Fundamentals of Image Processing

Spatial Filtering

Image Filtering

- Image filtering: computes a function of a *local neighborhood* at each pixel position
- Called "Local operator," "Neighborhood operator," or "Window operator"
- *f*: image → image
- Uses:
  - Enhance images
    - Noise reduction, smooth, resize, increase contrast, recolor, artistic effects, etc.
  - Extract features from images
    - Texture, edges, distinctive points, etc.
  - Detect patterns
    - Template matching, e.g., eye template

Slide credit: D. Hoiem

Filtering

- The name "filter" is borrowed from frequency domain processing (next week's topic)
- Accept or reject certain frequency components
- Fourier (1807): Periodic functions could be represented as a weighted sum of sines and cosines

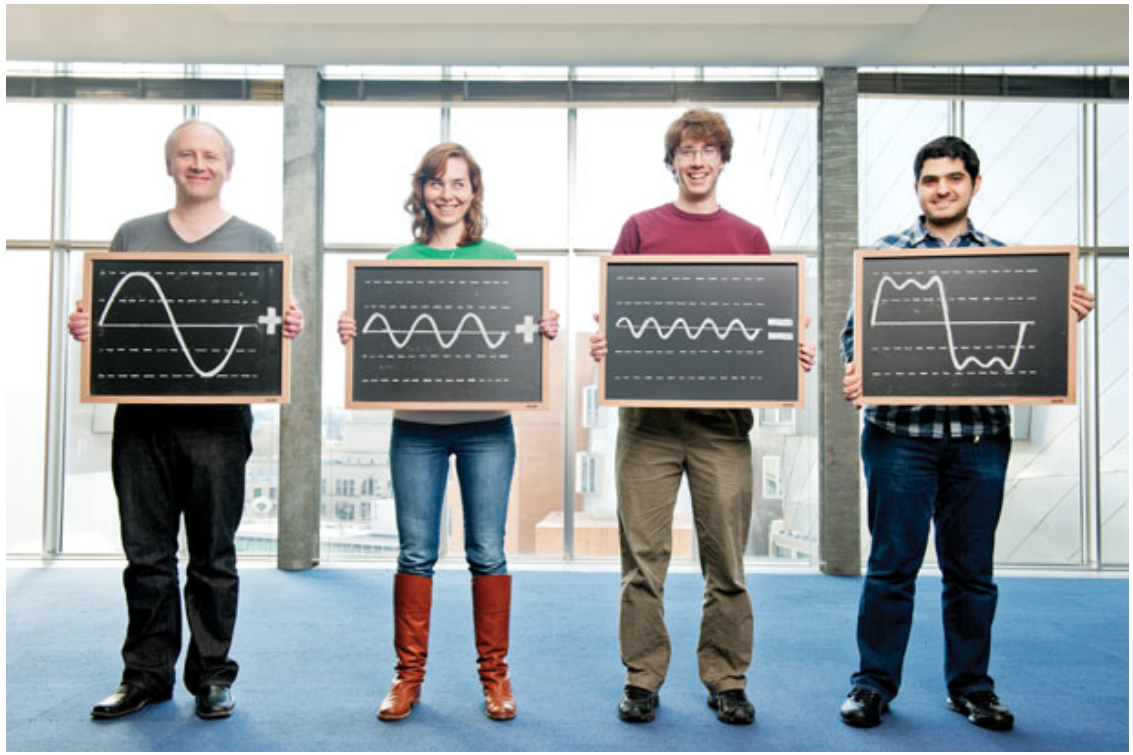
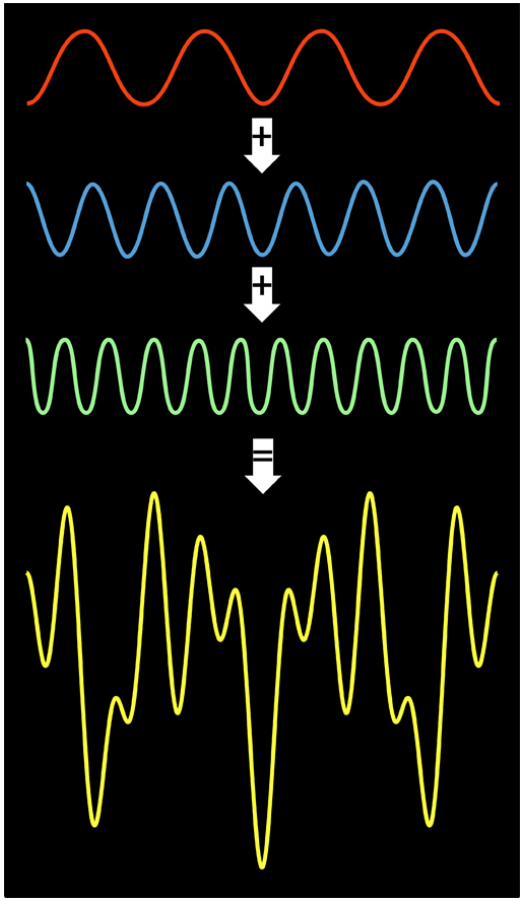


Image courtesy of Technology Review

Signals

- A signal is composed of low and high frequency components



low frequency components: smooth / piecewise smooth

Neighboring pixels have similar brightness values

You're within a region

high frequency components: oscillatory

Neighboring pixels have different brightness values

You're either at the edges or noise points

Low/high frequencies vs. fine/coarse-scale details

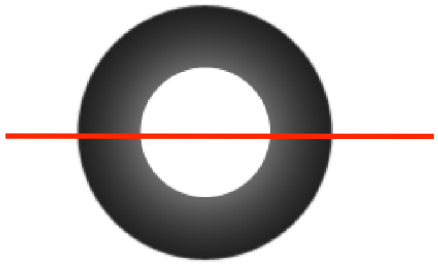
Original image

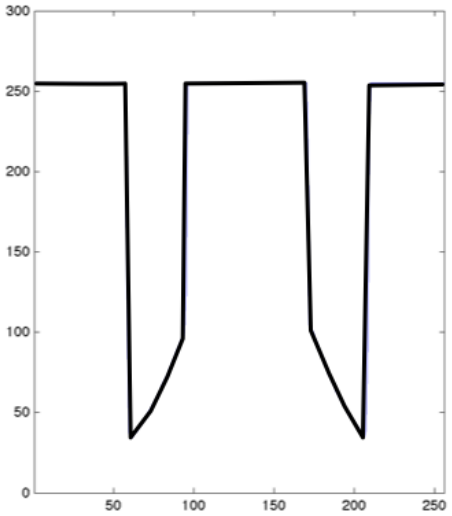
Low-frequencies
(coarse-scale details)
boosted

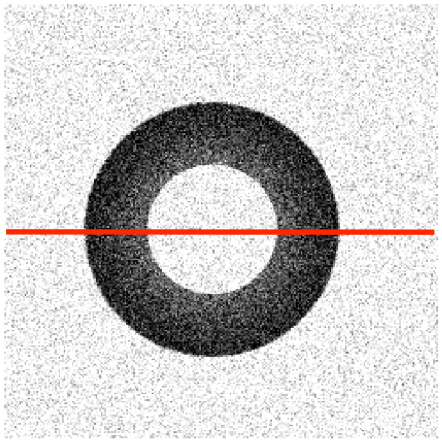
High-frequencies
(fine-scale details)
boosted

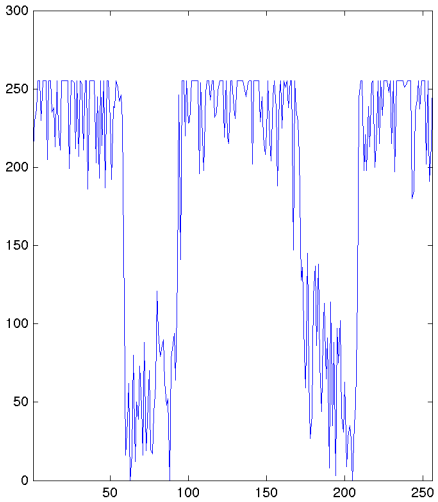
L. Karacan, E. Erdem and A. Erdem, Structure Preserving Image Smoothing via Region Covariances, TOG, 2013

Signals – Examples









Motivation: noise reduction

- Assume image is degraded with an additive model.
- Then,

Observation = True signal + noise

Observed image = Actual image + noise

low-pass filters

high-pass filters

smooth the image

Common types of noise

- **Salt and pepper noise**: random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Original

