

Under distribution assumption : Multinomial $\left(N, \frac{1}{2} - \frac{\theta}{2}, \frac{\theta}{4}, \frac{\theta}{4}, \frac{1}{2}\right)$, where θ is unknown.

Perform the EM algorithm for the exercise on page 25 of “Week 12_1 EM Algorithm.ppt”

$$\mathbf{x}_{obs} = (x_1, x_2, x_3 + x_4)^T = (38, 34, 125)^T$$

Do this under three different initial setup.

```
function mtn(data::Vector ; init = 1/2, ε = 1e-16)
    if length(data) != 3
        throw(DomainError())
    end
    θ₀ = init
    d = Inf
    while d > ε
        global x₃ = round(data[3] * (θ₀/4) / (1/2 + θ₀/4))
        d = abs(θ₀ - (data[2] + x₃) / (data[1] + data[2] + x₃))
        θ₀ = (data[2] + x₃) / (data[1] + data[2] + x₃)
    end
    return θ₀, x₃
end

> mtn

mtn([38, 34, 125], init = 0.2)  (0.6274509803921569, 30.0)
mtn([38, 34, 125], init = 0.5)  (0.6274509803921569, 30.0)
mtn([38, 34, 125], init = 0.8)  (0.6274509803921569, 30.0)
```

分別用三種初始值： $\theta_0 = 0.2, 0.5, 0.8$ 來做 EM algorithm

最後三種初始值所做出來的 $\hat{\theta}$ 皆為 0.6274509803921569, x_3 的填入值為 30