

# Introduction

- Last time: introduced matrix factorization
  - Singular Value Decomposition
- This video: details on how SVDs work
  - Algebraic understanding
  - Dealing with missing data
  - Updating with new data

## 7-2: Diving Deeper with SVD

## The Algebra of an SVD

- Rating matrix  $M$  decomposes to  $U\Sigma V^T$
- $U$ ,  $V$  orthogonal
  - translate vectors into & out of low-dim space
- $\Sigma$  diagonal matrix of singular values
- Truncate: keep  $k$  ‘most important’ dimensions (highest singular values)
  - Best rank- $k$  approximation (by RMSE/Frobenius norm)
  - de-noises the data

## Computing an SVD

- Well-known algorithms in many linear algebra packages
  - Matlab
  - LINPACK
  - ARPACK
- Very slow

## Missing Data

- SVD formulation (and many solvers) assume matrix is complete
  - But if it's complete, don't need recommender
- What to do with missing values?
  - 'Impute' — assume they are a mean
  - Normalize data first — assume they are 0
  - Next lecture: ignore them

Introduction to Recommender Systems

## Adding New Data (folding in)

- New user joins the system and rates some items
  - They weren't in last night's model build
- Vector spaces to the rescue!
  - Multiply user rating vector by item-feature matrix
  - This gives you user-feature vector

Introduction to Recommender Systems

## General SVD Practice

- Build models regularly
- Fold-in user's current ratings for live recommendation
- Impute means or pre-normalize data to handle missing data
  - Pre-normalizing lets you use standard sparse matrix and vector arithmetic

Introduction to Recommender Systems

## 7-2: Diving Deeper with SVD

Introduction to Recommender Systems

$m = \# \text{ users}$   
 $n = \# \text{ items}$   
 $m \times n$

$$R = U \Sigma V^T$$

$m \times k$   $U$  - user-feature matrix  
 $n \times k$   $V$  - item-feature matrix  
 $k \times k$   $\Sigma$  - weights diagonal

only keep  $k$  largest

- best rank- $k$  approx.  
 by global RMSE

$$p_{ij} = \sum_{f=1}^k u_{if} \sigma_f v_{jf}$$

$$R = U \Sigma V^T$$

$$\vec{u}_a = \Sigma V^T \vec{r}_a^T$$