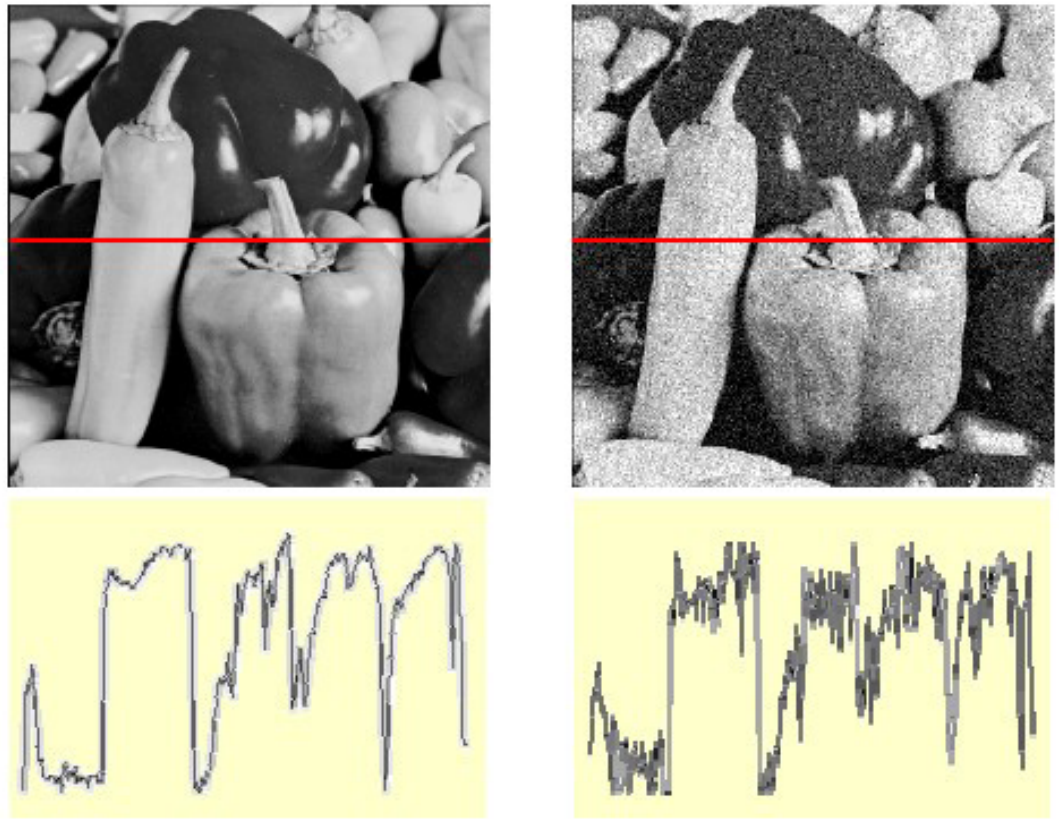
Salt and pepper noise

Impulse noise

Gaussian noise

Slide credit: S. Seitz

Gaussian noise



$$f(x,y) = \overbrace{\bar{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

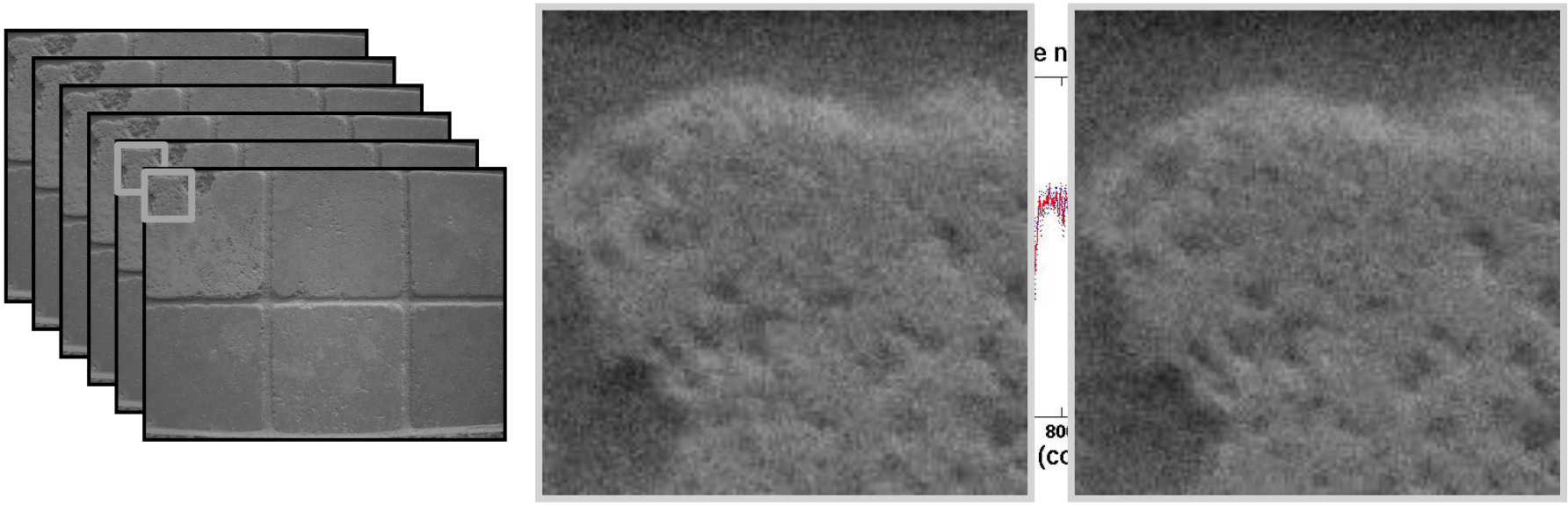
Gaussian i.i.d. ("white") noise:
$\eta(x,y) \sim \mathcal{N}(\mu,\sigma)$

```
>> noise = randn(size(im)).*sigma;
>> output = im + noise;
```

What is the impact of the sigma?

Slide credit: M. Hebert

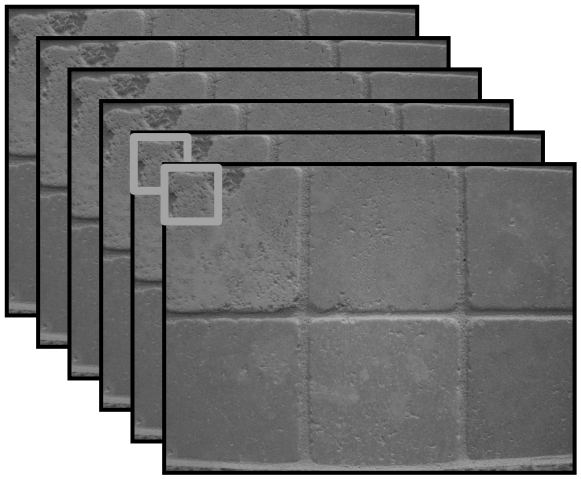
Motivation: noise reduction

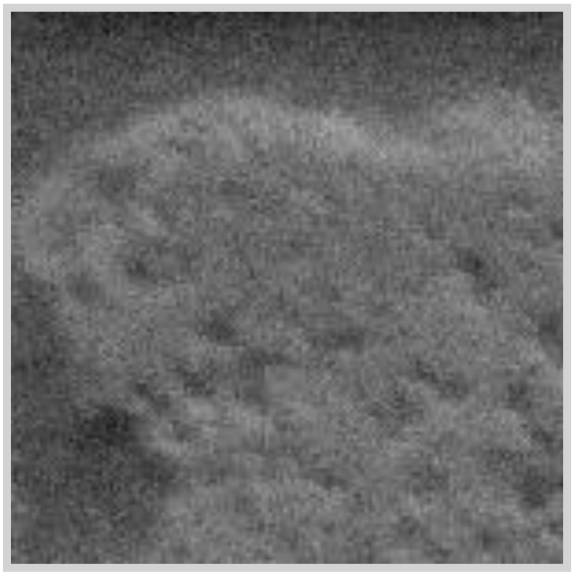


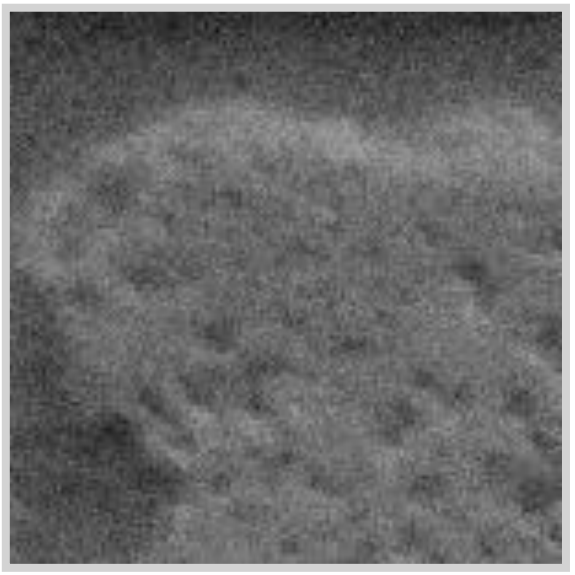
- Make multiple observations of the same static scene
- Take the average
- Even multiple images of the same static scene will not be identical.

Adapted from: K. Grauman

Motivation: noise reduction







- Make multiple observations of the same static scene
- Take the average
- Even multiple images of the same static scene will not be identical.
- What if we can't make multiple observations?
  What if there's only one image?

Adapted from: K. Grauman

Image Filtering

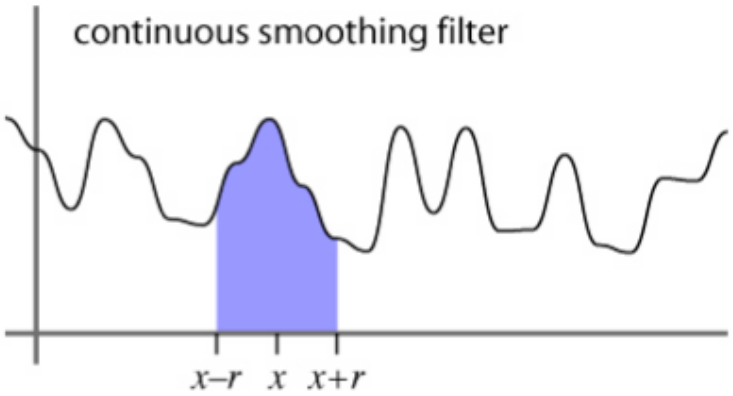
说明：像素的邻域内的像素可以视为是该像素的多个抽样

- Idea: Use the information coming from the neighboring pixels for processing
- Design a transformation function of the local neighborhood at each pixel in the image
  - Function specified by a "filter" or mask saying how to combine values from neighbors.
- Various uses of filtering:
  - Enhance an image (denoise, resize, etc)
  - Extract information (texture, edges, etc)
  - Detect patterns (template matching)

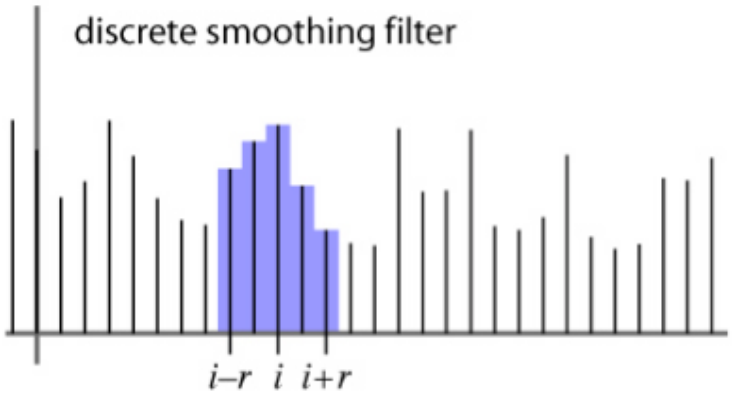
Adapted from: K. Grauman

Filtering

- Processing done on a function
  - can be executed in continuous form (e.g. analog circuit)
  - but can also be executed using sampled representation
- Simple example: smoothing by averaging



积分



求和

Slide credit: S. Marschner

Linear filtering

- Filtered value is the linear combination of neighboring pixel values.
- Key properties

– linearity: filter(f + g) = filter(f) + filter(g)

– shift invariance: behavior invariant to shifting the input

  - delaying an audio signal
  - sliding an image around

- Can be modeled mathematically by *convolution*

Adapted from: S. Marschner

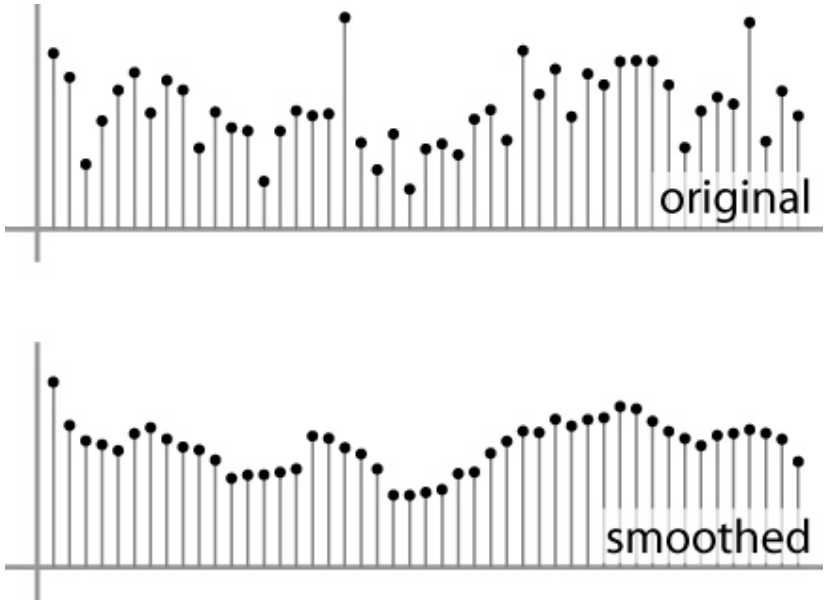
First attempt at a solution

- Let's replace each pixel with an average of all the values in its neighborhood
- Assumptions:
  - Expect pixels to be like their neighbors (spatial regularity in images)
  - Expect noise processes to be independent from pixel to pixel

Slide credit: S. Marschner, K. Grauman

First attempt at a solution

- Let's replace each pixel with an average of all the values in its neighborhood
- Moving average in 1D:



