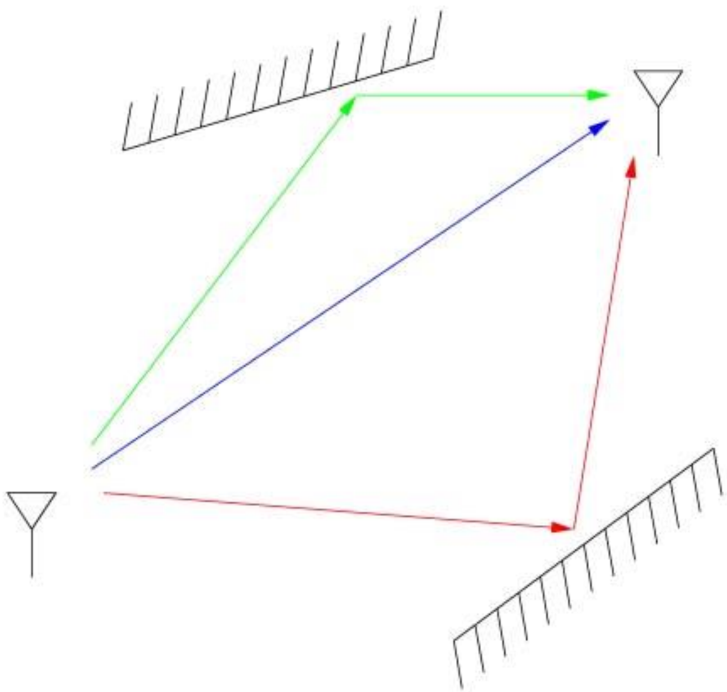
Lecture 2
Fundamentals

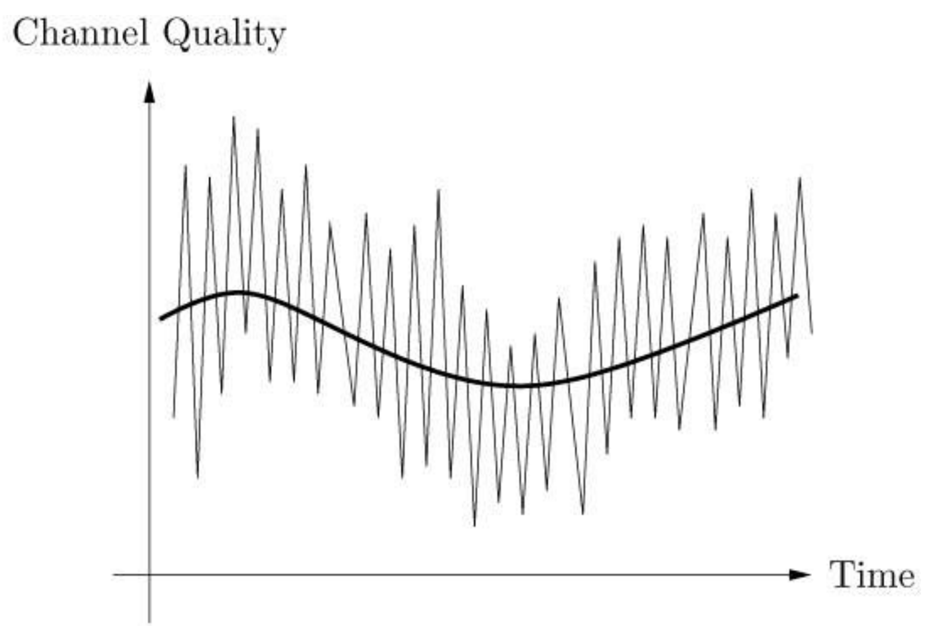
- Refer to <<Fundamentals of Wireless Communication>> Chapter 2&5&3

Outline

- The Wireless Channel (ch.2)
- Capacity of Wireless Channels (ch.5)
- Diversity & Error Probability (ch.3)

Wireless Mulipath Channel





- Channel varies at two spatial scales:
  - ✓ large scale fading
  - ✓ small scale fading

Wireless Mulipath Channel

Large-scale fading

- In free space, received power attenuates like $1/r^2$.
- With reflections (反射) and obstructions (障碍), can attenuate even more rapidly with distance. Detailed modelling complicated.
- Time constants associated with variations are very long as the mobile moves, many seconds or minutes.
- More important for cell site planning, less for communication system design.

Path Loss

- The ratio between the transmit power, Pt, and the receive power, Pr,

$$\mathrm{PL} = \frac{P_t}{P_r}$$

- The path loss is usually represented in the decibel scale, i.e.,

$$\mathrm{PL}_{(\mathrm{dB})} = 10\log_{10}\frac{P_t}{P_r}$$

A General Path-Loss Model

- To facilitate analytical studies on wireless communication systems, it is often convenient to use simpler path loss models,

$$P_r = P_t K \left(\frac{d}{d_0} \right)^{-\alpha}$$

- d_0 is the reference distance, K is a constant related to the antenna gain and the average channel attenuation, and α is the path-loss exponent.

$$P_{r(\mathrm{dBm})} = P_{t(\mathrm{dBm})} + K_{(\mathrm{dB})} - 10\alpha \log_{10}\left(\frac{d}{d_0}\right)$$

Path-loss exponent

- Free space: $\alpha = 2$
- Urban macrocells: $3.7 \le \alpha \le 6.5$
- Urban microcells: $2.7 \le \alpha \le 3.5$
- Office building (same floor): $1.6 \le \alpha \le 3.5$
- Office building (multiple floors): $2 \le \alpha \le 6$
- Store: $1.8 \le \alpha \le 2.2$
- Factory: $1.6 \le \alpha \le 3.3$
- Home: $\alpha \approx 3$

Shadowing Effect

- The radio waves may also be distorted by the obstacles that appear along the transmission paths.

- These obstacles may absorb part of the signal energy, resulting in signal strength degradation or cause random scattering.

- The effects may vary slowly over time. This slow-varying power variation is called the shadowing effect and is considered as a type of large-scale fading.

Shadowing Effect Model

- A log-normal random variable with probability density function (PDF)

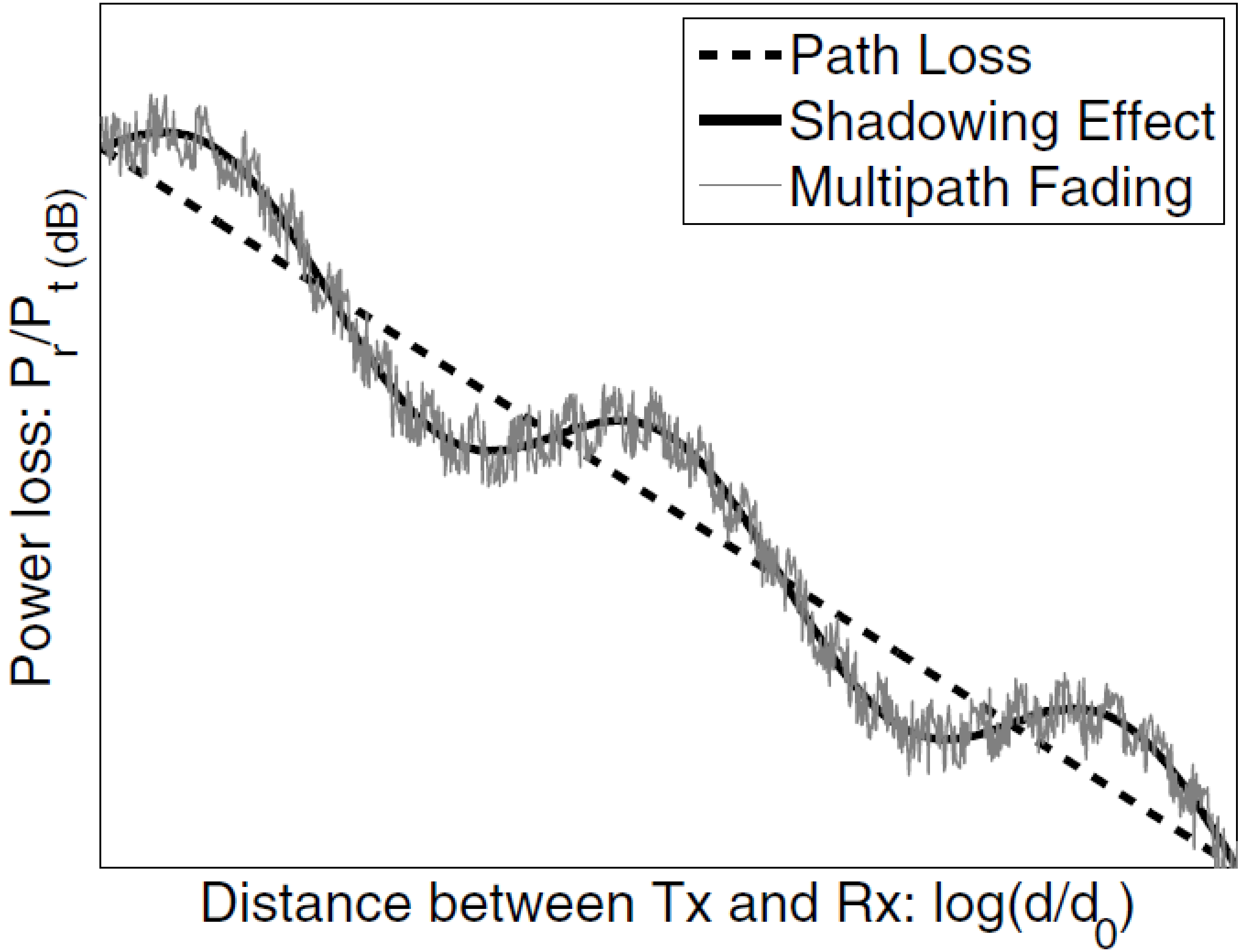
$$f_\psi(\psi) = \frac{\xi}{\sqrt{2\pi}\sigma_{\psi_{\mathrm{dB}}}\psi} \exp\left(-\frac{(10\log_{10}\psi - \mu_{\psi_{\mathrm{dB}}})^2}{2\sigma^2_{\psi_{\mathrm{dB}}}}\right), \quad \psi > 0,$$

$$\xi = 10/\ln(10)$$

- The log-normal distributed shadowing effect with the average path loss

$$\frac{P_r}{P_t}_{(\mathrm{dB})} = 10\log_{10} K - 10\alpha\log_{10}\left(\frac{d}{d_0}\right) - \psi_{\mathrm{dB}}$$

Illustration of path loss, shadowing effect and multipath fading



Small-scale multipath fading

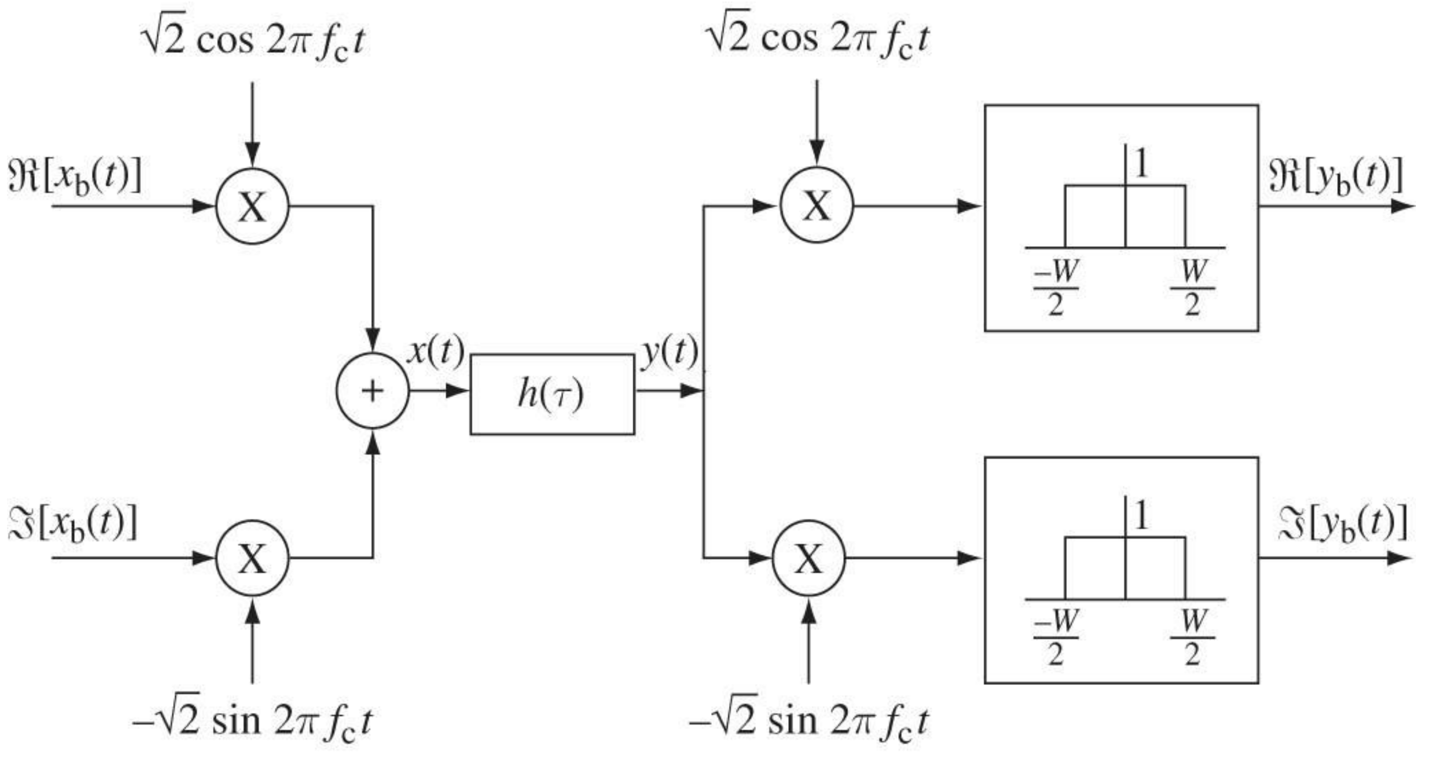
- Wireless communication typically happens at very high carrier frequency. (eg. fc = 900 MHz or 1.9 GHz for cellular)
- Multipath fading due to constructive and destructive interference of the transmitted waves.
- Channel varies when mobile moves a distance of the order of the carrier wavelength. This is about 0.3 m for 900 MHz cellular.
- Primary driver behind wireless communication system design.

