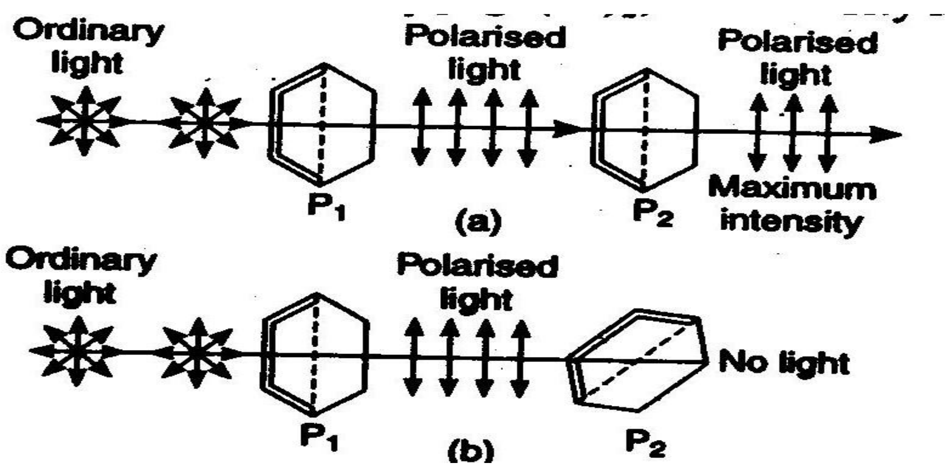
Polarization is a property of waves that describes the orientation of their oscillations. In a transverse wave if all the vibrations are confined in a single direction, it is said to be polarized.

Polarization: It is the process of converting ordinary light into polarized light.

Polarized wave: the wave which is unsymmetrical about the direction of propagation is called polarized wave.

Polarized light: The light which has acquired the property of **one sidedness** is called polarized light

POLARIZATION OF LIGHT WAVES



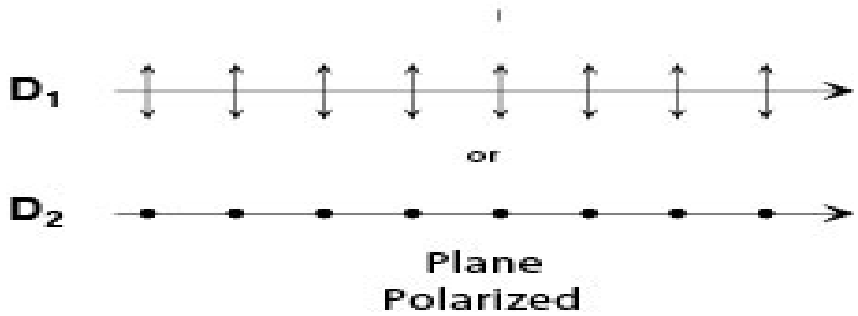
When a ordinary light is passed through a pair of tourmaline crystal plates with their planes parallel to each other, then the maximum intensity is obtained. When their planes perpendicular to each other, the intensity is zero. This shows that light is a transverse wave motion

TYPES OF POLARIZED LIGHT

There are three different types of polarized light.

1. Plane polarized light
2. Circularly polarized light.
3. Elliptically polarized light.

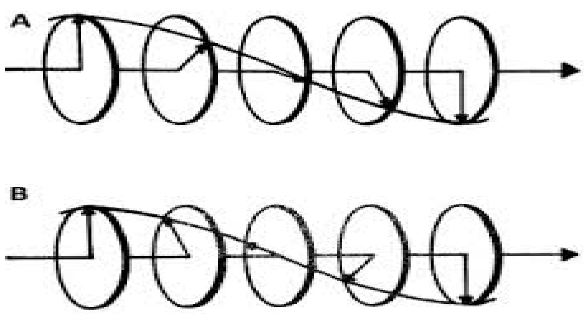
1. Plane polarized light: When the vibrations of light are confined along a single direction, the light is said to be plane polarized light. (Eighter in the direction along the plane of the paper (or) in the direction along the perpendicular to the plane of the paper)

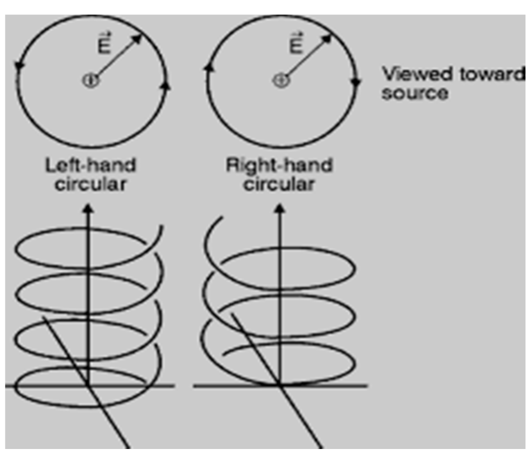


2. Circularly polarized light:

The projection of a wave on a plane intercepting the axis of propagating gives a circle with the amplitude vector remaining constant.

I.e. The vector rotates in the clock wise direction with respect to the direction of propagation; it results in right, circularly polarized light while the rotation anti-clock wise direction results in left circularly polarized light.

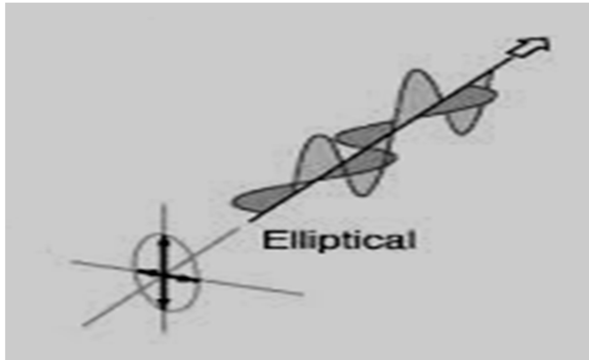




If the vibrations are along a circle, the light is said to be circularly polarized light.

3. Elliptically polarized light:

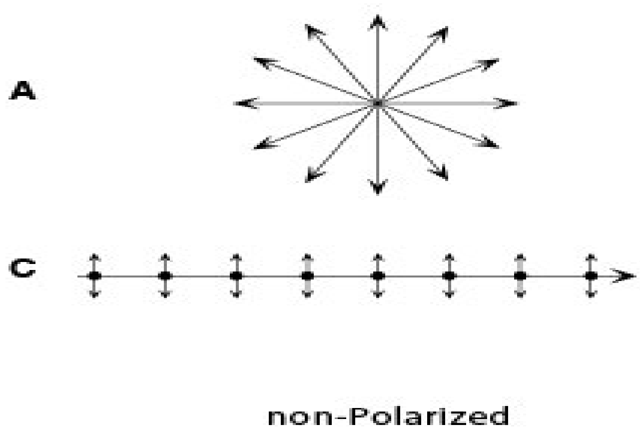
The projection of a wave on a plane intercepting the axis of propagating gives a ellipse and amplitude vector is not constant but various periodically.



If the vibrations are along an ellipse, the light is said to be elliptically polarized light.

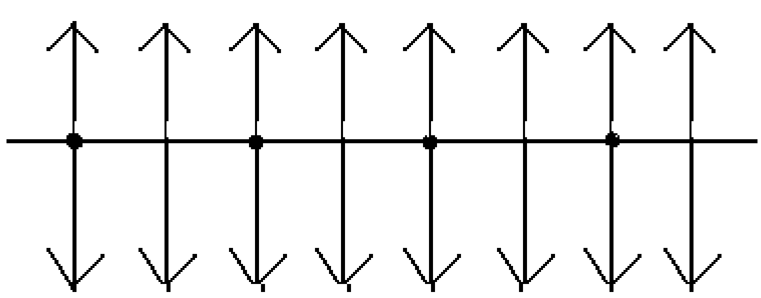
Unpolarized light:

Unpolarized light (or) ordinary light has vibrations both parallel and perpendicular to the plane of the paper.

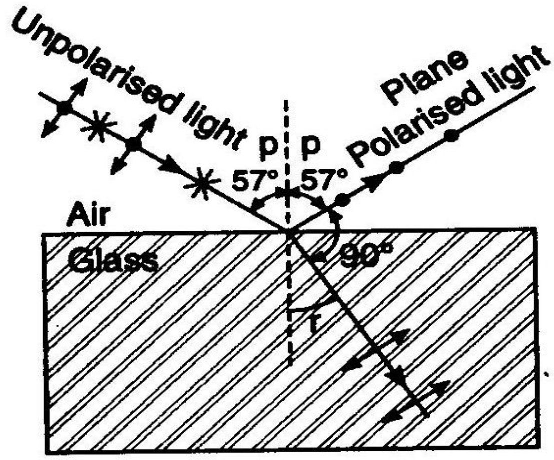


Partially polarized light:

If the linearly polarized light contains small additional component of unpolarised light it becomes partially plane polarized light.



1. POLARIZATION BY REFLECTION (BREWSTER'S LAW)



Brewster observed that for a particular angle of incidence is known as angle of polarization .The refracted light is completely plane polarized in the plane if incidence.

Brewster proved that the tangent of the angle of polarization (P) is numerically equal to refractive index of material.

$$\boxed{\mu = \tan P}$$

This is known as Brewster's law.

He also proved that the reflected and refracted rays are perpendicular to each other.

The angle between reflected and refracted rays

From Brewster's law

$$\mu = \tan p ....(1)$$

From Snell's law

$$\mu = \frac{\sin p}{\sin r} ....(2)$$

$$from (1) and (2)$$

$$\frac{\sin p}{\cos p} = \frac{\sin p}{\sin r}$$

$$\cos p = \sin r$$

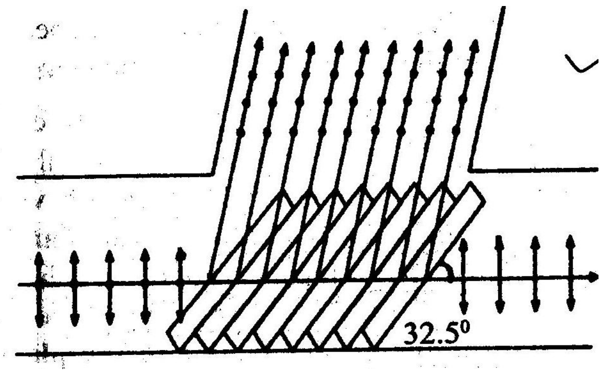
$$\cos p = \cos(90 - r)$$

$$p = 90 - r$$

$$p + r = 90$$

The angle between reflected and, refracted ray is 90^o

2. POLARIZATION BY REFRACTION (PILE OF PLATES)

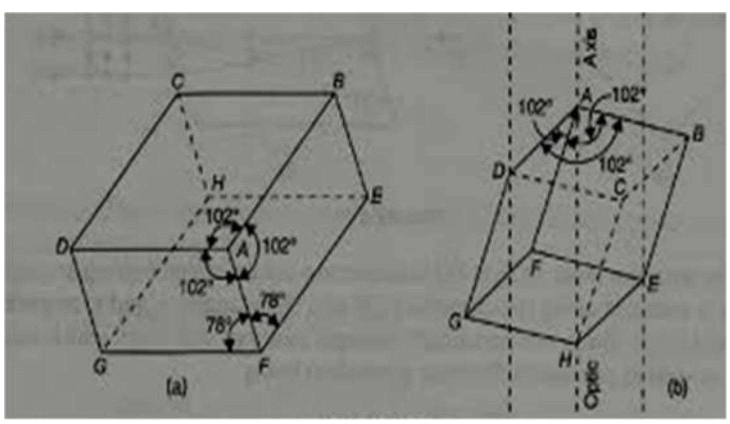


We know that when unpolarized light is incident at polarizing angle the reflected light is completely plane polarized and transmitted light contains a greater proportion of light vibrating parallel to the plane of incidence. if the process of reflection at polarizing angle is repeated using no. of plates all inclined at polarizing angle, finally the transmitted light become s purely plane polarized. Such an arrangement is known as pile of plates.

GEOMETRY OF CALCITE CRYSTAL

Calcite is a transparent color less crystal. Chemically it is hydrated calcium carbonate. It was at one time found in large quantities in ICELAND. Hence it is also known as ICELAND SPAR.

It consist six faces of parallegrams having angles of 102^0 and 78^0. The corners A and H are said to be blunt corners. The other corners of the crystal consist of two acute and one obtuse angle.



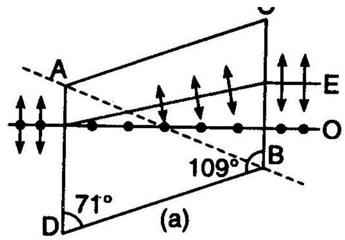
Blunt corner:

The corner where three obtuse angles meet .that corner is called Blunt corner

Principle axis:

It is the line passing through the any one of the blunt corner and making equal angles with the three faces which meet at this corner.

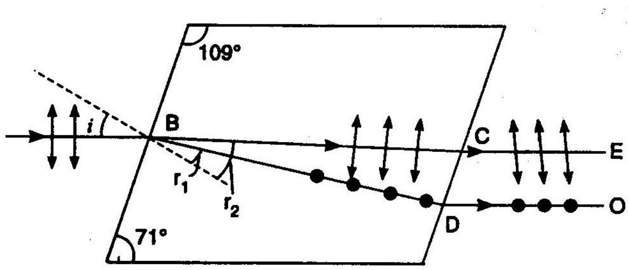
Principle section:



Any plane which contains principle axes and is perpendicular to two opposite faces is called a principle section.

3. BIREFRINGENCE (POLARIZATION BY DOUBLE REFRACTION):

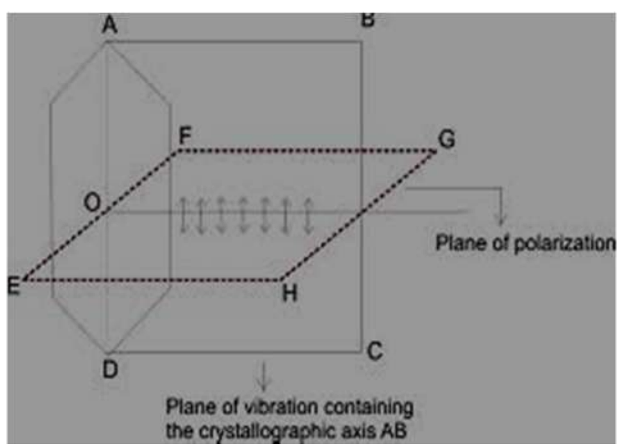
When a beam of ordinary un polarized light is passed through a calcite crystal, the refracted light split into two refracted rays .this phenomenon is called double refraction



Among the two rays one which always obey the ordinary laws of refraction and having vibrations perpendicular to the principle section is known as ordinary ray and The other which do not obey general laws of refraction and having vibrations in the principle section is called extraordinary ray .

The crystals showing this phenomenon are known as doubly refracting crystals or **Birefringent crystals**.

PLANE OF POLARIZATION & PLANE OF VIBRATION:



When ordinary light is passed through a tourmaline crystal, the light is polarized and the vibrations are confined only in one direction which is perpendicular to the direction of propagation of light

The plane in which the vibrations of polarized light are confined .this plane is known as plane of vibration. This plane contains the direction of vibration as well as direction of propagation

The plane which has no vibrations the plane is known as plane of polarization. Thus a plane passing through the direction of propagation and perpendicular to the plane of vibration is known as plane of polarization

HUYGEN'S THEORY OF DOUBLE REFRACTION

1. When any wave front strikes a double refracting crystal, every point of the crystal becomes a source of two wave fronts

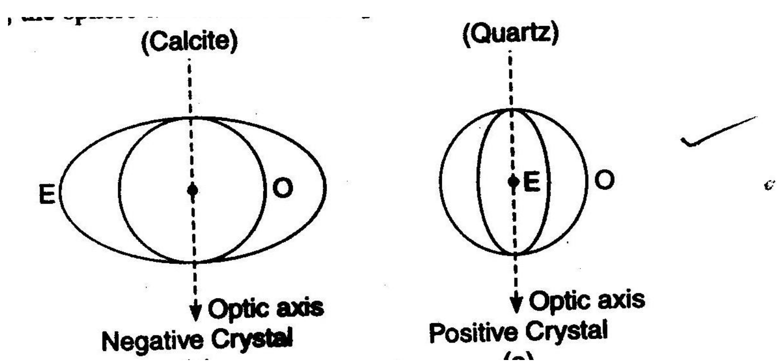
2. Ordinary wave front is spherical, because ordinary have same velocity in all directions

3. Extraordinary wave front is elliptical, because E-Ray has different velocities in different directions

4. The sphere and ellipsoid are touch each other along optic axis because the velocity of ordinary and extraordinary rays is same along optic axis

4. The crystal in which the velocity of ordinary is grater than extraordinary ray. That crystal are called Positive crystals

5. The crystals in which the velocity of extra ordinary is grater than ordinary ray .that crystals are called Negative crystal



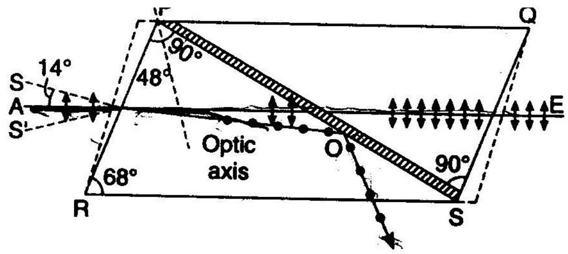
NICOL PRISM

When an ordinary light is transmitted through a calcite crystal, it splits into ordinary and extraordinary rays. Nicol eliminated the ordinary beam by utilizing the phenomenon of total internal reflection at Canada balsam separating the two pieces of calcite .this device is called NICOL PRISM

Construction:

A calcite crystal whose length is three times as that of its width is taken .The end faces of this crystal are grounded in such a way that the angle in the principle section becomes 68^0 and 112^0 .Then calcite crystal cut into two pieces . The cut surfaces are grounded and polished optically flat and then cemented together by Canada balsam. The refractive index of Canada balsam lies between refractive indices of O-ray and E-ray. i.e

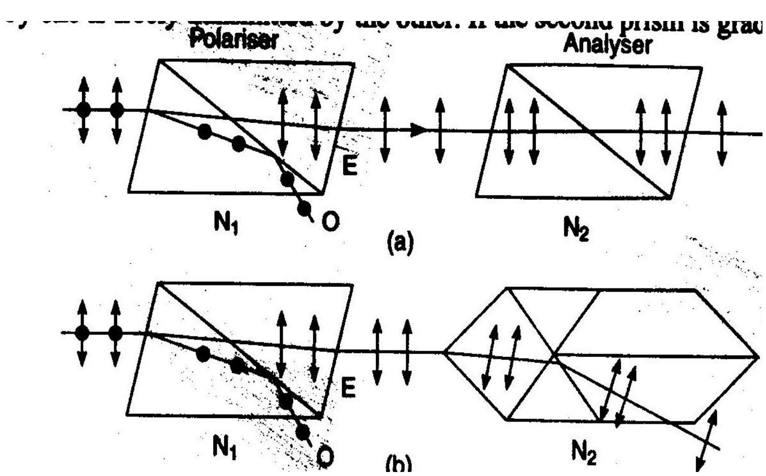
$$\mu_e < \mu_{cb} < \mu_o$$



Working:

When an ordinary beam of light incident on the Nicol prism, it split into ordinary plane polarized light and extraordinary plane polarized light. From the values of refractive indices the Canada balsam acts as a rarer medium for ordinary ray and denser medium for extraordinary ray. Moreover the dimensions of the crystal are so chosen that the angle of incidence of ordinary ray at the calcite-Canada balsam surface become grater than the corresponding critical angle .Under these conditions the ordinary ray under go total internal reflection and is eliminated. Only extraordinary transmitted

EXPLAIN HOW NICOL PRISM CAN BE USED BOTH AS POLARIZER AND ANALYZER



When two of Nicols placed co-axially then the first Nicol produces polarized light is known as polarizer while the second which analyzes the polarized light is known as analyzer.

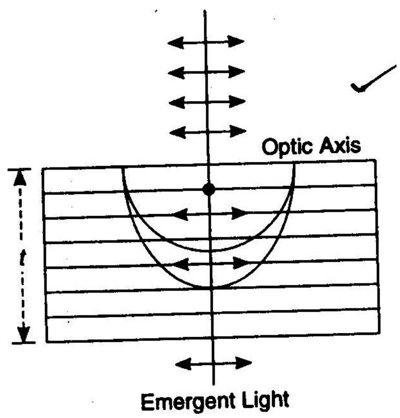
When two Nicols are placed with their planes parallel to each other. Then the extra ordinary Plane polarized transmitted by one is freely transmitted by the other. .if the second nicol is rotated gradually, then the intensity of E-ray gradually decreases and when the two nicols are at right angles to each other, no light comes from the second prism.

Thus first Nicol produces plane polarized light and second nicol detects it.

WAVE PLATES:

The wave plates are introduced specified path difference between o-ray and e-ray for particular wavelength.

HALF WAVE PLATE:



If the thickness of a crystal is taken such that it introduces a path difference of $\frac{\lambda}{2}$ or phase difference of π,then that crystal is called half wave plate.

Let μ_o, μ_e are the refractive indices of ordinary, extraordinary rays and t is the thickness of the calcite crystal

Then the path difference between ordinary and extraordinary ray $= \mu_e t - \mu_o t$

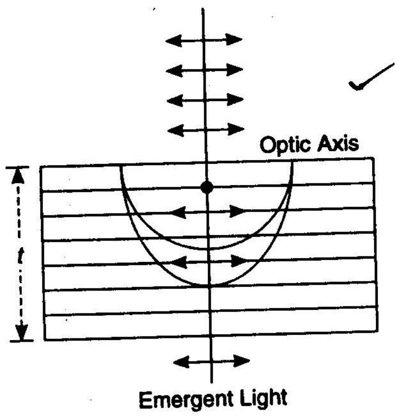
But for half wave plate $path.difference = \frac{\lambda}{2}$

$$\therefore \mu_e t - \mu_o t = \frac{\lambda}{2}$$

$$(\mu_e - \mu_o)t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

QUARTER WAVE PLATE



If the thickness of a crystal is taken such that it introduces a path difference of $\frac{\lambda}{4}$ or phase difference of $\frac{\pi}{2}$, then that crystal is called quarter wave plate.

Let μ_o, μ_e are the refractive indices of ordinary, extraordinary rays and t is the thickness of the calcite crystal

Then the path difference between ordinary and extraordinary ray $= \mu_e t - \mu_o t$

But for half wave plate $path.difference = \frac{\lambda}{4}$

$$\therefore \mu_e t - \mu_o t = \frac{\lambda}{4}$$

$$(\mu_e - \mu_o)t = \frac{\lambda}{4}$$

$$\boxed{t = \frac{\lambda}{4(\mu_e - \mu_o)}}$$

UNIT I - WAVE OPTICS

INTERFERENCE

Introduction

In the 17th century, the properties of light were explained by Sir Isaac Newton and Christian Huygens. Sir Isaac Newton was explained the properties of light by introducing Corpuscular theory in 1675. It explains reflection, refraction, and dispersion properties of light. It fails to explain interference, diffraction, polarization, photo electric effect, and double refraction.

In 1679, Christian Huygens proposed the wave theory of light. According to Huygens wave theory, each point on the wave front is to be considered as a source of secondary wavelets. It explains reflection, refraction, dispersion, double refraction, diffraction, interference, and polarization properties of light. It fails to explain, photo electric effect, black body radiation etc.

Interference of light

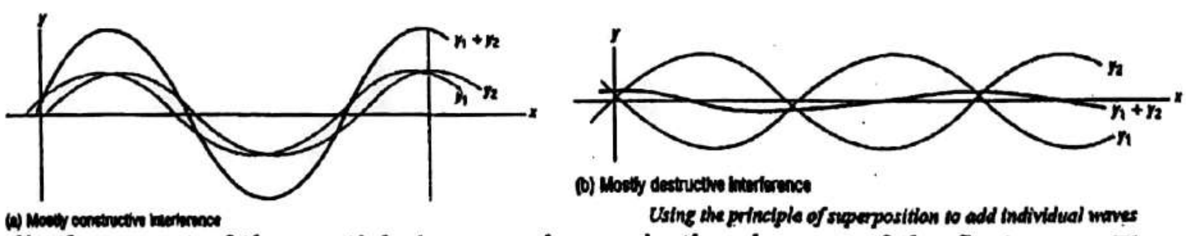
The best evidence for the wave nature of light is interference phenomenon. This was experimentally demonstrated by Thomas Young in 1801, through double slit experiment. Due to interference, we will observe many observations in our day today life, such as multiple colours on soap bubbles as well as on oil film when viewed under sun light. Interference concept is explained on the basis of superposition of wave's concept. When two light waves superimpose, then the resultant amplitude or intensity in the region of superposition is different than the amplitude of individual waves.

1. What is the principle of superposition?

Principle of Superposition of waves:

When two or more waves travel simultaneously in a medium, the resultant displacement at any point is due to the algebraic sum of the displacements due to individual waves.

Let y_1 is the displacement of the particle of first wave in a given direction and y_2 is



Using the principle of superposition to add individual waves

the displacement of the particle in second wave in the absence of the first wave. Therefore according to principle of superposition, the resultant displacement is

$$R = y_1 \pm y_2$$

- If the displacements are in the same direction then $R = y1+y2$.
- If the displacements are in opposite direction then $R = y1-y2$.

$$R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

$$I = R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

This represents the equation of resultant intensity.