Slide credit: S. Marschner

Convolution warm-up

- Same moving average operation, expressed mathematically:

$$b_{\text{smooth}}[i] = \frac{1}{2r+1} \sum_{j=i-r}^{i+r} b[j]$$

Slide credit: S. Marschner

Discrete convolution

- Simple averaging:

$$b_{\text{smooth}}[i] = \frac{1}{2r+1} \sum_{j=i-r}^{i+r} b[j]$$

– every sample gets the same weight

- Convolution: same idea but with *weighted* average

$$(a \star b)[i] = \sum_{j} a[j]b[i-j]$$

– each sample gets its own weight (normally zero far away)

- This is all convolution is: it is a **moving weighted average**

Slide credit: S. Marschner

Filters

- Sequence of weights $a[j]$ is called a *filter*
- Filter is nonzero over its *region of support*

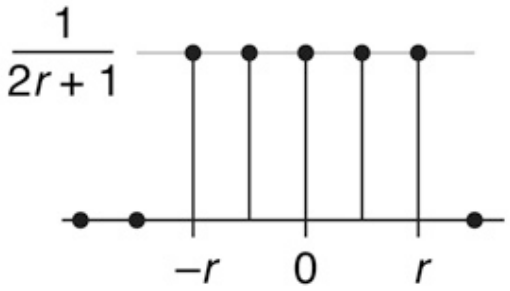
– usually centered on zero: support radius r

- Filter is *normalized* so that it sums to 1.0

– this makes for a weighted average, not just any old weighted sum

- Most filters are symmetric about 0

– since for images we usually want to treat left and right the same

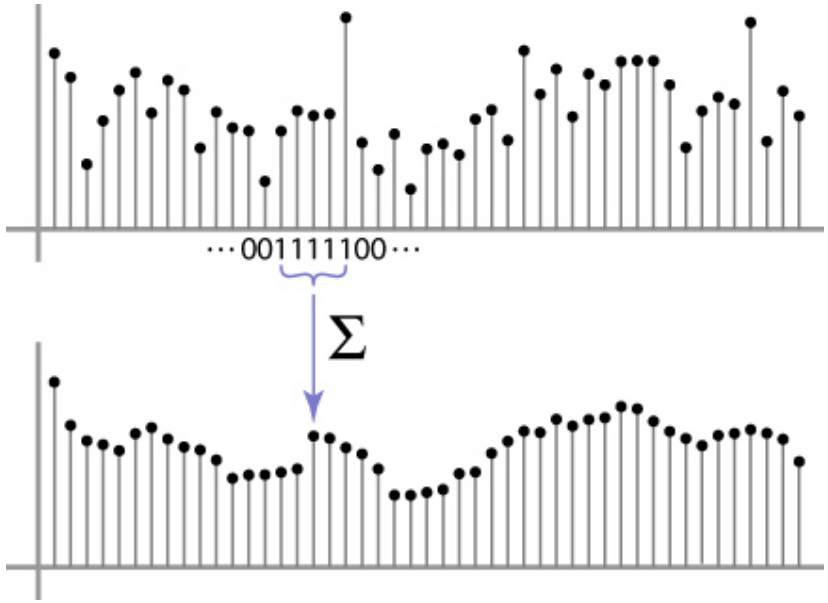


a box filter

Slide credit: S. Marschner

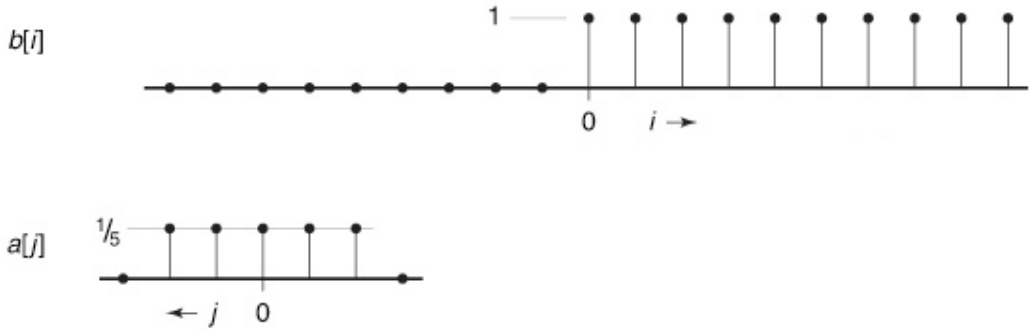
Convolution and filtering

- Can express sliding average as convolution with a *box filter*
- a_{box} = […, 0, 1, 1, 1, 1, 1, 0, …]



Slide credit: S. Marschner

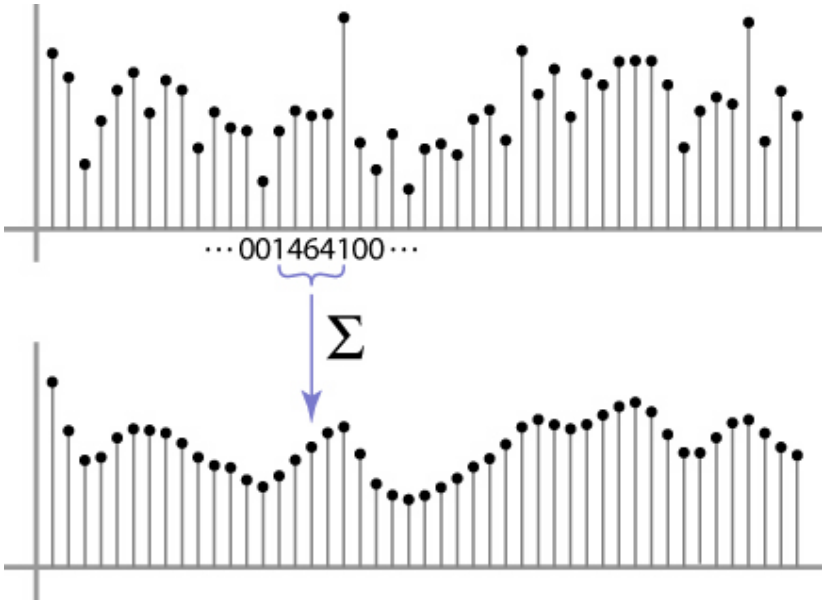
Example: box and step



Slide credit: S. Marschner

Convolution and filtering

- Convolution applies with any sequence of weights
- Example: bell curve (gaussian-like) […, 1, 4, 6, 4, 1, …]/16



Slide credit: S. Marschner

And in pseudocode...

```
function convolve(sequence a, sequence b, int r, int i )
    s = 0
    for j = −r to r
        s = s + a[j]b[i − j]
    return s
```

Slide credit: S. Marschner

Key properties

- **Linearity:** filter($f_1 + f_2$) = filter(f_1) + filter(f_2)
- **Shift invariance:** filter(shift(f)) = shift(filter(f))
  - same behavior regardless of pixel location, i.e. the value of the output depends on the pattern in the image neighborhood, not the position of the neighborhood.
- Theoretical result: any linear shift-invariant operator can be represented as a convolution

Slide credit: S. Lazebnik

Properties in more detail

- Commutative: $a * b = b * a$
  - Conceptually no difference between filter and signal
- Associative: $a * (b * c) = (a * b) * c$
  - Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - This is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
- Scalars factor out: $ka * b = a * kb = k (a * b)$
- Identity: unit impulse $e = [\ldots, 0, 0, 1, 0, 0, \ldots]$,
  $a * e = a$

Slide credit: S. Lazebnik

A gallery of filters

- Box filter

– Simple and cheap

- Tent filter

– Linear interpolation

- Gaussian filter

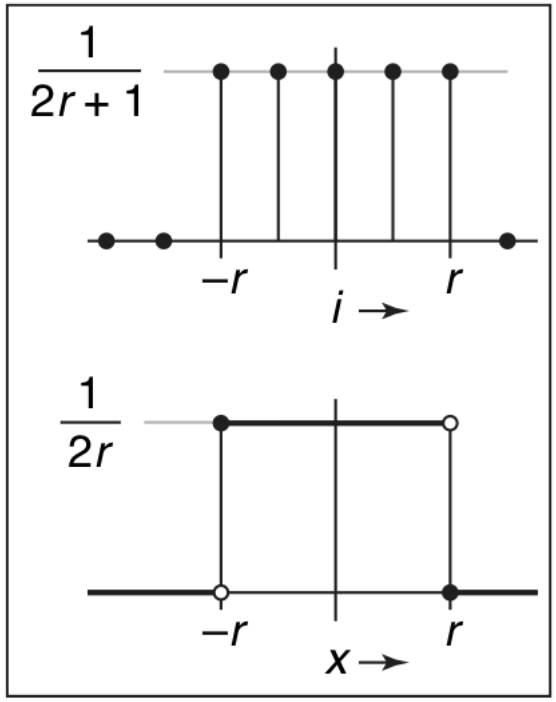
– Very smooth antialiasing filter

Slide credit: S. Marschner

Box filter

$$a_{\text{box},r}[i] = \begin{cases} 1/(2r+1) & |i| \le r, \\ 0 & \text{otherwise.} \end{cases}$$

$$f_{\text{box},r}(x) = \begin{cases} 1/(2r) & -r \le x < r, \\ 0 & \text{otherwise.} \end{cases}$$

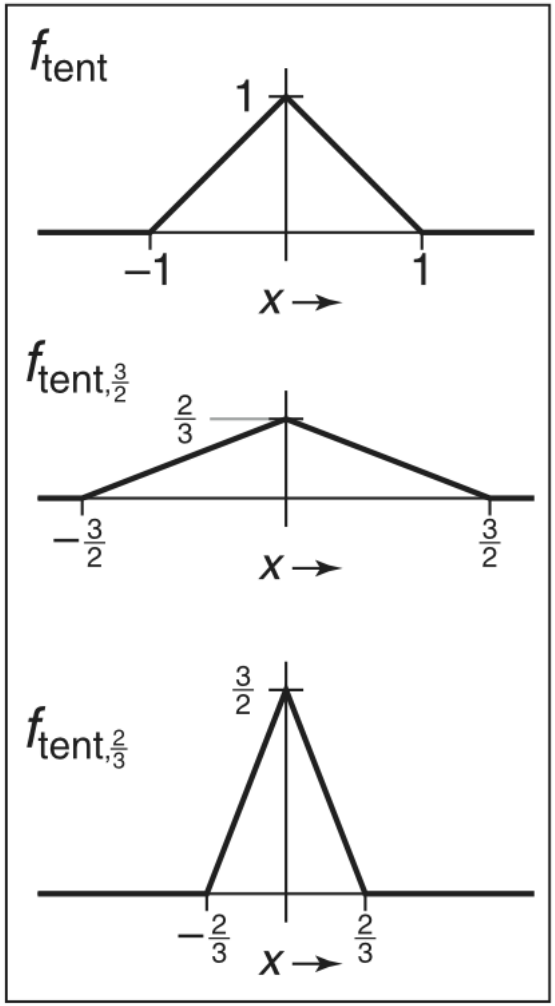


Slide credit: S. Marschner

Tent filter

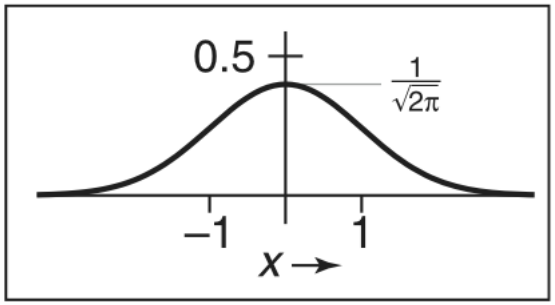
$$f_{\text{tent}}(x) = \begin{cases} 1 - |x| & |x| < 1, \\ 0 & \text{otherwise;} \end{cases}$$

$$f_{\text{tent},r}(x) = \frac{f_{\text{tent}}(x/r)}{r}.$$



Slide credit: S. Marschner

Gaussian filter



$$f_g(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Slide credit: S. Marschner