Small-scale multipath fading

- How to describe?

Passband to Baseband Conversion

- Communication takes place at $[f_c - W/2, f_c + W/2]$
- Processing takes place at baseband $[-W/2, W/2]$



Complex Baseband Equivalent Channel

- Sampled baseband-equivalent channel model:

$$y[m] = \sum_{\ell} h_{\ell}\, x[m-\ell]$$

- where h_l is the l-th complex channel tap (抽头).

$$h_{\ell} \approx \sum_{i} a_i e^{-j2\pi f_c \tau_i}$$

- Each path is associated with a delay and a complex gain.
- Note: l denotes time slot, and i denotes path.

Flat and Frequency-Selective Fading

- Fading occurs when there is destructive interference of the multipaths that contribute to a tap.

$$h_\ell \approx \sum_i a_i e^{-j2\pi f_c \tau_i}$$

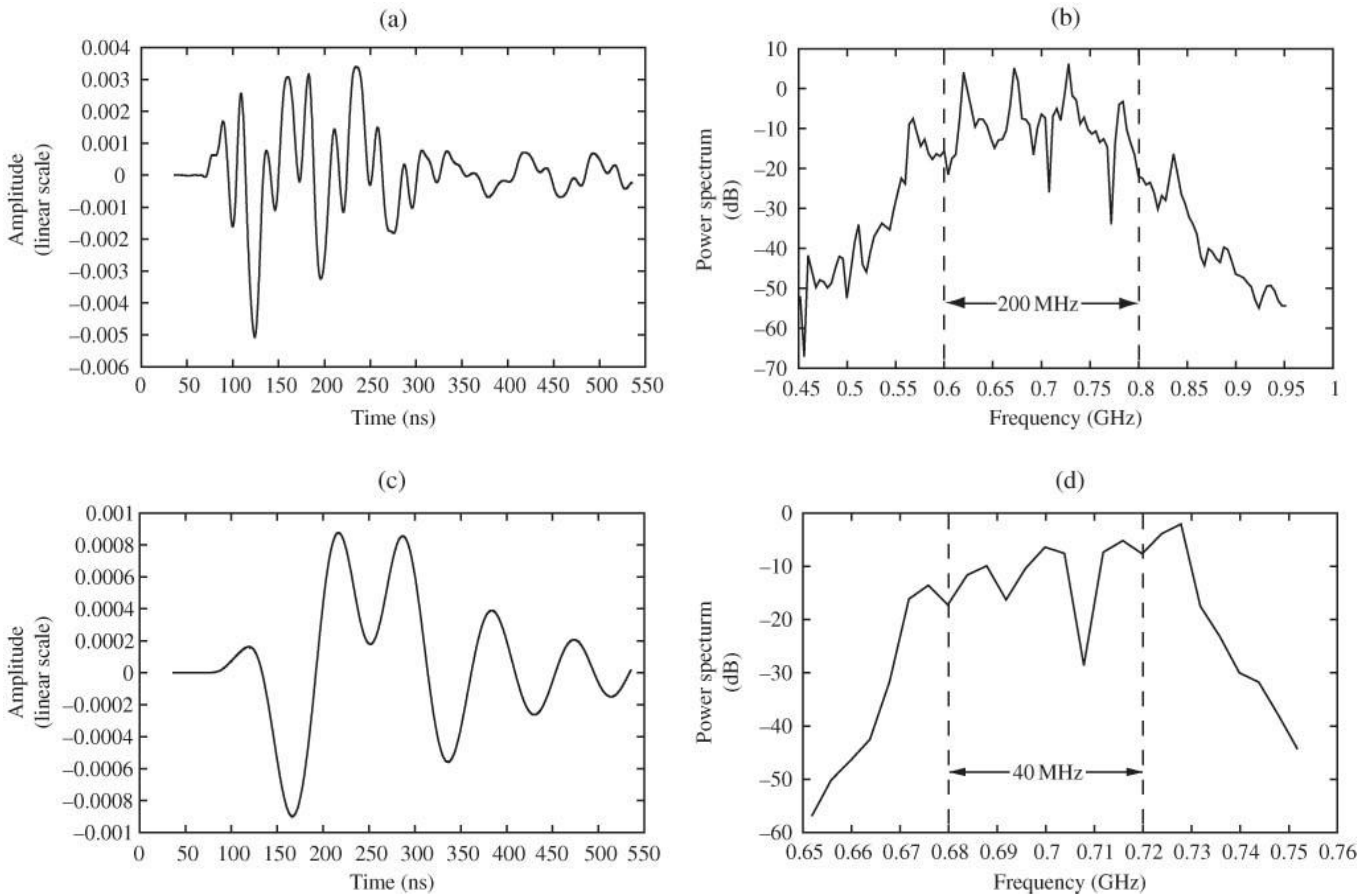
–Delay spread $T_d := \max_{i,j} |\tau_i(t) - \tau_j(t)|$

–Coherence bandwidth $W_c := \frac{1}{T_d}$

$T_d \ll \frac{1}{W}, W_c \gg W \Rightarrow$ –single tap, flat fading

$T_d > \frac{1}{W}, W_c < W \Rightarrow$ –multiple taps, frequency selective

Flat and Frequency-Selective Fading



Statistical Models

- Design and performance analysis based on statistical ensemble of channels rather than specific physical channel.

$$h_\ell[m] \approx \sum_i a_i e^{-j2\pi f_c \tau_i}$$

- Rayleigh flat fading (瑞利平衰落) model: many small scattered paths

$$h[m] \sim \mathcal{N}(0, \frac{1}{2}) + j\mathcal{N}(0, \frac{1}{2}) \sim \mathcal{CN}(0, 1)$$

Complex circular symmetric Gaussian .

Rayleigh flat fading model

- The *magnitude* $|h_\ell[m]|$ *is a* *Rayleigh* random variable with density

$$\frac{x}{\sigma_\ell^2}\exp\left\{\frac{-x^2}{2\sigma_\ell^2}\right\}, \quad x \geq 0,$$

- The *squared magnitude* $|h_\ell[m]|^2$ is exponentially distributed with density

$$\frac{1}{\sigma_\ell^2}\exp\left\{\frac{-x}{\sigma_\ell^2}\right\}, \quad x \geq 0$$

Rician model

- **Rician model: 1 line-of-sight plus scattered paths**

$$h_\ell[m] = \sqrt{\frac{\kappa}{\kappa+1}}\sigma_\ell e^{j\theta} + \sqrt{\frac{1}{\kappa+1}}\mathcal{CN}(0, \sigma_\ell^2)$$

- The parameter K (so-called K-factor) is the ratio of the energy in the specular path to the energy in the scattered paths; the larger K is, the more deterministic is the channel.

Additive Gaussian Noise

- Complete baseband-equivalent channel model:

$$y[m] = \sum_{\ell} h_\ell[m] x[m-\ell] + w[m]$$

$$w[m] \sim \mathcal{CN}(0, N_0)$$

- Special case: flat fading:

$$y[m] = h[m] x[m] + w[m]$$

- Will use this throughout the course.

Additive Gaussian White Noise Chanel （AWGN Chanel）

- AWGN Chanel

$$y[m] = h[m]x[m] + w[m]$$

- h[m] is a constant, w[m] is additive Gaussian noise.

Common Complete Channel Models

- Combining large scale fading and small scale fading

$$h[m] = K\left(\frac{d}{d_0}\right)^{-\alpha} \tilde{h}[m]$$

Capacity of Wireless Channels

- **Information Theory**
  - ✓ Information theory provides a fundamental limit to (coded) performance.
  - ✓ It succinctly identifies the impact of channel resources on performance as well as suggests new and cool ways to communicate over the wireless channel.
  - ✓ It provides the basis for the modern development of wireless communication.

Capacity of AWGN Channel

- Capacity of AWGN channel

$$
\begin{aligned}
C_{\mathsf{awgn}} &= \log(1+\mathsf{SNR}) \quad \mathsf{bits/s/Hz} \\
&= W \log(1+\mathsf{SNR}) \quad \mathsf{bits/s}
\end{aligned}
$$

- If average transmit power constraint is $\bar{P}$ watts and noise psd is N_0 watts/Hz,

$$
C_{\mathsf{awgn}} = W \log\left(1 + \frac{\bar{P}}{N_0 W}\right) \quad \mathsf{bits/s.}
$$

Power and Bandwidth Limited Regimes

$$C_{\mathsf{awgn}} = W \log\left(1 + \frac{\bar{P}}{N_0 W}\right)$$

$$\mathsf{SNR} = \frac{\bar{P}}{N_0 W}$$

- Bandwidth limited regime $\mathsf{SNR} \gg 1$: capacity logarithmic in power, approximately linear in bandwidth.
- Power limited regime $\mathsf{SNR} \ll 1$: capacity linear in power, insensitive to bandwidth.

$$\log_2(1+x) \approx x \log_2 e \qquad \text{when} x \approx 0,$$

$$\log_2(1+x) \approx \log_2 x \qquad \text{when} x \gg 1.$$

Bandwidth Limited Regimes

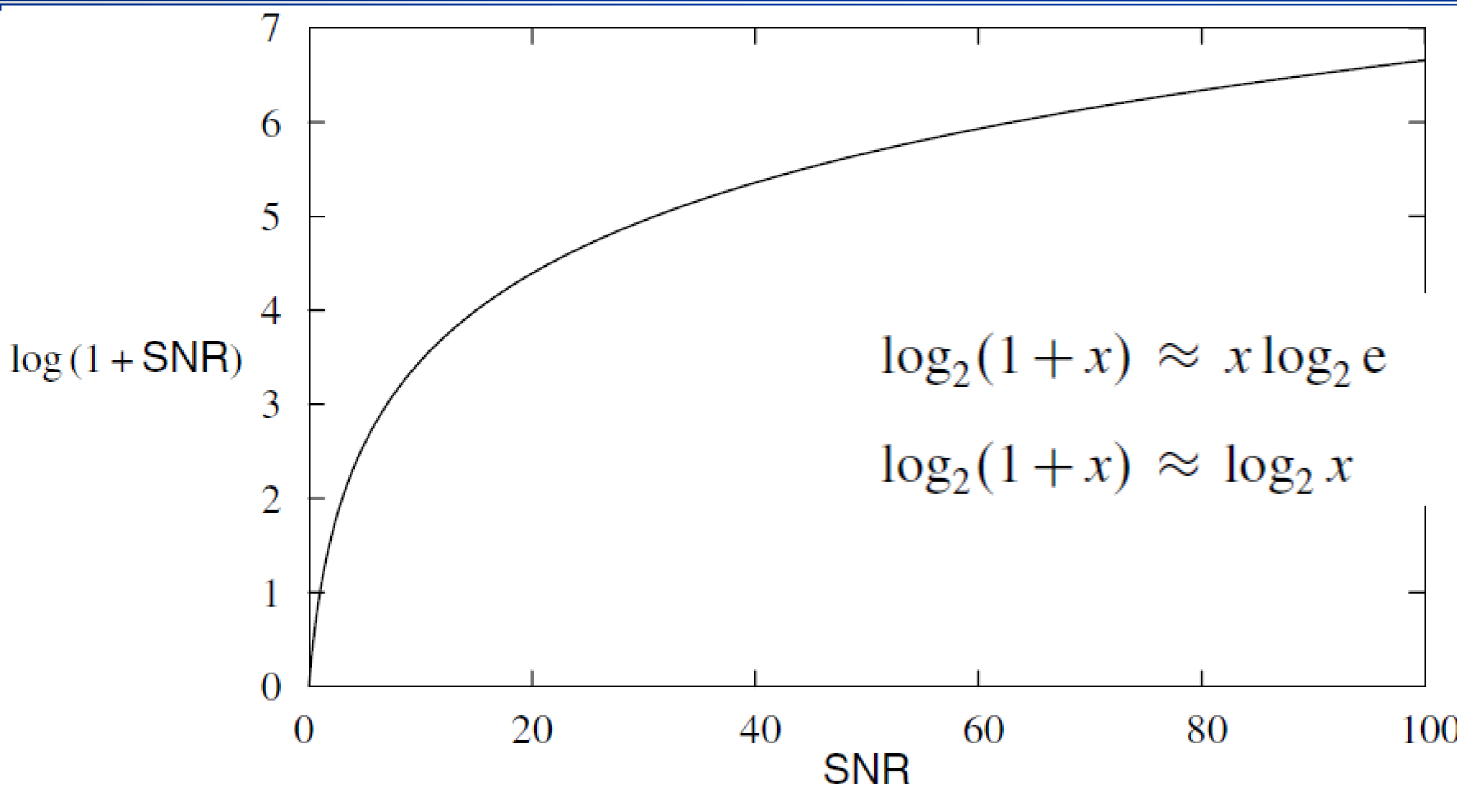
- Let us first see how the capacity depends on the received power. To this end, a key observation is that the function