Case (i): Condition for maximum intensity:

I is maximum when $\cos\delta = +1$ => $\delta = 2n\pi$, $n = 0, 1, 2, 3 \ldots$

$$2n\pi = (2\pi/\lambda) \text{ path difference}$$

path difference = $n\lambda$

$$\mathbf{I_{max} = a1^2 + a2^2 + 2a1a2}$$

$$= (a1 + a2)^2$$

If $a1 = a2$ then $\mathbf{I_{max} = 4a^2}$

Case (ii): Condition for minimum intensity:

I is minimum when $\cos\delta = -1$ => $\delta = (2n+1)\pi$, $n = 0, 1, 2, 3 \ldots$

$$(2n+1)\pi = (2\pi/\lambda) \text{ path difference}$$

=> Path difference = $(2n+1)\lambda/2$

$$\mathbf{I_{min} = a_1^2 + a_2^2 - 2a_1a_2}$$

$$= (a_1 - a_2)^2$$

If $a1 = a2$ then $I_{min} = 0$

2. What is coherence?

Coherence: Two waves are said to be coherent if they have same phase or maintaining constant phase difference between them. Hence coherence is a measure of the correlation between the phases of the wave measured at different points.

Methods of producing coherent sources for interference:-

For the formation of interference pattern, two coherent light sources are required. To get two coherent sources form a single light source, two techniques are used. They are

i. Division of wave front

ii. Division of amplitude

i. Division of wave front

The wave front from a single light source is divided into two parts using the phenomenon of reflection, refraction, or diffraction. Young's double slit experiment is belongs to this class of interference.

ii. Division of amplitude

The amplitude of a single light beam is divided into two parts by parallel reflection or refraction. Newton's ring experiment, Michelson's interferometer belongs to this class of interference.

3. What is interference?

Interference: Modification or redistribution of light energy due to superposition of light waves from two coherent sources is known as interference. The phenomenon of interference obeys law of conservation of energy.

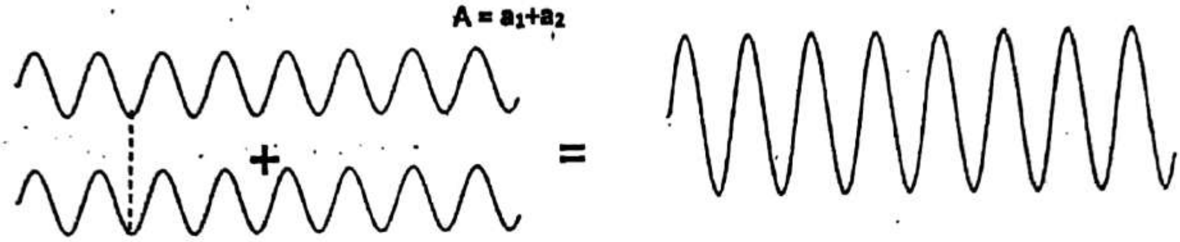
4. What are the types of interference?

Types of Interference: i. Constructive Interference ii. Destructive Interference

i) Constructive Interference: When crest of one wave falls on the crest of another wave, the resultant amplitude is the sum of the amplitudes of two waves and intensity is increased. Hence bright fringe is formed and it is known as constructive interference.

Condition: The path difference between the two waves is equal to the integral multiple of wave length (λ) the constructive interference occurs.

path diference$=n\lambda$ Where n = 0, 1, 2, 3, 4 ...



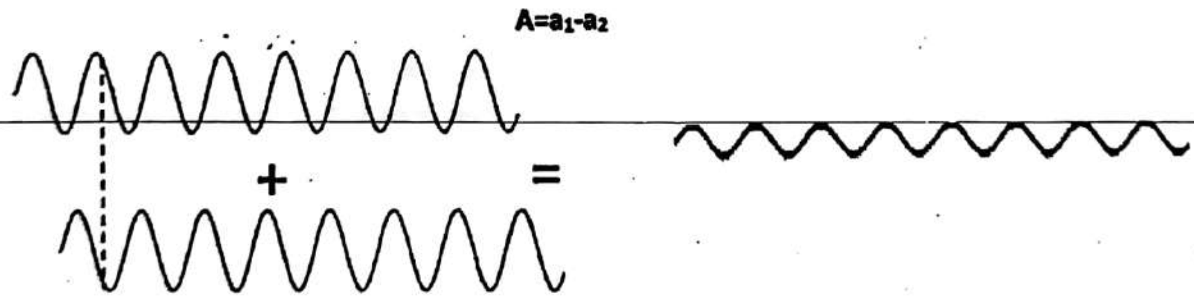
(Waves are in phase) Constructive interference

ii) Destructive Interference: When crest of one wave falls on the trough of another wave, the resultant amplitude is the difference of the amplitudes of two waves and intensity is decreased. Hence dark fringe is formed and it is known as destructive interference.

Condition: The path difference between the two waves is equal to the odd integral multiple of $\lambda/2$ destructive interference occurs

path diference $= (2n-1)\lambda/2$ Where n = 1,2,3,4 ... *or*

path diference $= (2n+1)\lambda/2$ Where n = 0,1,2,3,4 ...



(Close to π out of phase) *Destructive interference*

5. What are the conditions for sustained interference?

Conditions for sustained interference:

i. The two sources should be coherent.
ii. The two sources must emit continuous waves of same wavelength and frequency.
iii. The background should be dark.
iv. The two sources should be monochromatic.
v. The amplitude of interfering waves should be equal.
vi. The two sources should be narrow.
vii. The distance between the two sources should be small.
viii. The distance between sources and screen should be large.

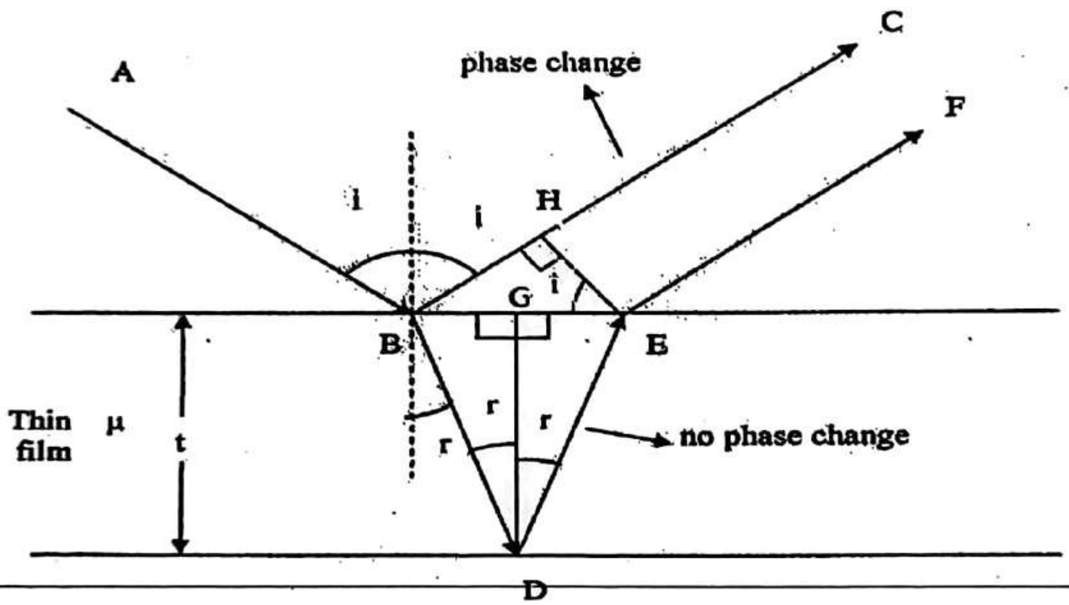
6. Explain the interference due to uniform thin films by reflected light or derive cosine law?

Interference in thin films by reflection: (Cosine law):

Principle:-

The formation of colours in thin films can explained using the phenomenon of interference. In this example, the formation of interference pattern is by the division of amplitude.

Consider a thin film of uniform thickness 't' and refractive index 'μ'. Let a monochromatic light ray AB is incident on the upper surface of the film at point 'A' with an angle 'i'. The incidence light ray AB is divided into two light rays ray 1 (BC) and ray 2 (EF) by the division of amplitude principle. These two light rays BC and EF are parallel and superimpose and produce interference. The intensity of interference fringe depends up on the path difference between the ray 1 and ray 2.



The path difference between the light rays (1) and (2) is

$$path\ difference = \mu(BD + DE)\ in\ film - BH\ in\ air \quad (1)$$

From ΔBDG $\quad \cos r = \frac{DG}{BD} = \frac{t}{BD} \Rightarrow BD = \frac{t}{\cos r}$

Similarly from ΔDEG $\quad \cos r = \frac{DG}{DE} = \frac{t}{DE} \Rightarrow DE = \frac{t}{\cos r}$

$$\therefore BD = DE = \frac{t}{\cos r} \quad (2)$$

From $\quad \Delta BEH \quad \sin i = \frac{BH}{BE} = \frac{BH}{BG+GE}$

$\therefore BH = (BG + GE).\sin i$

From ΔBDG *and* ΔDEG $\quad BG = GE = t\tan r$

$BH = (2t\tan r).\sin i$

From Snell's law at point B

$$\sin i = \mu \sin r$$

$$\therefore BH = 2\mu t \tan r . \sin r \quad (3)$$

Substituting the equations (2) and (3) in equation (1), we get

$$Path\ difference = \frac{2\mu t}{\cos r} - 2\mu t \tan r . \sin r$$

$$= \frac{2\mu t}{\cos r} - 2\mu t.\frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$= \frac{2\mu t}{\cos r}\cos^2 r$$

$$= 2\mu t \cos r$$

At point B the light ray (1) is reflected at the surface of thin film (denser medium). So the light ray (1) undergoes a phase change π or an additional path difference $\lambda/2$.

Total path difference $= 2\mu t \cos r - \frac{\lambda}{2}$

Constructive interference (or Bright fridge)

General condition: $pathdifference = n\lambda$

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = n\lambda + \frac{\lambda}{2}$$

$$2\mu t \cos r = \frac{(2n+1)\lambda}{2}$$

Destructive interference (or Dark fridge)

General condition: $pathdifference = (2n+1)\frac{\lambda}{2}$

$$2\mu t \cos r - \frac{\lambda}{2} = \frac{(2n-1)\lambda}{2}$$

$$2\mu t \cos r = n\lambda$$

Note: In case of transmitted light, the conditions for bright and dark fringes are reversed than that of in reflected light.

- For bright fringe, $\quad$ **$2\mu t \cos r = n\lambda$**
- For dark fringe, $\quad$ **$2\mu t \cos r = (2n \pm 1)\lambda/2$**

7. Write short note on colour in thin films?

Colours of thin films:

i. When a thin film (Soap bubble) is exposed to a white light source beautiful colours are observed.

ii. The incident light will split up by reflection at the top and bottom surfaces of the film.

iii. These splitted rays interfere with each other and produces interference pattern and is responsible for colours.

iv. We have **$2\mu t \cos r = (2n \pm 1) \lambda/2$** for bright fringe

v. **$2\mu t \cos r = n \lambda$** for dark fringe.

vi. Hence bright and dark fringes depend on μ, t and r. Here t and r are made constant but μ changes with wavelength.

vii. At a particular point of the film and at a particular position of the eye, only certain wavelengths (colours) satisfy the condition for bright fringe. Hence only those colours appear on thin film.

viii. The colours which satisfy dark fringe condition are absent. If position of the eye changes, different set of colours are observed.

ix. We know that the conditions for bright and dark fringes in transmitted light are reversed than that in reflection. Hence colours which appear in reflected light disappears in transmitted light.

8. What are Newton's rings? Explain the formation of Newton's rings. Write the conditions for maxima and minima?

Newton's Rings: When a Plano-convex lens with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between the two. At the point of contact, the thickness of the film is zero. If monochromatic light is incident normally and the film is viewed in reflected light we observe alternate bright and dark rings around the point of contact. These rings are known as Newton's rings.

Principle:-

The formation of Newton's rings is due to the phenomenon of interference. In this example, the formation of interference pattern is obtained by the division of amplitude.

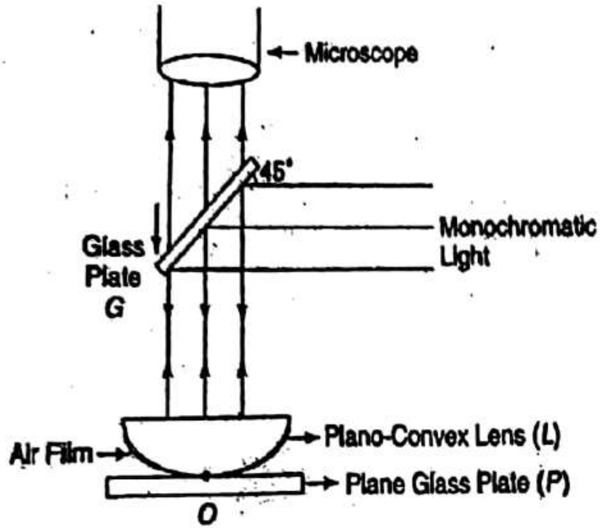
Experimental arrangement

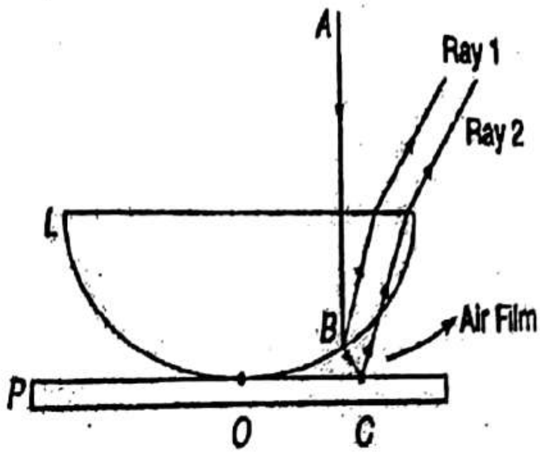
- The experimental arrangement of Newton's rings is shown in figure.
- The Plano -convex lens (L) of large radius of curvature is placed with its convex surface on the glass plate (P). The Plano convex lens touches the glass plate at O.
- A monochromatic light is allowed to fall normally on the lens with the help of glass plate M kept at 45^0 to the incident monochromatic light.
- A part of light is reflected by the curved surface of the lens 'L' and a part of light is transmitted and partly reflected back by the upper surface of the plane glass plate P. These reflected rays interfere and give rise to an interference pattern in the form of circular fringes. These rings are seen through a travelling microscope.

Explanation of Newton's rings

Newton's rings are formed due to the interference between the light rays reflected from the lower surface of the lens and the upper surface of the glass plate (or top and bottom surfaces of the air film).

Let a vertical light ray AB be partially reflected from the curved surface of plano convex lens without phase change and partially transmitted light ray BC is again reflected at C on the glass plate with additional phase change of π or path difference λ/2.





The path difference between the two rays = $2\mu t \cos r+\lambda/2$

For air film $\mu=1$ and for normal incidence $r=0$, so

The path difference $=2t+\lambda/2$

At the point of contact $t=0$, path difference is $\lambda/2$ i.e., the reflected and incidence light are out of phase and destructive interference occur. So the center fringe is always dark.

Constructive interference (or Bright fringe)

General condition: $pathdifference=n\lambda$

$$2t+\lambda/2=n\lambda$$

$$2t=(2n-1)\lambda/2 \text{ , Where } n=0,1,2,\ldots\ldots\ldots$$

Destructive interference (or Dark fringe)

General condition: $pathdifference=(2n+1)\lambda/2$

$$2t+\lambda/2=(2n+1)\lambda/2$$

$$2t=n\lambda \text{ , Where } n=0,1,2,\ldots\ldots\ldots$$

Theory of Newton's rings

To find the diameters of a dark and bright rings construct a circle with the radius of curvature R of a lens L. Let us choose a point P at a distance 'r' from the center of lens and t be the thickness of air film at point p.

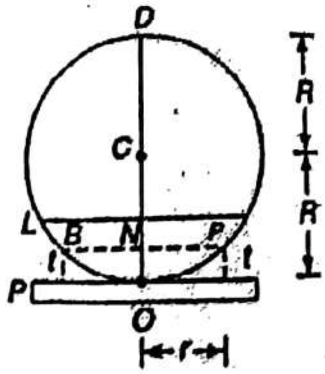
From the property of a circle $NP.NB = NO.ND$

$$r.r - t.(2R - t)$$
$$r^2 = 2Rt - t^2$$

If t is small t^2 is negligible.

$$r^2 = 2Rt$$

$$t - \frac{r^2}{2R}$$



Bright rings

For bright ring, the condition is $2t = (2n-1)\frac{\lambda}{2}$

$$\frac{2r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

By replacing r by D/2, the diameter of the bright ring is

$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$
$$D^2 = 2(2n-1)\lambda R$$
$$D = \sqrt{2(2n-1)\lambda R}$$
$$D = \sqrt{(2n-1)}\sqrt{2\lambda R}$$
$$D \propto \sqrt{(2n-1)}$$

$D \propto \sqrt{odd\ natural\ number}$

Dark rings

For dark rings, the condition is

$$2t = n\lambda$$
$$\frac{2r^2}{2R} = n\lambda$$
$$r^2 = n\lambda R$$

By replacing r by D/2, the diameter of the dark ring is

$$\frac{D^2}{4} = n\lambda R$$
$$D = \sqrt{4n\lambda R}$$
$$D - 2\sqrt{n\lambda R}$$
$$D \propto \sqrt{n}$$
$$D \propto \sqrt{natural\ number}$$

Note: suppose a liquid is taken in between the lens and glass plate having refractive index μ, then the diameter of the dark n^{th} dark ring can be written as

$$D = \frac{\sqrt{4n\lambda R}}{\mu}$$

9. What are the applications of Newton's rings?

Applications of Newton's rings:

1. Determination of wave length of sodium light using Newton's rings:

By forming Newton's rings and measuring the radii of the rings formed, we can calculate the wavelength of the light used if the radius of curvature of the lens is known. Let R be the radius of curvature of the lens and λ is the wavelength of the light used.

So the diameter of the m[th] dark ring can be written as

$$D_m^2=4\,m\,\lambda\,R \quad \text{.......................... (1)}$$

Similarly the diameter of the n[th] dark ring is

$$D_n^2=4\,n\,\lambda\,R \quad \text{........................ (2)}$$

Subtracting equation (1) from (2)

we get $D_n^2-D_m^2=(4\,n\lambda\,R)-(4\,m\,\lambda\,R)$

$$D_n^2-D_m^2=4\,(n-m)\,\lambda\,R$$

$$\lambda=\frac{D_n^2-D_m^2}{4(n-m)R}$$

Using the above relation wavelength can be calculated

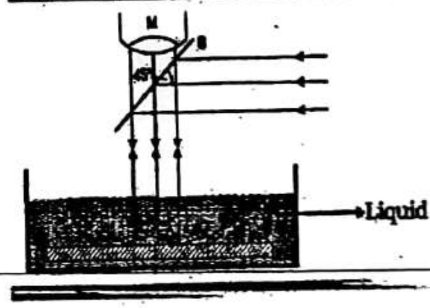
2. Determination of refractive index of a liquid using Newton's rings:

By forming Newton's rings and measuring the diameter of the rings formed, we can calculate the refractive index of the liquid.

In air film, the diameters of the m[th] and n[th] dark rings are D_m and D_n are measured with the help of travelling microscope.

Newton Rings...

Determination of refractive index of liquid:



The diameter of the n[th] dark ring is

$$D_n^2=4\,n\,\lambda\,R \quad \text{................ (1)}$$

The diameter of the m[th] dark ring is

$$D_m^2=4\,m\lambda\,R \quad \text{............ (2)}$$

Subtracting equation (1) from (2) we get

$$D_n^2-D_m^2=[4\,(n-m)\,\lambda\,R] \quad \text{......... (3)}$$

The Newton's rings setup is taken in a liquid. Now the air film is replaced by liquid film. In liquid film, the diameters of the same n[th] and m[th] dark rings are D'_n and D'_m are measured with the help of travelling microscope.

$$D'^2_n=\frac{4n\lambda R}{\mu}$$

And $D'^2_m=\frac{4m\lambda R}{\mu}$

So $$D'^2_n-D'^2_m=\frac{4\,(n-m)\,\lambda R}{\mu} \qquad (4)$$

Dividing equation (3) by (4)

$$\frac{D_n^2-D_m^2}{D'^2_n-D'^2_m}=\frac{4\,(n-m)\,\lambda\,R}{\frac{4\,(n-m)\,\lambda\,R}{\mu}}$$

$$\frac{D_n^2-D_m^2}{D'^2_n-D'^2_m}=\mu$$

Using the above relation μ can be calculated.

10. What are the applications of interference?

Applications of interference: Interference phenomenon is used to

i. Determine the wavelength of light.
ii. Find the difference in wavelengths of two spectral lines having small separation.
iii. Find the thickness of transparent materials.
iv. Determine the refractive index of transparent solids, liquids and gases.
v. Find the velocity of light (Michelson interferometer experiment).
vi. Test the optical flatness of surfaces.
vii. Find the reflecting power of the lens and prism surfaces.

Assignment Questions

1. What is the principle of superposition?
2. What is interference? What are the types of interference?
3. What are coherent sources?
4. What are the conditions for sustained interference?
5. What are the applications of interference?
6. Explain the interference of light due to thin films or derive cosine law.
7. Write a short note on colors in thin films.
8. Discuss the theory of Newton's Rings with relevant diagram and discuss its applications.

Problems

1. Newton's rings are observed in the reflected light of wave length 5900 A^0. The diameter of 10^{th} dark ring is 0.5 cm. Find the radius of curvature of the lens used. (Feb 2011, Set No.4), (July 2011, Set No.3; Ans: R=1.059m)
2. A parallel beam of light (λ=5890A^0) is incident on a glass plate (μ=1.5) such that angle of refraction into plate is 60^0, calculate the smallest thickness of the plate which will make it appear dark by reflection. (Feb 2011, Set No.2, Jan 2012, Set No.2, Ans: $t=3.926 \times 10^{-4}$mm)
3. In Newton's rings experiment, the diameters of the 4^{th} and 12^{th} dark rings are 0.40 cm and 0.70 cm respectively. Find the diameter of the 20^{th} dark ring. (Jan 2012, Set No.3, Ans: D_{20} = 0.905m)
4. In Newton's rings experiment, the diameter of the 10^{th} ring changes from1.40 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.(Jan 2012, Set No.4, Ans: μ=1.215)
5. In Newton's rings experiment, the diameter of the 15^{th} ring was found to be 0.59 cm and that of the 5^{th} ring was 0.336 cm. If the radius of the plano convex lens is 100 cm, compute the wavelength of light used. (June 2012, set No.1, Set No.4, Ans: λ = 588 nm)
6. Calculate the thickness of a soap film (μ=1.463) that will result in constructive interference in the reflected light, if the film is illuminated normally with light whose wave length in free space is 6000 A^0.(June 2012, Set No.2, Ans: $t=1.025 \times 10^{-4}$mm)

Objective Questions

1. The wave nature of light is evidenced by
a) Photo electric effect **b) interference**
c) Black Body Radiation d) emission

2. In interference the intensity of light gets
a) **Modified** b) Remains same
c) both (a) &(b) d) None

3. Path difference between coherence wave is
a) Constant b) Zero **c) Both (a) & (b)** d) None

4. Two light sources are said to be coherent if their waves have
a) same frequency b) constant phase difference
c) same wavelength **d) All the above**

5. In super position of waves of constant phase difference, the resultant amplitude is maximum when Ø=
a) **$2n\pi$** b) $n\pi$ c) $(2n-1)\pi$ d) none

6. In super position of waves of constant phase difference, the resultant amplitude is minimum when Ø=
a) $2n\pi$ b) $n\pi$ **c) $(2n-1)\pi$** d) none

7. The resultant intensity of the superposition of waves of constant phase difference is
a) $I = 2a^2 \cos^2\varphi/2$ **b) $I=4a^2$** c) $I= 2a^2 \sin^2 \varphi/2$ d) $I= 4a^2 \sin^2 \varphi/2$

8. The resultant amplitude of waves of equal phase and frequency is
a) sum of the amplitudes of individual waves
b) difference of amplitudes of individual waves
c) sum of the squares of amplitudes of individual waves
d) none

9. Two light beams interfere have their amplitudes in the ratio 2:1 then the intensity ratio of bright and dark fringes is
a)2:1 b) 1:2 c) **9:1** d) 4:1

10. In super position of waves of constant phase difference , the phase angle φ
a) **$\tan^{-1}[b \sin \varphi/ a+b \cos \varphi]$** b) $\tan^{-1} [b \cos \varphi/a+b \sin \varphi]$
c) $\tan^{-1} [b \sin \varphi/a \sin \varphi +b \cos \varphi]$ d) $\tan^{-1}[b / a+b \cos \varphi]$

11. Newton's rings and Michelson's interferometer experiments are example for
a) Division of amplitude b) Division of wave front
c) Both a & b d) none

12. In Newton's rings experiment the condition for bright fringes in the case of reflected light
a) $2t = (n+1)\lambda/2$ b) $2t = (2n-1)\lambda$ c) $2t = (n-1)\lambda$ d) $2t = n\lambda$

13. In Newton's rings experiment the condition for dark fringes in the case of reflected light

a) $2t = n\lambda$ b) $2t = (2n-1)\lambda$ c) $2t = (n-1)\lambda/2$ d) none

14. In Newton's rings experiment, wave length

a) $D^2_{n+p} - D^2_n$ /4PR b) $D^2_n - D^2_{n+p}/4PR$

c) $D^2_{n+p} + D^2_n$ /4PR d) $D^2_{n+p} - D^2_n/4\lambda R$

15. The refractive index of the liquid from Newton's rings μ=

a) $D^2_{n+p} - D^2_n / D^2_{n+p} - D^2_n$ b) $D'^2_{n+p} - D'^2_n$ / $D^2_{n+p} - D^2_n$

c) $D^2_{n+p} + D^2_n / D^2_{n+p} - D^2_n$ **d) $D^2_{n+p} - D^2_n / D'^2_{n+p} - D'^2_n$**

16. In Newton's rings experiment the diameter of dark ring is proportional to

a) Odd number b) natural number

c) Even natural number **d) square root of natural number**

17. When light wave suffers reflection at interface between glass and air, the change of phase of reflected wave is equal to

a) $\pi/2$ b) π **c) zero** d) 2π

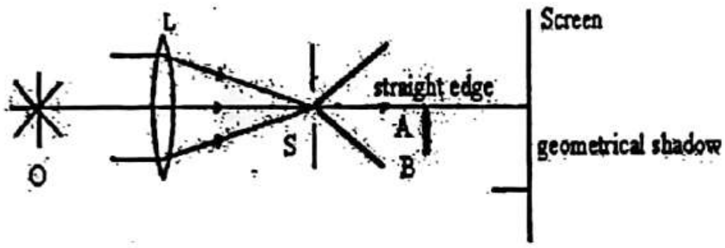
DIFFRACTION

Introduction:

The wave nature of light is first confirmed by the phenomenon of interference. Further it is confirmed by the phenomenon of diffraction. The word 'diffraction' is derived from the Latin word diffractus which means break to piece. When the light waves encounter an obstacle, they bend round the edges of the obstacle. The bending is predominant when the size of the obstacle is comparable with the wavelength of light. The bending of light waves around the edges of an obstacle is diffraction. It was first observed by Francesco Gremaldi.