Discrete filtering in 2D

- Same equation, one more index

$$(a \star b)[i,j] = \sum_{i',j'} a[i',j']b[i-i',j-j']$$

– now the filter is a rectangle you slide around over a grid of numbers

- Usefulness of associativity

– often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$

– this is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$

Slide credit: S. Marschner

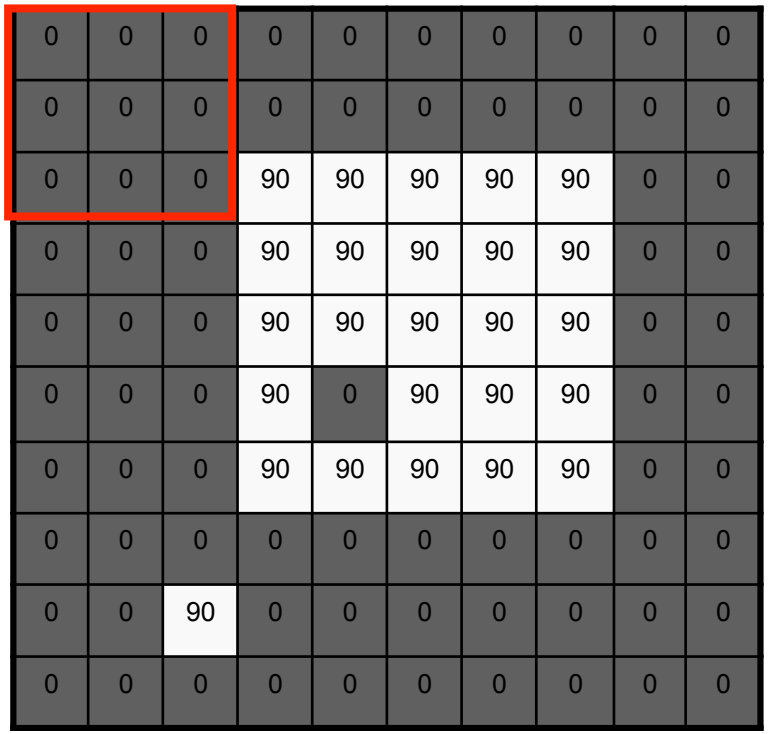
And in pseudocode…

function convolve2d(filter2d a, filter2d b, int i, int j)
$s = 0$
$r = a$.radius
for $i' = -r$ to r **do**
  for $j' = -r$ to r **do**
    $s = s + a[i'][j']b[i - i'][j - j']$
return s

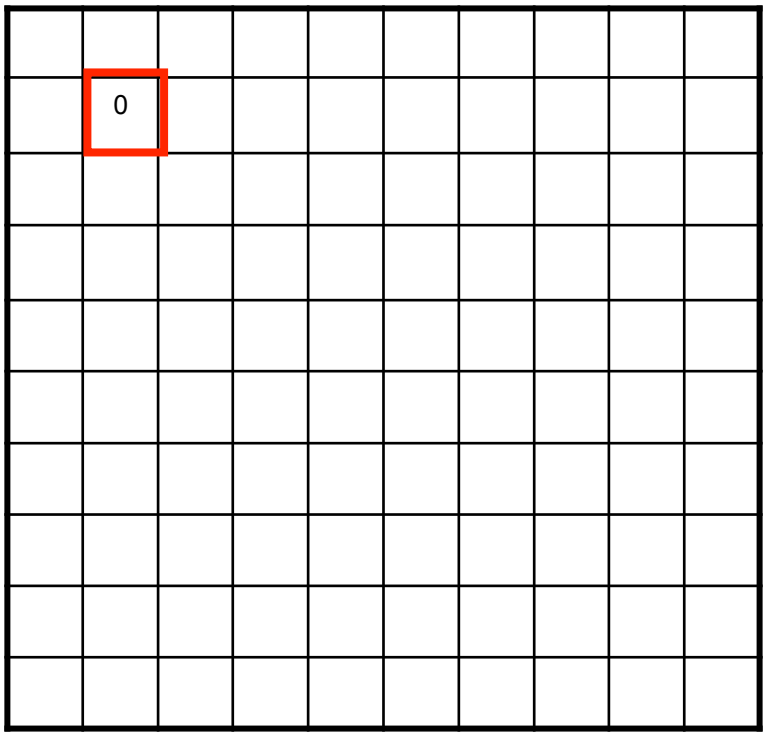
Slide credit: S. Marschner

Moving Average In 2D

$F[x, y]$



$G[x, y]$



Slide credit: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10							

Slide credit: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20						

Slide credit: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30					

Slide credit: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30				

Slide credit: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Slide credit: S. Seitz

Image Correlation Filtering

图像相关过滤

- Center filter *g* at each pixel in image *f*
- Multiply weights by corresponding pixels
- Set resulting value in output image *h*
- *g* is called a ***filter***, ***mask***, ***kernel***, or ***template***
- Linear filtering is sum of dot product at each pixel position
- Filtering operation called ***cross-correlation***

Slide credit: C. Dyer

Correlation filtering

1. 如果滤波器对称，即h(u,v)=h(-u,-v)，则卷积滤波和相关滤波等价
2. 将相关滤波器h(u,v)按中心逆时针旋转180度再作卷积，和相关滤波具有等价效果
3. 使用卷积是为了应用有关频域处理的卷积定理（第四章）

Say the averaging window size is 2k+1 x 2k+1:

$$G[i,j] = \underbrace{\frac{1}{(2k+1)^2}}_{\text{Attribute uniform weight to each pixel}} \underbrace{\sum_{u=-k}^{k}\sum_{v=-k}^{k} F[i+u, j+v]}_{\text{Loop over all pixels in neighborhood around image pixel F[i,j]}}$$

Now generalize to allow different weights depending on neighboring pixel's relative position:

对比：1）卷积：H(u,v)->F(i-u,j-v)

2）相关：H(u,v)->F(i+u,j+v)

$$G[i,j] = \sum_{u=-k}^{k}\sum_{v=-k}^{k} \underbrace{H[u,v]}_{\text{Non-uniform weights}} F[i+u, j+v]$$

Slide credit: K. Grauman

Correlation filtering

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

This is called cross-correlation, denoted $G = H \otimes F$

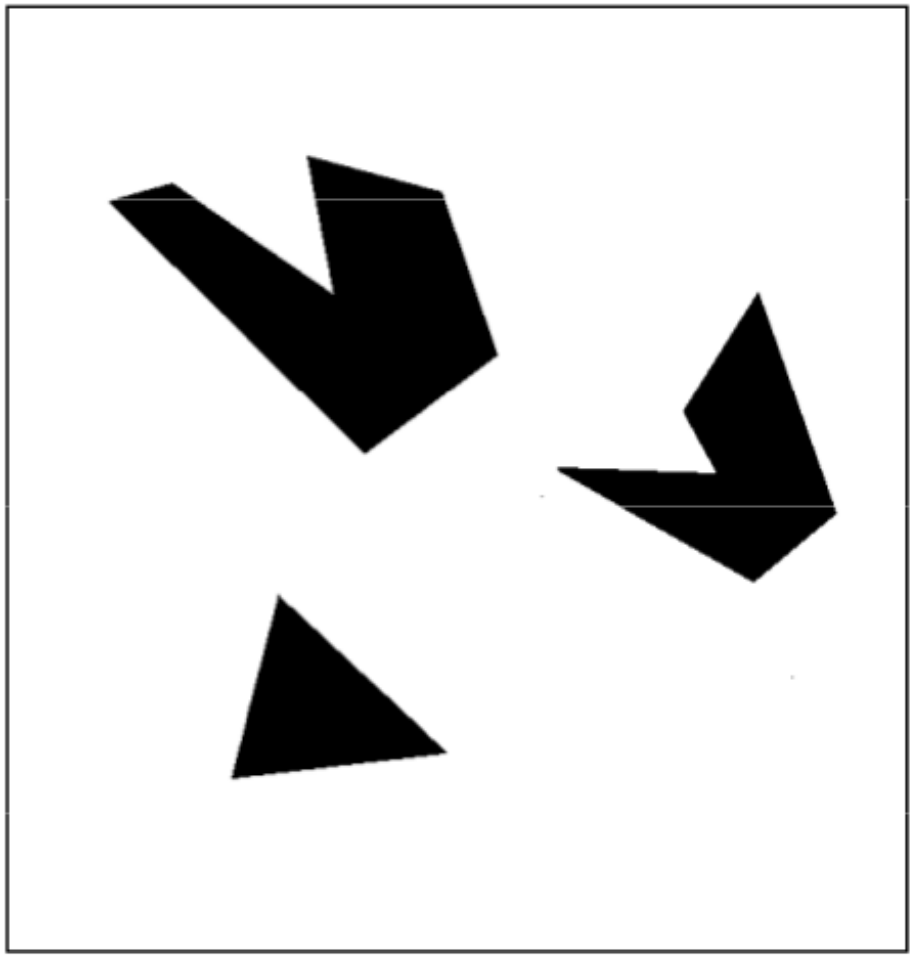
互相关

Filtering an image: replace each pixel with a linear combination of its neighbors.

The filter "kernel" or "mask" *H*[*u*,*v*] is the prescription for the weights in the linear combination.