$$f(\mathrm{SNR}) := \log(1+\mathrm{SNR})$$

is concave, i.e., $f''(x) \leq 0$ for all $x \geq 0$.

- This means that increasing the power $\bar{P}$ suffers from a law of diminishing marginal returns (边际收益递减规律): the higher the SNR, the smaller the effect on capacity.

Bandwidth Limited Regimes



$$\log_2(1+x) \approx x\log_2 e \qquad \text{when} x \approx 0,$$
$$\log_2(1+x) \approx \log_2 x \qquad \text{when} x \gg 1.$$

- When the SNR is low, the capacity increases linearly with the received power $\bar{P}$: every 3 dB increase in (or, doubling) the power doubles the capacity.
- When the SNR is high, the capacity increases logarithmically with $\bar{P}$: every 3 dB increase in the power yields only one additional bit per dimension.

Power-Limited Regime

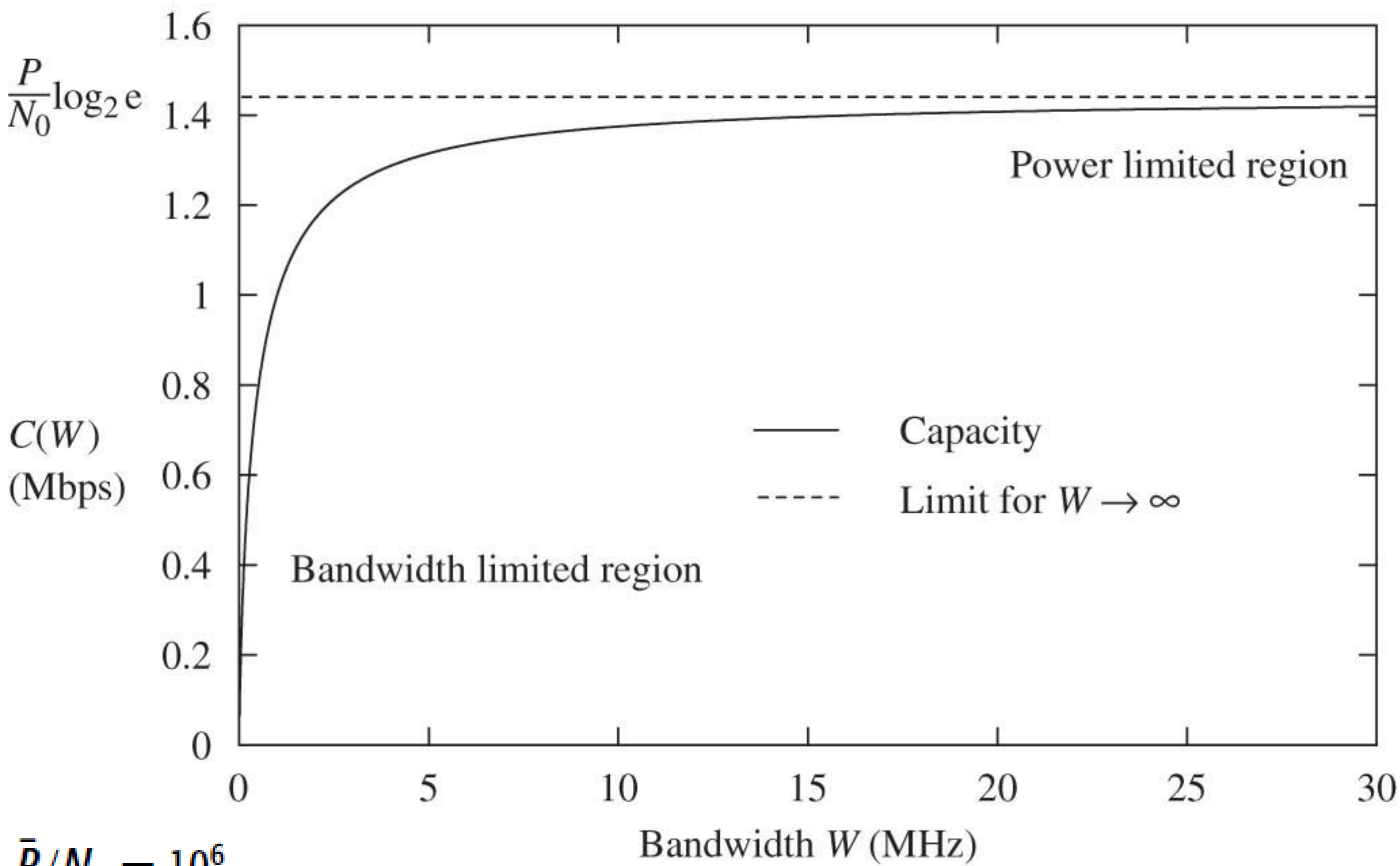
- When the bandwidth is large such that the SNR per degree of freedom is small,

$$W \log\left(1+\frac{\bar{P}}{N_0 W}\right) \approx W\left(\frac{\bar{P}}{N_0 W}\right)\log_2 \mathrm{e} = \frac{\bar{P}}{N_0}\log_2 \mathrm{e}.$$

$$C_\infty = \frac{\bar{P}}{N_0}\log_2 \mathrm{e} \ \text{bits/s}$$

- In this regime, the capacity is proportional to the total received power across the entire band.
- It is insensitive to the bandwidth, and increasing the bandwidth has a small impact on capacity.

Power and Bandwidth Limited Regimes



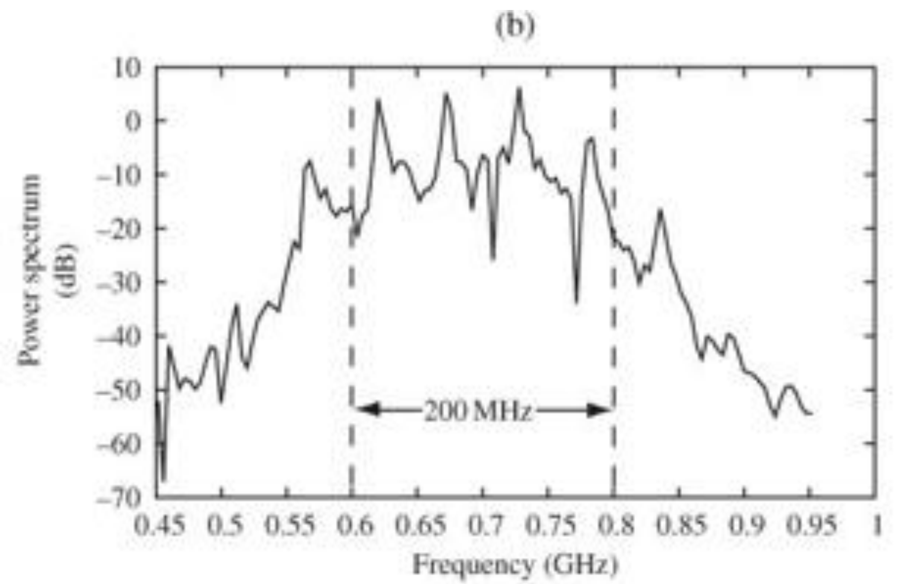
$\bar{P}/N_0 = 10^6$

$\log_2 e \approx 1.442695$

Frequency-selective Channel

- Consider a time-invariant L-tap frequency-selective AWGN channel,

$$y[m] = \sum_{\ell} h_{\ell} x[m-\ell] + w[m]$$



- OFDM converts it into a parallel channel:

$$\tilde{y}_n = \tilde{h}_n \tilde{d}_n + \tilde{w}_n, \qquad n = 1, \ldots, N_c.$$

Frequency-selective Channel

- Consider a time-invariant L-tap frequency-selective AWGN channel,

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- OFDM converts it into a parallel channel:

$$\tilde{y}_n = \tilde{h}_n \tilde{d}_n + \tilde{w}_n, \qquad n = 1, \ldots, N_c.$$

- The maximum rate of reliable communication：

$$\sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |\tilde{h}_n|^2}{N_0}\right) \text{ bits/OFDM symbol.}$$

Optimal Power Allocation

- The power allocation can be chosen appropriately, so as to maximize the rate

$$C_{N_c} := \max_{P_0,\ldots,P_{N_c-1}} \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n|\tilde{h}_n|^2}{N_0}\right)$$

- subject to

$$\sum_{n=0}^{N_c-1} P_n = N_c P, \qquad P_n \geq 0, \quad n = 0,\ldots,N_c - 1$$

Waterfilling Power Allocation

- Consider the Lagrangian

$$\mathcal{L}(\lambda, P_0, \ldots, P_{N_c-1}) := \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n|\tilde{h}_n|^2}{N_0}\right) - \lambda \sum_{n=0}^{N_c-1} P_n$$

- where λ is the Lagrange multiplier. The Kuhn–Tucker condition for the optimality of a power allocation is

$$\frac{\partial \mathcal{L}}{\partial P_n} \begin{cases} = 0 & \text{if } P_n > 0 \\ \leq 0 & \text{if } P_n = 0. \end{cases}$$