1. Define diffraction. Give examples of diffraction.

Definition of Diffraction: The phenomenon of bending of light round the corners of obstacles and spreading of light waves into the geometrical shadow of an obstacle placed in the path of light is called Diffraction.



The effects of diffraction can be seen in everyday life. The most colourful examples of diffraction of light are

1. The closely spaced tracks on a CD or DVD act as diffraction grating to form a rainbow pattern when looking at a disk.
2. The hologram on a book or debit card.
3. Diffraction in the atmosphere by small particles can cause a bright ring to be visible around the sun or the moon.
4. A shadow of a solid object using light from a compact source shows small fringes near its edges.

Explanation: Consider light waves diverging from a narrow slit 'S' illuminated by a mono chromatic source 'O' and passes towards an obstacle AB. A small portion of light bends around the edge and forms a geometrical shadow on the screen which is not sharp. Outside the shadow parallel to its edge several bright and dark bands are observed. Thus when light falls on obstacles whose size is comparable with wavelength of light, the light bends round the corners of the obstacles or aperture and enter in to the geometrical shadow. It was found that diffraction produces bright and dark fringes known as diffraction bands or fringes. According to Fresnel, the diffraction phenomenon is due to mutual interference of secondary wavelets originating from various points of the same primary wave front which are not blocked off by the obstacle. Hence diffraction is also known as self interference.

2. What are the types of diffraction?

Types of diffraction: The diffraction phenomena are classified into two ways

i. Fresnel diffraction

ii. Fraunhofer diffraction.

Fresnel diffraction:-

In this diffraction the source and screen are separated at finite distance. To study this diffraction lenses are not used because the source and screen separated at finite distance. This diffraction can be studied in the direction of propagation of light. In this diffraction the incidence wave front must be spherical of cylindrical.

Fraunhofer diffraction:-

In this diffraction the source and screen are separated at infinite distance. To study this diffraction lenses are used because the source and screen separated at infinite distance. This diffraction can be studied in any direction. In this diffraction the incidence wave front must be plane.

3. Write the differences between Fresnel diffraction and Fraunhofer diffraction.

Fresnel Diffraction	**Fraunhofer Diffraction**
1. The source and the screen are placed at finite distances from the obstacle producing diffraction.	1. The source and the screen are placed at infinite distances from the obstacle producing diffraction.
2. No lenses are used for making the rays parallel or convergent.	2. Lenses are used for making the rays parallel or convergent.
3. The incident wave front is either spherical or cylindrical.	3. The incident wave front is plane.
4. Either a point source (or) an illuminated narrow slit is used.	4. Extended source at infinite distance is used.
5. This is also called near-field diffraction.	5. This is also called far-field diffraction.
6. It is general approach.	6. It is simplified approach.
7. Mathematical treatment is quite complicated.	7. Mathematical treatment is simple
8. Examples: Diffraction at a straight edge, thin wire, narrow slit, a small hole etc.,	8. Examples: Diffraction at a single slit, double slit and n slits (grating) etc.,

4. Write the differences between interference and diffraction.

Differences between Interference and Diffraction:

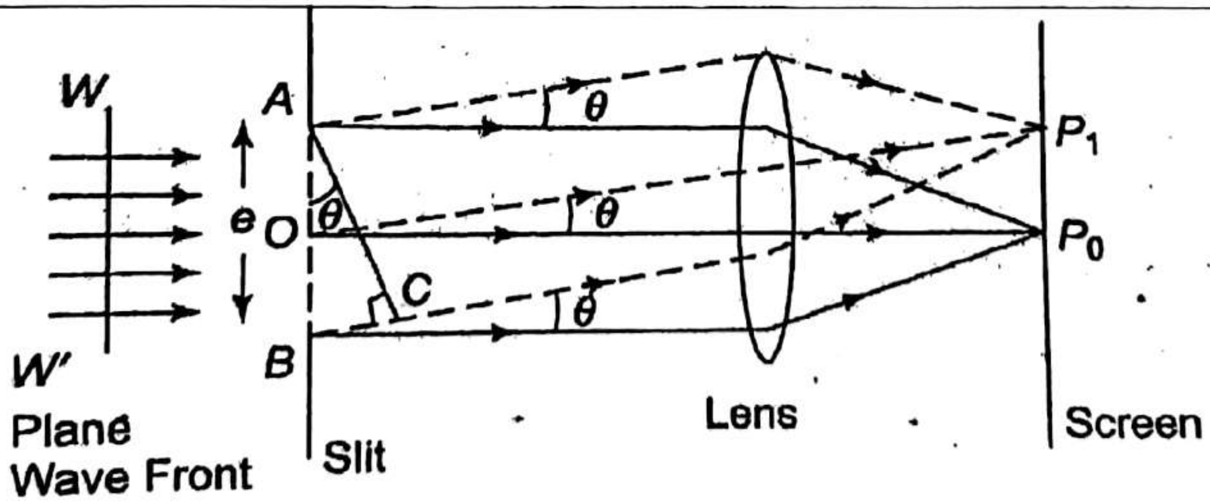
Diffraction	Interference
1. It is due to interaction of light waves coming from different parts of the same wave front. It is also called as self interference.	1. It is due to interaction of light waves coming from two different wave fronts originating from the same source (i.e. coherent sources).
2. Diffraction bands decrease in their width as the order increases.	2. Interference bands are of equal width i.e. all are equally spaced.
3. The bright fringes are of varying intensity.	3. All the bright fringes are of the same intensity.
4. Points of minimum intensity are not perfectly dark. Hence fringes will not appear with contrast.	4. Points of minimum intensity are perfectly dark. Hence fringes will appear with contrast.

5. Explain Fraunhofer diffraction due to single slit.

Fraunhofer single slit diffraction:

Let us consider a slit AB of width 'e'. Let a plane wave front *ww'* of monochromatic light of wavelength λ is incident on the slit AB.

According to Huygens principle, every point on the wave front is a source of secondary wavelets. The wavelets spread out to the right in all directions. The secondary wavelets which are travelling normal to the slit are brought to focus at point P_0 on the screen by using the lens. These secondary wavelets have no path difference. Hence at point P_0 the intensity is maxima and is known as central maximum. The secondary wavelets travelling at an angle θ with the normal are focused at point P_1.



Intensity at point P1 depends up on the path difference between the wavelets A and B reaching to point P1. To find the path difference, a perpendicular AC is drawn to B from A.

The path difference between the wavelets from A and B in the direction of θ is

$$path\ difference = BC = AB\sin\theta$$
$$= e\sin\theta$$

$$phase\ difference = \frac{2\pi}{\lambda}(path\ difference)$$
$$= \frac{2\pi(e\sin\theta)}{\lambda}$$

Let the width of the slit is divided into 'n' equal parts and the amplitude of the wave front each part is 'a'. Then the phase difference between any two successive waves from these parts would be

$$\frac{1}{n}(phase\ difference) = \frac{1}{n}\left(\frac{2\pi e\sin\theta}{\lambda}\right) = d$$

Using the vector addition method, the resultant amplitude R is

$$R = \frac{a\sin\frac{nd}{2}}{\sin\frac{d}{2}}$$

$$R = A\frac{\sin\alpha}{\alpha} \qquad \because na = A\ and\ \alpha = \frac{\pi e\sin\theta}{\lambda}$$

Therefore resultant intensity $I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$

Principal maximum:-

The resultant amplitude R can be written as

$$R = \frac{A}{\alpha}\left(\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots\right)$$
$$= \frac{A\alpha}{\alpha}\left(1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\right)$$
$$= A\left(1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\right)$$

In the above expression for $\alpha=0$ values the resultant amplitude is maximum $R=A$, then $I_{max}=R^2=A^2$

$$\alpha = \frac{\pi e\sin\theta}{\lambda} = 0$$
$$\sin\theta = 0$$
$$\theta = 0$$

For $\theta=0$ and $\alpha=0$ value the resultant intensity is maximum at P0 and is known as principal maximum.

Minimum intensity positions: I Will be minimum when $\sin\alpha=0$

$$\alpha = \pm m\pi \qquad m = 1,2,3,4,5 \ldots\ldots$$

$$\alpha = \frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

$$\pi e \sin\theta = \pm m\lambda$$

So we obtain the minimum intensity positions on either side of the principal maxima for all $\alpha = \pm m\pi$ values.

Secondary maximum

In between these minima secondary maxima positions are located. This can be obtained by differentiating the expression of I w.r.t α and equation to zero

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left(A^2\left[\frac{\sin\alpha}{\alpha}\right]^2\right) = 0$$

$$A^2\frac{2\sin\alpha}{\alpha}\frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right) = 0$$

$$A^2\frac{2\sin\alpha}{\alpha}\cdot\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

In the above expression α can never equal to zero,

so either $\sin\alpha=0$ or $\alpha\cos\alpha-\sin\alpha=0$

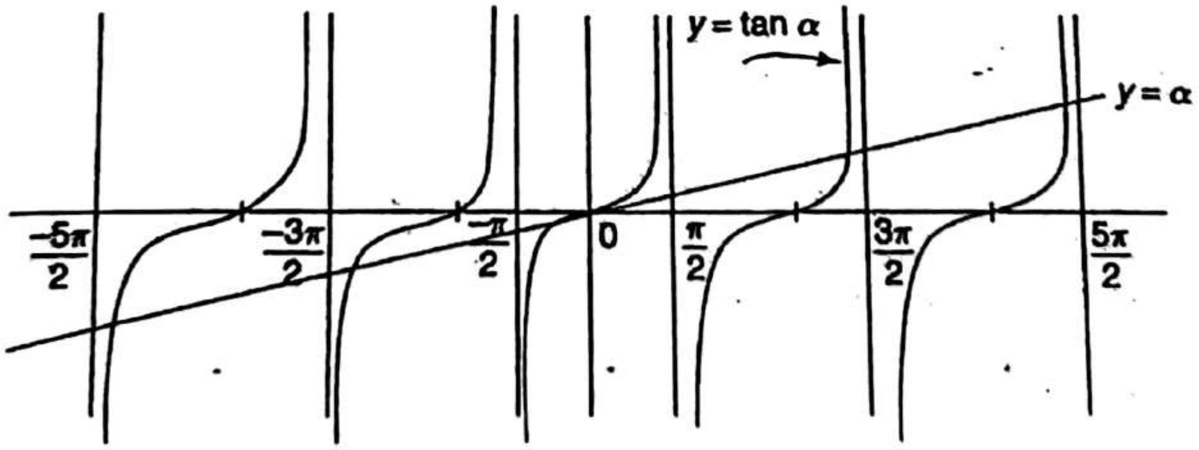
$\sin\alpha=0$ gives the positions of minima

The condition for getting the secondary maxima is $\alpha\cos\alpha-\sin\alpha=0$

$$\alpha\cos\alpha = \sin\alpha$$

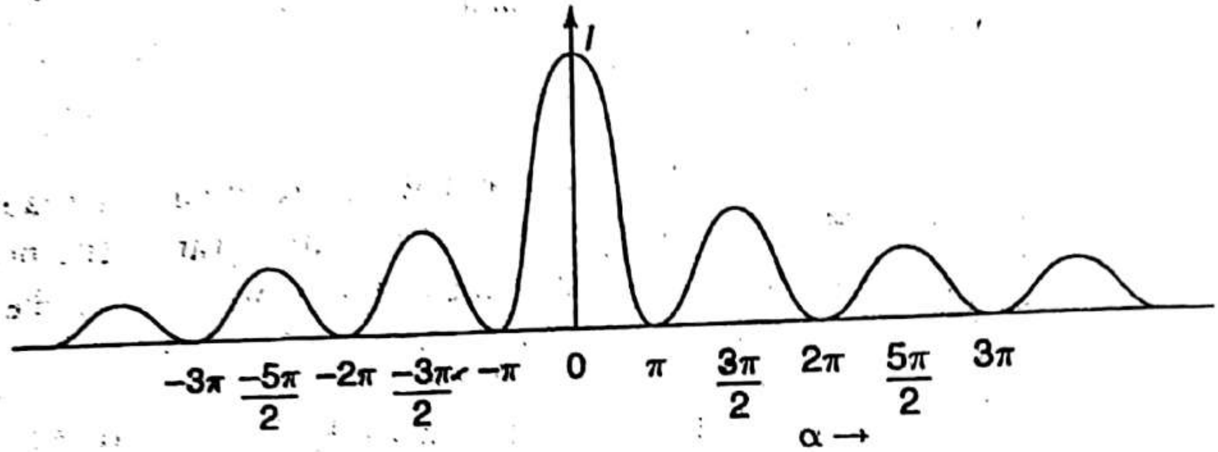
$$\alpha = \tan\alpha$$

The values of α satisfying the above equation are obtained graphically by plotting the curves $Y=\alpha$ and $Y=\tan\alpha$ on the same graph. The plots of $Y=\alpha$ and $Y=\tan\alpha$ is shown in figure.



In the graph the two curves intersecting curves gives the values of satisfying of α satisfying the above equation. From the graph intersecting points are $\alpha=0, \pm 3\pi/2, \pm 5\pi/2, \pm 7\pi/2$...........

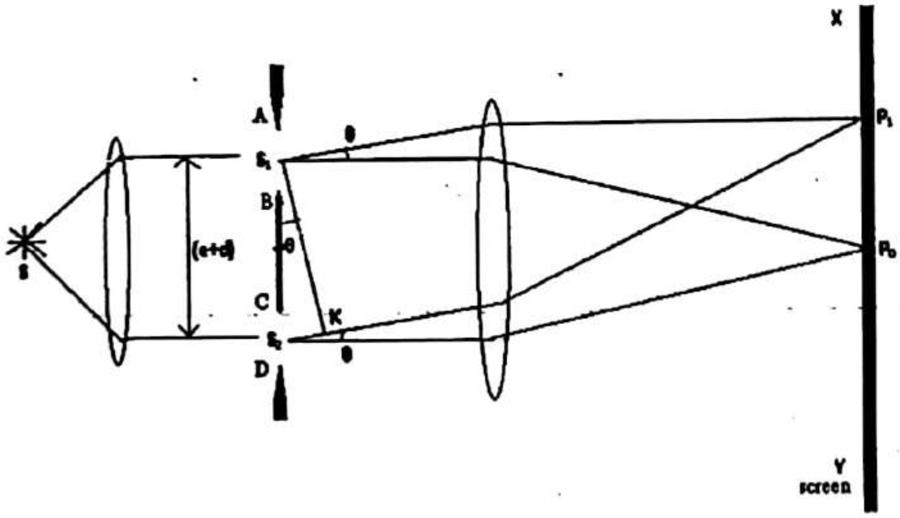
From the above concepts the intensity distribution curve verses α is shown in figure.



6. Explain Fraunhofer diffraction due to double slit and explain the intensity distribution.

Fraunhofer diffraction by a double slit:

Description: Consider two parallel slits AB and CD of equal width 'e' and separated by distance 'd'. The distance between the midpoints of the two slits is (e+d). Let a parallel beam of monochromatic light incident on the two slits normally. Then the light will be focused on the screen XY placed at the focal plane of the lens. The diffraction at two slits is the combination of diffraction as well as interference.



Explanation: When a plane wave front is incident normally on both slits, the secondary wavelets come to focus at P0 and the secondary wavelets traveling at an angle θ with normal come to a focus at P1.

Theory: For simplicity let us assume the two slits equivalent to two coherent sources S1 and S2 each sending a wavelet of amplitude **A** in a direction θ. The resultant amplitude at P1 will be the result of interference between two waves of amplitude (**A**) and having phase difference δ between them. To find δ, draw a perpendicular S_1K on S_2K.

Path difference between wavelets from S_1 and S_2 = S_2K = $(e+d)\sin\theta$

Phase difference $\delta = (e+d)\sin\theta$.

Resultant amplitude $R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

$$= \{A \frac{\sin\alpha}{\alpha}\}^2 + \{A\frac{\sin\alpha}{\alpha}\}^2 + 2\{A \frac{\sin\alpha}{\alpha}\}\{A \frac{\sin\alpha}{\alpha}\}\cos\delta$$

$$= \{A \frac{\sin\alpha}{\alpha}\}^2 \ [1+1+2\cos\delta] = \{A \frac{\sin\alpha}{\alpha}\}^2 \ 2[1+\cos\delta]$$

$$= \{A \frac{\sin\alpha}{\alpha}\}^2 \ 2[2\cos^2\delta/2]$$

$$R^2 = 4A^2\{\frac{\sin\alpha}{\alpha}\}^2 \cos^2\delta/2$$

$$R^2 = 4A^2\{\frac{\sin\alpha}{\alpha}\}^2\cos^2\left(\frac{\pi(e+d)\sin\theta}{\lambda}\right)$$

$$R^2 = 4A^2\{\frac{\sin\alpha}{\alpha}\}^2\cos^2\beta \qquad \text{where } \beta = \frac{\pi(e+d)\sin\theta}{\lambda}$$

$$\boxed{I = R^2 = 4A^2 \ \{\frac{\sin\alpha}{\alpha}\}^2\cos^2\beta}$$

Intensity distribution: The resultant intensity depends upon two factors

1. $4A^2 \{\frac{\sin\alpha}{\alpha}\}^2$ which is same as that of single slit diffraction. This gives the intensity distribution in the diffraction pattern due to single slit.
2. $\cos^2\beta$ which gives the interference pattern due to waves starting from two parallel slits.

Therefore the resultant intensity at any point on the screen is the product of these two factors.

In the diffraction pattern,

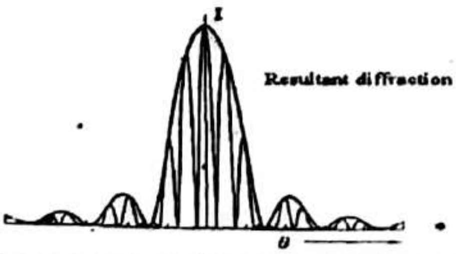
- The central maximum is obtained in the direction $\theta = 0$.
- The minima are obtained in the direction given by $e \sin\theta = \pm m\lambda$, m = 1, 2, 3, ----
- The positions of secondary maxima approaches to $\alpha = 0, \pm 3\pi/2, \pm 5\pi/2$ and so on.

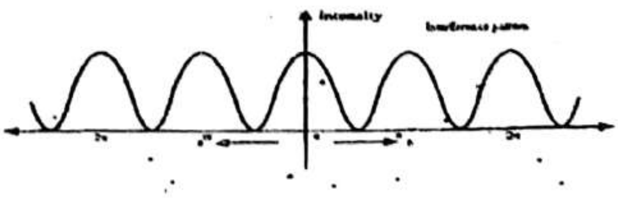
In the interference pattern,

The maxima are obtained in the direction given by $\cos^2\beta = 1 \implies \beta = \pm n\pi$

$$\implies \frac{\pi(e+d)\sin\theta}{\lambda} = \pm n\pi \implies (e+d)\sin\theta = \pm n\lambda, \ n=0, 1, 2...$$

The intensity distribution in diffraction pattern, interference pattern and the resultant pattern are as shown respectively.

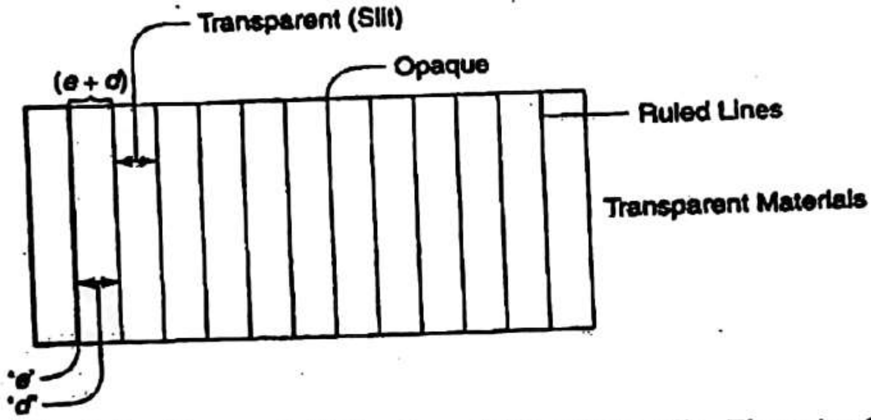




7. What is plane diffraction grating? Explain grating spectrum.

Plane diffraction grating:

Construction: An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer constructed grating by placing large no. of parallel wires closely side by side at regular intervals. Now gratings are constructed by ruling equidistant parallel lines on a transparent material glass with a fine diamond point. The ruled lines are opaque to light and the space between the lines is transparent to light and acts as slit. This is known as plane transmission grating. If the lines are drawn on silvered surface then it forms plane reflection grating. Commercial gratings are produced by taking the cast of an actual grating on a transparent film like that of cellulose acetate. Solution of cellulose acetate is poured on the ruled surface and allowed to dry to form a thin film, detachable from the surface. These impressions of a grating are preserved by mounting the film between two glass sheets.



Let 'e' be the width of the line and 'd' be the width of the slit. Then (e+d) is known as grating element. If N is the number of lines per inch on the grating, then

$$N\,(e+d) = 1'' = 2.54 \text{ cm}$$

$$e+d = 2.54/N \text{ cm}$$

There will be nearly 30,000 lines per inch of a grating. Due to the above fact, the width of the slit is very narrow and is comparable to the wavelength of light. When light falls on the grating, the light gets diffracted through each slit. As a result, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as diffraction spectrum.

Grating Spectrum

The condition to form the principal maxima in a grating is given by $(e+d) \sin\theta = n\lambda$

Where (e+d) is the grating element and the above equation is known as grating equation.

From the grating equation, the following is clear.

1. For a particular wavelength λ, the angle of diffraction θ is different for principal maxima of different orders.
2. As the number of lines in the grating is large, maxima appear as sharp, bright parallel lines and are termed as spectral lines.
3. For white light and for a particular order of n, the light of different wavelengths will be diffracted in different directions.
4. At the center, $\theta=0$ gives the maxima of all wavelengths which coincides to form the central image of the same colour as that of the light source. This forms zero order (Fig.)

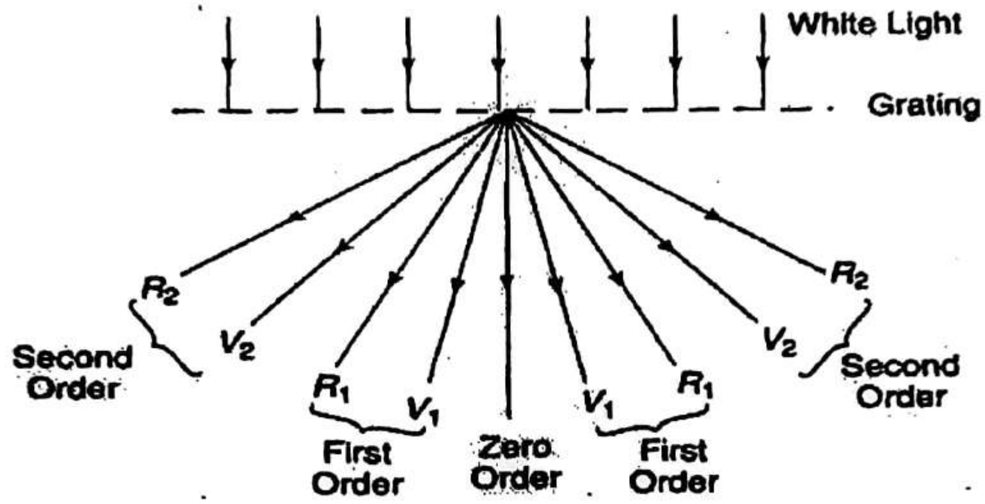
5. The principal maxima of all wavelengths forms the first, second,... order spectra for n=1,2,...

6. The longer the wavelength, greater is the angle of diffraction. Thus, the spectrum consists of violet being in the innermost position and red being in the outermost positions.

7. Most of the intensity goes to zero order and the rest is distributed among other orders.

8. Spectra of different orders are situated symmetrically on both sides of zero order.

9. The maximum number of orders available with the grating is $n_{max} = (e+d)/\lambda$



8. Explain diffraction due to N-Slits?

Diffraction Grating-Normal incidence-(Diffraction at N parallel slits)

Construction

An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed very closely side by side at regular intervals. The diameter of the wires was of the order of 0.05mm and their spacing varied from0.0533 mm to 0.687 mm. Now gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit. This is known as *Plane transmission grating*. On the other hand, if the lines are drawn on a silvered surface (plane or concave) then the light is reflected from the positions of mirrors in between any two lines and it forms a *plane or concave reflection grating*. When the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of light is produced.