Slide credit: K. Grauman

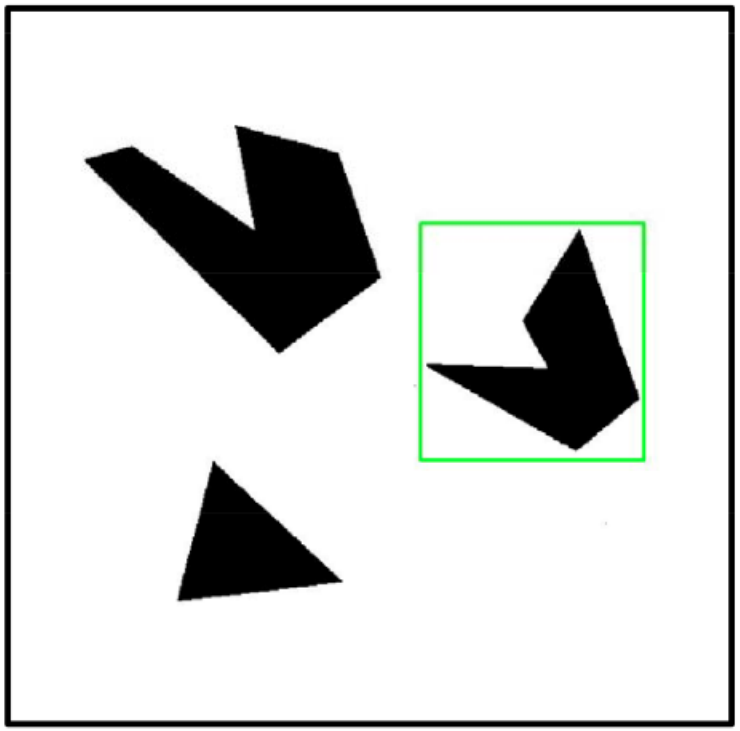
Correlation filtering



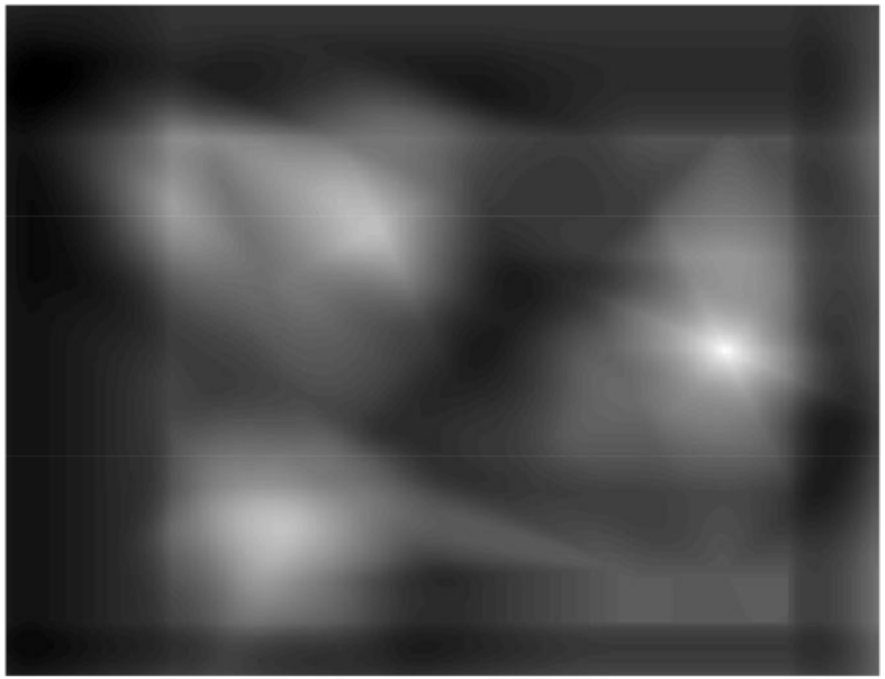
Scene

Template (mask)

Correlation filtering



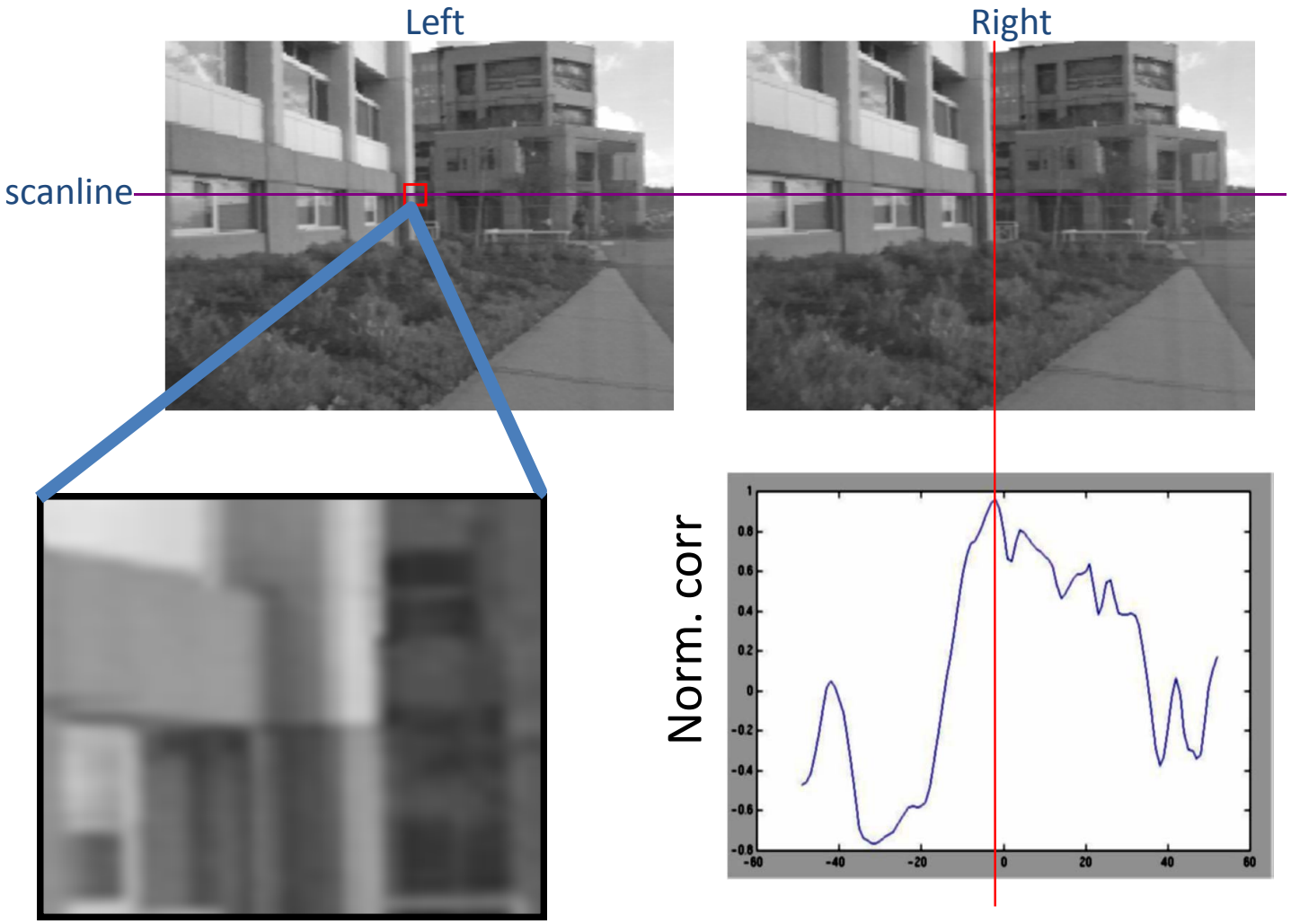
Detected template



Correlation map

可用于检测和定位一个由kernel表示的pattern，被检测pattern处响应强

Cross correlation example



Slide credit: Fei-Fei Li

有关模板匹配（1）

可以直接使用 $G(i,j)=\sum_{u=-k}^{k}\sum_{v=-l}^{l}H(u,v)F(i+u,j+v)$ 吗？

考虑下图在车图中匹配车轮的例子，如果车轮匹配到一个各像素灰度值都较大的区域，可能会有较大的相应输出，得到错误结果。

有关模板匹配（2）

相关匹配公式需要改造，可以使用教材中的相关计算，或

$$G(i,j)=\frac{\sum_{u=-k}^{k}\sum_{v=-l}^{l}H(u,v)F(i+u,j+v)}{\sum_{u=-k}^{k}\sum_{v=-l}^{l}F^{2}(i-u,j-v)}$$

其中 $H(u,v)$ 为匹配模板，而分母为图像内容　要对匹配的部分作归一化

Averaging filter

- What values belong in the kernel *H* for the moving average example?

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$\otimes$

$H[u, v]$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & ? & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

"box filter"

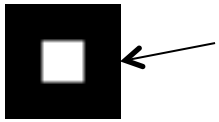
$G[x, y]$

	0	10	20	30	30				

$$G = H \otimes F$$

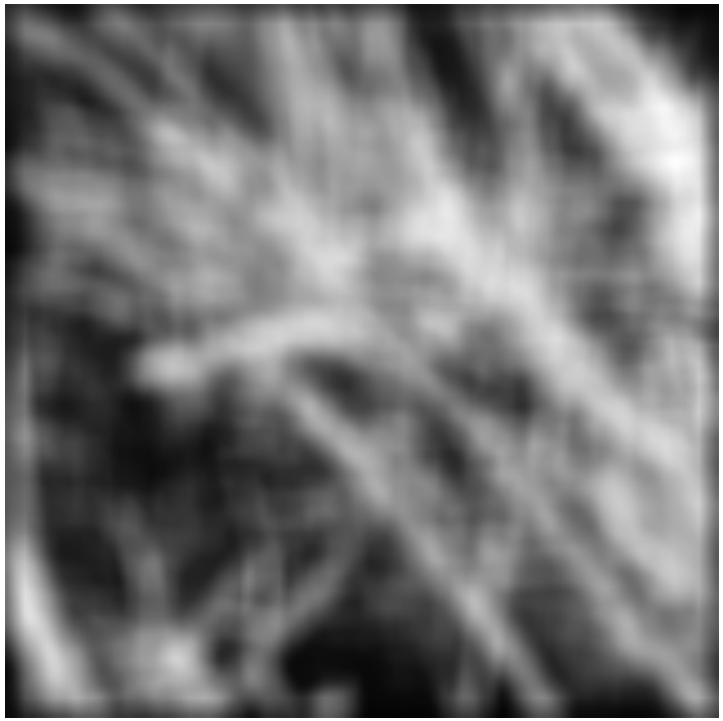
Slide credit: K. Grauman

Smoothing by averaging



depicts box filter:
white = high value, black = low value

original



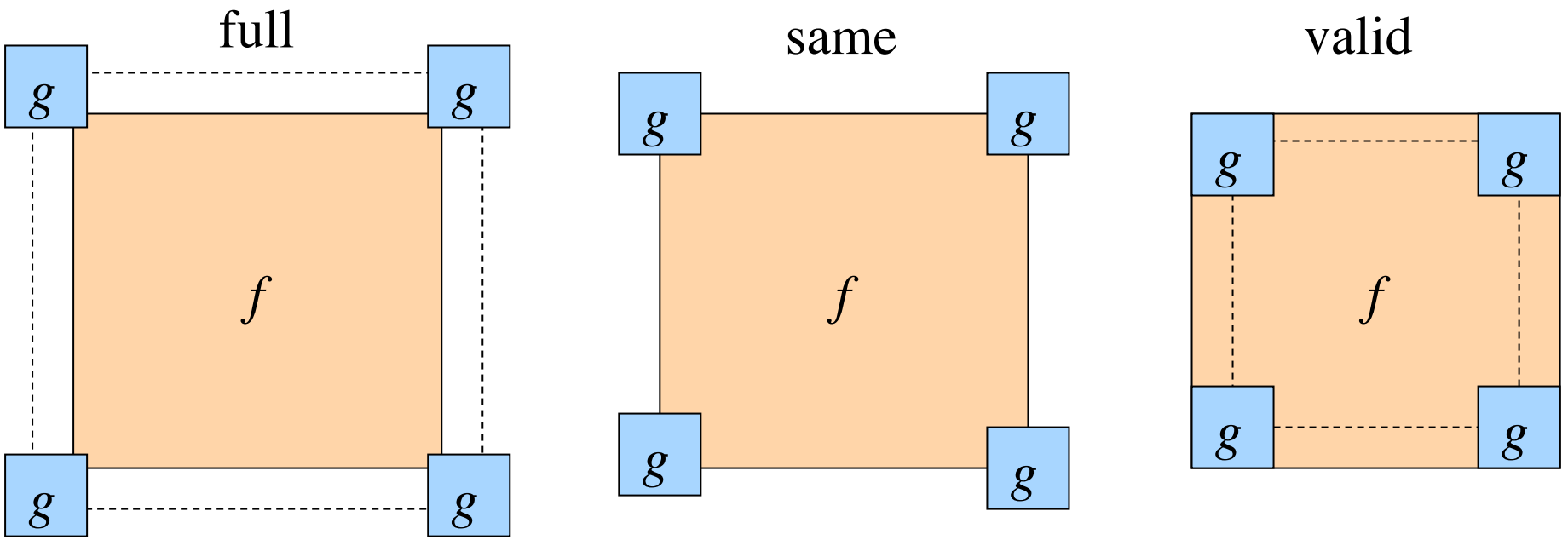
filtered

What if the filter size was 5 x 5 instead of 3 x 3?