Waterfilling Power Allocation

- Define x+ = max{x,0}. The power allocation

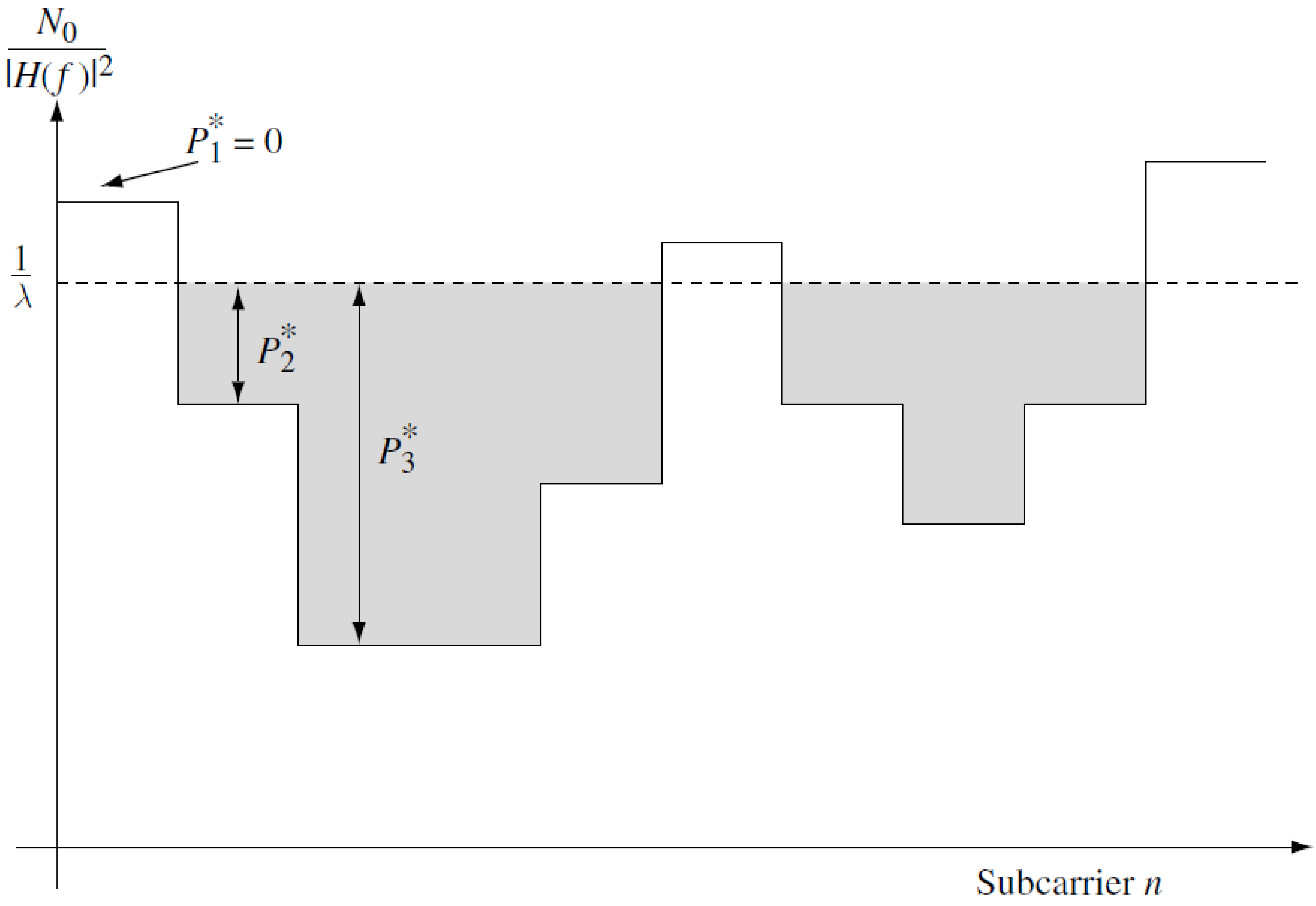
$$P_n^* = \left(\frac{1}{\lambda} - \frac{N_0}{|\tilde{h}_n|^2}\right)^+$$

- where λ is the Lagrange multiplier and satisfies the power constraint

$$\frac{1}{N_c}\sum_{n=0}^{N_c-1}\left(\frac{1}{\lambda} - \frac{N_0}{|\tilde{h}_n|^2}\right)^+ = P$$

Waterfilling Power Allocation

- $N_0/|\tilde{h}_n|^2$ versus index $n = 0, \ldots, N_c - 1$



Waterfilling Power Allocation

- If P units of water per sub-carrier are filled into the vessel, the depth of the water at sub-carrier n is the power allocated to that sub-carrier, and $1/\lambda$ is the height of the water surface.

- There are some sub-carriers where the bottom of the vessel is above the water and no power is allocated to them. In these sub-carriers, the channel is too poor for it to be worthwhile to transmit information.

- In general, the transmitter allocates more power to the stronger sub-carriers, taking advantage of the better channel conditions, and less or even no power to the weaker ones.

Slow Fading Channel

- The channel gain is random but remains constant for all time, i.e.,

$$h[m] = h \text{ for all } m.$$

which models the slow fading situation. This is also called the quasi-static scenario.

- The system is said to be in outage, if

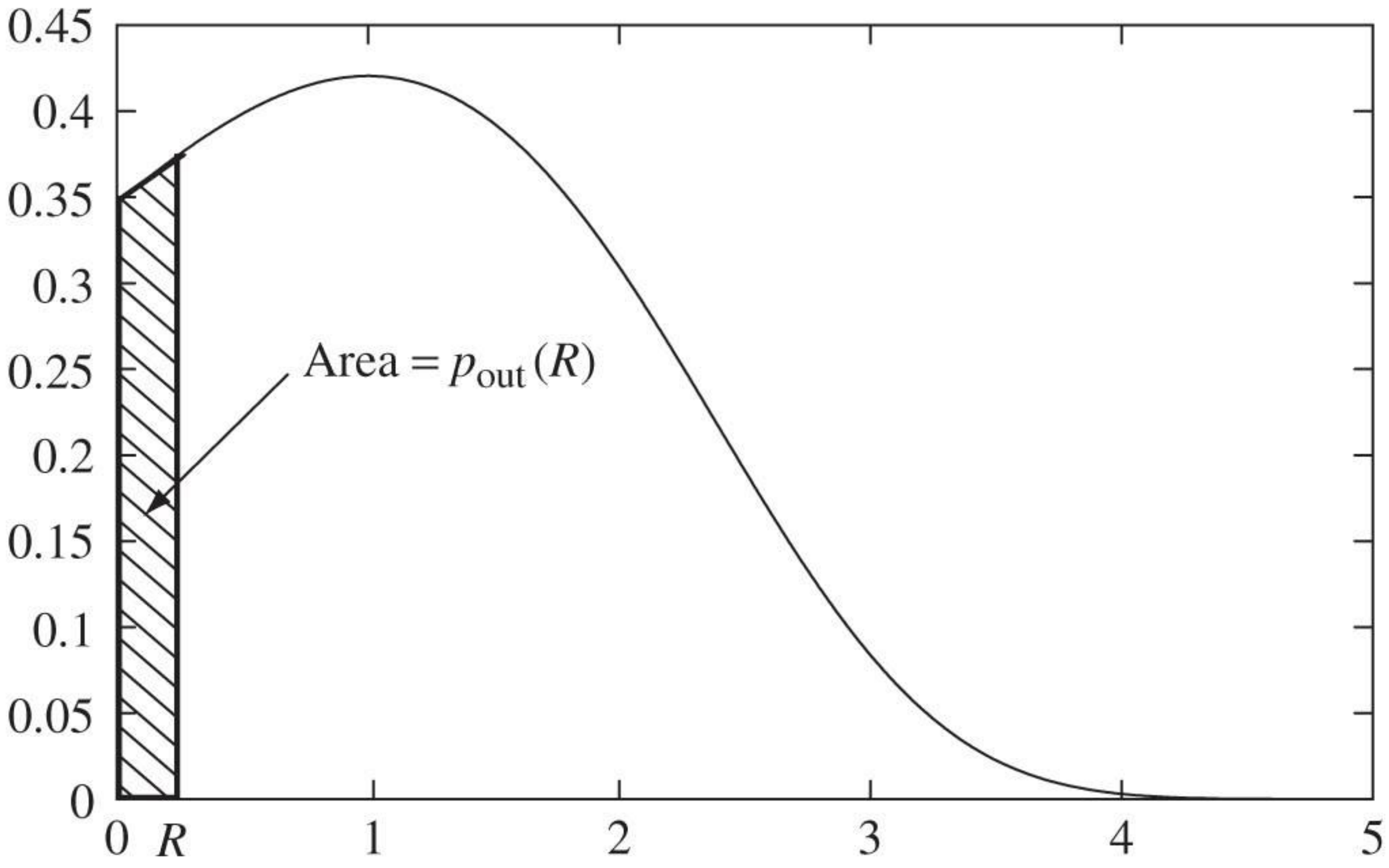
$$\log(1 + |h|^2 \mathsf{SNR}) < R$$

- The outage probability is

$$p_{\text{out}}(R) := \mathbb{P}\{\log(1 + |h|^2 \mathsf{SNR}) < R\}$$

Outage for Rayleigh Channel

- Pdf of log(1+|h|²SNR), Rayleigh fading and SNR = 0 dB.



- For any target rate R, there is a non-zero outage probability.

Outage for Rayleigh Channel

- For Rayleigh fading (i.e., h is CN(0,1)), the outage probability is

$$p_{\text{out}}(R) = 1 - \exp\left(\frac{-(2^R - 1)}{\text{SNR}}\right)$$

- At high SNR,

$$p_{\text{out}}(R) \approx \frac{(2^R - 1)}{\text{SNR}}$$

AWGN channel vs fading channel

- For AWGN channel, one can send data at a positive rate (in fact, any rate less than C) while making the error probability as small as desired.

- This cannot be done for the slow fading channel as long as the probability that the channel is in deep fade is non-zero.

- Thus, the capacity of the slow fading channel in the strict sense is zero. An alternative performance measure is the ϵ-outage capacity C_ϵ

ϵ-outage capacity C_ϵ

- $\epsilon - outage\ capacity\ C_\epsilon$ is the largest rate of transmission R such that the outage probability $p_{\text{out}}(R)$ is less than ϵ.

$$C_\epsilon = \log(1 + F^{-1}(1-\epsilon)\,\mathsf{SNR})\ \text{bits/s/Hz}$$

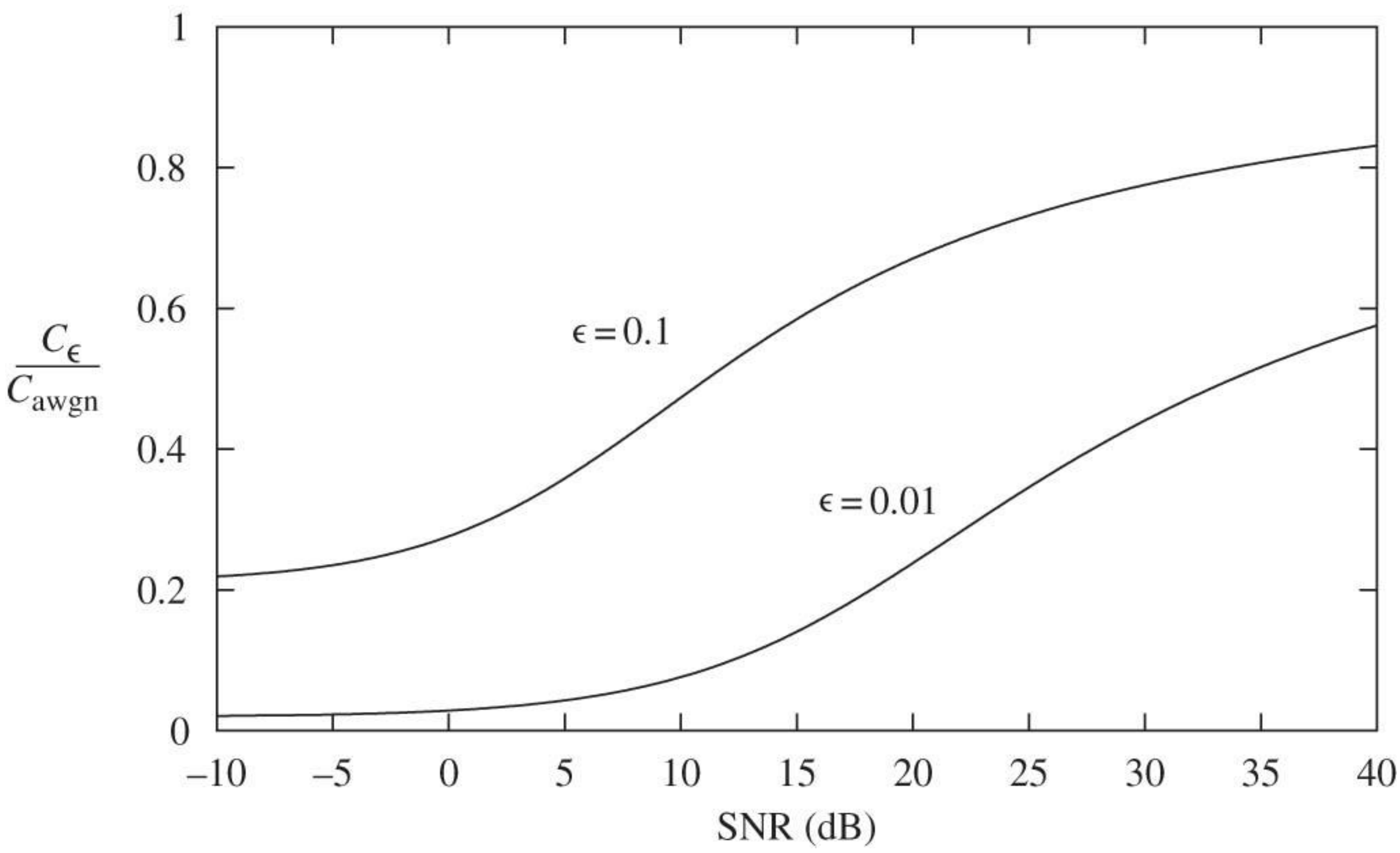
$$F(x) := \mathbb{P}\{|h|^2 > x\}$$

- Recall for AWGN channel, capacity is

$$C_{\text{awgn}} = \log(1 + \mathsf{SNR}) \quad \text{bits/s/Hz}$$

- To achieve the same rate as the AWGN channel, an extra $10\log(1/F^{-1}(1-\epsilon))$ dB of power is needed.