Theory

Fig. represents the section of a plane transmission grating placed perpendicular to the plane of the paper. Let „e" be the width of each slit and "d" be the width of each opaque part. Then (e+d) is known as grating element. XY is the screen placed perpendicular to the plane of a paper. Suppose a parallel beam of monochromatic light of wavelength λ be incident normally on the grating. By Huygen"s principle, each of the slit sends secondary wavelets in all directions. The secondary wavelets travelling in the same direction of incident light will come to a focus at a point P_0 of the screen as the screen is placed at the focal plane of the convex lens. The point P_0 will be the central maximum. Now, consider the secondary waves travelling in a direction inclined at an angle θ with the direction of the incident light. These waves reach the point P_1 on passing through the convex lens in different phases. As a result, dark and bright bands on both sides of the central maximum are obtained.

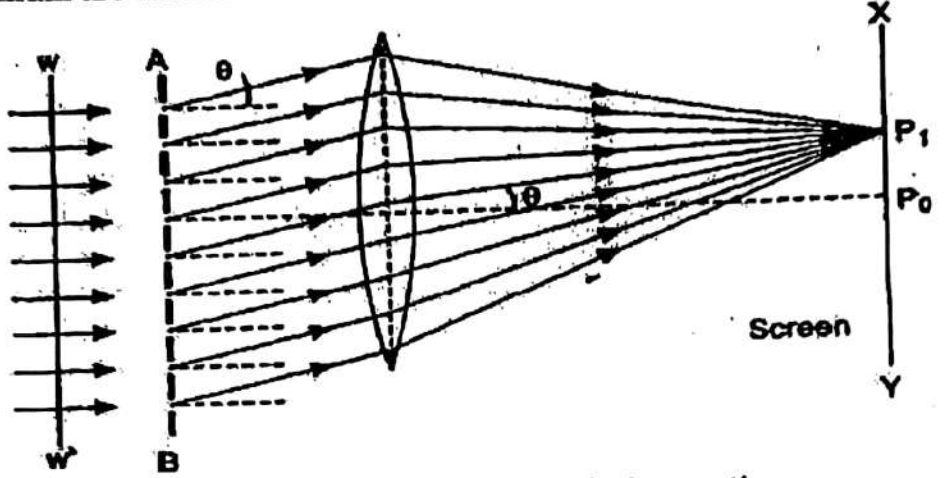


Fig. Section of a Plane transmission grating

The intensity at point P_1 may be considered by applying the theory of Fraunhofer diffraction at a single slit. The wavelets proceeding from all points in a slit along the direction Θ are equivalent to a single wave of amplitude ($A \sin\alpha/\alpha$) starting from the middle point of the slit, where

$$\alpha = (\pi e \sin\Theta/\lambda).$$

If there are N slits, then there will be N diffracted waves, one each from the middle points of the slits. The path difference between two consecutive slits is $(e+d)\sin\Theta$. Therefore, there is a corresponding phase difference of $(2\pi/\lambda)(e+d)\sin\Theta$ between the two consecutive waves. The phase difference is constant and it is 2β.

Hence, the problem of determining the intensity in the direction Θ reduces to finding the resultant amplitude of N vibrations each of amplitude ($A \sin\alpha/\alpha$) and having a common phase difference

$$(2\pi/\lambda)(e+d)\sin\Theta = 2\beta \rightarrow (1)$$

Now, by the method of vector addition of amplitudes, the direction of Θ will be

$$R^* = (A\sin\alpha/\alpha)(\sin N\beta/\sin\beta)$$

And $I = R^2 = (A\sin\alpha/\alpha)^2 (\sin N\beta/\sin\beta)^2 = I_o (\sin\alpha/\alpha)^2 (\sin^2 N\beta/\sin^2\beta) \rightarrow (2)$

The factor $(A\sin\alpha/\alpha)^2$ gives the distribution of intensity due to single slit while the factor $(\sin^2 N\beta/\sin^2\beta)$ gives the distribution of intensity as a combined effect of all the slits.

Intensity distribution in N-Slits

Principle maxima

The intensity would be maximum when $\sin\beta = 0$. or $\beta = \pm n\pi$ where, n=0, 1, 2, 3,...

but at the same time $\sin N\beta = 0$, so that the factor $(\sin N\beta/\sin\beta)$ becomes indeterminate.

By applying the Hospital's rule

The resultant intensity is $I = R^2 = (A \sin\alpha/\alpha)^2 . N^2$

The maxima are most intense and are called as principal maxima.

The maxima are obtained for

$$\beta = \pm n\pi$$

$$[\pi\,(e+d)\sin\Theta]/\lambda = \pm n\pi$$

$$\text{or } e+d\ \sin\theta = \pm n\lambda \quad \text{Where, } n = 0, 1, 2, 3\ldots.$$

n=0 corresponds to zero order maximum. For n=1, 2, 3...etc., the first, second, third, etc., principal maxima are obtained respectively. The ± sign shows that there are two principal maxima of the same order lying on the either side of zero order maximum.

Minima

A series of minima occur, when Sin Nβ = 0 but sinβ≠0

For minima, Sin Nβ = 0

$$N\beta = \pm m\pi$$

$$[N\pi\,(e+d)\sin\Theta]/\lambda = \pm m\pi$$

$$N\,(e+d)\sin\Theta = \pm m\lambda$$

Where m has all integral values except 0, N, 2N ...nN, because for these values sinβ becomes zero and the principal maxima is obtained. Thus, m = 1, 2, 3... (N-1). Hence, they are adjacent principal maxima.

Secondary maxima

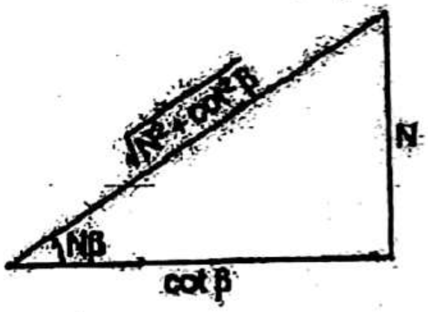
To find out the position of these secondary maxima, equation (2) must be differentiated with respect to β and then equate it to zero. Thus, $dI/d\beta = 0$ and on solving

$$N\tan\beta = \tan N\beta$$

The roots of this equation other than those for which β = ±$n\pi$ give the positions of secondary maxima. To find out the value of ($\sin^2 N\beta/\sin^2\beta$) from equation N tanβ = tan Nβ; a triangle shown below, in the figure is used.

$$\sin N\beta = \frac{N}{\sqrt{(N^2+\cot^2\beta)}}$$

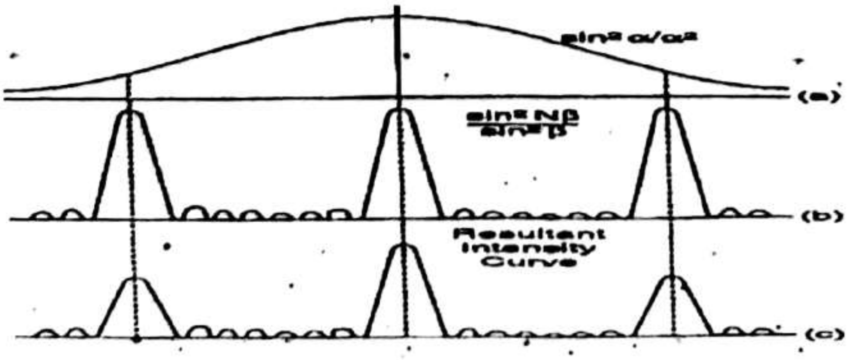
$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{(N^2+\cot^2\beta)\times\sin^2\beta} = \frac{N^2}{N^2\sin^2\beta+\cos^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$



$$\text{Therefore, } \frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

As N increases, the intensity of secondary maxima relative to principal maxima decreases and becomes negligible when N becomes large.

Figures (a) and (b) show the graphs of variation of intensity due to the factors $\sin^2\alpha/\alpha^2$ and $\sin^2 N\beta/\sin^2\beta$ respectively. The resultant is shown in figure (c).



9. What are the characteristics of grating spectra?

Characterization of grating spectra:

i. Spectra of different orders are situated symmetrically on the both sides of zero order image.

ii. Spectral lines are almost straight and quite sharp.

iii. Spectral lines are in the order from violet to red.

iv. The spectral lines are more and more dispersed as we go to higher orders.

v. Most of the incident intensity goes to zero order and rest is distributed among the other orders.

10. Explain the condition for maximum number of orders available with grating?

Maximum no. of orders available with a grating:

For principal maxima, $(e+d)\sin\theta = n\lambda$

$$\Longrightarrow n = \frac{(e+d)\sin\theta}{\lambda}$$

The maximum angle of deflection is $\theta = 90^0$ $\therefore (n)_{max} \leq \frac{(e+d)}{\lambda} \leq \frac{1}{N\lambda}$

Where $N=\frac{1}{e+d}$ = Number of lines per unit distance of grating.

$$(n)_{max} \leq \frac{(e+d)}{\lambda} \leq \frac{1}{N\lambda}$$

This condition gives the maximum number of orders possible with a grating.

Ex: If N=5000 lines/cm and λ = 420 nm then $(n)_{max} \leq 4.76$

Here n has to be an integer, so maximum number of orders possible is only 4.

11. Explain dispersive power of a grating.

Dispersive power of grating:

Dispersive power of grating is defined as the ratio of variation of angle of diffraction with wavelength.

If θ_1 & θ_2 are the angles of diffraction in a particular order for wavelengths λ_1& λ_2 Respectively, then $\frac{\theta_1-\theta_2}{\lambda_1-\lambda_2}$ is called dispersive power. If λ_1-λ_2 is very small then it was represented by $\frac{d\theta}{d\lambda}$

We have $(e+d)\sin\theta = n\lambda$ differentiating w.r.t λ we have $(e+d)\cos\theta\, d\theta = n\, d\lambda$ ==>

$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$$

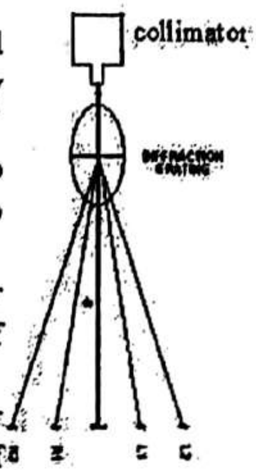
$$\boxed{\text{Therefore } \frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}}$$

Therefore $\frac{d\theta}{d\lambda}\ \alpha\ n$; $\frac{d\theta}{d\lambda}\ \alpha\ \frac{1}{(e+d)}$; $\frac{d\theta}{d\lambda}\ \alpha\ \frac{1}{\cos\theta}$

12. Explain the determination of wavelength of light by a diffraction grating.

Determination of wavelength:

To determine the wavelength of mono chromatic light using diffraction grating and spectrometer, the procedure is as follows.



1. The collimator C of the spectrometer is adjusted to produce parallel rays and the telescope T is adjusted to receive the parallel rays by focusing the distant object.
2. The grating G is placed on the grating table such that it is normal to the axis of collimator. It is illuminated by source whose wavelength is to be determined.
3. The telescope is slowly turned to one side and coincide the first order diffracted image with vertical cross wire of eyepiece. The reading of telescope is noted.
4. Now the telescope is turned to another side and coincide the first order diffracted image with the vertical cross wire of eyepiece. The reading of telescope is noted. The difference of these two readings gives 2θ.

By substituting the value of θ in the given equation λ can be determined.

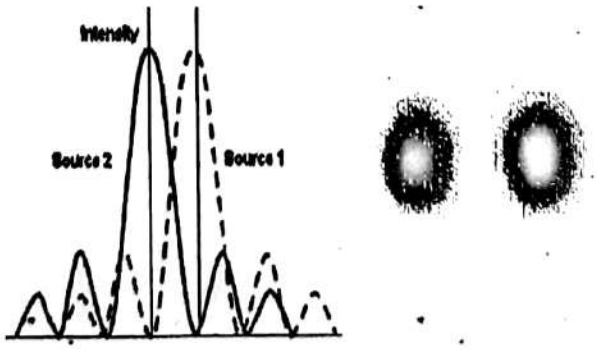
$$\lambda = \frac{\sin\theta}{nN}$$

Where n is order of the spectrum and N is no. of lines per unit width of the grating.

13. What is resolving power? Derive the expression for resolving power of grating.

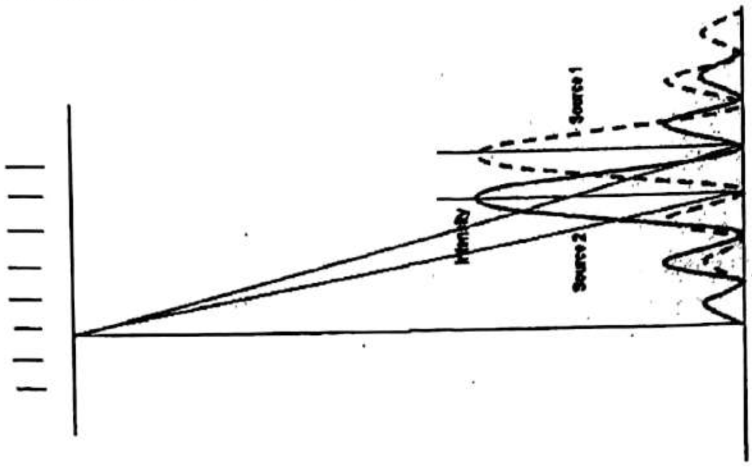
Resolving power: When the two objects are very near to each other or at very large distance from our eye it cannot able to see them separately. To see them separately, optical instruments like telescope, microscope, prism and grating etc. are employed. Thus an optical instrument is said to be able to resolve two point objects if the corresponding diffraction patterns are distinguishable from each other.

Definition of resolving power: The ability of the instrument to produce two separate images of very close objects is known as resolving power.



Resolving power of a grating: The resolving power of a diffraction grating is its ability to separate spectral lines which have nearly the same wavelength. This is measured by $(\lambda/d\lambda)$ where $d\lambda$ is the smallest difference in two wavelengths which are just resolvable by grating. λ is the wavelength of either (or) mean wavelength.

Expression for resolving power of grating:



Let AB be the plane transmission grating having grating element (e+d), N be the total no. of slits. Let a beam of light having two wavelengths λ and $\lambda+d\lambda$ incident normally on the grating. P1 is n^{th} primary maximum of spectral line of λ at an angle of diffraction θ_n and P2 is the n^{th} primary maximum of $\lambda+d\lambda$ at an angle of diffraction $(\theta_n+d\theta_n)$. According to Rayleigh criterion, the two wavelengths will be resolved if the position P2 corresponds to first minimum of P1 and vice versa. Consider the first minimum of P1 in the direction of $(\theta_n+d\theta_n)$.

The principal maximum of λ in the direction θ_n is given by

$(e+d) \sin \theta_n = n\lambda$ -----------(1)

Similarly the principal maximum of $\lambda+d\lambda$ in the direction $(\theta_n+d\theta_n)$ is given by

$(e+d) \sin(\theta+d\theta_n) = n(\lambda+d\lambda)$ ----------------------(2)

The path difference between the rays corresponding to the angle of diffraction θ_n and $(\theta_n+d\theta_n)$ is (λ/N), where N is the total number of lines on the grating surface.

$$n(\lambda+d\lambda) - n\lambda = \lambda/N$$

$$n.d\lambda = \lambda/N$$

$$\boxed{\Rightarrow \frac{\lambda}{d\lambda} = nN \qquad \Rightarrow \frac{\lambda}{d\lambda}\ \alpha\ n, \quad \frac{\lambda}{d\lambda}\ \alpha N}$$

This is the required equation.

15. What are the applications of diffraction?

Applications of diffraction:

1. The wavelength of spectral lines can be measured by using diffraction grating.
2. The wavelength of x-rays can be determined by x-ray diffraction.
3. The structures of the crystal can be determined by the x-ray diffraction.
4. The velocity of sound in liquids can be determined by using ultrasonic diffraction.
5. The size and shape of tumors, ulcers etc., inside the human body can be assessed by ultrasound scanning.

Assignment Questions

1. What are the differences between interference and diffraction?
2. What are the types of diffraction and give differences between them.
3. What is meant by diffraction of light and explain Fraunhofer diffraction due to single slit.
4. Give the theory of Fraunhofer diffraction due to double slit. Using this obtain intensity distribution curve.
5. Explain with the necessary theory of Fraunhofer diffraction due to n slits.
6. What is resolving power? Derive the expression for resolving power of grating.
7. Explain the determination of wavelength of light by a diffraction grating.
8. What are the applications of diffraction?
9. Explain dispersive power of a grating

Problems

1. How many orders will be visible, if the wave length of light is 5000 A^0. Given that the number of lines per centimeter on the grating is 6655 (Ans: 3)
2. Show that the grating with 500 lines / cm cannot give a spectrum in the 4^{th} order for the light of wavelength 5890A^0.
3. A plane transmission grating having 4250 lines per cm is illuminated with sodium light normally. In second order spectrum the spectral lines are divided by 30^0 are observed. Find the wavelength of the spectral line. (Ans: λ = 5882 A^0)
4. A diffraction grating having 4000 lines / cm is illuminated normally by light of wavelength 5000A^0. Calculate its resolving power in the third order spectrum. (Ans: 1mm, 2mm)
5. A grating has 6000 lines/cm. Find the angular separation b/w two wave lengths 500 mm and 510 mm in the 3^{rd} order. (Ans: $d\theta = 2.48^0$)
6. Find the highest order that can be seen with a grating having 15000 lines / inches. The wave length of the light used is 600 mm. (Ans: 2)
7. A plane transmission grating having 6000 lines / cm is used to obtain a spectrum of light from a sodium lamp in the second order. Calculate the angular separation between two sodium lines D1 and D2 of wave length 5890A^0 and 5896 A^0. ($d\theta = 0.06^0$)

Objective Questions

1. When white light is incident on a diffraction grating, the light that will be deviated more from central image will be

a. Yellow b. Violet c. Indigo **d. Red**

2. The diffraction phenomenon is

a. Bending of light around an obstacle b. rectilinear propagation of light
c. Oscillation of light wave in one direction d. None of them

3. To find prominent diffraction, the size of the diffracting objects should be

a. Greater than the wavelength of light used **b. Of the order of wavelength of light**
c. Less than the wavelength of light d. None of them

4. In Fresnel diffraction
   a. Source of light is kept at infinite distance from the aperture
   b. Source of light is kept at finite distance from the aperture
   c. Convex lens is used
   d. Aperture width is selected so that, it can act as a point source.

5. In Fraunhofer diffraction, the incident wave front should be
   a. Elliptical **b. Plane** c. Spherical d. Cylindrical

6. Significant diffraction of X-rays can be obtained
   a. By a single slit b. By a double slit
   c. By a diffraction **d. By an atomic crystal**

7. Rising and setting sun appears to be reddish because
   a. Diffraction sends red rays to the earth at these times
   b. Scattering due to dust particles and air molecules is responsible for this effect
   c. Refraction is responsible for this effect
   d. Polarization is responsible for this effect

8. The penetration of waves into the regions of the geometrical shadow is
   a. Interference **b. Diffraction** c. Polarization d. Dispersion

9. In a single slit experiment if the slit width is reduced
   a. The fringes become brighter b. The fringes becomes narrower
   c. The fringes become wider d. The colour of the fringes changes

10. In diffraction due to double slit we observe
    a. Wider interference fringes and narrower diffraction bands
    b. Interference and diffraction fringes of equal width
    c. Wider diffraction bands and within that narrower interference fringes
    d. Diffraction pattern due to both the slits independently

11. In double slit diffraction if the width of the slit is equal to the spacing between the slits then
    a. Even order interference maxima will be missing
    b. All interference maxima will be present
    c. All interference maxima will be missing
    d. Diffraction fringes & interference fringes exactly coincide and hence totally disappear.

12. A parallel beam of mono chromatic light falls normally on a plane diffraction grating having 5000 lines/cm. A second order spectral line is diffracted through an angle of 30^0. The wavelength of light is
    a. 5×10^{-7}cm b. 5×10^{-6}cm c. 5×10^{-5}cm d. 5×10^{-4}cm

$$HINT: \lambda = \frac{(e+d)\sin\theta}{n}, (e+d) = \frac{1}{5000cm}, \theta = 30^0, n = 2)$$

13. Maximum number of orders possible with a grating is
    a. Independent of grating element **b. directly proportional to grating element**
    c. Inversely proportional to grating element d. directly proportional to wave length

14. If 1000 is the resolving power of a grating in it first order, its resolving power in 2nd order is

a. 500 b. 1000 **c. 2000** d. None

15. The class of diffraction in which lenses required is
a. Fresnel **b. Fraunhofer** c. Both a & b d. None

16. A diffraction grating is
a. Large number of equidistant slits b. Large number of random distant slits
c. More than two slits d. None

17. In diffraction grating, the conditions for principal maxima
a. $(e+d)\sin\theta = n\lambda$ b. $d\sin\theta = n\lambda$ c. $\sin\theta = n\lambda$ d. $e\sin\theta = n\lambda$

18. Resolving power of a grating is
a. Directly proportional to N b. Inversely proportional to N
c. Independent of N d. Directly proportional to N^2

19. The expression for number of orders possible in diffraction grating is
a. $n_{max}=1/N\lambda$ b. $n_{max}=(e+d)/\lambda$ **c. Both a & b** d. None

20. The intensity of various fringes in the diffraction pattern is
a. Constant **b. Varies** c. Zero d. None

21. When the number of lines on the grating surface is large, the grating spectrum becomes
a. Bands b. Continuous colors **c. Line** d. None

22. The expression of resolving power of grating is R=
a. $\lambda/d\lambda$ b. n N **c. Both a & b** d. None

Wave Optics

Principle of Super position:-

When two or more waves travel simultaneously in a medium then the resultant displacement of the individual waves then they are super imposed with each other is equal to the algebraic sum of the displacements of individual waves. This principle is called 'Super Position.' (or) 'Principle of Super Position'.

Explanation:-

Let y_1, y_2 be the displacements of any two waves travels simultaneously in a medium.

When the two waves super imposed with each other then according to the principle of super position. The resultant displacement (y) can we written as.

$$\boxed{y = y_1 \pm y_2}$$

Case (i):-

If the resultant displacement

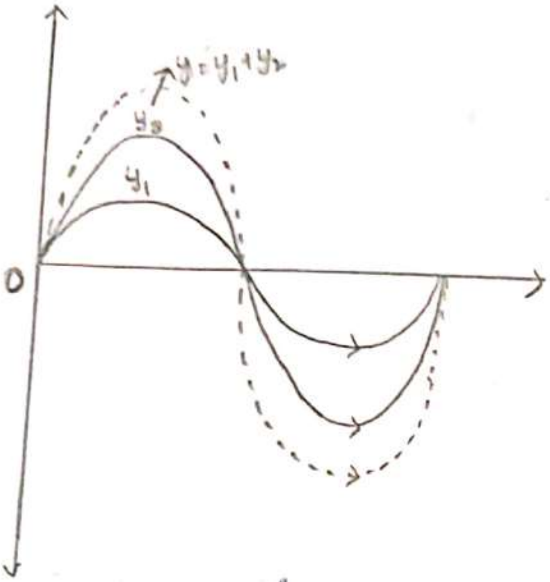
$$\boxed{y = y_1 + y_2}$$

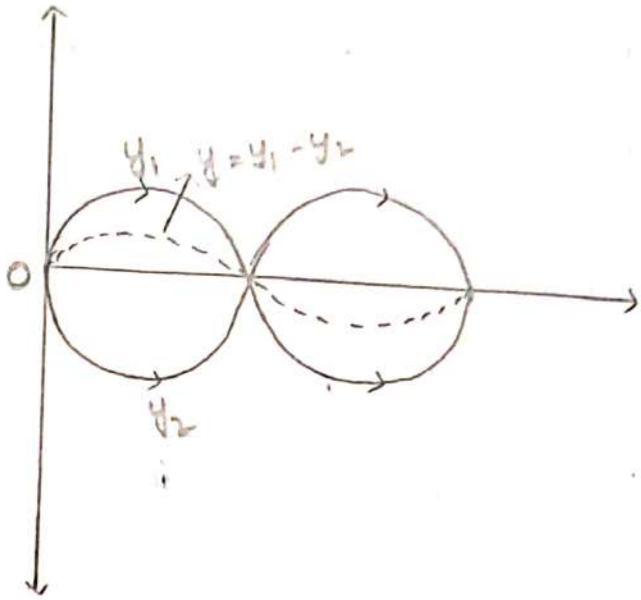
i.e., the two waves are travel in same direction.

Case-(ii) :-

If suppose the two waves travel in opposite direction then the resultant displacement.

$$\boxed{y = y_1 - y_2}$$





Coherent Waves :-

The waves which are having same amplitude same wavelength and constant phase difference are called 'coherent Waves'.

Coherent Sources :-

The light sources which can exhibits coherent waves are called 'Coherent Sources'.

Interference :-

When the two coherent waves super imposed with each other than the formation of maximum intensity and minimum intensity (or) bright fringe and drak fringe in the region of super position. This phenomenon is called 'Interference'

(or)

When the two coherent waves super imposed with each other then the intensity or amplitude can be modify in the region of super position. This phenomenon is called 'Interference'.

Types of Interference :-

The interference pattern is mainly two types.

They are : ① Constructive interference.

② Destructive interference.

① Constructive Interference :-

In which the resultant amplitude is sum of the individual amplitudes of the waves then the interference is called constructive interference.

Let a_1, a_2, be the amplitudes of the any two waves. If 'A' be the resultant amplitude of the waves then according to the constructive interference.