Slide credit: K. Grauman

Boundary issues

- What is the size of the output?
- MATLAB: output size / "shape" options
  - *shape* = 'full': output size is sum of sizes of f and g
  - *shape* = 'same': output size is same as f
  - *shape* = 'valid': output size is difference of sizes of f and g



Slide credit: S. Lazebnik

Boundary issues

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge

Slide credit: S. Marschner

Boundary issues

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods (MATLAB):
    - clip filter (black): `imfilter(f, g, 0)`
    - wrap around: `imfilter(f, g, 'circular')`
    - copy edge: `imfilter(f, g, 'replicate')`
    - reflect across edge: `imfilter(f, g, 'symmetric')`

Slide credit: S. Marschner

规则：1）低通滤波器 - Σweights = 1
2）高通滤波器 - Σweights = 0

Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

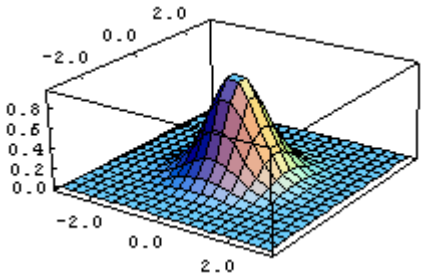
$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

$H[u, v]$

This kernel is an approximation of a 2d Gaussian function:

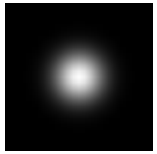
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

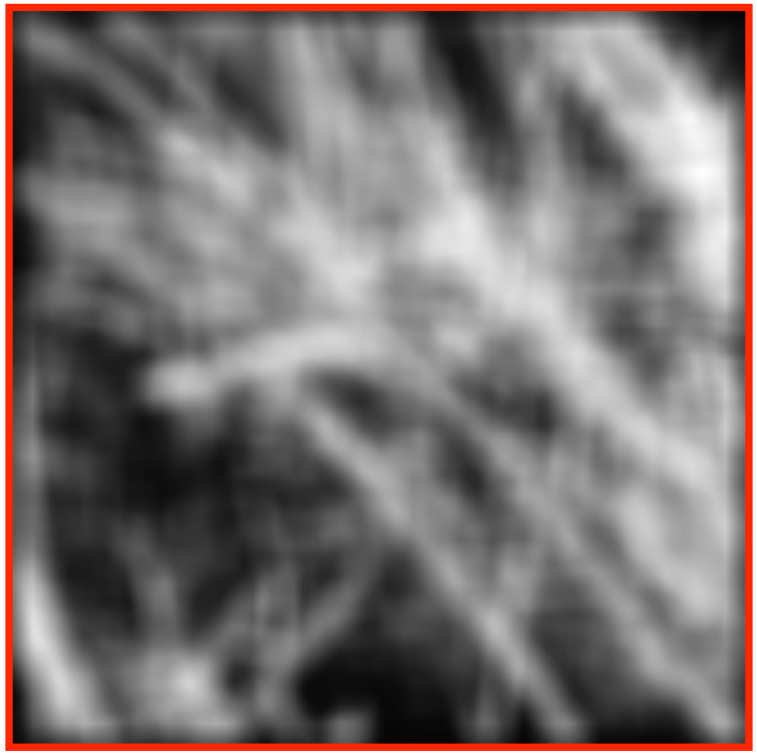


- Removes high-frequency components from the image ("low-pass filter").

Slide credit: S. Seitz

Smoothing with a Gaussian

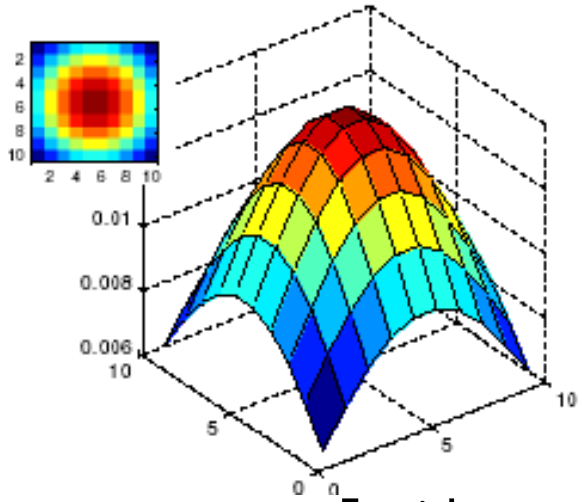




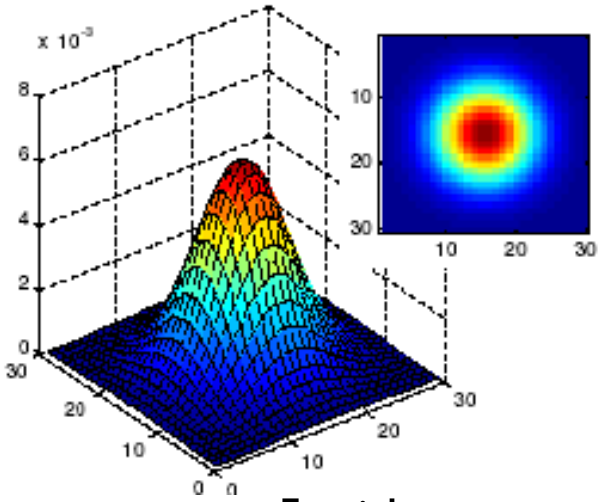
Slide credit: K. Grauman

Gaussian filters

- What parameters matter here?
- **Size** of kernel or mask
  - Note, Gaussian function has infinite support, but discrete filters use finite kernels



σ = 5 with
10 x 10 kernel

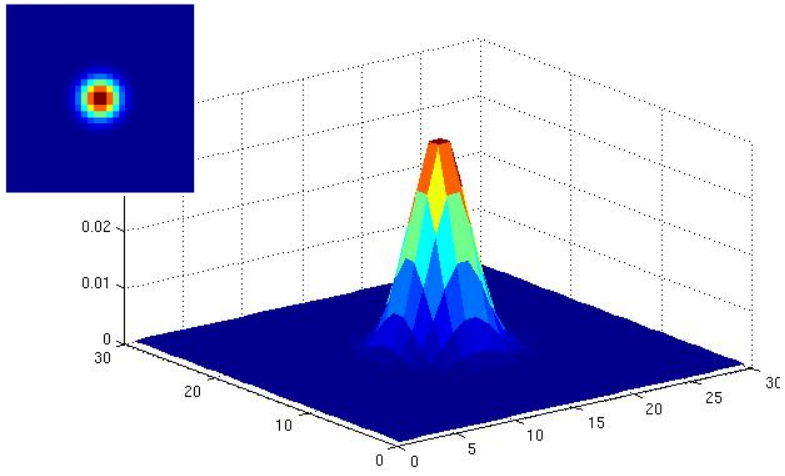


σ = 5 with
30 x 30 kernel

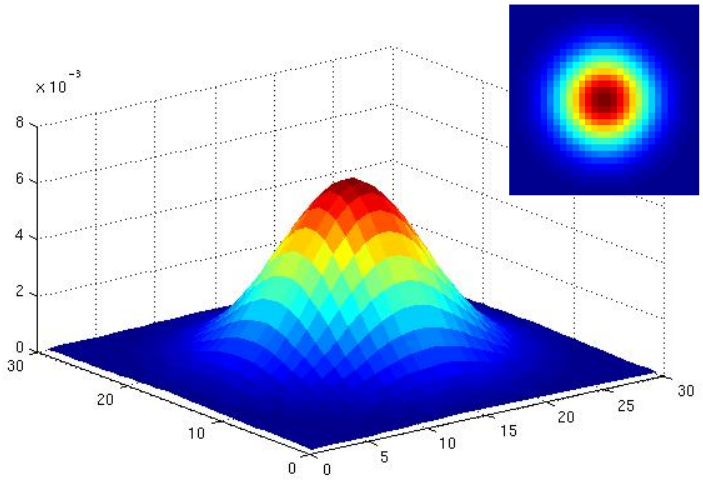
Slide credit: K. Grauman

Gaussian filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing



σ = 2 with
30 x 30 kernel



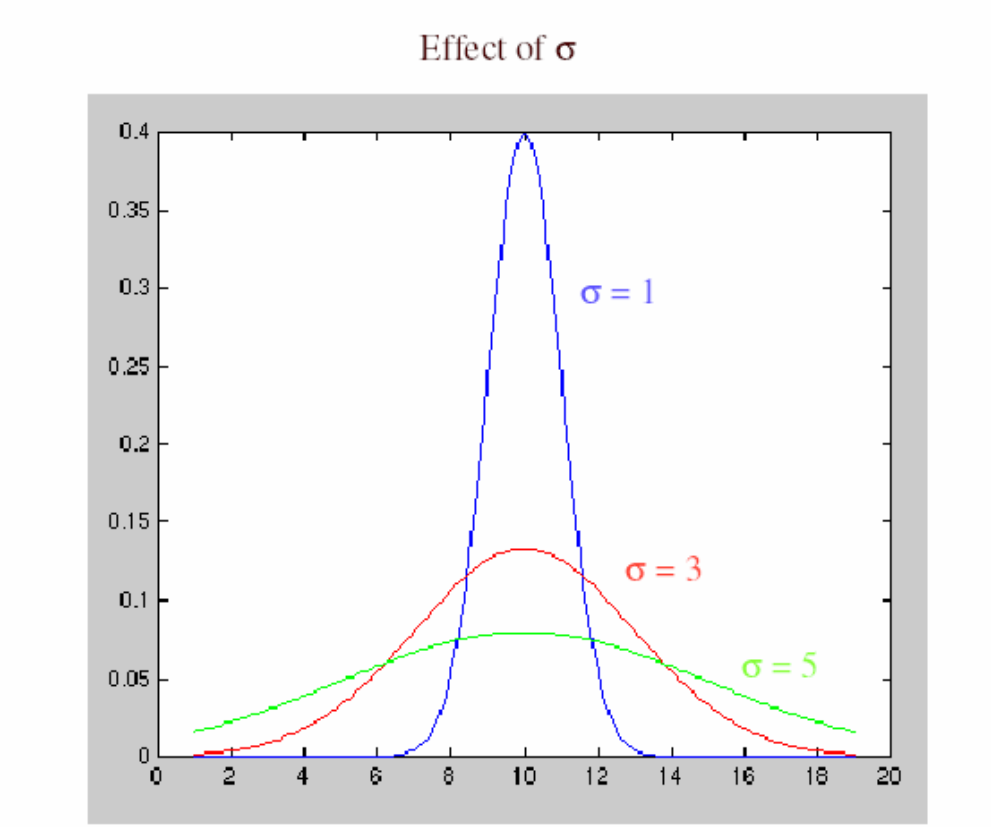
σ = 5 with
30 x 30 kernel

Slide credit: K. Grauman

Choosing kernel width

- Rule of thumb: set filter half-width to about 3σ

经验公式

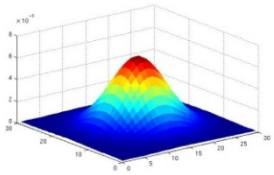


Slide credit: S. Lazebnik

Matlab

```
>> hsize = 10;
>> sigma = 5;
>> h = fspecial('gaussian' hsize, sigma);
```

```
>> mesh(h);
```



```
>> imagesc(h);
```