AWGN channel vs fading channel (2)



AWGN channel vs fading channel (3)

- At high SNR,

$$C_\epsilon \approx \log \mathsf{SNR} + \log(F^{-1}(1-\epsilon))$$
$$\approx C_{\text{awgn}} - \log\left(\frac{1}{F^{-1}(1-\epsilon)}\right)$$

- At low SNR,

$$C_\epsilon \approx F^{-1}(1-\epsilon)\mathsf{SNR}\log_2 \mathrm{e}$$
$$\approx F^{-1}(1-\epsilon)C_{\text{awgn}}.$$

- For Rayleigh fading, $F^{-1}(1-\epsilon) \approx \epsilon$ for small ϵ and the impact of fading is very significant!

Fast Fading Channel

- Ergodic Capacity:

$$C = \mathbb{E}[\log(1 + |h|^2 \mathsf{SNR})] \text{ bits/s/Hz}$$

- At low SNR,

$$C = \mathbb{E}[\log(1 + |h|^2 \mathsf{SNR})] \approx \mathbb{E}[|h|^2 \mathsf{SNR}] \log_2 e = \mathsf{SNR} \log_2 e \approx C_{\text{awgn}}$$

- At high SNR,

$$C \approx \mathbb{E}[\log(|h|^2 \mathsf{SNR})] = \log \mathsf{SNR} + \mathbb{E}[\log |h|^2] \approx C_{\text{awgn}} + \mathbb{E}[\log |h|^2],$$

- This difference is −0.83 bits/s/Hz for the Rayleigh fading channel. Equivalently, 2.5 dB more power is needed.

Fast fading: waterfilling

- Ergodic Capacity:

$$C = \mathbb{E}[\log(1 + |h|^2 \mathsf{SNR})] \text{ bits/s/Hz}$$

- How to optimize power with transmitter channel knowledge?

$$\max_{P_1,\dots,P_L} \frac{1}{L} \sum_{\ell=1}^{L} \log\left(1 + \frac{P_\ell |h_\ell|^2}{N_0}\right)$$

- Satifying

$$\frac{1}{L} \sum_{\ell=1}^{L} P_\ell = P,$$

- Optimal power allocation is waterfilling $P_\ell^* = \left(\frac{1}{\lambda} - \frac{N_0}{|h_\ell|^2}\right)^+,$

Fast fading with Full CSI

- The capacity of the fast fading channel with transmitter channel knowledge (Full CSI) is

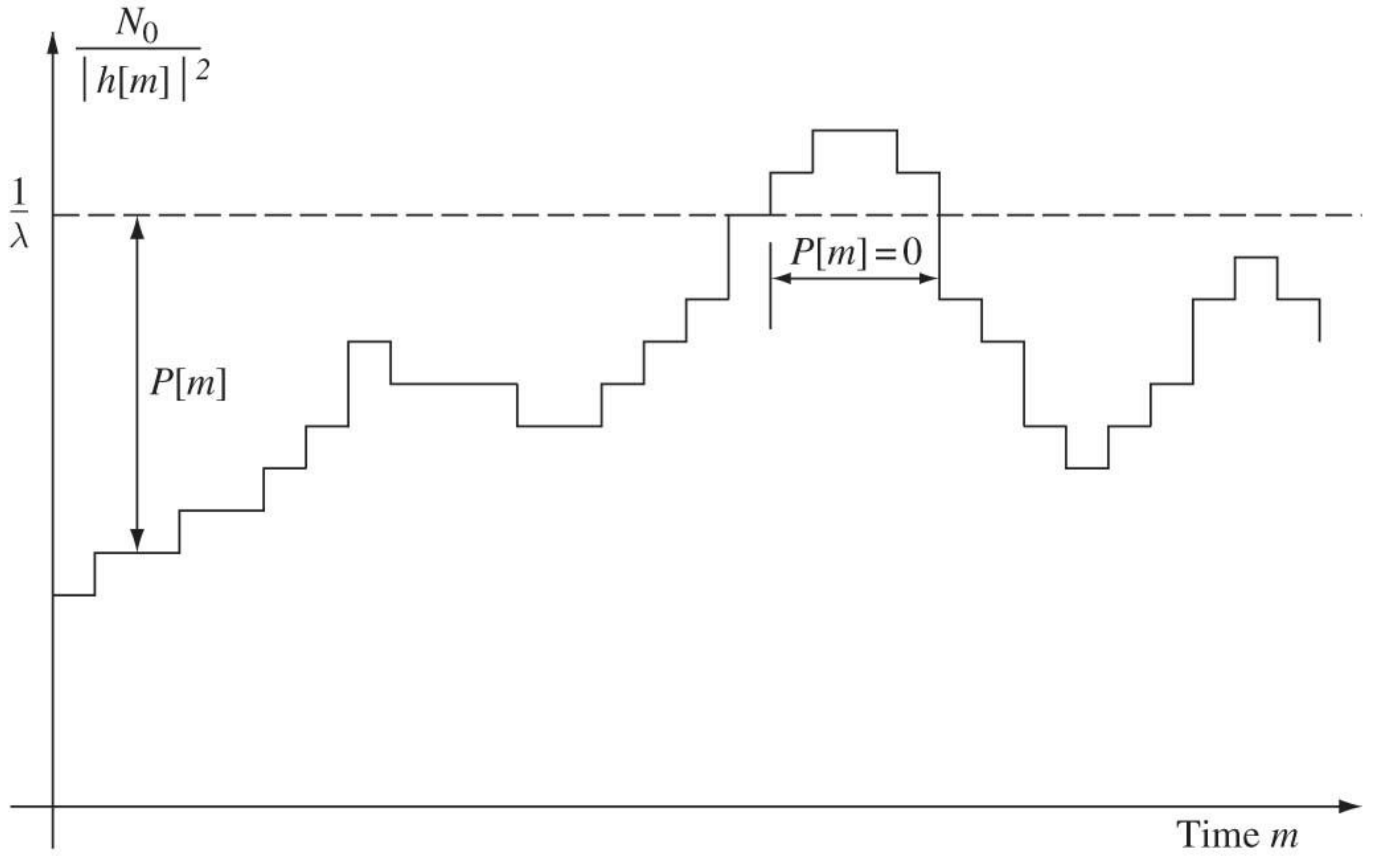
$$C = \mathbb{E}\left[\log\left(1 + \frac{P^*(h)|h|^2}{N_0}\right)\right] \text{ bits/s/Hz}$$

- Recall that with only receiver tracking the channel only (CSIR),

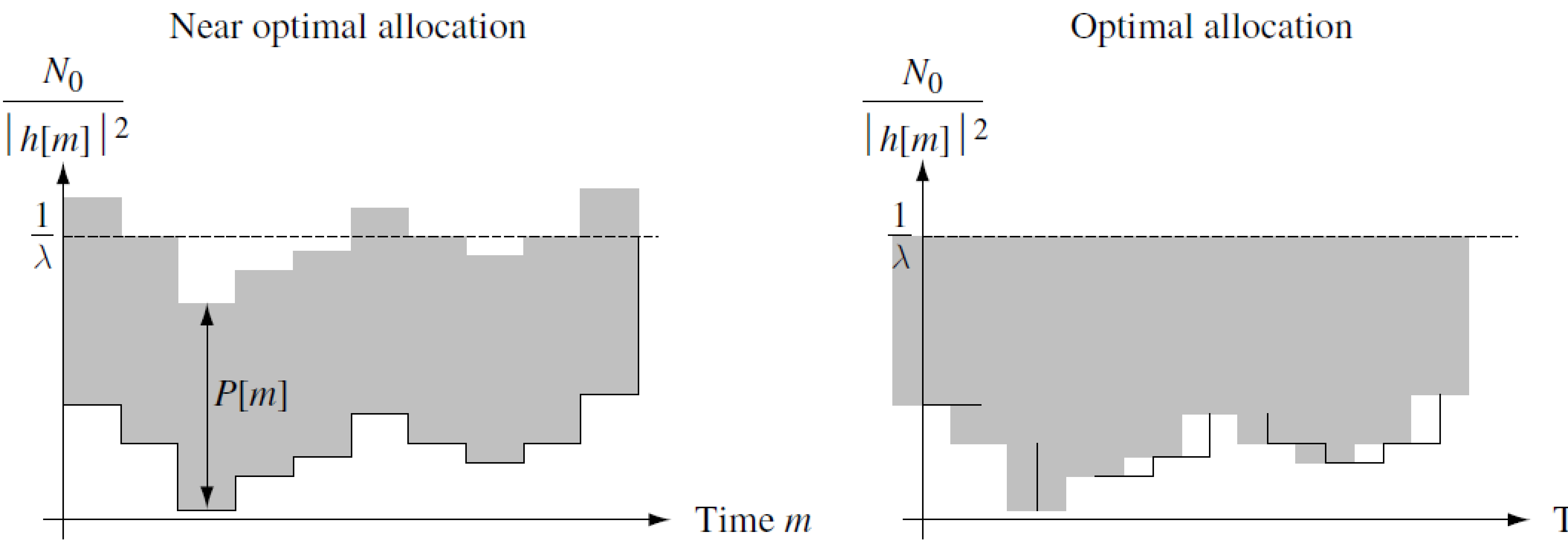
$$C = \mathbb{E}[\log(1 + |h|^2 \mathsf{SNR})] \text{ bits/s/Hz}$$