Resultant Amplitude $A = a_1 + a_2$

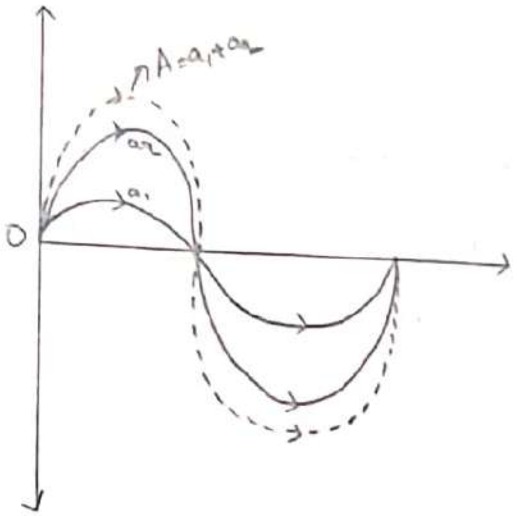
$a_1 = a_2 \approx a$

$I = (a+a)^2$

$\boxed{I_{max} = 4a^2}$

$A^2 = (a_1+a_2)^2$

$\boxed{I = (a_1+a_2)^2}$



② <u>Destructive</u> <u>Interference</u>:-

The Interference in which the resultant amplitude is difference of the individual amplitudes of the waves then then the interference is called Destructive Interference.

Let a_1, a_2 be the amplitude of any two waves. If 'A' be the resultant amplitude of the waves then according to the destructive interference

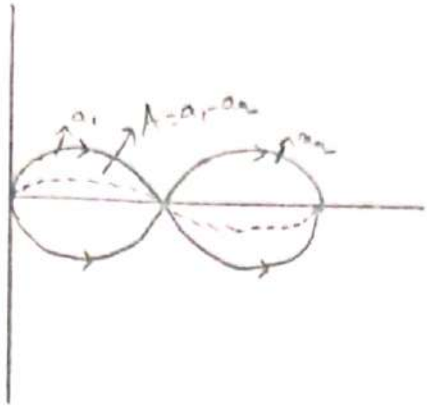
$$A = a_1 - a_2$$

$$A^2 = (a_1 - a_2)^2$$

$$I = (a_1 - a_2)^2$$

$$I = (a - a)^2$$

$$\boxed{I_{min} = 0}$$



Conditions when the two waves super impose with each other:-

let y_1, y_2 be the displacements of any two waves which can be taken as $y_1 = a_1 \sin \omega t \longrightarrow (1)$

$$y_2 = a_2 \sin(\omega t + \phi) \text{ --- } (2)$$

If suppose 'y' be the resultant displacement of above two waves then we can be written as

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 (\sin \omega t \cdot \cos\phi + \cos \omega t \sin\phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cdot \cos\phi + a_2 \cos \omega t \sin\phi$$

$$= \sin \omega t (a_1 + a_2 \cos\phi) + a_2 \cos \omega t \sin\phi$$

let $a_1 + a_2 \cos\phi = R \cos\theta$ --- (3)

$$a_2 \sin\phi = R \sin\theta \text{ --- } (4)$$

eq^n $(3)^2 + (4)^2$

$$[a_1 + a_2 \cos\phi]^2 + [a_2 \sin\phi]^2 = R^2 \cos^2\theta + R^2 \sin^2\theta$$

$$a_1^2 + a_2^2 \cos^2\phi + 2a_1a_2\cos\phi + a_2^2\sin^2\phi = R^2\cos^2\theta + R^2\sin^2\theta$$

$$a_1^2 + a_2^2\left[\cos^2\phi + \sin^2\phi\right] + 2a_1a_2\cos\phi = R^2\left[\cos^2\theta + \sin^2\theta\right]$$

$$\boxed{a_1^2 + a_2^2 + 2a_1a_2\cos\phi = R^2} \quad \text{—— (5)}$$

if $a_1 = a_2 \approx a$

$$a^2 + a^2 + 2a^2\cos\phi = R^2$$

$$2a^2 + 2a^2\cos\phi = R^2$$

$$R^2 = 2a^2\left[1 + \cos\phi\right]$$

$$= 2a^2\left[2\cos^2\frac{\phi}{2}\right]$$

$$\boxed{R^2\ I = 4a^2\cos^2\left(\frac{\phi}{2}\right)}$$

The above equation can represents the resultant intensity of two waves when they are super imposed with each other.

Case (i):- For the constructive interference (or) maximum intensity (or) Bright Fringe.

The phase difference values should be $0, 2\pi, 4\pi, 6\pi, \ldots 2n\pi$

i.e., $\boxed{\phi = 2n\pi}$ Where $n = 1, 2, 3, \ldots$

$\therefore$ For the constructive interference the values of path difference $= 0, \lambda, 2\lambda, 3\lambda, 4\lambda, \ldots n\lambda$.

i.e., for the constructive interference

$$\boxed{\text{path different} = n\lambda}$$

Case (ii):- For the constructive interference (or) minimum intensity (or) dark fringe.

The phase destructive interference the values of path difference

$$\frac{\lambda}{2}, \frac{3}{2}\lambda, \frac{5}{2}\lambda, \frac{7}{2}\lambda, \ldots\ldots (2n+1)\frac{\lambda}{2}.$$

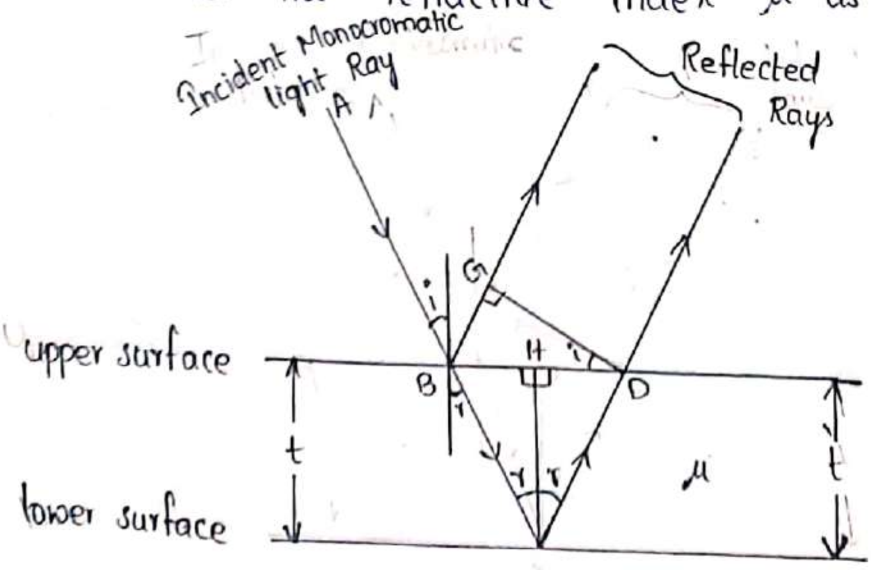
$$\text{path difference} = (2n+1)\frac{\lambda}{2}.$$

Relation between Path difference and phase difference:-

$$\boxed{\text{Phase difference} = \frac{2\pi}{\lambda}(\text{path difference})}$$

Interference in Thin films:-

Let us consider a thin film whose thickness is 't' and the refractive index 'μ' as shown in the figure.



When a monocromatic light ray ($\overrightarrow{AB}$) incident on the thin film the some portion of incident light ray reflects from the upper surface of the thin film as BGF.

The remaining portion of incident light ray refracts into the film as $\overrightarrow{BC}$ and again it will reflects from the lower surface of the thin film emerges through the film as $\overrightarrow{CDE}$ as shown in the figure.

Here the two reflected rays reflecting from the upper and lower surface of the thin film super imposed with each other than from the interference pattern.

From the figure 'i' be the incident angle.

'r' be the refracted angle.

$\overline{CH}$ be the perpendicular line to the upper surface drawn from the lower surface at the pt 'C'.

'$\overline{DG}$' be the perpendicular line drawn to the reflected ray $\overrightarrow{BF}$ from the point 'D'.

From the figure the path difference b/w to the two affected rays can be written as

$$\boxed{\Delta = (BC + CD)\mu - BG \quad (1)}$$

Where,

(BC + CD) be the path travelled by the second ray in the film.

From the ΔBCH,

$$\cos r = \frac{CH}{BC}$$

$$\cos r = \frac{t}{BC} \qquad [\because CH = t]$$

$$\boxed{BC = \frac{t}{\cos r}} \quad \text{—— (2)}$$

Similarly, from the ΔCHD,

$$\cos r = \frac{CH}{CD}$$

$$\cos r = \frac{t}{CD}$$

$$\boxed{CD = \frac{t}{\cos r}} \quad \text{—— (3)}$$

$$(2) + (3) \Longrightarrow BC + CD = \frac{t}{\cos r} + \frac{t}{\cos r}$$

$$BC + CD = \frac{2t}{\cos r}$$

$$\boxed{(BC + CD)\mu = \frac{2\mu t}{\cos r}} \quad \text{—— (4)}$$

For the calculation for BG first we should calculate

$$\boxed{BD = BH + HD} \quad \text{—— (5)}$$

From the ΔBCH

$$\tan r = \frac{BH}{CH}$$

$$\tan r = \frac{BH}{t}$$

$$\boxed{BH = t\ \text{Tan}\, r} \quad \text{--- } ⑥$$

From the ΔCHD,

$$\text{Tan}\, r = \frac{HD}{CH}$$

$$\text{Tan}\, r = \frac{HD}{t}$$

$$\boxed{HD = t\ \text{Tan}\, r} \quad \text{--- } ⑦$$

⑥ + ⑦ $\Rightarrow$ $BH + HD = BD$

$$BD = t\ \text{Tan}\, r + t\ \text{Tan}\, r$$

$$\boxed{BD = 2t\ \text{Tan}\, r} \quad \text{--- } ⑧$$

From the ΔBDG,

$$\sin i = \frac{BG}{BD}$$

$$\sin i = \frac{BG}{2t\ \text{Tan}\, r}$$

$$\boxed{BG = \sin i\ (2t\ \text{Tan}\, r)} \quad \text{--- } ⑨$$

We know that,

Snell's law $\mu = \frac{\sin i}{\sin r}$

$$\boxed{\sin i = \mu \sin r} \quad \text{--- } ⑩$$

Substitute the eq ④ & ⑪ in ① eq^n

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r} \sin^2 r$$

$$= \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$

$$= \frac{2\mu t}{\cos r}[\cos^2 r]$$

$$\boxed{\Delta = 2\mu t \cos r} \quad \text{——— (12)}$$

When the light travel from rarer to denser medium then 'π' phase difference (or) $\lambda/2$ path difference will occur in b/w the reflected and refracted rays.

∴ The perfect path difference b/w to effective rays can be written as.

$$\boxed{\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}}$$

Condition for Constructive Interference (or) Maximum Interference (or) Bright fringe

For the constructive interference the path difference b/w the two effective rays should be equal to '$n\lambda$'.

i.e., $2\mu t \cos r + \frac{\lambda}{2} = n\lambda$

$$2\mu t \cos r = n\lambda - \frac{\lambda}{2}$$

$$= \frac{2n\lambda - \lambda}{2}$$

$$\boxed{2\mu t \cos r = (2n-1)\lambda/2} \quad \text{——— (13)}$$

Condition for Destructive Interference (or) Minimum Intensity (or) Dark fringe

For the destructive interference the path difference b/w the two effective rays should be equal to $(2n+1)\frac{\lambda}{2}$.

i.e., $2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2} - \frac{\lambda}{2}$$

$$= \left(2n\left(\frac{\lambda}{2}\right) + \left(\frac{\lambda}{2}\right) + \frac{\lambda}{2} - \frac{\lambda}{2}\right)$$

$$\boxed{2\mu t \cos r = n\lambda} \quad \text{——} \ (14)$$

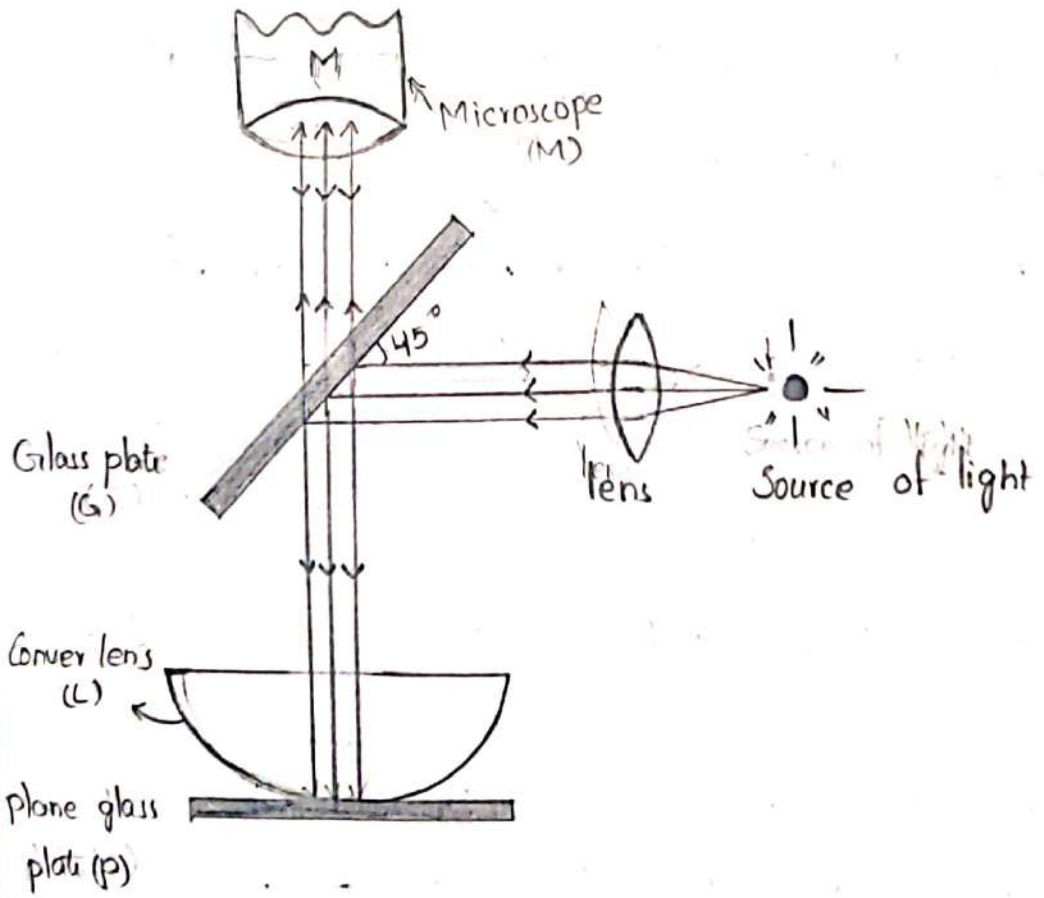
Condititions for sustainable Interference :-

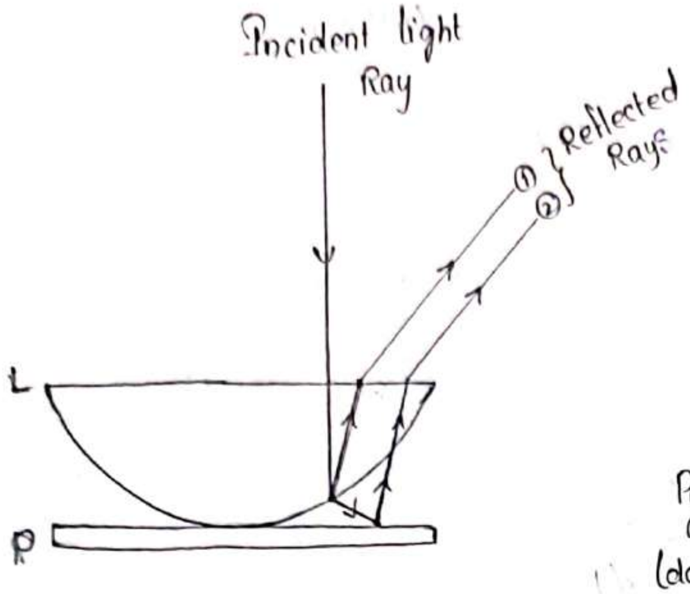
1. The light wave should be coherent (i.e., the light waves have same wavelength, same frequency, same amplitude, same Intensity and constant phase difference).
2. The width of the slit should be narrow. The distance b/w the two slits (d) should be minimum.
3. The distance b/w the slits and the screen should be maximum.
4. The background of the screen should be dark.

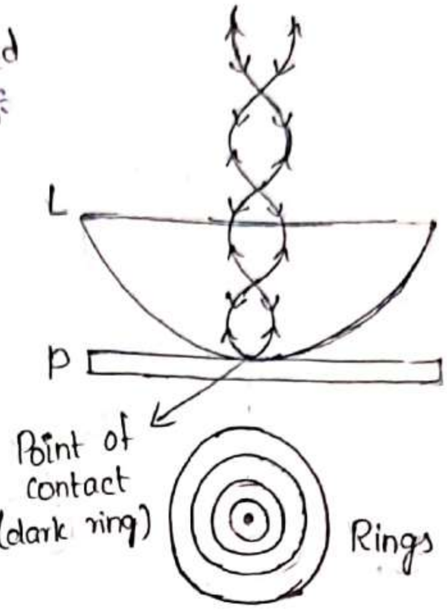
Newton Rings (or) Interference pattern in uniform thin films :-

Newton Rings are the alternating bright and dark rings which are formed due to the interference of two light rays. First the formation of these interfered alternating bright and dark rings was observe by newton. Hence these are called Newton Rings.

"Newton Rings are formed due to two reflected light rays reflecting from the upper and lower surface of the air film which is formed in between plano convex lens and plane glass plate."







Experimental Arrangement:-

The experimental arrangement of formation of Newton Rings as shown in the figure.

Where 's' is the light source which can emit light rays. These light rays incident on the 45° arrangement of glass plate then the light rays falls normally on the plano convex lens from the glass plate.

Due to two reflected rays reflecting from the upper and lower surface of air film ($\mu = 1$) which is formed in between plano convex lens (L) and plane Glass plate (P), then form interference pattern. Due to circular symmetry we can observe alternating bright and dark rings as shown in the figure.

The entire formation of Newton Rings can be observe by the microscope (M) which is place top of the experimental arrangement

Theory :-

We know that the path difference between the two reflected rays in the case of thin film interference

It can be written as

$$\Delta = 2\mu t \cos r \pm \lambda/2 \quad \text{—} (1)$$

In the case of Newton Rings experiment the thin film is air film. For the air film $\mu = 1$

For the normal reflection $r = 0$

Substitute these two conditions in eq^n (1)

$$\Delta = 2(1)t \cos 0 \pm \lambda/2$$

$$\Delta = 2t \pm \lambda/2 \quad \text{—} (2)$$

Condition for Bright Ring :-

$$2t + \lambda/2 = n\lambda$$

$$2t = n\lambda - \frac{\lambda}{2}$$

$$2t = \frac{2n\lambda - \lambda}{2}$$

$$\boxed{2t = (2n-1)\frac{\lambda}{2}} \quad \text{—} (3)$$

Condition for Dark Ring:-

$$2t \pm \frac{\lambda}{2} = (2n+1)\lambda/2$$

$$2t = (2n)\frac{\lambda}{2} + \lambda/2 - \lambda/2$$

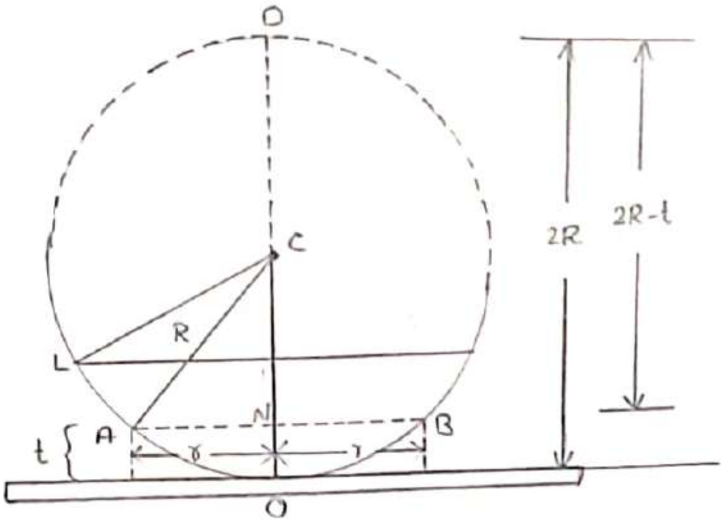
$$\boxed{2t = n\lambda} \quad \text{--- (4)}$$

Where 't' variable thickness of air film

<u>Calculation of thickness of air film:-</u>

let us consider 't' be the variable thickness of air film, 'r' be the radius of any one of the consider ring.

Let us draw a circle by using the radius of curvature of plano convex lens.



According to the property of the circle from figure.

$$ON \times ND = NA \times NB$$

$$t \times (2R-t) = r \times r$$

$$2Rt - t^2 = r^2$$

for $R >>>> t$

Neglect t^2 from above term

$$2Rt = r^2$$

$$t = \frac{r^2}{2R} \quad \text{—— (5)}$$

(or)

$$t = \frac{(D/2)^2}{2R}$$

$$t = \frac{D^2}{8R} \quad \text{—— (6)}$$

Substitute eq^n (6) in eq^n (3)

$$\cancel{2}\left(\frac{D^2}{\cancel{8}_4 R}\right) = (2n-1)\frac{\lambda}{2}$$

$$D^2 = (2n-1)\frac{\cancel{4}^{2}\lambda R}{\cancel{2}}$$

$$\boxed{D^2 = (2n-1)\,2\lambda R} \quad \text{—— (7)}$$

$$\boxed{D_{n_{bright}} = \sqrt{2n-1}\,\sqrt{2\lambda R}} \quad \text{—— (8)}$$

The above equation can be represents the diameter of the n^{th} bright ring.

$$\boxed{D_{n_{bright}} \propto \sqrt{2n-1}}$$

$$\boxed{D_{n_{bright}} \propto \sqrt{\text{odd natural numbers}}}$$

Now substitute eq^n (6) in (4)

$$r\left(\frac{D^2}{\cancel{8}R}\right) = n\lambda$$
$\quad 4$

$$\boxed{D^2_{n\,dark} = 4n\lambda R} \quad — (9)$$

$$\boxed{D_{n\,dark} = \sqrt{4n\lambda R}} \quad — (10)$$

$$D_{n\,dark} \propto \sqrt{n}$$

$$\boxed{D_{n\,dark} \propto \sqrt{\text{natural numbers}}}$$

Applications of Newton Rings Experiment:-

1. Determination of wavelength of light (λ) :-

let us consider the condition for the n^{th} dark ring.

$$\boxed{D_n^2 = 4n\lambda R} \quad — (1)$$

Similarly, let us consider the diameter of m^{th} dark ring

$$\boxed{D_m^2 = 4m\lambda R} \quad — (2)$$

$$(1) - (2) \Longrightarrow D_n^2 - D_m^2 = 4n\lambda R - 4m\lambda R$$

$$= 4\lambda R(n-m)$$

$$\boxed{\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}} \quad — (3)$$

2. Determination of radius of curvature of plano convex lens:-

We know that the wavelength of light

$$\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}$$

$$\boxed{R = \frac{D_n^2 - D_m^2}{4(n-m)}}$$

3. Determination of Refractive index of a liquid (μ):-

If suppose the entire experiment formation of a Newton rings will be represent in a liquid whose refractive index is 'μ', then the diameter of n^{th} & m^{th} dark rings will be change $D_n'^2$ and $D_m'^2$.

We know that $\boxed{D_n^2 - D_m^2 = 4(n-m)\lambda R}$ —— ①

When the liquid is introduced then

$$\boxed{\mu \{D'^2_n - D'^2_m\} = 4(n-m)\lambda R} \quad \text{—— ②}$$

$$\frac{②}{①} \Rightarrow \frac{\mu(D'^2_n - D'^2_m)}{D_n^2 - D_m^2} = \frac{4(n-m)\lambda R}{4(n-m)\lambda R}$$

$$\boxed{\mu = \frac{D_n^2 - D_m^2}{D_n'^2 - D_m'^2}}$$

Difraction :-

The bending property of light is called Diffraction.

→ The Scientist grimaldi was first observed the diffraction pattern in ~~1665~~. 1665.

When the light is incident on an obstacle or Small aperture whose sizes are comparable with the wave-length of light then the light can bends around the edges or corners of the obstacle then enter into the geometrical shadow. This phenomenon of bending property of light is called "diffraction of light".

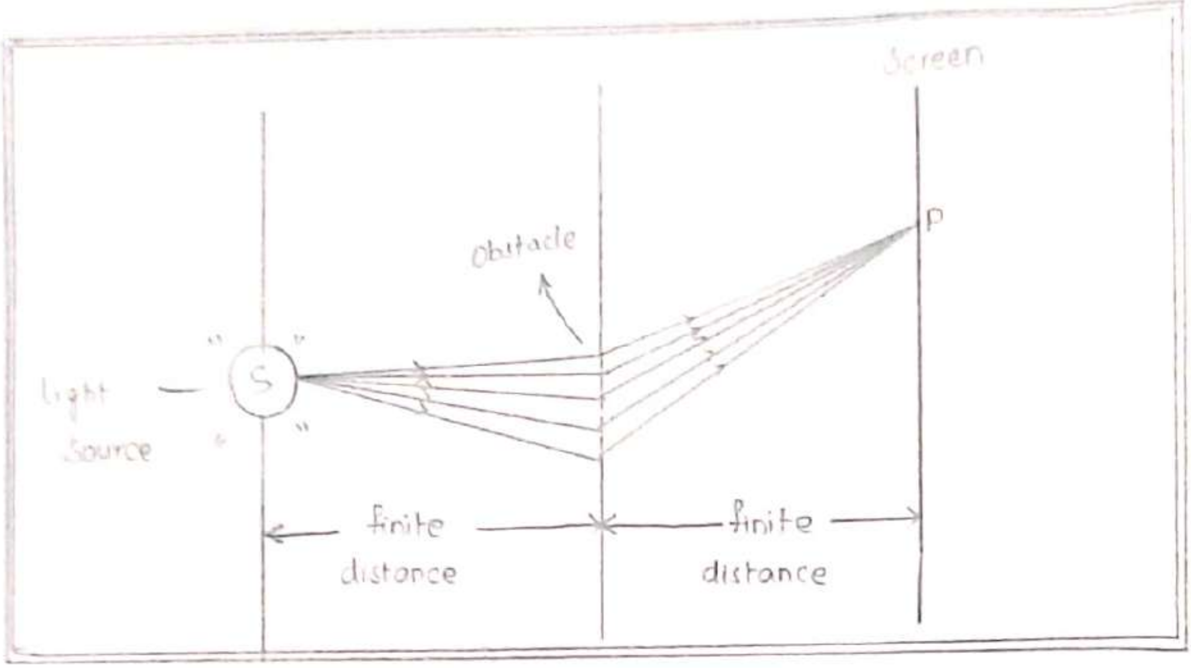
Types of diffraction:-

There are two types of diffraction pattern. They are :- ① Fresnel's diffraction.