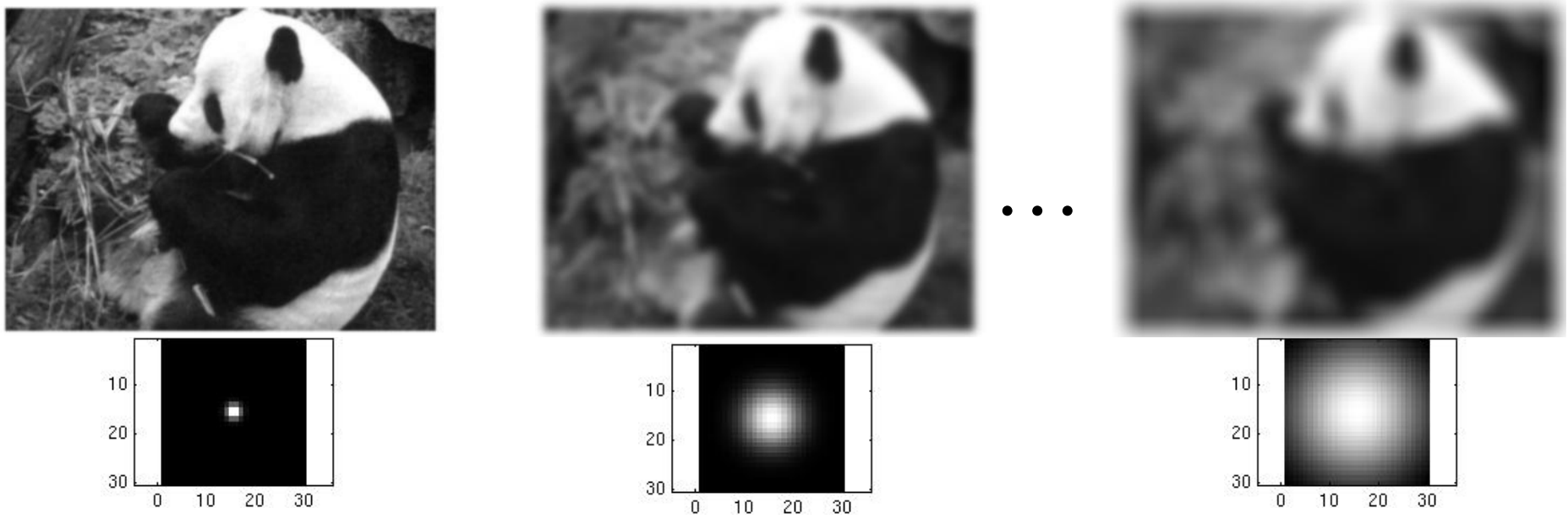
```
>> outim = imfilter(im, h); % correlation
>> imshow(outim);
```