Fast fading with Full CSI

Transmit More when Channel is Good

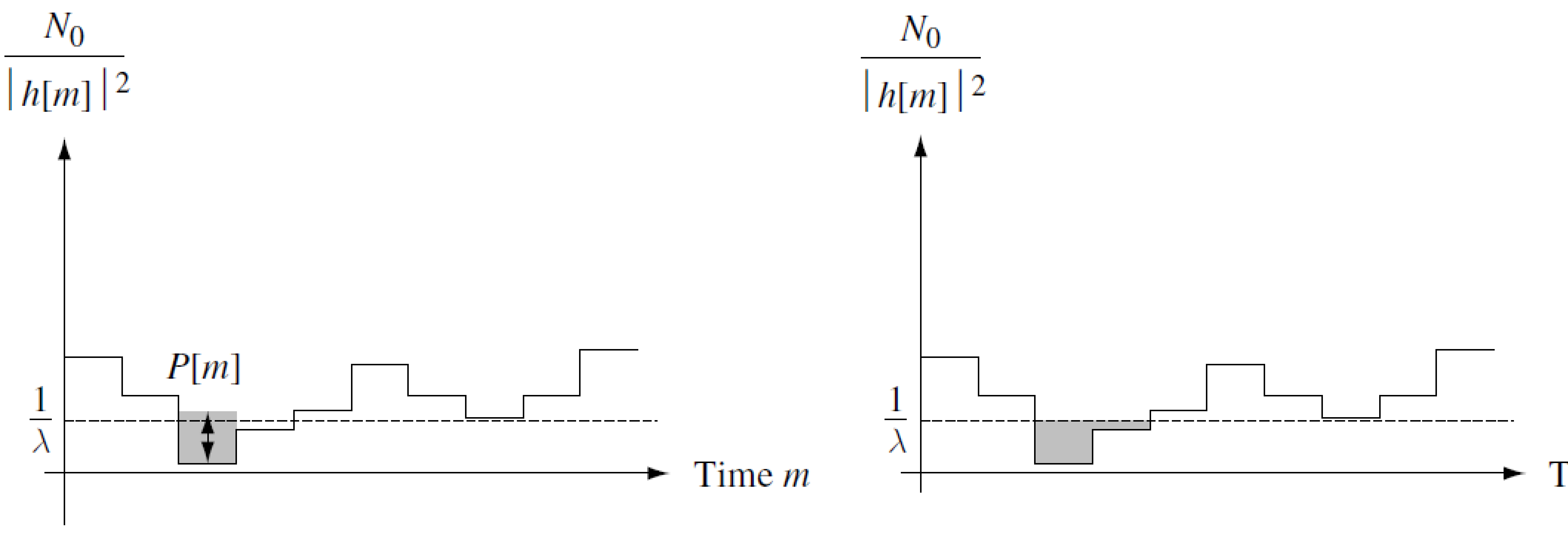


Fast fading with Full CSI, High SNR



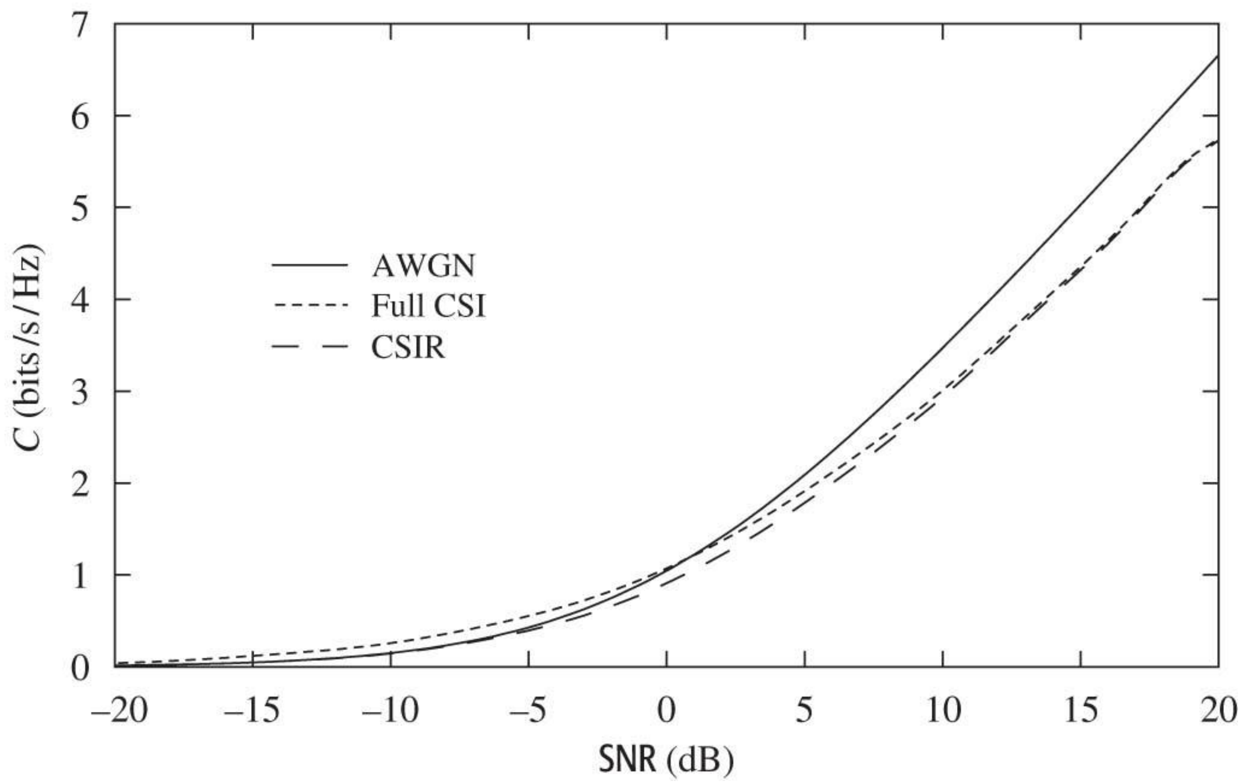
Fast fading with Full CSI, Low SNR

Near optimal allocation Optimal allocation



Fast fading with Full CSI vs CSIR

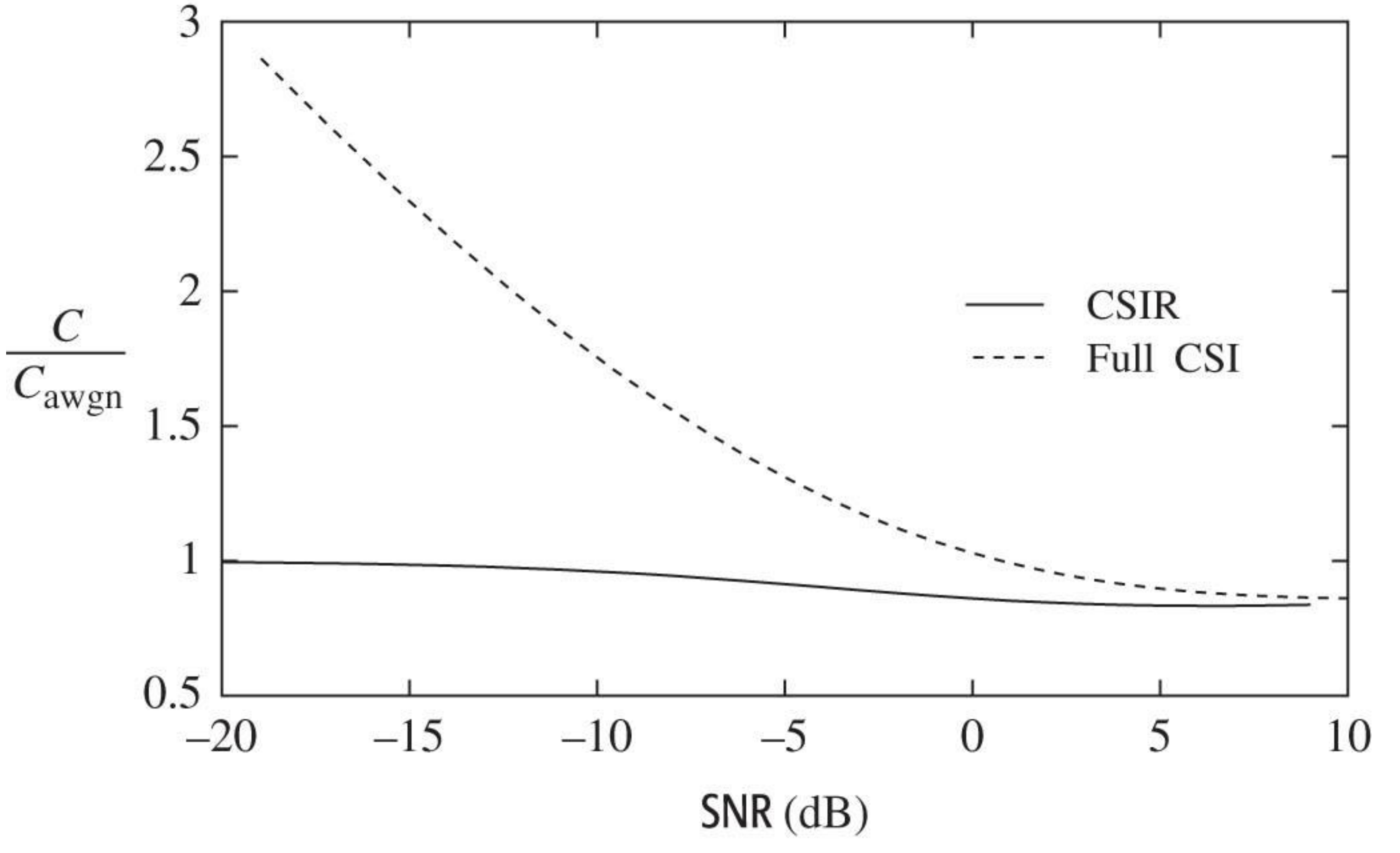
Performance



- ✓ At high SNR, waterfilling does not provide any gain.
- ✓ But transmitter knowledge allows rate adaptation and simplifies coding.

Fast fading with Full CSI vs CSIR

Performance: Low SNR



- Waterfilling povides a significant power gain at low SNR.

Error probability

Baseline: AWGN Channel

$$y = x + w$$

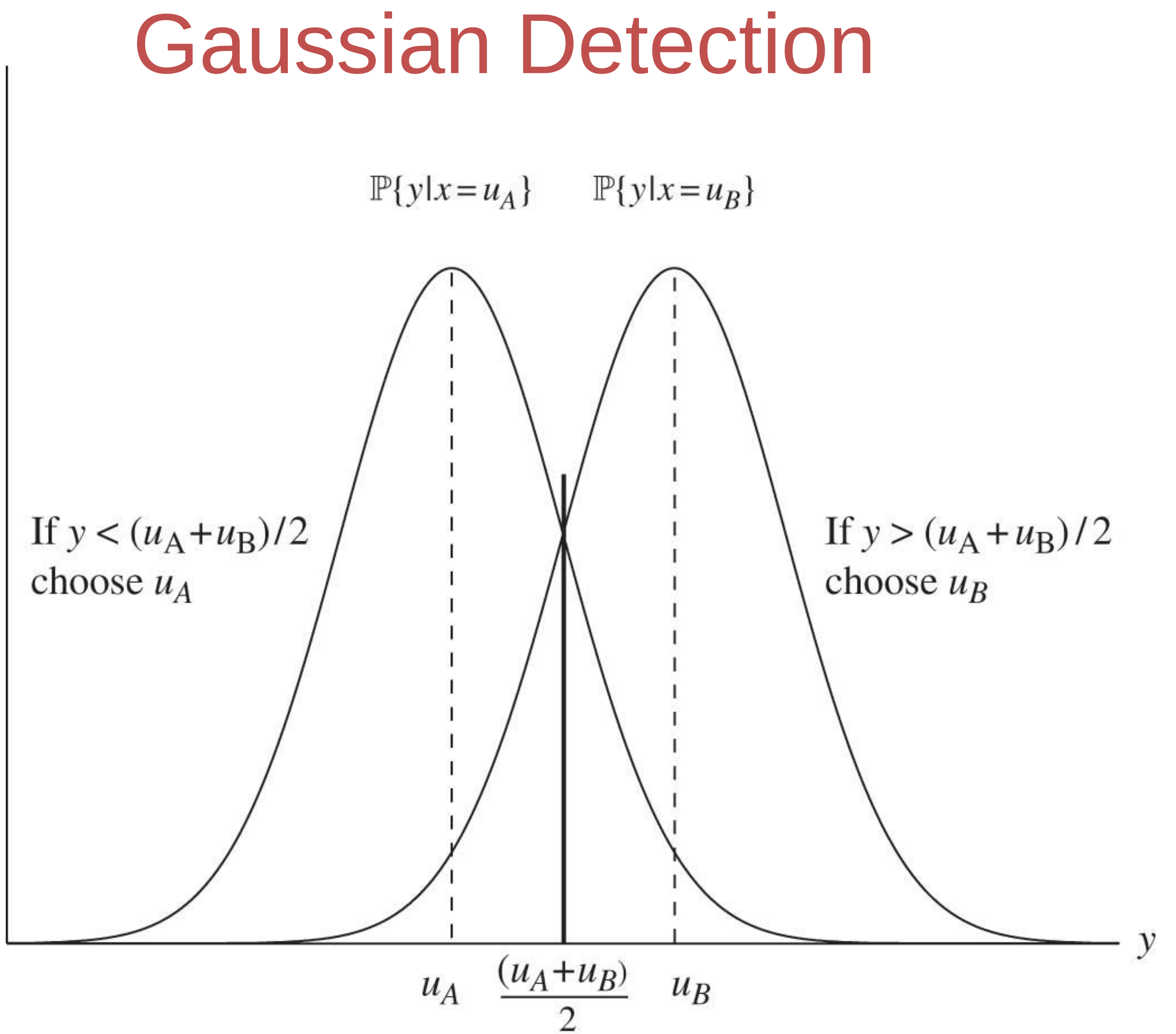
BPSK modulation $x = \pm a$

$$p_e = Q\left(\frac{a}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{2\mathsf{SNR}}\right)$$

$$\mathsf{SNR} := \frac{a^2}{N_0}$$

Error probability decays exponentially with SNR.

Error probability



Q-function

- Q-function is the complementary cumulative distribution function (CCDF) of an N(0,1) random variable,

$$Q(x) = \int_{x}^{+\infty} \frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}t^2) dt$$

- This function decays exponentially with x2

$$Q(x) < \mathrm{e}^{-x^2/2}, \qquad x > 0$$

$$Q(x) > \frac{1}{\sqrt{2\pi}x}\left(1 - \frac{1}{x^2}\right)\mathrm{e}^{-x^2/2}, \qquad x > 1$$

Rayleigh Flat Fading Channel

$$y = hx + w$$

$$h \sim \mathcal{CN}(0, 1)$$

BPSK: $x = \pm a.$ Coherent detection.

Conditional on h,

$$p_e = Q\left(\sqrt{2|h|^2 \mathsf{SNR}}\right)$$

Averaged over h,

$$p_e = \frac{1}{2}\left(1 - \sqrt{\frac{\mathsf{SNR}}{1 + \mathsf{SNR}}}\right) \approx \frac{1}{4\mathsf{SNR}}$$

at high SNR.