② Fraunhofor diffraction.

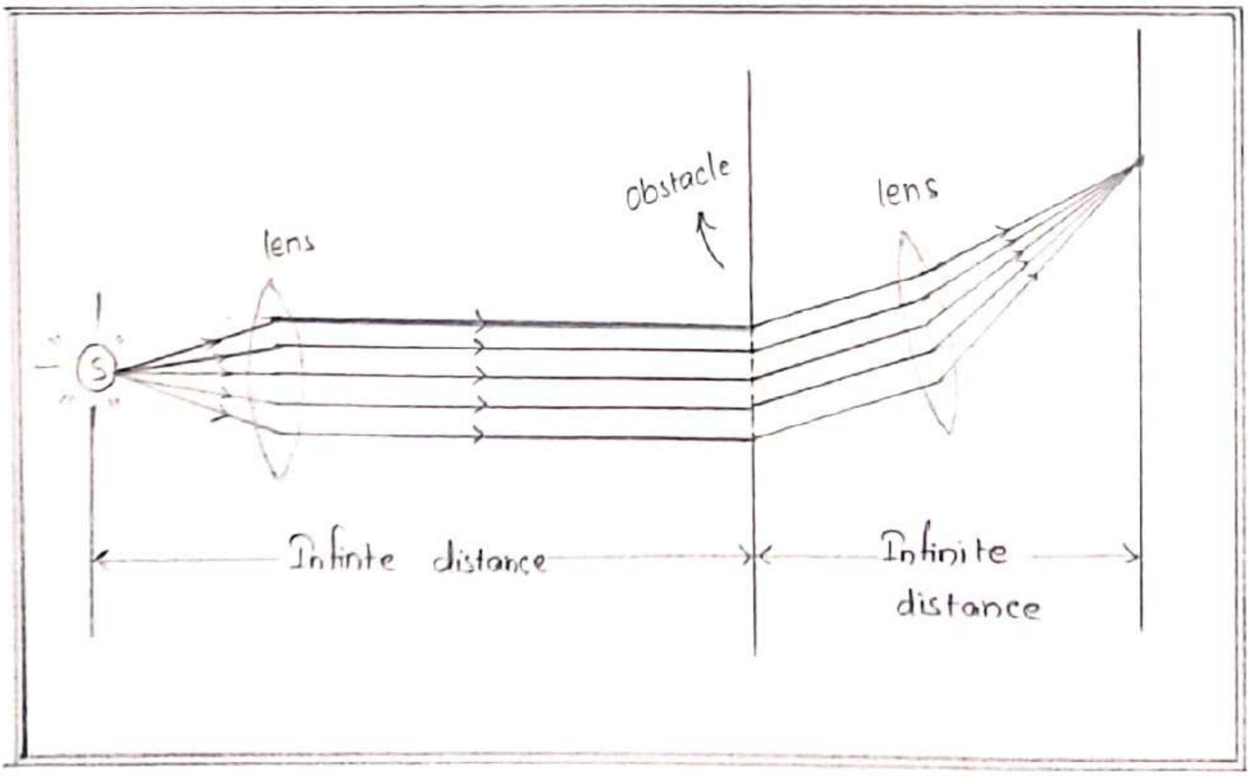
① Fresnel's diffraction :-

The diffraction in which the obstacle is finite distance from the source and the Screen then the diffraction is called Fresnel's diffraction.

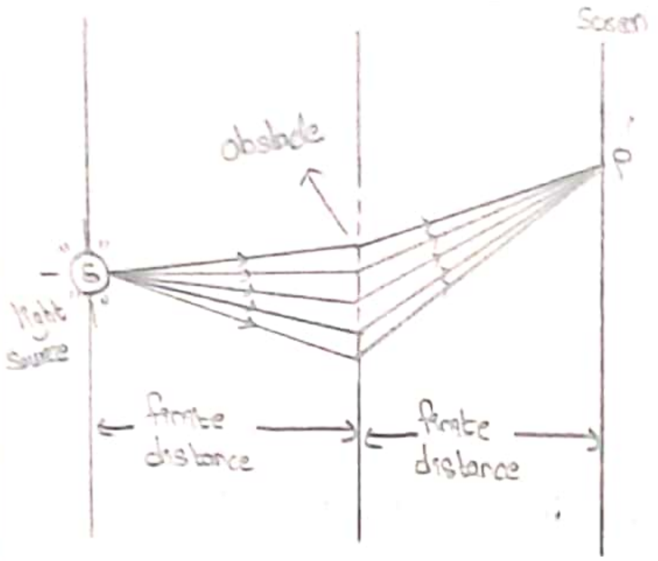
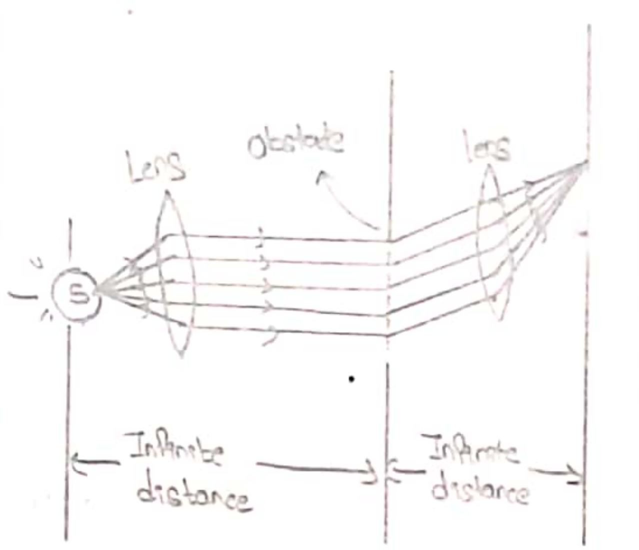


② fraunhofor diffraction :-

The diffraction in which the obstacle is infinite distance from the source and the screen then the diffraction is called fraunhofor diffraction.



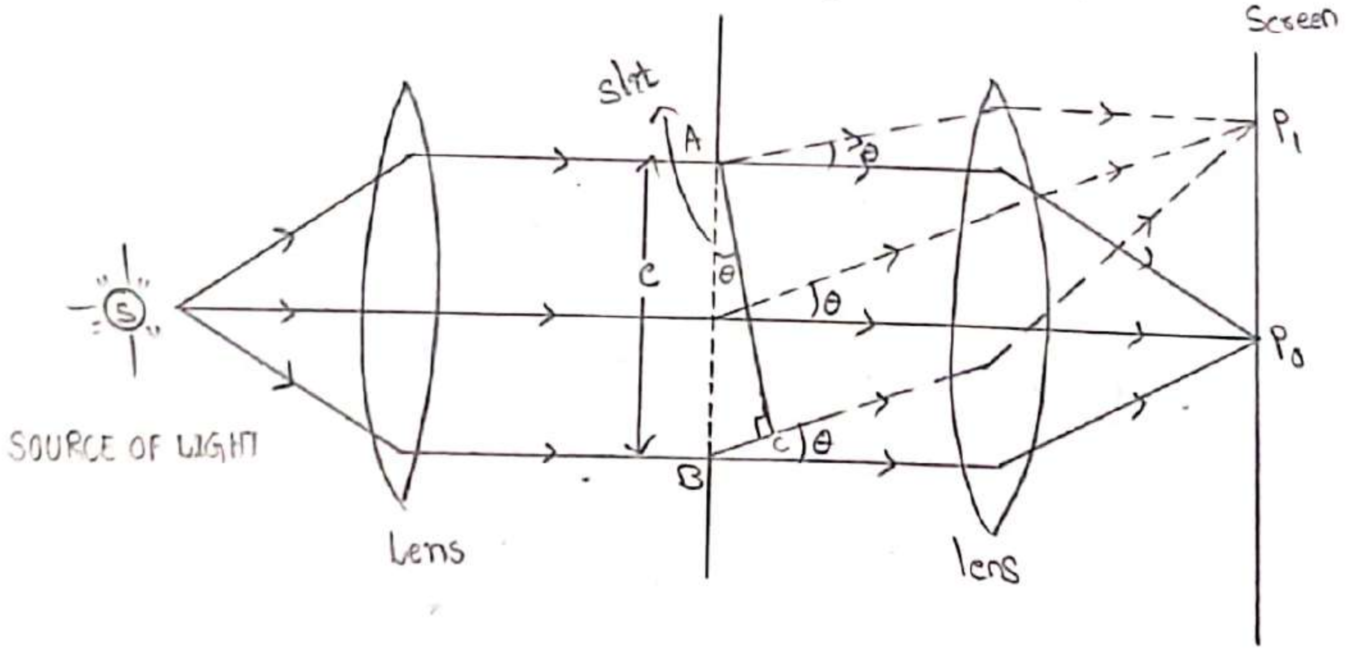
Difference between Fresnel's and Fraunhofor diffraction:-

Fresnel's diffraction	Fraunhofor diffraction
	
1. In this diffraction the obstacle is present finite distance from the source and the screen.	1. In this diffraction the obstacle is present infinite distance from the source and the screen
2. In this diffraction point sources or light from narrow slits can be used as light source.	2. Extended sources (or) highly illuminated sources are used as source of light.
3. Spherical (or) Cylindrical wave fronts are participate in this diffraction pattern.	3. Plane wave fronts are partici-pate in this diffraction pattern
4. No lenses is used in this diffraction pattern	4. Lenses are used in this diffraction pattern.
5. We can observe diffraction pattern in only one direction.	5. We can observe diffraction pattern in all directions.

Differences between Interference and Diffraction:

Interference	Diffractions
1. The interaction takes place b/w two sperate wave fronts ~~organi~~ Originating from the coherent sources.	1. The interaction takes place between the secondary wave lets originating different points of the exposed paths of the same wave front.
2. The fringe width may (or) may not be equal.	2. The fringe width of various fringes are never equal.
3. All the bright fringes have the same intensity.	3. The bright fringes or have various of intensity.
4. The dark fringes are perfectly dark.	4. There ~~are~~ is no perfect dark fringe.

Fraunhofar diffraction due to Single slit :-



let us consider a slit (i.e., Single slit) AB whose width is e

$$\boxed{AB = e}$$

When a plane wave length front ww' incident on the single slit then we can observe the resultant intensity of the non deviated rays at P. On the screen that can represents principle maxima (or) central maxima.

At the same time we can observe the resultant diffraction pattern of diffracted rays with the diffraction angle 'θ'. at P_1 on the screen as shown in figure.

From the figure the path difference b/w the two effective rays is BC.

From ΔABC, $\sin\theta = \frac{BC}{AB}$

$$\sin\theta = \frac{BC}{e}$$

$$\boxed{BC = e\sin\theta} \longrightarrow (1)$$

∴ The phase difference $\delta = \frac{2\pi}{\lambda}$ (path difference)

$$= \frac{2\pi}{\lambda}(BC)$$

$$\boxed{\delta = \frac{2\pi}{\lambda}(e\sin\theta)} \longrightarrow (2)$$

If suppose 'n' number of waves are passing through the single slit then the phase difference b/w the any two successive light rays can be written as

$$d = \frac{\text{Total phase difference}}{n}$$

$$= \frac{1}{n} \times \frac{2\pi}{\lambda} (e \sin\theta)$$

$$\boxed{d = \frac{2\pi}{n\lambda} (e \sin\theta)} \quad \text{--- (3)}$$

If suppose 'a' be the amplitude of a wave, 'n' be the no. of waves, 'd' be the phase difference b/w two light rays then according to vector addition method the resultant amplitude (R) can be written as

$$R = \frac{a \sin\left(\frac{nd}{2}\right)}{\sin\left(d/2\right)} \quad \text{--- (4)}$$

Now substitute eq^n (3) in (4)

$$R = \frac{a \sin\left[\frac{n}{2} \times \frac{2\pi}{n\lambda} (e \sin\theta)\right]}{\sin\left[\frac{1}{2} \times \frac{2\pi}{n\lambda} (e \sin\theta)\right]}$$

$$R = \frac{a \sin\left[\pi/\lambda\, (e \sin\theta)\right]}{\sin\left[\frac{\pi}{n\lambda} (e \sin\theta)\right]} \quad \text{--- (5)}$$

let

$$\boxed{\alpha = \frac{\pi}{\lambda} (e \sin\theta)} \quad \text{--- (6)}$$

Substitute eq^n (6) in (3)

$$R = \frac{a \sin\alpha}{\sin(\alpha/n)}$$

let us assume $\sin\left(\frac{\alpha}{n}\right) \approx \frac{\alpha}{n}$

$$R = \frac{a \sin\alpha}{\alpha/n}$$

$$R = \frac{(na) \sin\alpha}{\alpha}$$

We know that $\boxed{na = A}$

$$\boxed{R = A\left(\frac{\sin\alpha}{\alpha}\right)} \quad \text{—— (7)}$$

The above equation can be represents the resultant amplitude of the waves.

The resultant intensity of the waves.

$$I = R^2$$

$$I = \left[A\left(\frac{\sin\alpha}{\alpha}\right)\right]^2$$

$$\boxed{I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2} \quad \text{—— (8)}$$

Condition for principle maxima (or) Central maxima:-

let us consider the resultant amplitude

$$R = A\left[\frac{\sin\alpha}{\alpha}\right]$$

$$R = \frac{A}{\alpha}[\sin\alpha]$$

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \ldots\ldots\right].$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \ldots\ldots\right]$$

∴ For the maximum value of 'R' the negative terms in the above expansion should be equal to '0'.

i.e., $\boxed{\alpha = 0}$ —— (9)

$$\frac{\pi}{\lambda}(e \sin\theta) = 0$$

$$\sin\theta = 0$$

$$\boxed{\theta = 0}$$

i.e., The central maxima (or) principle maxima occurs at $\boxed{\theta = 0}$.

∴ The resultant amplitude $R = A$

$$R^2 = A^2$$

$$\boxed{I_{max} = A^2}$$

Condition for minima (or) Secondary minima :-

let us consider the resultant amplitude

$$R = A\left[\frac{\sin\alpha}{\alpha}\right]$$

For the minimum value of R, $\sin\alpha = 0$

i.e., $\boxed{\alpha = \pm m\pi}$

$$\boxed{\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\ldots \pm m\pi} \quad —— (10)$$

$\therefore$ We know that $\alpha = \frac{\pi}{\lambda}(e \sin\theta)$

$$\frac{\pi}{\lambda}(e \sin\theta) = \pm m\pi$$

$$\boxed{e \sin\theta = \pm m\lambda}$$

Condition for Secondary maxima :-

For the secondary maxima $\frac{dI}{d\alpha} \approx 0$

$$\frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$A^2 \frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right)^2 = 0$$

$$A^2 \; 2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\sin\alpha}{\alpha}\right] = 0$$

$$2A^2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\alpha\cos\alpha - \sin\alpha(1)}{\alpha^2}\right] = 0$$

$$2A^2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

$$\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha\cos\alpha = \sin\alpha$$

$$\alpha = \frac{\sin\alpha}{\cos\alpha}$$

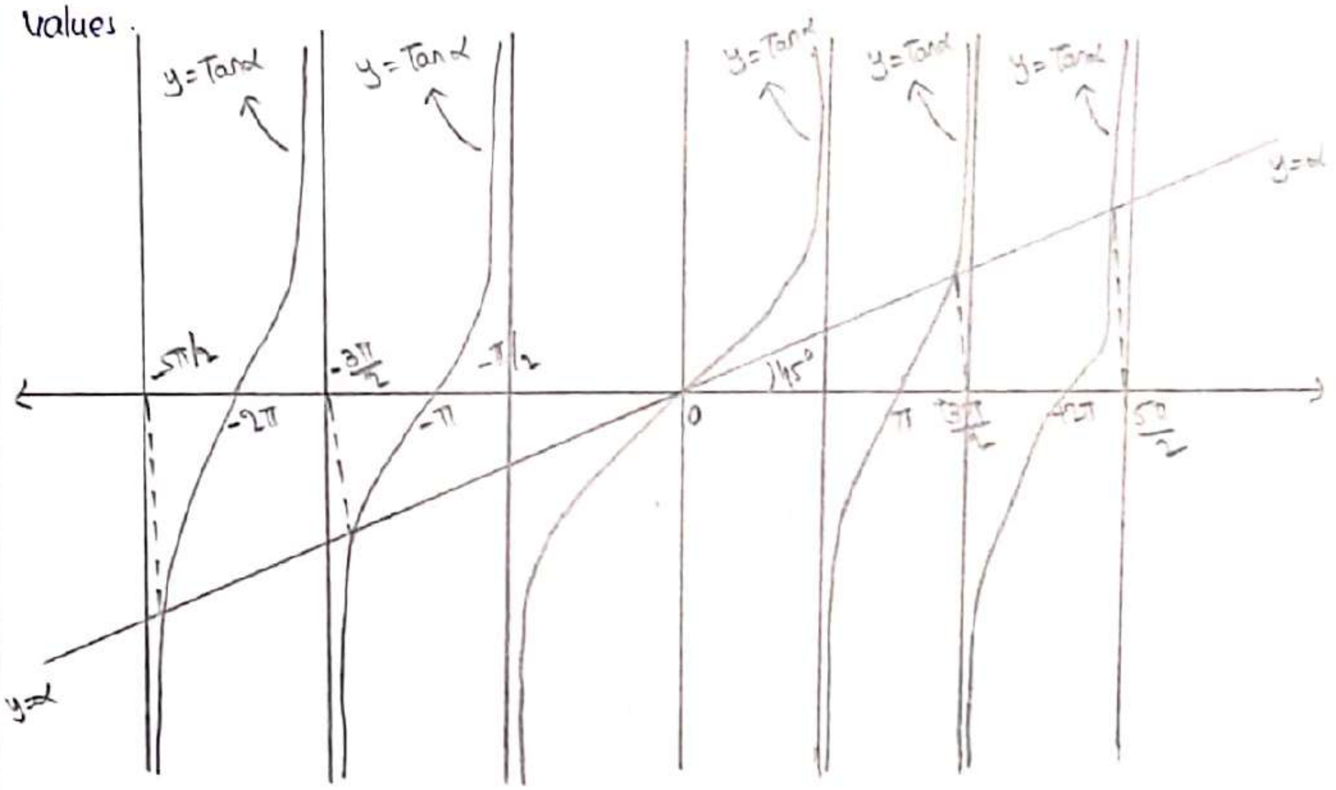
$$\boxed{\alpha = \tan\alpha}$$

The above equation can be written as

$$\boxed{y = \alpha}$$

$$\boxed{y = \mathrm{Tan}\alpha}$$

Now plot a graph between $\boxed{y=\alpha}$ and $y = \tan\alpha$ then the intersecting points in the graph gives the secondary maxima values.



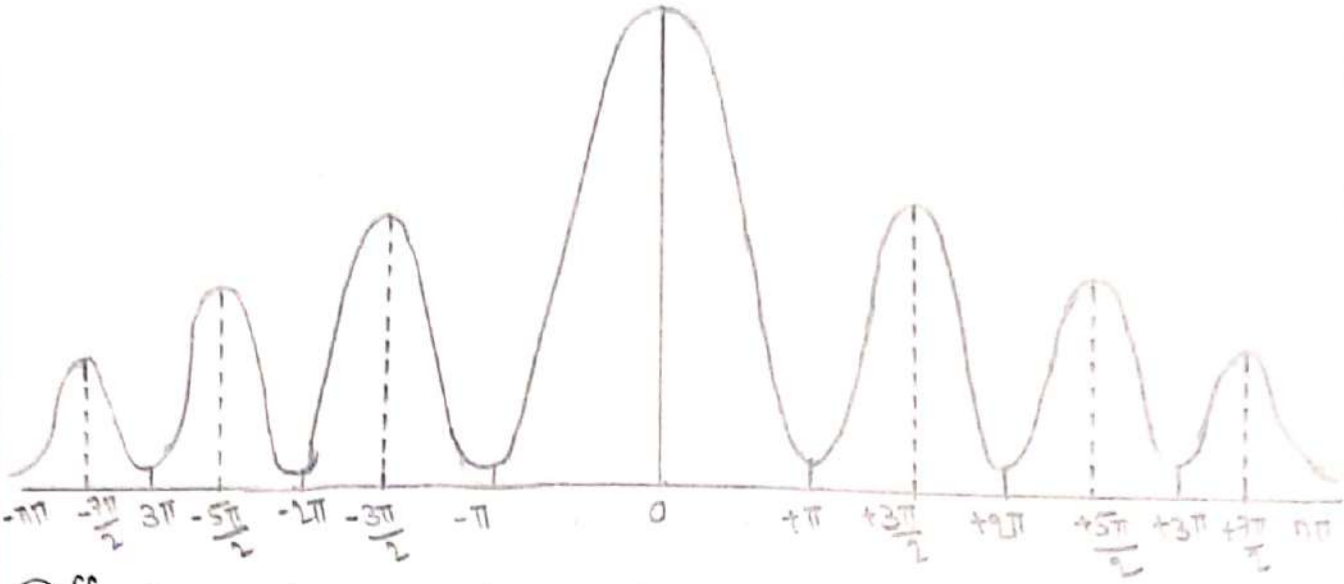
From the graph the intersecting points are $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots \pm(2n+1)\frac{\pi}{2}$ gives secondary maxima.

<u>Resultant graph</u>:-

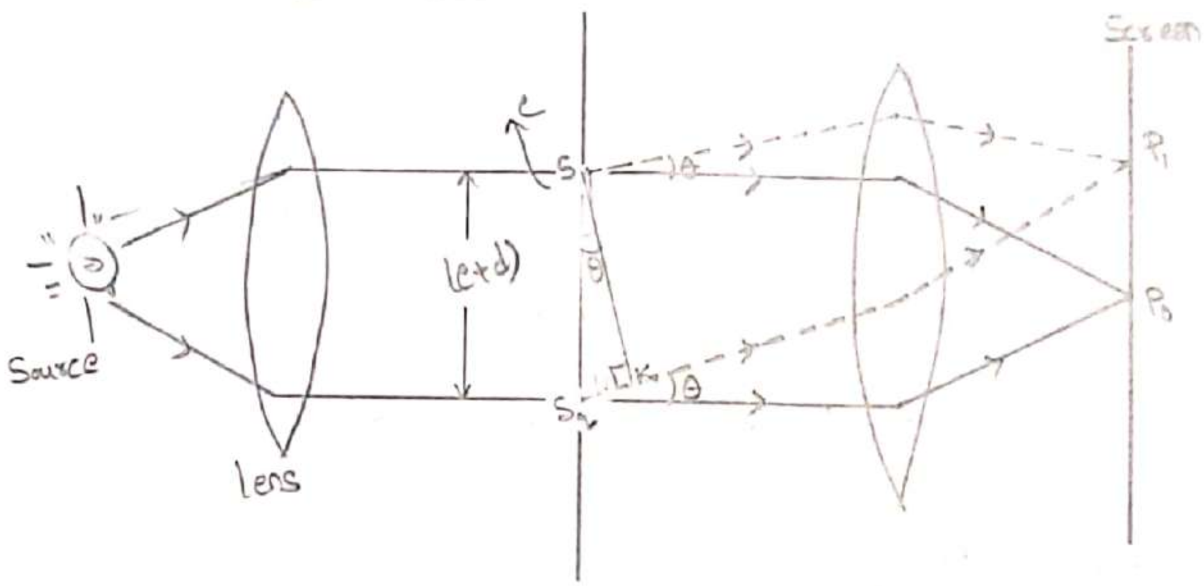
We know that the condition for principle maxima $\alpha = 0$ (or) $\theta = 0$.

The secondary maxima occurs at $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots \pm(2n+1)\frac{\pi}{2}$.

The minima occurs at $\pm\pi, \pm 2\pi, \pm 3\pi, \ldots, \pm n\pi$.



Diffraction due to double slit :-



Let us consider two single slit S_1 & S_2 each single slit width is 'e'. Which are seperated by a distance 'd'.

∴ The mean distance b/w the two slits.

$$S_1 S_2 = e + d \quad \text{--- (1)}$$

When the light passing through the two single slits then we can observe the both interference and diffraction pattern.

On the screen the point P_0 can represents the resultant intensity of non deviated light rays through the single slits S_1 & S_2 repectively.

At the same time the point P_1 on the screen can represents resultant intensity of the diffracted rays with the angle of diffraction 'θ'.

From the figure the path difference b/w the two effective rays equal to S_2k.

From $\Delta\, S_1 S_2 k$.

$$\sin\theta = \frac{S_2 k}{S_1 S_2}$$

$$\sin\theta = \frac{S_2 k}{e+d}$$

$$\boxed{S_2 k = e+d \ \sin\theta} \quad \text{—— ②}$$

∴ Phase difference can be written as

$$\delta = \left(\frac{2\pi}{\lambda}\right) \text{path difference}$$

$$= \frac{2\pi}{\lambda}\,(S_2 k)$$

$$\delta = \frac{2\pi}{\lambda}\,(e+d \ \sin\theta) \quad \text{—— ③}$$

Let us assume $\overline{OG}$ be the resultant amplitude of through the single slit S_1.