Coherent detection

- The decision is now based on the sign of the real sufficient statistic,

$$r := \Re\{(h/|h|)^* y\} = |h|x + z$$

$$z \sim N(0, N_0/2)$$

- For a given value of h,

$$Q\left(\frac{a|h|}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{2|h|^2 \mathsf{SNR}}\right)$$

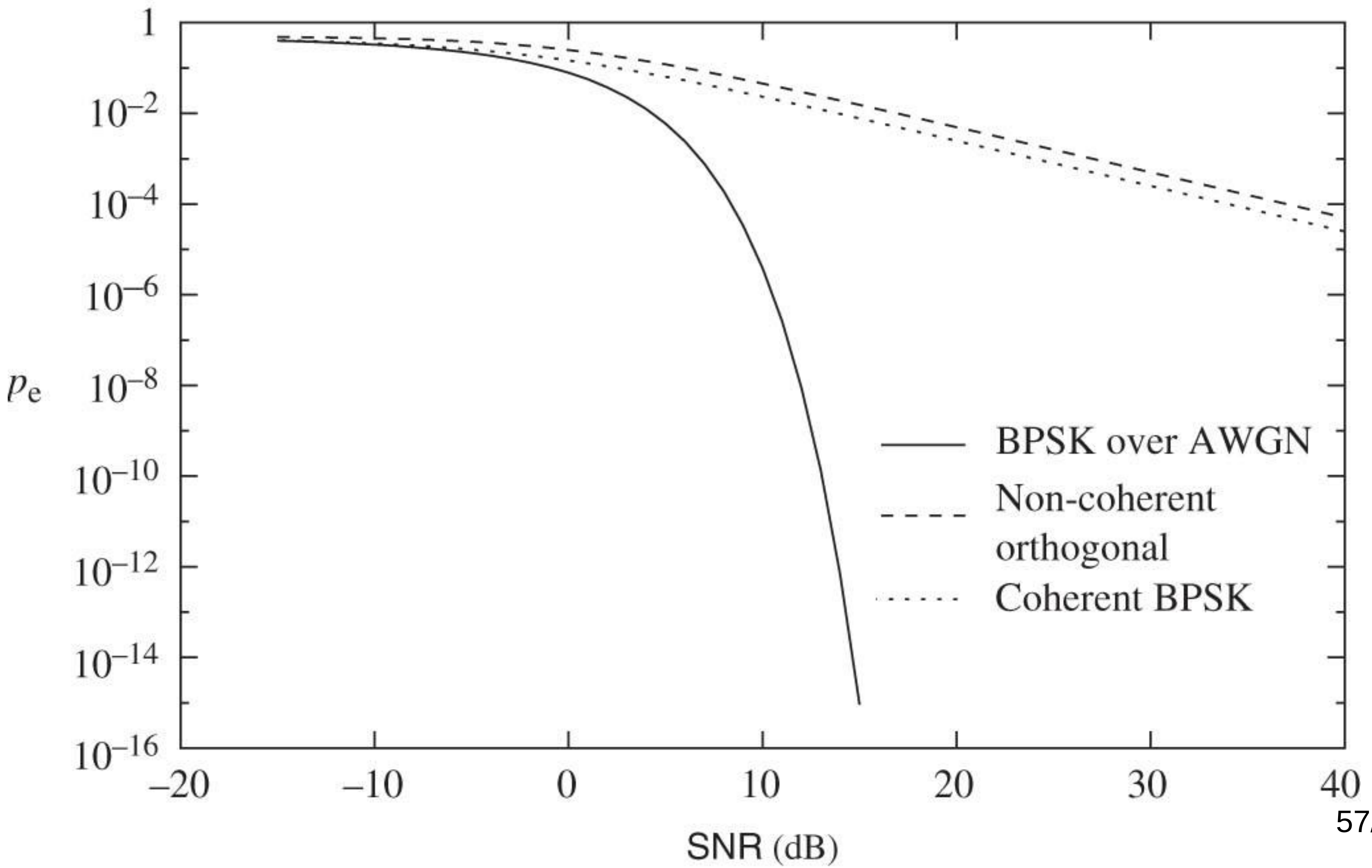
- Average over the random gain h,

$$p_e = \mathbb{E}\left[Q\left(\sqrt{2|h|^2 \mathsf{SNR}}\right)\right] = \frac{1}{2}\left(1 - \sqrt{\frac{\mathsf{SNR}}{1+\mathsf{SNR}}}\right)$$

$$\sqrt{\frac{\mathsf{SNR}}{1+\mathsf{SNR}}} = 1 - \frac{1}{2\mathsf{SNR}} + O\left(\frac{1}{\mathsf{SNR}^2}\right)$$

Error probability

Rayleigh vs AWGN



Coherent detection: Rayleigh vs AWGN

- There is only a 3 dB difference in the required SNR between the coherent and non-coherent schemes.

- At an error probability of 10^−3, there is a 17 dB difference between the performance on the AWGN channel and coherent detection on the Rayleigh fading channel.

Typical Error Event

Conditional on h,

$$p_e = Q\left(\sqrt{2|h|^2 \mathsf{SNR}}\right)$$

When $|h|^2 >> \frac{1}{\mathsf{SNR}}$, error probability is very small.

When $|h|^2 < \frac{1}{\mathsf{SNR}}$, error probability is large:

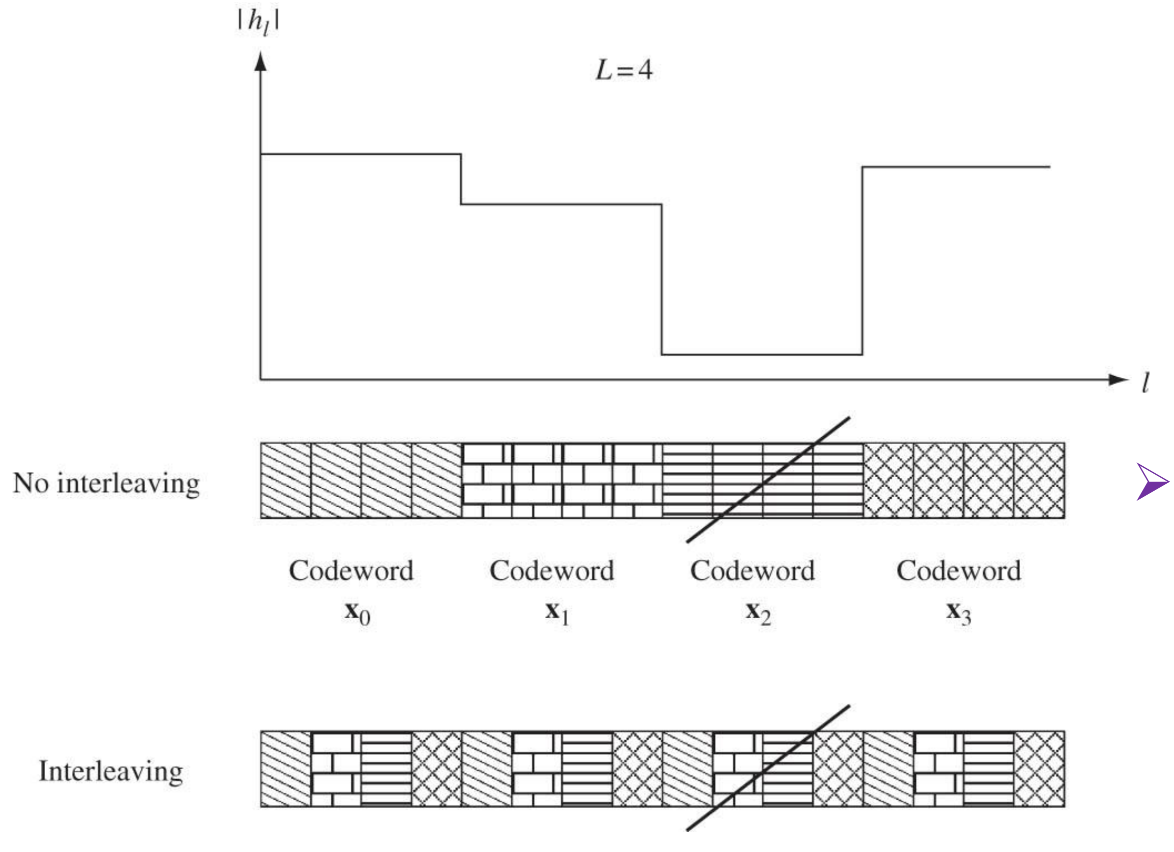
$$p_e \approx P\left(|h|^2 < \frac{1}{\mathsf{SNR}}\right) \approx \frac{1}{\mathsf{SNR}}$$

$$|h|^2 \sim \exp(1).$$

Typical error event is due to channel being in deep fade rather than noise being large.

Time Diversity

- Time diversity can be obtained by interleaving and coding over symbols across different coherent time periods.



- **Repetition coding**

Coding alone is not sufficient!

Simplest Code: Repetition

After interleaving over L coherence time periods,

$$y_\ell = h_\ell x_\ell + w_\ell, \qquad \ell = 1, \ldots, L$$

Repetition coding: $x_\ell = x$ for all ℓ.

$$\mathbf{y} = \mathbf{h}x + \mathbf{w}$$

where $\mathbf{y} = [y_1, \ldots, y_L]^t$, $\mathbf{h} = [h_1, \ldots, h_L]^t$, and $\mathbf{w} = [w_1, \ldots, w_L]^t$.

- This is classic vector detection in white Gaussian noise.

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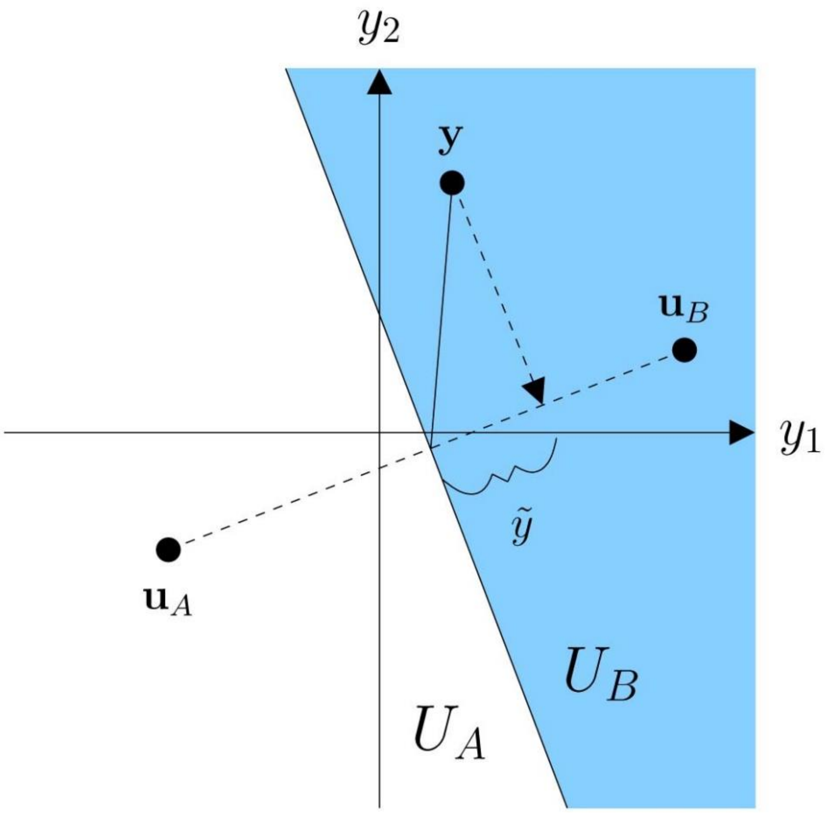
Simplest Code: Repetition

Geometry



For BPSK $x = \pm a$,

$$\mathbf{u}_A = +a\mathbf{h}, \mathbf{u}_B = -a\mathbf{h}.$$

$$\tilde{y} = \frac{\mathbf{h}^*}{\|\mathbf{h}\|}\mathbf{y}$$

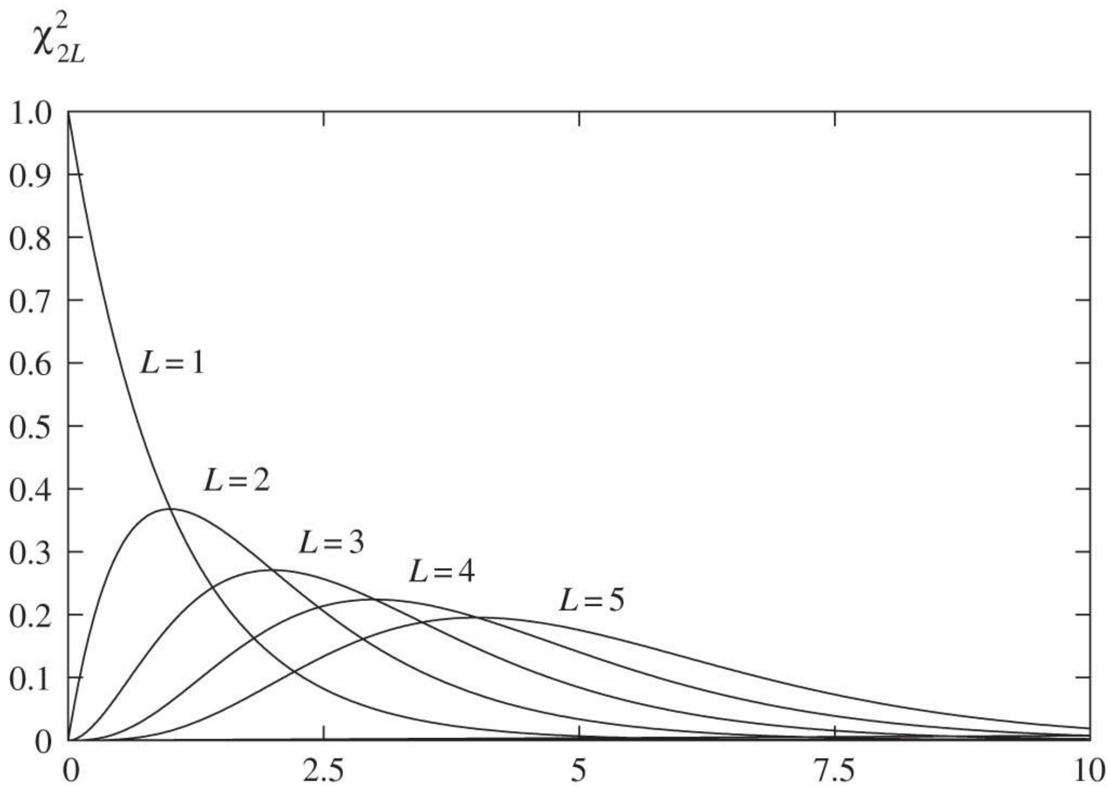
Is a sufficient statistic (match filtering/maximal ratio combiner).

Reduces to scalar detection problem:

$$\tilde{y} = \|\mathbf{h}\|x + \tilde{w}$$

Simplest Code: Repetition

Deep Fades Become Rarer



$$P\left(\|\mathbf{h}\|^2 < \epsilon\right) \approx \frac{1}{L!}\epsilon^L$$

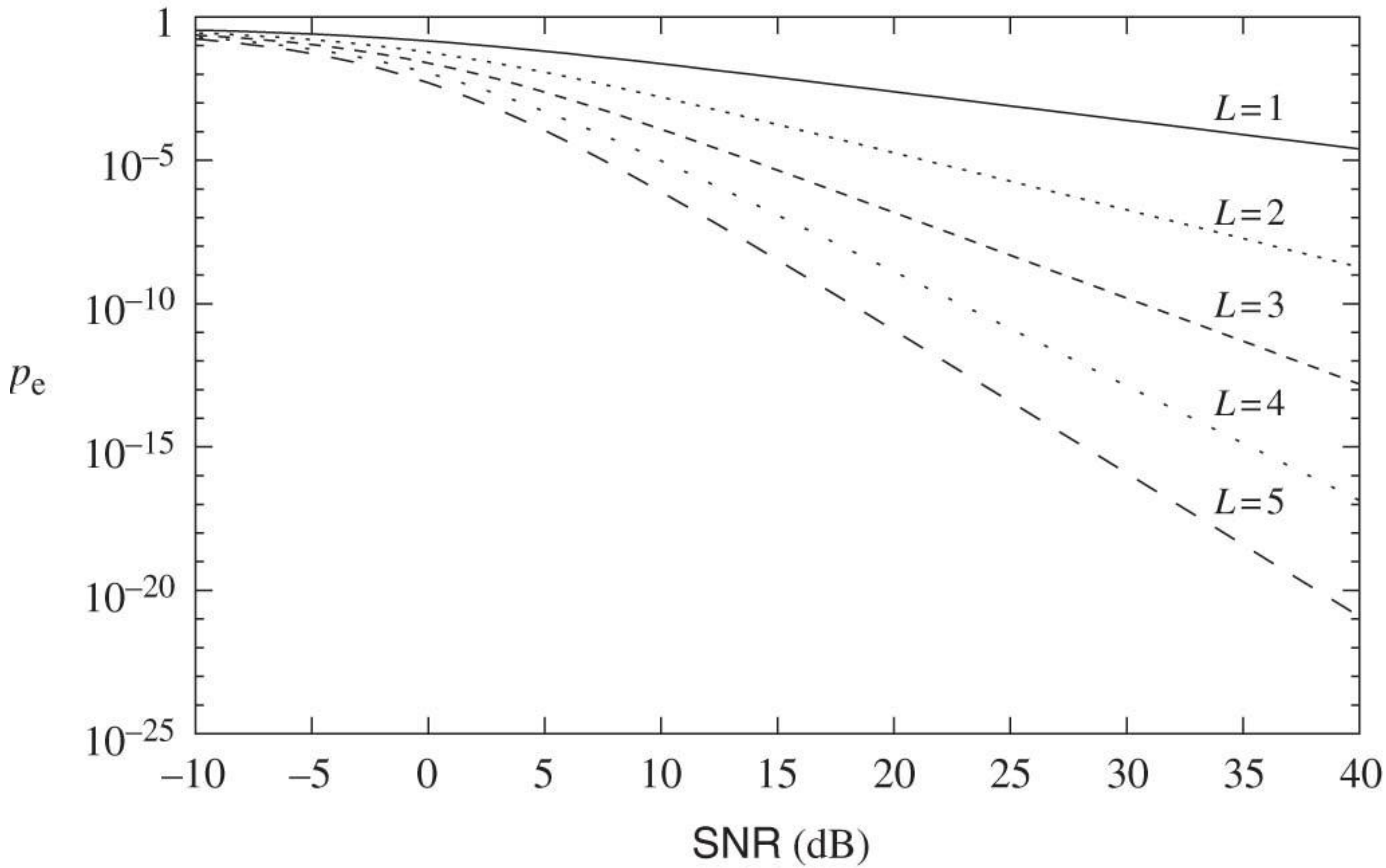
$$\begin{aligned} p_e &\approx P\left(\|\mathbf{h}\|^2 < \frac{1}{\mathsf{SNR}}\right) \\ &\approx \frac{1}{L!}\frac{1}{\mathsf{SNR}^L} \end{aligned}$$

–chi-square distribution:卡方分布/西格玛分布

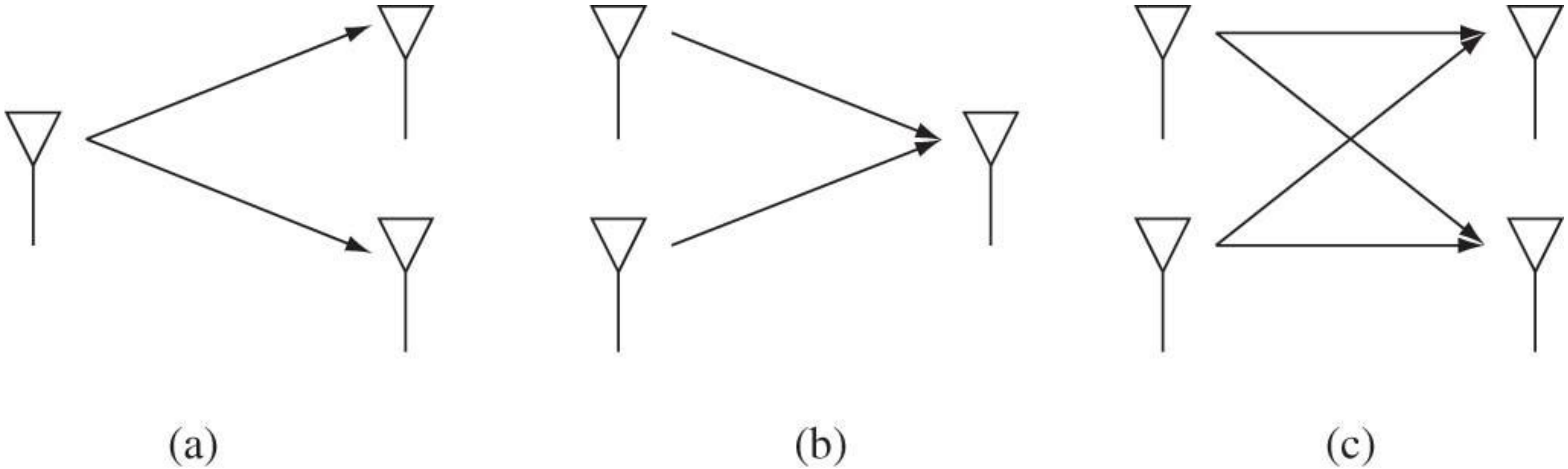
Simplest Code: Repetition

Performance

$$p_e \approx \binom{2L-1}{L} \frac{1}{(4\mathsf{SNR})^L}$$



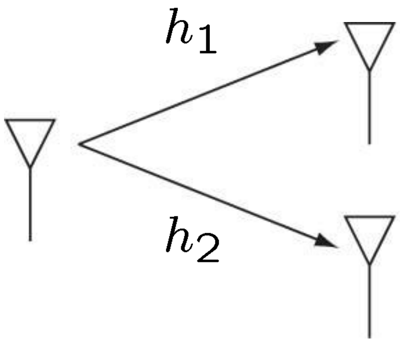
Antenna Diversity



Receive Transmit Both

Receive Diversity

$$\mathbf{y} = x\mathbf{h} + \mathbf{w}$$

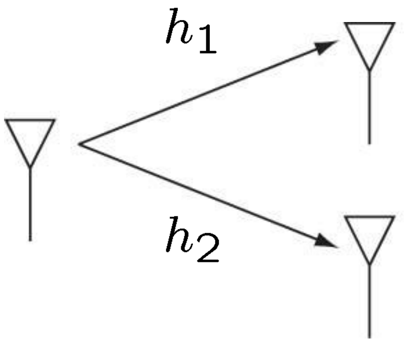


- Same as repetition coding in time diversity, except that there is a further power gain.

- Optimal reception is via match filtering (receive beamforming).

Receive Diversity

$$\mathbf{y} = x\mathbf{h} + \mathbf{w}$$



- Total received SNR: $\|\mathbf{h}\|^2 \mathsf{SNR} = L\mathsf{SNR} \cdot \frac{1}{L}\|\mathbf{h}\|^2$

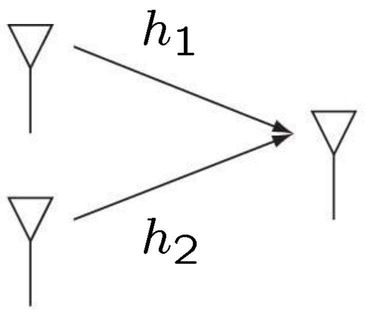
  $L\mathsf{SNR}$: *power gain* (also called *array gain*)

  $\frac{1}{L}\|\mathbf{h}\|^2$: *diversity gain, converges to 1 with increasing L*

- A 3-dB gain is obtained for every doubling of the number of antennas.

Transmit Diversity

$$y = \mathbf{h}^* \mathbf{x} + w$$



If transmitter knows the channel, send:

$$\mathbf{x} = x \frac{\mathbf{h}}{\|\mathbf{h}\|}.$$

maximizes the received SNR by in-phase addition of signals at the receiver (transmit beamforming).

Reduce to scalar channel:

$$y = \|\mathbf{h}\| x + w,$$

same as receive beamforming.

What happens if transmitter does not know the channel?

Space-time Codes

- Transmitting the same symbol simultaneously at the antennas doesn't work.

- Using the antennas one at a time and sending the same symbol over the different antennas is like repetition coding.

- More generally, can use any time-diversity code by turning on one antenna at a time.

- Space-time codes are designed specifically for the transmit diversity scenario.

Alamouti Scheme

$$y[m] = h_1[m]x_1[m] + h_2[m]x_2[m] + w[m]$$

Over two symbol times:

$$\begin{bmatrix} y[1] & y[2] \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix} + \begin{bmatrix} w[1] & w[2] \end{bmatrix}.$$

$$\begin{bmatrix} y[1] \\ y[2]^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} w[1] \\ w[2]^* \end{bmatrix}$$

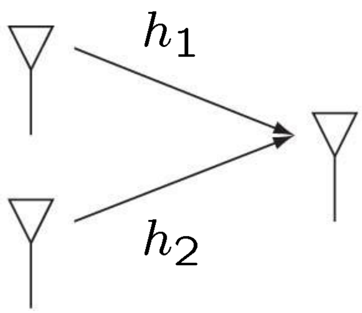
Projecting onto the two columns of the H matrix yields:

$$r_i = \|\mathbf{h}\| u_i + w_i, \qquad i = 1, 2.$$

- ✓ double the symbol rate of repetition coding.
- ✓ 3dB loss of received SNR compared to transmit beamforming.

Homework

- **1. If K=0dB, transmit power is 1 w(watt), reference distance is 1 m, path-loss exponent is 3, and the distance between the transmitter and receiver is 10 m, what is the received power?**
- **2. In the right Figure, h1=3, h2=4, what is the diversity gain ?**



- **3. Compute the Capacity of an AWGN channel when SNR=20dB.**

- **Tips: For Q1, see P6; For Q2, see P67-69; for Q3, see P24.**
- **Requirements: Submitted next week.**