According to fraunhofer single slit diffraction.

The resultant amplitude $\overrightarrow{OG} = \left[A \ \frac{\sin\alpha}{\alpha}\right]$

Similarly let us consider $\vec{GH}$ be the resultant vector of waves through the single slit S_2.

$$\therefore \vec{GH} = \left[A \frac{\sin\alpha}{\alpha}\right]$$

Let $\vec{OH}$ be the resultant vector (R) of both $\vec{OG}$ and $\vec{GH}$.

According to the triangle law of vectors from triangle.

The resultant vector $(\vec{OH})^2 = (\vec{OG})^2 + (\vec{GH})^2 + 2(\vec{OG})(\vec{GH})\cos\delta$

$$R^2 = \left[A \frac{\sin\alpha}{\alpha}\right]^2 + \left[A \frac{\sin\alpha}{\alpha}\right]^2 + 2\left[\frac{A \sin\alpha}{\alpha}\right]\left[\frac{A \sin\alpha}{\alpha}\right]\cos\delta$$

$$= 2\left(\frac{A \sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos\delta$$

$$= 2\left(\frac{A \sin\alpha}{\alpha}\right)^2 \left[1 + \cos\delta\right]$$

$$= 2\left(\frac{A \sin\alpha}{\alpha}\right)^2 \left(2\cos^2 \delta/2\right)$$

$$R^2 = 4\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos^2(\delta/2) \quad \text{--- (4)}$$

$$R^2 = 4\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos^2\left[\frac{2\pi/\lambda\,(e+d)\sin\theta}{2}\right]$$

$$= 4\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos^2\left[\frac{\pi}{\lambda}(e+d)\sin\theta\right] \quad \text{--- (5)}$$

let, $\boxed{\frac{\pi}{\lambda}(e+d)\sin\theta = \beta}$ --- (6)

eq^n (6) in eq^n (5)

$$\boxed{I = 4\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cdot \cos^2\beta} \quad \text{--- (7)}$$

The above equation can be represents the resultant intensity of both interference pattern and diffraction pattern.

The above equation the term $4\left(\frac{A\sin\alpha}{\alpha}\right)^2$ can represents diffraction pattern.

The term $\cos^2\beta$ can represents interference pattern.

Diffraction pattern:-

The above equation $\left(\frac{A\sin\alpha}{\alpha}\right)^2$ can represents the diffraction pattern.

Condition for principle maxima:-

The principal maxima occurs at $\boxed{\alpha = 0}$ or $\boxed{\theta = 0}$

Condition for Secondary maxima:-

The secondary maxima occurs at $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots \pm (2n+1)\pi/2$.

Condition for minima:-

The minima occurs at

$$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots \ldots \pm m\pi.$$

Interference Pattern:-

In the resultant intensity the factor $\cos^2\beta$ represents interference pattern.

Condition for bright:-

let us consider resultant intensity $I = \cos^2\beta$

For the maximum intensity $\cos^2\beta = 1$

$$\therefore \beta = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots \pm n\pi$$

$$\frac{\cancel{\pi}}{\lambda}(e+d)\sin\theta = \pm n\cancel{\pi}$$

$$(e+d)\sin\theta = n\lambda$$

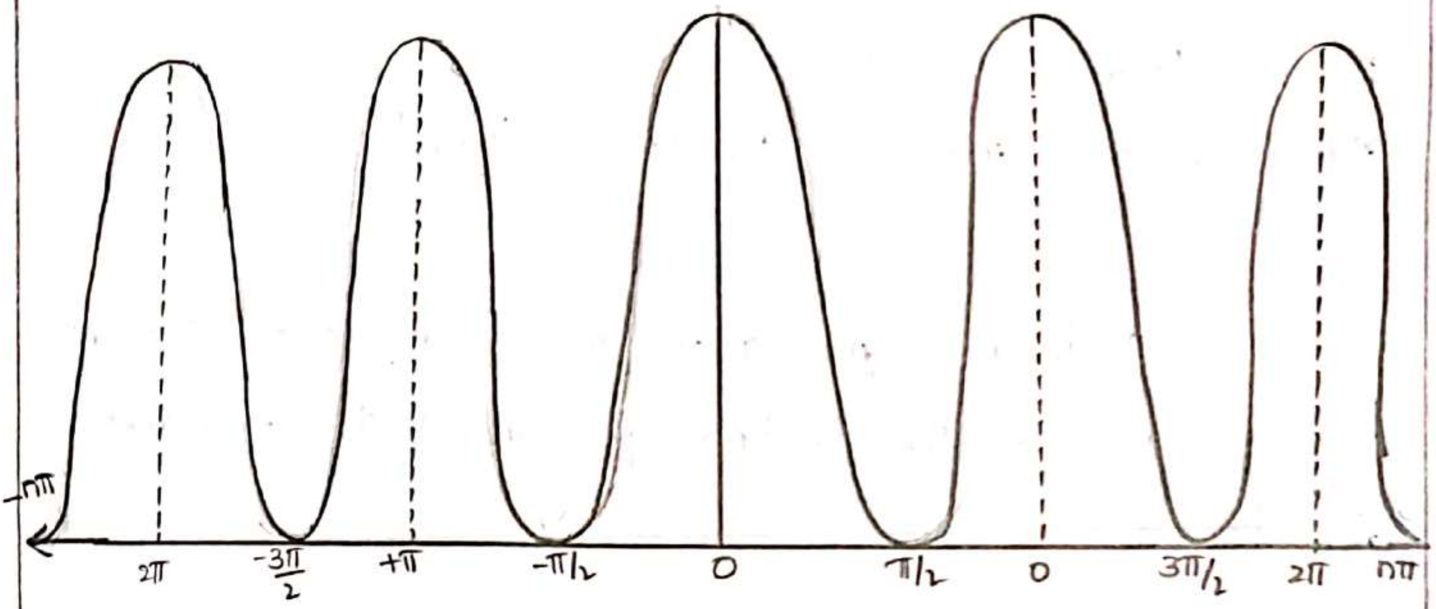
Condition for dark:-

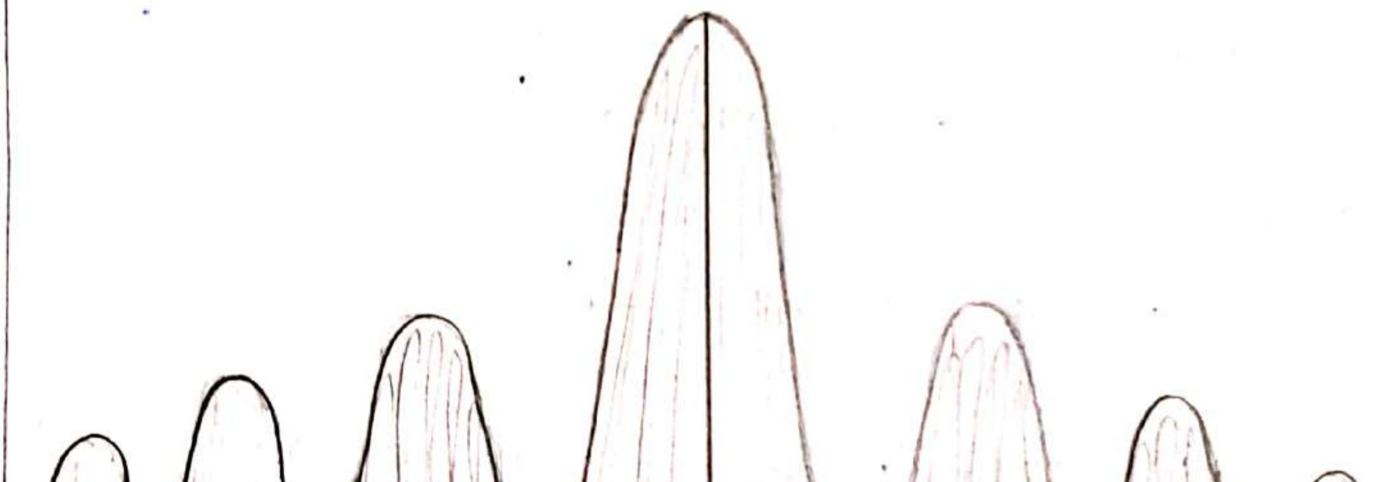
For the dark fringe $\cos^2\beta = 0$

$$\therefore \beta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots \pm(2n+1)\pi/2 .$$

$$\frac{\cancel{\pi}}{\lambda}(e+d)\sin\theta = (2n+1)\frac{\cancel{\pi}}{2} .$$

$$(e+d)\sin\theta = (2n+1)\lambda/2$$


-nπ
2π
-3π/2
+π
-π/2
0
π/2
0
3π/2
2π
nπ



Diffraction Gratting :-

"Diffraction Gratting is a small glass plate which contains large number of equally spaced transparent parallel slits which are seperated by opaque lines."

(or)

Diffraction gratting is an optical device which is used to produce the gratting spectrum by the phenomenon of diffraction.

In the diffraction gratting 'e' be the width of the opaque line.

'd' be the width of the slit.

'e+d' be the distance between the two lines which is called gratting element.

$$\boxed{\therefore \text{Gratting element} = e+d}$$

If suppose 'N' be the no. of lines per inch on the gratting.

$$\therefore N(e+d) = 1''$$

$$= 2.54 \text{ cm.}$$

$$\boxed{e+d = \frac{2.54}{N} \text{ cm}} \quad \text{(or)}$$

$$\boxed{N = \frac{2.54}{(e+d)}} \text{ cm}$$

If suppose 'N' be the no.of lines per unit on the grating.

$$\therefore N(e+d) = 1$$

$$\boxed{(e+d) = \frac{1}{N}}$$

(or)

$$\boxed{N = \frac{1}{(e+d)}}$$

Generally a diffraction grating consists of 5000 to 30,000 lines per inch on the grating.

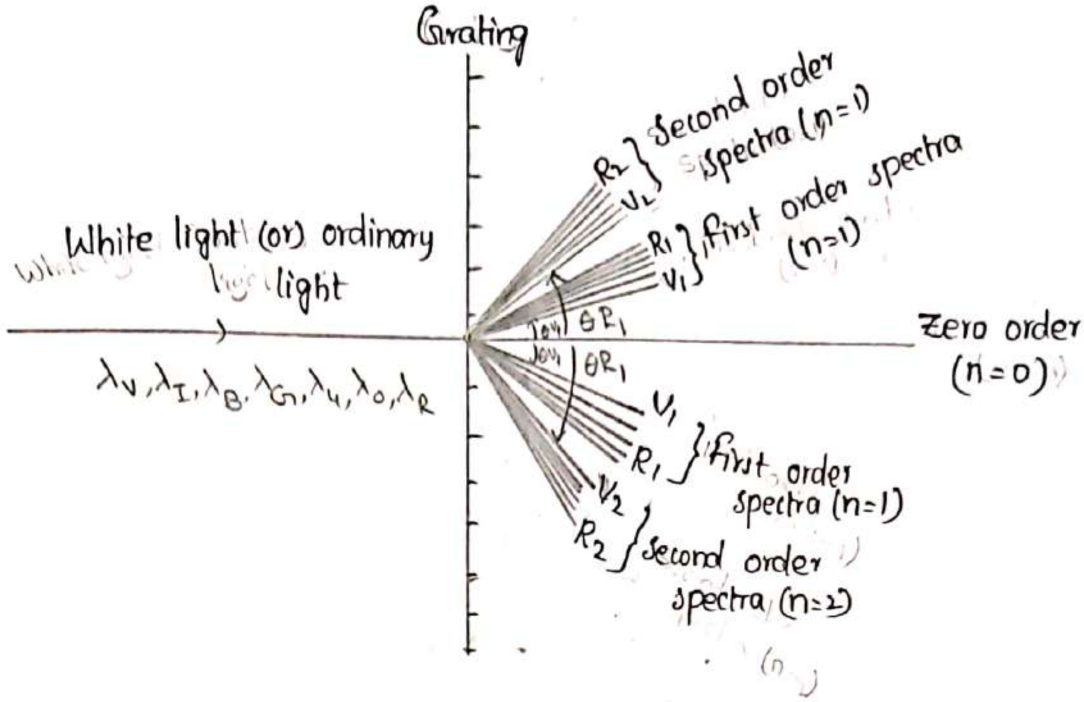
The lines on the grating can be rulled by a diamond point.

Diffraction Spectrum (or) Grating Spectrum:-

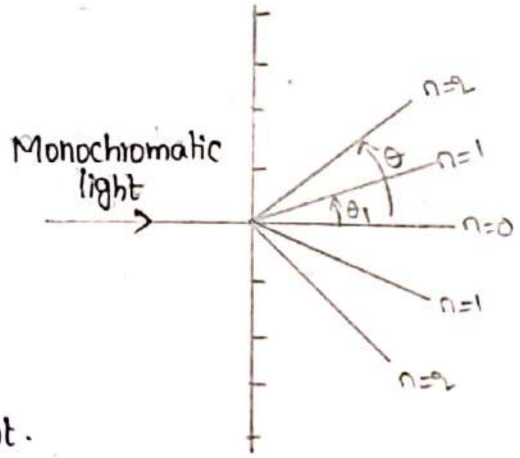
"When the light falls on the grating then the light can diffract and from the spectra (collection of 7 colours) due to diffraction which is called Diffraction spectrum or Gratting spectrum."

* The intensity of the spectral lines can be enplain by using the equation.

$$\boxed{(e+d)\sin\theta = n\lambda}$$



Which is called grating equation.



Where (e+d) = Grating element

θ = Angle of diffraction

n = order of diffraction (or) Order of spectra

λ = Wavelength of light.

* For a particular order of spectra (n is constant) the wavelength of the spectral line will be change with the angle of diffraction.

i.e., $\boxed{\theta \propto \lambda}$, Where n = constant

* For a particular wavelength of light (i.e., monochromatic light) the order of the spectrum will be change with respect to the angle of diffraction.

i.e., $\boxed{\theta \propto n}$ wher λ = constant.

* In any order of spectra the violet spectral line is in inner most position and the red colour spectral line is in outer most position to the zero order.

* The maximum intensity of light concentrated at the zero order and the remaining intensity of light.

* If the width of the slit in the grating is small then the spectral lines will be appear as sharp.

* <u>Maximum order of diffraction possible (n_{max})</u>:-

For the maximum order of diffraction (n_{max}).

$$\boxed{\theta = 90^\circ}$$

i.e., If $\boxed{\theta = 90^\circ}$ then $\boxed{n = n_{max}}$

$$(e+d)\sin\theta = n\lambda$$

$$\boxed{\theta = 90^\circ} \Rightarrow n = n_{max}$$

$$(e+d)\sin 90^\circ = n_{max}\lambda$$

$$(e+d) = n_{max}\lambda$$

$$n_{max} = \frac{(e+d)}{\lambda} \qquad \boxed{\because (e+d) = \frac{1}{N}}$$

$$\boxed{n_{max} = \frac{1}{N\lambda}}$$

* **Dispersive power of grating:-**

The change in the angle of diffraction with respect to the change in wavelength is called dispersive power of grating.

i.e., $D = \frac{d\theta}{d\lambda}$

let us consider $(e+d)\sin\theta = n\lambda$

Differentiating on both ~~di~~ sides

$$(e+d)\cos\theta \cdot d\theta = n\,d\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$$

$$\boxed{D = \frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta}} \qquad \left[\because \frac{1}{e+d} = N\right]$$

* **Determination of Wave length:-**

$$(e+d)\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{1}{(e+d)} n\lambda$$

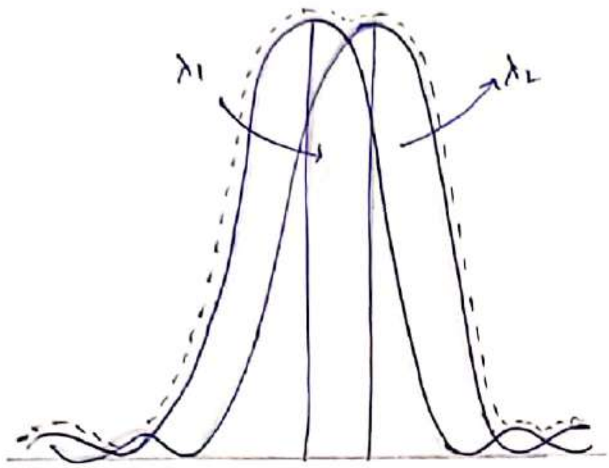
$$\sin\theta = nN\lambda$$

$$\boxed{\lambda = \frac{\sin\theta}{nN}}$$

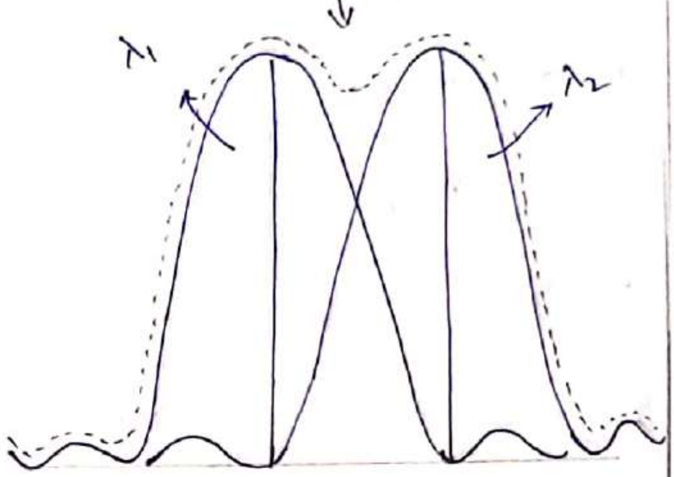
Resolving power of an optical instrument:-

"The capacity of an optical instrument to seperate two images or objects or wavelengths when they are near by each other is called Resolving Power of an optical instruments."

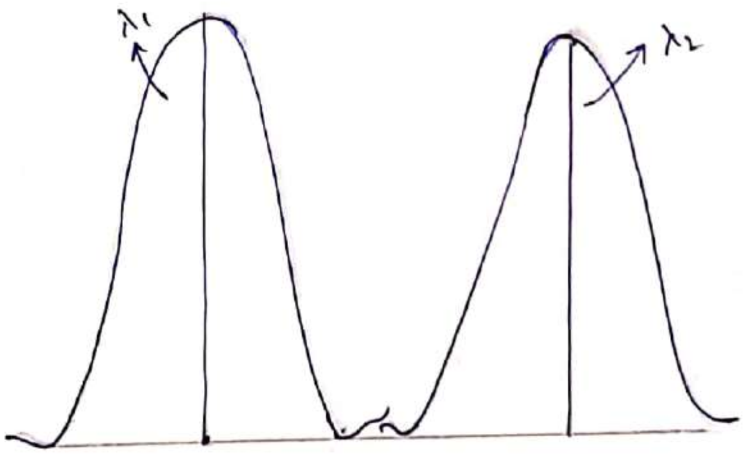
Rayleigh's creterian for resolving power of an optical instrument:-



(a) Not Resolved condition



(b) Just Resolved condition
(or)
Rayleigh's limit



(c) Well resolved condition

let us consider any two images whose wavelengths are λ_1 and λ_2 respectively. Figure 'a' shows the two images or in not resolved condition. i.e., The two images are merged with each other. Figure 'b' shows the Rayleigh's crêterian (or) Rayleigh's limit of an optical instrument. ~~Accor~~

According to Rayleigh's creteria two images are

Said to be just resolved. " The principle maxima of one image coincides with the minima of the other image. Similarly the central maxima of other image and coincid with the minima of first image.

Fig (c) shows the two images are called well resolved condition.

Resolving Power of gratting:-

" The ability of an grating to seperate two images. When they are nearby each other is called Resolving Power of grating."

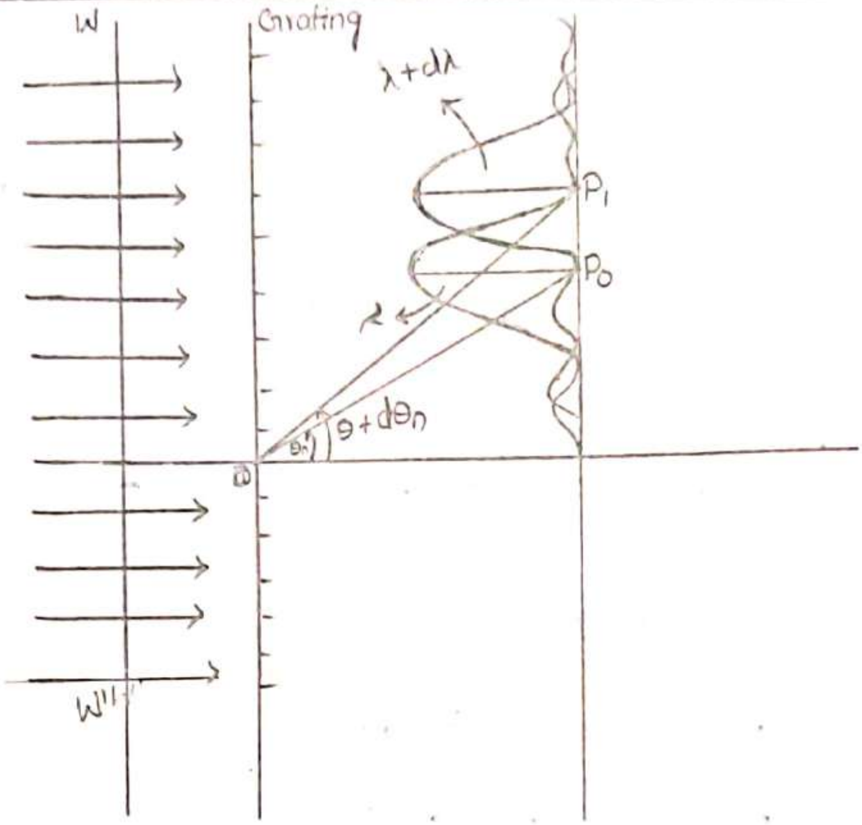
(Or)

" The ratio between the wavelength of one image(λ) and the wavelength of difference b/w the two images ($d\lambda$) is called Resolving Power of grating".

Resolving power of gratting

$$R = \frac{\lambda}{d\lambda}$$

let us consider any two images whose wavelengths are λ, $\lambda + d\lambda$ on the screen at P_0, P_1 respectively where θ_n, $\theta_n + d\theta_n$ are the diffraction angles to the central maxima's of both images from the point 'o'. Where ww' is the incident wave.



Let us consider any two images whose wavelengths are $\lambda, \lambda+d\lambda$.

Now let us consider the condition for principle maxima of the image whose wavelength is 'λ' at the angle of diffraction θ_n.

$$\text{i.e., } \boxed{(e+d) \sin\theta_n = n\lambda} \quad \text{——} \ (1)$$

If 'N' be the number of slits on the grating then the above equation can be written as

$$N(e+d) \sin\theta_n = nN\lambda \quad \text{——} \ (2)$$

i.e., $N, 2N, 3N, \ldots\ldots nN$ gives maxima.

Now let us consider the equation for minima of the image whose wavelength is 'λ' at the angle of diffraction $\theta_n+d\theta_n$.

$$\boxed{N(e+d)\sin(\theta_n + d\theta_n) = (nN+1)\lambda} \quad \text{— (3)}$$

Now let us consider the condition for central maxima of the image whose wavelength is '$\lambda + d\lambda$' at the angle of diffraction '$\theta_n + d\theta_n$':

$$\boxed{N(e+d)\sin(\theta_n + d\theta_n) = Nn(\lambda + d\lambda)} \quad \text{— (4)}$$

From eq^{n}s (3) & (4)

$$(nN+1)\lambda = nN(\lambda + d\lambda)$$

$$nN\lambda + \lambda = nN\lambda + nNd\lambda$$

$$\lambda = nNd\lambda$$

$$\boxed{\frac{\lambda}{d\lambda} = nN}$$

(or)

Resolving power of grating

$$\boxed{R = \frac{\lambda}{d\lambda} = nN}$$

i.e., The resolving power of grating is depends on the

order of spectrum (n)

The Number of lines on grating (N).

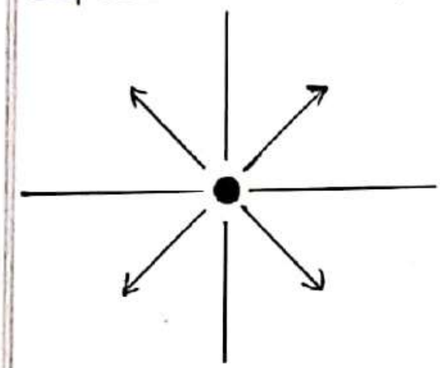
THE END

POLARIZATION

Polarization:-

The process of converting the unpolarised light into polarised light is called "polarisation".

Representation of unpolarised light (or) ordinary light :-



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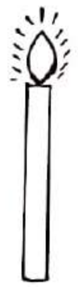

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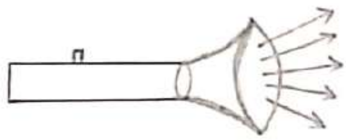

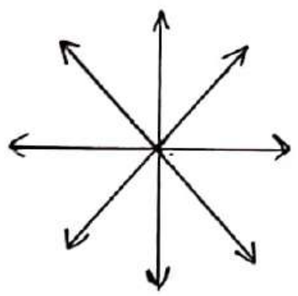

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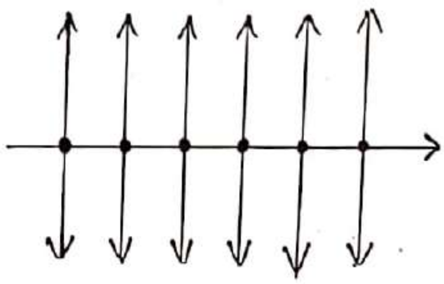


Types of polarised light :-

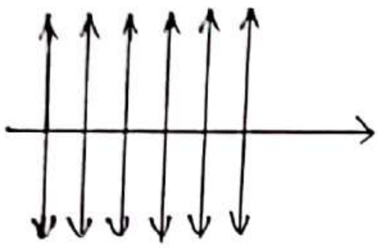
There are three types of polarised light.

They are :-

1. Plane polarized light
2. Circularly polarized light.
3. Eliptical polarized light.

Plane polarised light:-

If the electrical vibrations are belongs to one plane then it is called plane polarised light.

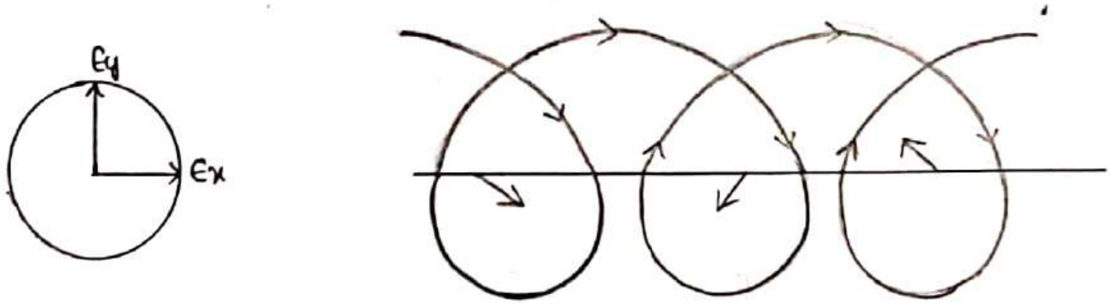


Vertical

Horizontal

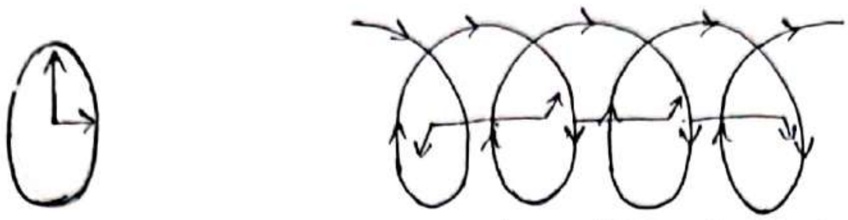
Circularly polarized light:-

If two plane polarised lights with equal magnitude and which are mutually perpendicular to each other (i.e., phase difference is $\pi/2$) super impose with each other, then from the circularly polarized light.



Representation of circularly polarised light.

Eliptical polarized light:- If two polarised lights with unequal magnitude and which are mutually perpendicular to each other (i.e., phase difference in $\pi/2$) Superimpose with each other then form elliptical polarised light.



Representation of elliptically polarization

Methods of production of polarised light:-

There are six types of methods to produce polarized light.

1. Polarization by reflection
2. Polarization by transition.
3. Polarization by refraction.
4. Polarization by selective absorption.
5. Polarization by scattering
6. Polarization by double refraction.

Polarization by reflection:-

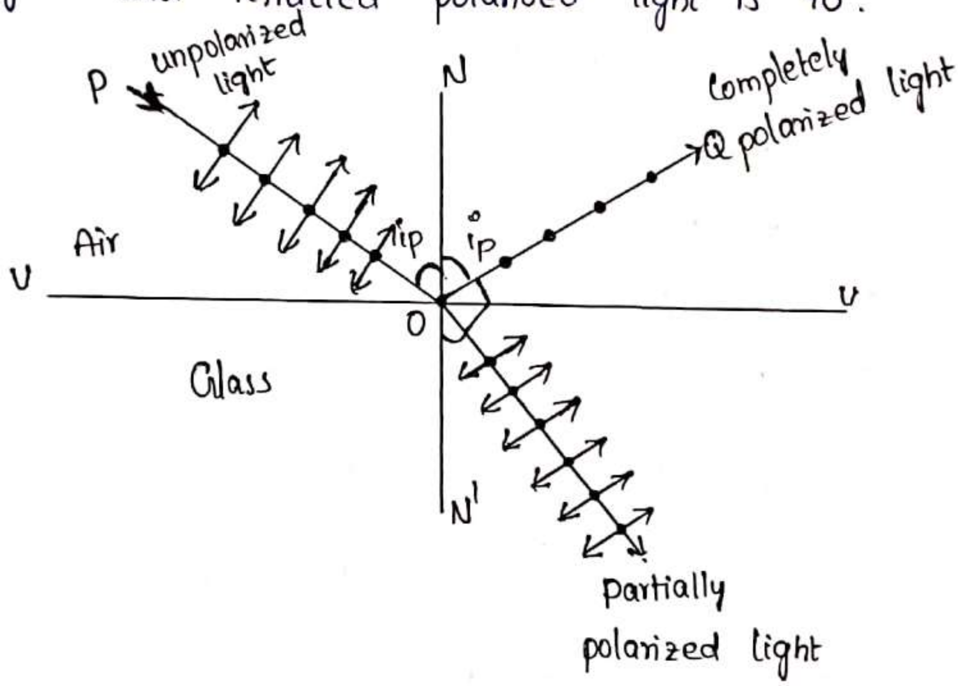
When an unpolarized light was reflected at the surface of some transparent medium such as glass, water etc.... The reflected medium such as light was found to be partially plane polarized. The degree of polarization changed with the angle of incidence. For the particular angle of incidence the reflected light was found to be completely plane polarized." The angle of incedence for which the reflected beam is completely plane polarized is known as polarizing angle or angle of polarization" This angle is also known as Brewester's angle.

Brewester's law:-

A tangential value of polarised angle is equal to the refractive index of the medium is known as Brewester's law.

$$\boxed{\mu = \tan i_p}$$

It can proves that the angle between the reflected polarised light and refracted polarised light is 90°.



Polarization by reflection

From the figure,

$\angle PON = i$ (angle of incidence)

$\angle N'OR = r$ (angle of refraction)

from Snell's law,

$$\mu = \frac{\sin i}{\sin r} \quad \text{——— ②}$$

from Brewester's law,

$$\mu = \tan i = \frac{\sin i}{\cos i} \quad \text{——— ③}$$

from equations ② and ③ we get

$$\cos i = \sin r$$

$$\sin(90 - i) = \sin r$$

$$90 - i = r$$

$$i + r = 90^\circ$$

From the figure,

$$\angle NOQ + \angle QOR + \angle NOR = 180^\circ$$

$$i + \angle QOR + r = 180$$

$$\angle QOR = 180^\circ - (i + r)$$

$$= 180^\circ - 90^\circ$$

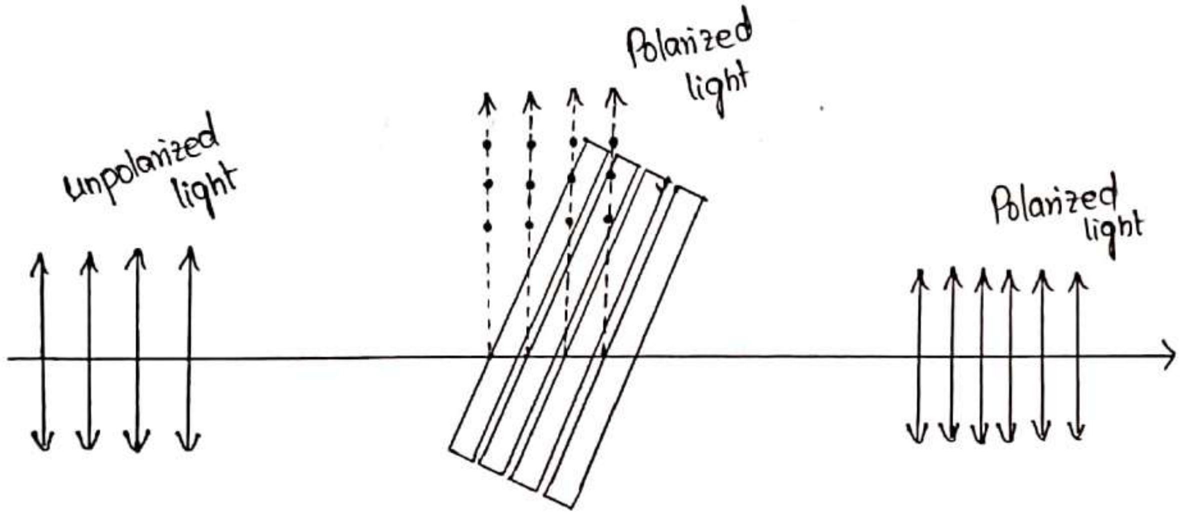
$$= 90^\circ$$

$$\boxed{\angle QOR = 90^\circ}$$

Hence it is proved that the reflected and refracted rays are at right angles.

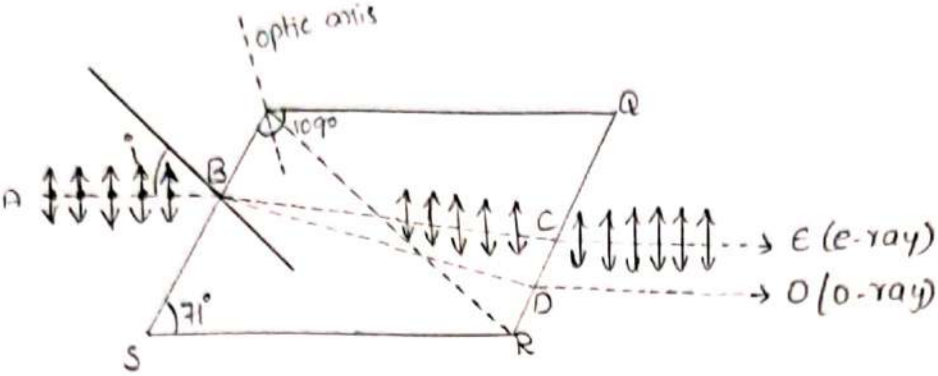
Polarization by Refraction:-

When an unpolarized light is incident at polarizing angle the reflected light is completely plane. Polarised and refracted light is partially plane polarized. The transmitted contains a greater proportion of light vibrating parallel to the plane of incidence. If the reflection at polarized angle is repeated using number of plates all inclined at polarizing angle, finally the transmitted light becomes purely plane polarized. Such an arrangement is known as pile of plates. The polarized light is perpendicular to the plane of incidence.



Double refraction:-

The phenomenon of splitting into two plane polarised refracted light rays when the unpolarised light is incident on the



Calcite crystal is called double refraction.

The resultant refracted rays are extraordinary ray or ordinary ray.

Extra ordinary ray:-

The ray which can not obeys the laws of refraction is called extra ordinary ray.

Ordinary ray:-

The ray which can be obey the laws of refraction is called ordinary ray.

Differences between ordinary ray and Extra ordinary ray:-

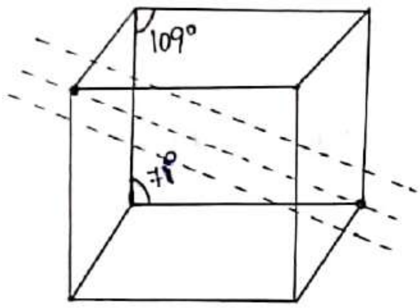
Ordinary ray	Extra Ordinary ray
① It can obey the class of fraction	① It cannot obey the class of refractions.
② It obeys snell's law $\mu = \frac{\sin i}{\sin r}$	② It cannot obey snell's law $\mu \neq \frac{\sin i}{\sin r}$
③ The velocity or speed of the ordinary ray is same in all direction	③ The velocity or speed of the extraordinary ray is different in different directions.
④ The vibrations are perpendicular to optic axis.	④ The vibrations are perpendicular to principal section.

⑤ If $\mu_o > \mu_e$, then crystal is called Negative crystal.	⑤ If $\mu_e > \mu_o$, then the crystal is called positive crystal.

Where, μ_o = refractive index of ordinary ray

μ_e = refractive index of extra ordinary ray.

Structure of calcite crystal:-



The shape of calcite crystal is rhombohedral. The common angle is 71°, 109°

Blunt corner:-

"A corner which has three obtuse angle is called Blunt corner."

Optic axis:-

"A line joining of two blunt corner is called optic axis."

Any line which is parallel to optic axis is also treated as optic axis.

Uniaxial Crystal :-

The crystal which has only one optic axis is called uniaxial crystals.

Ex: calcite, tourmaline.

Biaxial Crystal:-

The crystal which has two optic axis are called Biaxial crystal.

Ex: Borax

Nicol prism:-

Nicol prism is a optical device which is made from the calcite crystal which is used to produce the plane polarised light.

⇒ The Nicol prism is invented by the scientist Nicol in 1828.

⇒ The Nicol prism can be used as a polarizer and analyser.

Principle:-

When an ordinary light is incident on a calcite crystal, it splits into O-ray and E-ray having orthogonal projections. The O-ray can undergo total internal reflection with in the Nicol prism and E-ray is transmitted through prism is a linearly polarized light.

Construction :-

A calcite crystal whose length is three times its breadth is taken for the construction of a Nicol prism. The unit cell of calcite crystal is a rhombohedron having principal section ABCD with angles 71° and 109° as shown in fig (a).

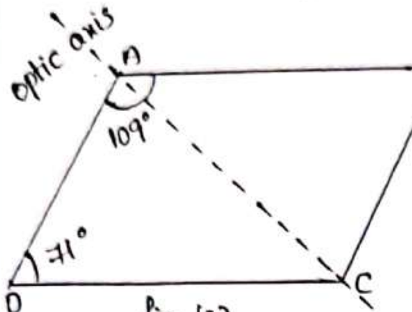


fig (a)

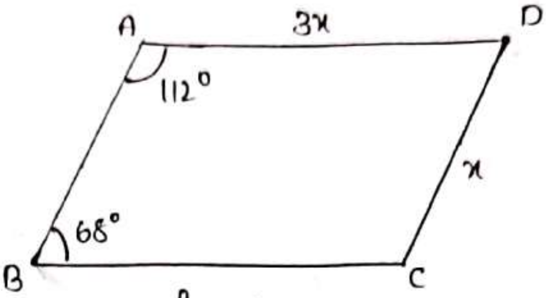
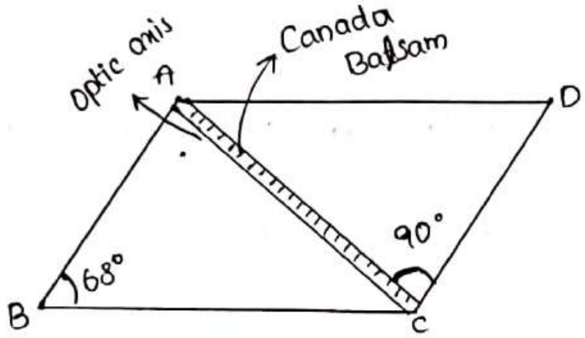


fig (b)



Construction of Nicol Prism.

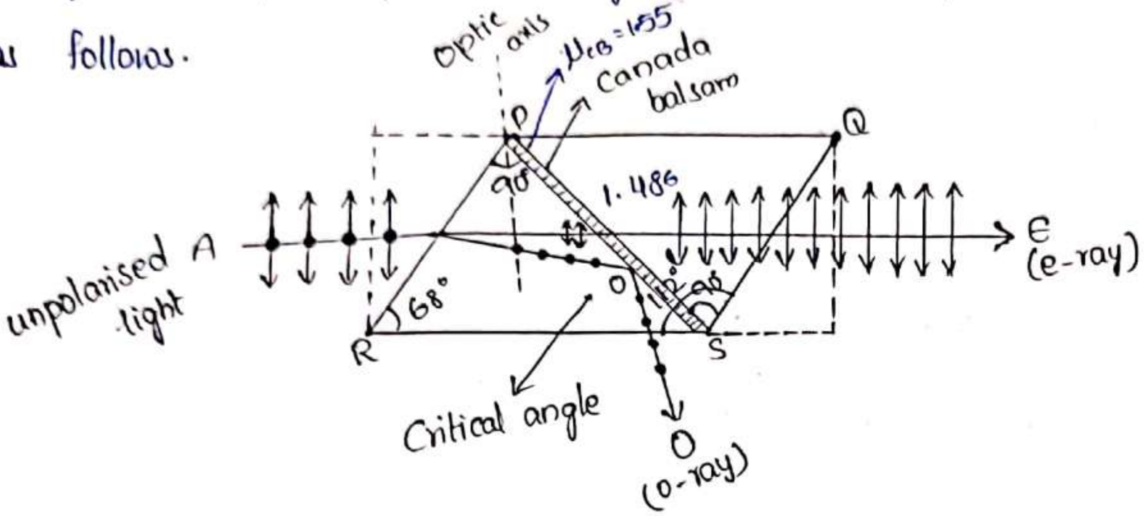
The end faces of the principal section AB and CD is cut. Such that the angles in the principal plane ABCD become 68° and 112° in place of 71° and 109° as shown in fig (b)

⇒ The resulting crystal is then cut diagonally i.e., along AC into two parts. The surface of each part is made optically flat and then these are polished.

⇒ The polished surfaces are joined together by 'canada balsam' as shown in fig. The resulting device is called as 'Nicol prism' which produces plane polarized light.

Working of a Nicol prism:-

A schematic diagram of Nicol prism is as shown in figure. The plane polarized light from Nicol prism can be produced as follows.



Working of a Nicol prism.

When a beam of plane polarized light is incident on the face PR with angle of 15°, it splits into two refracted rays: o-ray and e-ray. These two rays are plane polarized rays whose vibrations are at right angles to each other.

The refractive index of canada balsam is 1.55 whereas it is 1.658 for ordinary ray and 1.486 for extraordinary ray. Hence canda balsam layer can act as an optically rarer medium for o-ray and optically denser medium for e-ray.

The O-ray travels in the Nicol prism from denser to rarer medium and is incident at the canada balsam with an angle greater than the critical angle 69°, the O-ray is totally internally reflected from the crystal.

$$\boxed{\theta_c : \sin^{-1}\left(\frac{n_2}{n_1}\right) : \sin^{-1}\left(\frac{1{\cdot}55}{1{\cdot}656}\right) = 69^\circ}$$

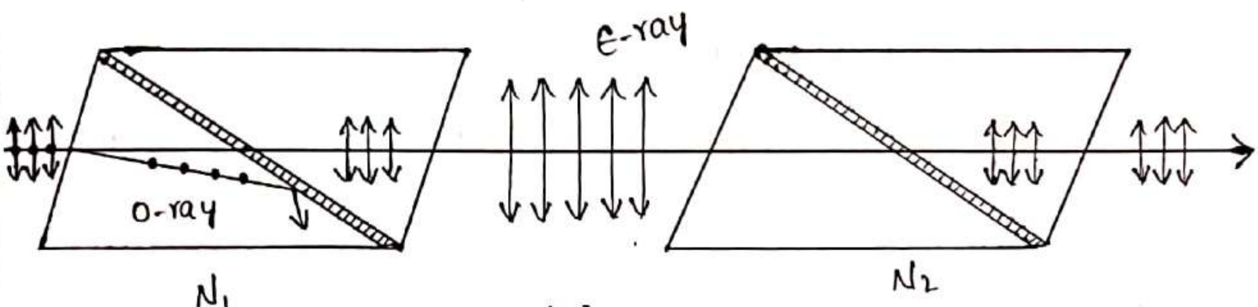
The E-ray is going from a rarer medium to a denser medium and is transmitted to the Nicol prism which is a plane polarized light.

Therefore, using the phenomenon of total internal reflection. The plane polarized light is generated from the Nicol prism.

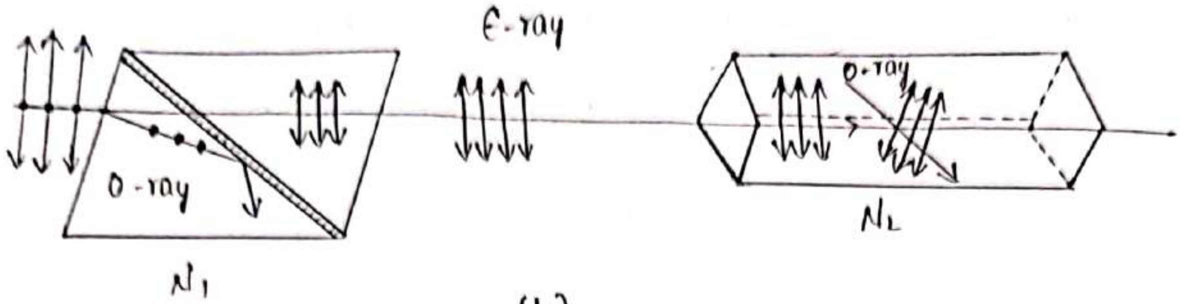
Thus, Nicol prism act as a polarizer and analyzer.

<u>Nicol prism as a polarizer and Analyser</u>:-

When a beam of use polarized light is incident on the Nicol prism N_1 the emergent beam from N_1 is plane polarized light. Therefore, the prism N_1 is called polarizer.



(a)



(b)

a) Principal planes parallel

b) Principal planes inclined at right angles to each other.

If the polarized beam from N_1 falls on another Nicol prism N_2, whose principal plane is parallel to that of N_1, E-ray is transmitted through N_2 with maximum intensity.

If the Nicol prism N_2 is slowly rotated with respect to N_1, the intensity of E ray gradually decreases.

When the principal plane of N_2 is perpendicular to the principal plane of N_1, there is no light from N_2. Because E-ray behaves as O-ray in the Nicol prism. N_2 and it is totally internally reflected by canada balsam layer.

If the Nicol prism N_2 is further rotated about its axis, the intensity of light coming out N_2, gradually increases and becomes maximum when the principal plane N_2 is again parallel to that of N_1.

This property can be used for detecting the plane polarized light, hence the Nicol prism N_2 acts as an analyzer

Thus, Nicol prisms N_1 and N_2 are known as polarizer and analyzer respectively.

Wave plates:-

Wave plates are the uniaxial doubly refracting positive or negative crystals which can introduce path difference or phase difference between the ordinary ray and extraordinary ray.

Wave plates are mainly two types. They are

1. Half Wave plate
2. Quater Wave plate

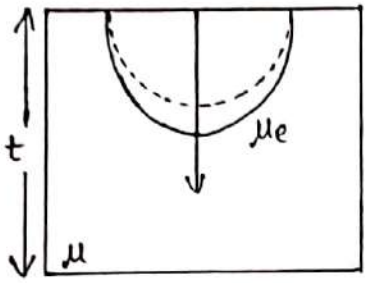
Half Wave Plate:-

Half wave plates are the uniaxial doubly refracting positive or negative crystals which can introduce the path difference $\lambda/2$ or phase difference π between the ordinary ray and extraordinary ray.

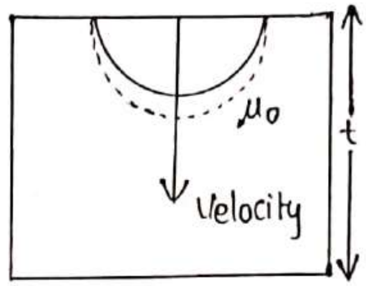
Calculation of thickness (t) of Half wave plate:-

Let us consider a glass plate whose refractive index is μ and thickness is 't'.

let 'μ_o' be the refractive index of ordinary ray and 'μ_e' be the refractive index of extraordinary ray.



Negative crystal



Positive crystal

The total path difference travelled by ordinary ray $= \mu_o t$

The total path difference travelled by extraordinary ray $= \mu_e t$

The path difference between two rays

$$\Delta = \mu_o t - \mu_e t$$

$$\Delta = (\mu_o - \mu_e) t$$

We know that the path difference introduced by Half wave plate, $\Delta = \frac{\lambda}{2}$

i.e., $\frac{\lambda}{2} = t(\mu_o - \mu_e)$

$$\boxed{t = \frac{\lambda}{2(\mu_o - \mu_e)}} \quad \text{—} (1)$$

The above eqⁿ represents the thickness of Half-wave plate for negative crystal.

Similarly:

$$t = \frac{\lambda}{2(\mu_e - \mu_o)} \quad \text{——— (2)}$$

It represents the thickness of Half-wave plate for positive crystal.

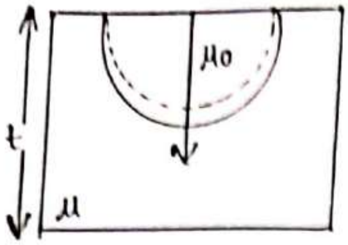
Quarter wave plate:-

Quarter wave plates are the uniaxial doubly refracting positive or negative crystals which can introduce the path difference $\frac{\lambda}{4}$ or phase difference $\frac{\lambda}{2}$ between the ordinary ray and extraordinary ray.

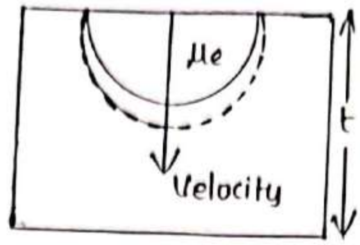
Calculation of thickness of Quater wave plate:-

let us consider a glass plate whose refractive index is 'μ' and thickness is 't'.

let 'μ_o' be the refractive index of o-ray and 'μ_e' be the refractive index of E-ray.



Negative crystal



positive crystal

The total path travelled by O-ray : $\mu_o t$

The total path travelled by E-ray - $\mu_e t$

The path difference between two rays

$$\Delta = \mu_o t - \mu_e t$$

$$= (\mu_o - \mu_e) t$$

We know that the path difference introduced by Quarter wave plate is $\Delta = \frac{\lambda}{4}$

$$\frac{\lambda}{4} = (\mu_o - \mu_o) t$$

$$t = \frac{\lambda}{4(\mu_o - \mu_e)} \quad \text{——} \quad (1)$$

It represents thickness of quarter wave plate for negative crystal.

Similarly, $t = \frac{\lambda}{4(\mu_o - \mu_e)}$ —— (2)

It represents thickness of Quarter wave plate for positive crystal.