outim

Slide credit: K. Grauman

Smoothing with a Gaussian

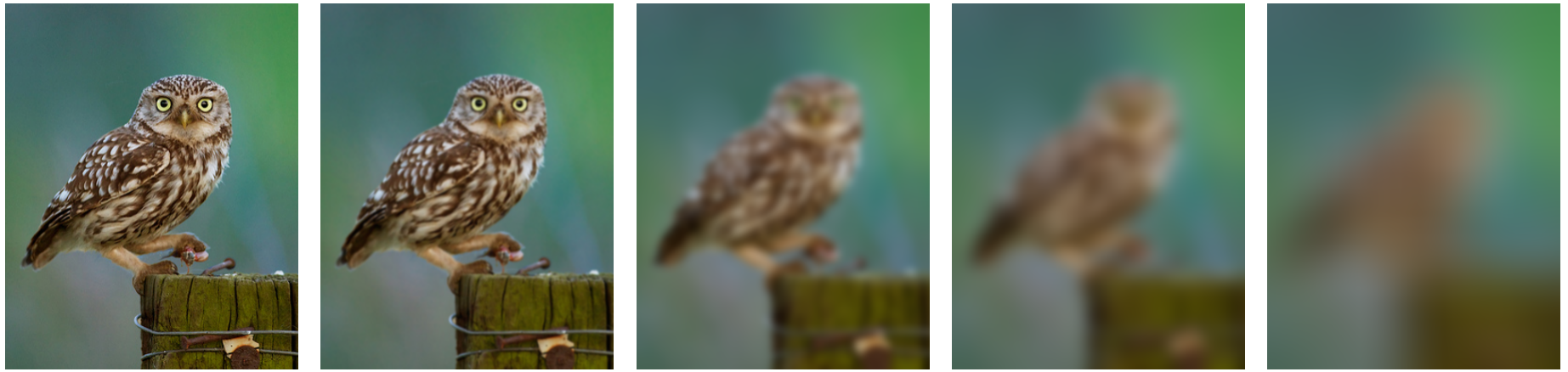
Parameter σ is the "scale" / "width" / "spread" of the Gaussian kernel, and controls the amount of smoothing.




```
for sigma=1:3:10
   h = fspecial('gaussian', fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

Slide credit: K. Grauman

Gaussian Filters



σ = 1 pixel σ = 5 pixels σ = 10 pixels σ = 30 pixels

Slide credit: C. Dyer

Spatial Resolution and Color

original

R

G

B

Slide credit: C. Dyer

Blurring the G Component （对G影响较大）

original

processed

R

G

B

Slide credit: C. Dyer

Blurring the R Component

original

processed

R

G

B

Slide credit: C. Dyer

Blurring the B Component

original

processed

R

G

B

Slide credit: C. Dyer

“Lab” Color Representation

L

a

b

A transformation of the colors into a color space that is more perceptually meaningful:
L: luminance,
a: red-green,
b: blue-yellow

Slide credit: C. Dyer

Blurring L

original

processed

L

a

b

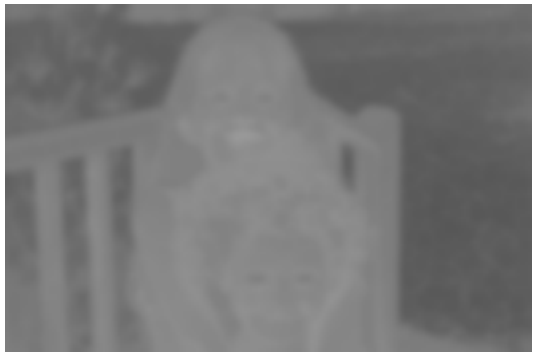
Slide credit: C. Dyer

Blurring a

original

processed

L



a

b

Slide credit: C. Dyer

Blurring b

original

processed

L

a

b

Slide credit: C. Dyer

Separability

滤波器可分离的条件：
对于kernel h(u,v)，有：
$h(u,v) = h(u)h(v)$

- In some cases, filter is separable, and we can factor into two steps:
  - Convolve all rows 用h(v)
  - Convolve all columns 用h(u)

Slide credit: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Slide credit: D. Lowe

Separability example

2D convolution (center location only)

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix}$$

The filter factors into a product of 1D filters:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Perform convolution along rows:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix} = \begin{bmatrix} & 11 & \\ & 18 & \\ & 18 & \end{bmatrix}$$

Followed by convolution along the remaining column:

Slide credit: K. Grauman

Why is separability useful?

- What is the complexity of filtering an $n \times n$ image with an $m \times m$ kernel?
  - $O(n^2 m^2)$
- What if the kernel is separable?
  - $O(n^2 m)$

Slide credit: S. Lazebnik

Properties of smoothing filters

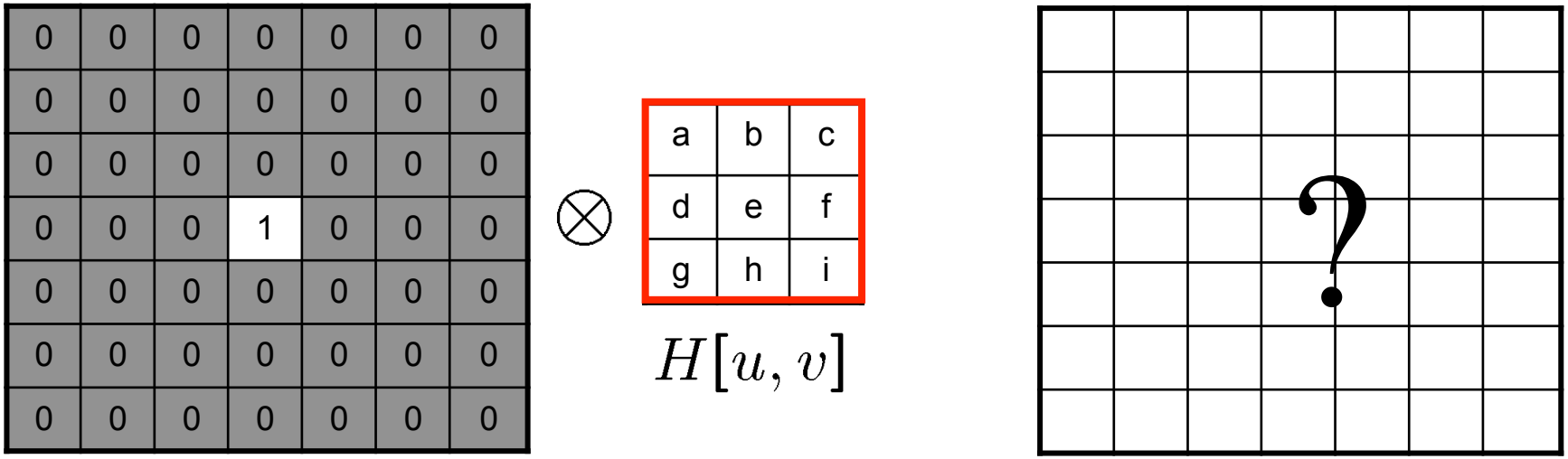
- Smoothing
  - Values positive
  - Sum to 1 → constant regions same as input
  - Amount of smoothing proportional to mask size
  - Remove "high-frequency" components; "low-pass" filter

Slide credit: K. Grauman

Filtering an impulse signal

对于脉冲图像F：
1. 和H相关滤波—》H旋转180度
2. 和H卷积滤波—》H本身

What is the result of filtering the impulse signal (image) *F* with the arbitrary kernel *H*?



Slide credit: K. Grauman

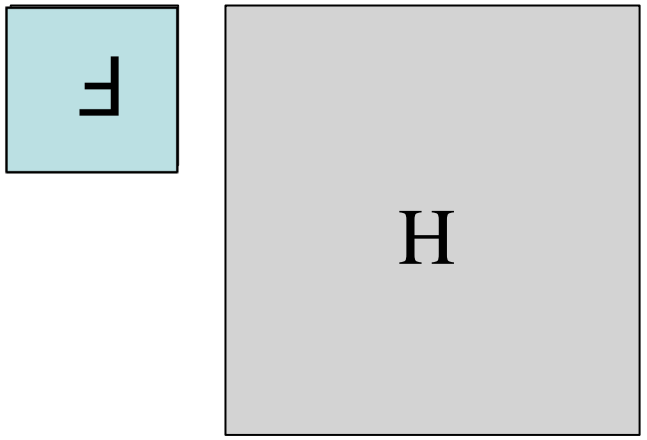
Convolution

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left)
  - Then apply cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H \star F$$

Notation for convolution operator



Slide credit: K. Grauman

Convolution vs. Correlation

- A **convolution** is an integral that expresses the amount of overlap of one function as it is shifted over another function.
  - convolution is a filtering operation
- **Correlation** compares the ***similarity*** *of* ***two*** *sets of* ***data.*** Correlation computes a measure of similarity of two input signals as they are shifted by one another. The correlation result reaches a maximum at the time when the two signals match best .
  - correlation is a measure of relatedness of two signals

Slide credit: Fei-Fei Li

Convolution vs. correlation

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H \star F$$

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

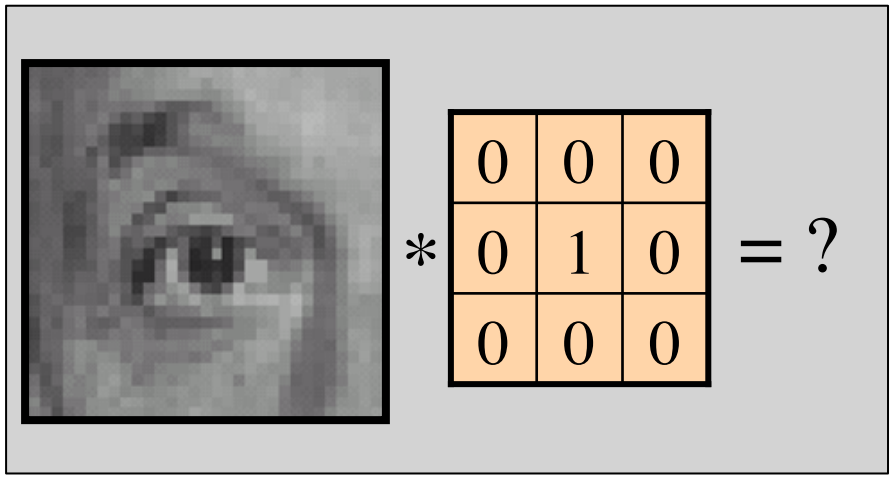
$$G = H \otimes F$$

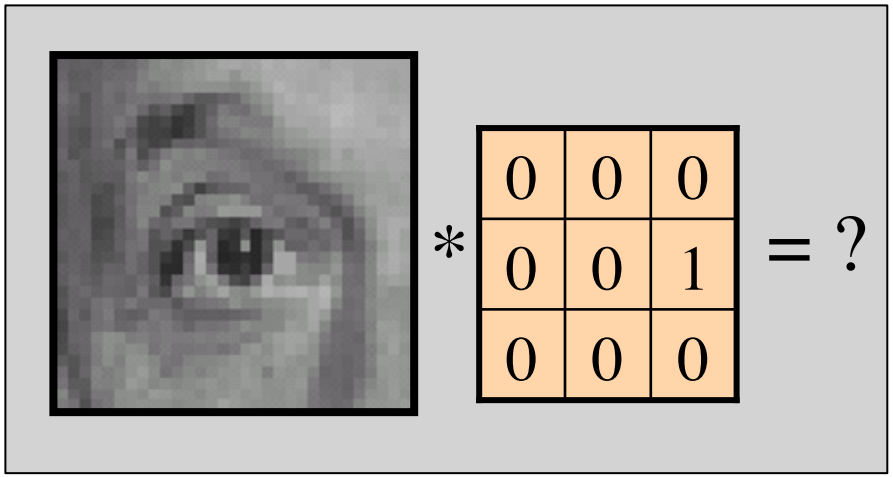
For a Gaussian or box filter, how will the outputs differ?

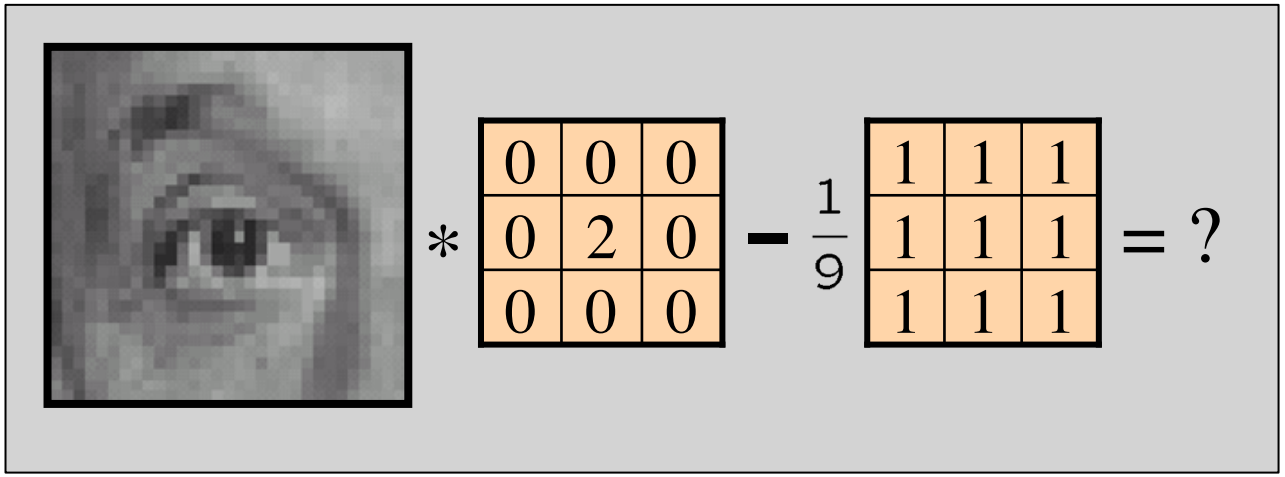
If the input is an impulse signal, how will the outputs differ?

Slide credit: K. Grauman

Predict the outputs using correlation filtering

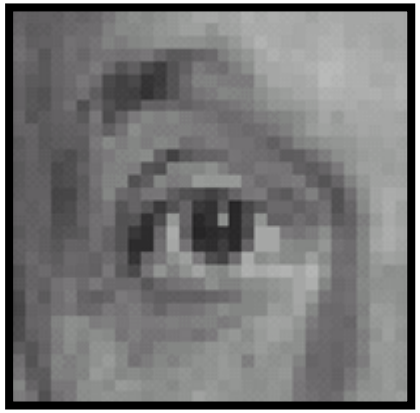






Slide credit: K. Grauman

Practice with linear filters



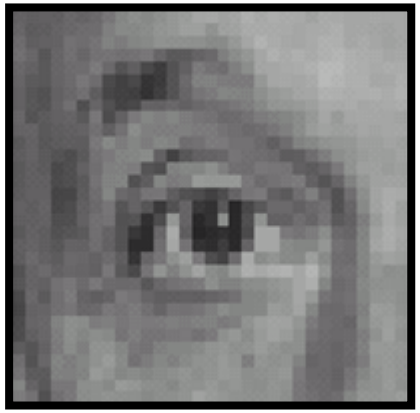
Original

0	0	0
0	1	0
0	0	0

?

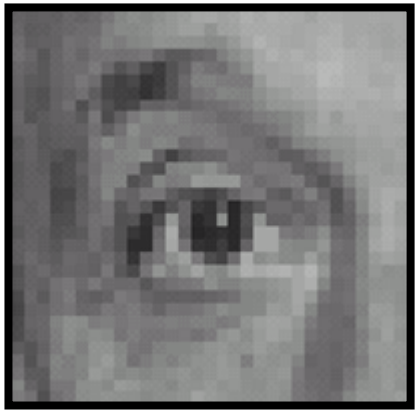
Slide credit: D. Lowe

Practice with linear filters



Original

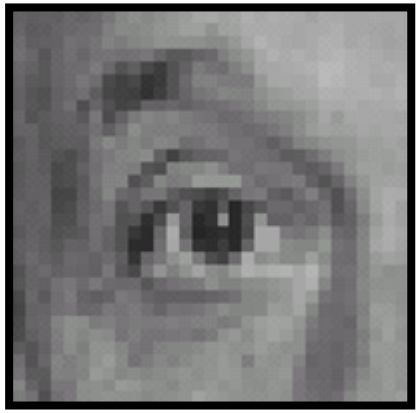
0	0	0
0	1	0
0	0	0



Filtered
(no change)

Slide credit: D. Lowe

Practice with linear filters



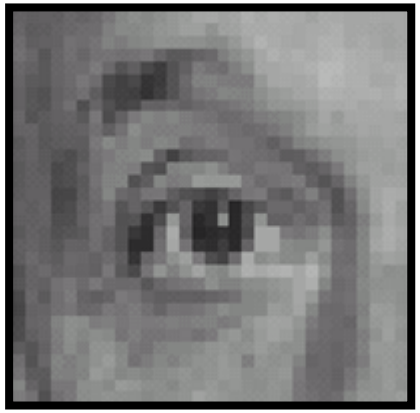
Original

0	0	0
0	0	1
0	0	0

?

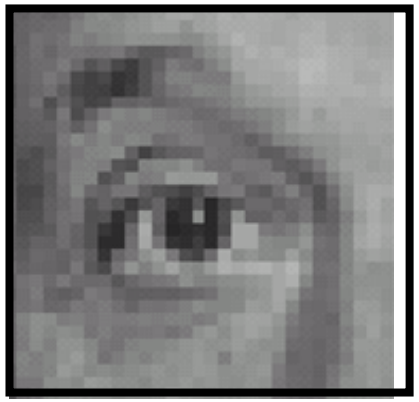
Slide credit: D. Lowe

Practice with linear filters



Original

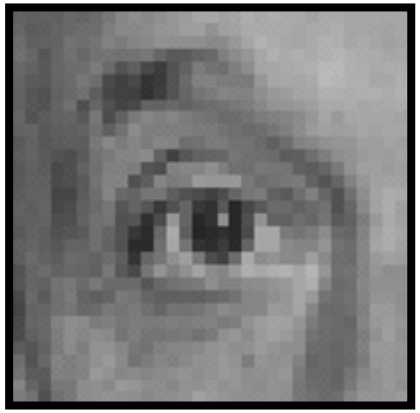
0	0	0
0	0	1
0	0	0



Shifted left
by 1 pixel with
correlation

Slide credit: D. Lowe

Practice with linear filters



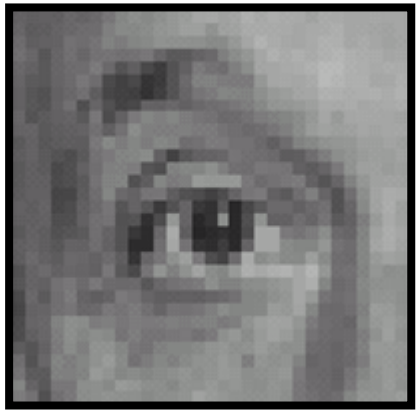
Original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

?

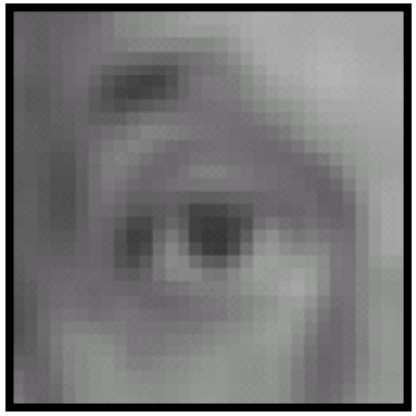
Slide credit: D. Lowe

Practice with linear filters



Original

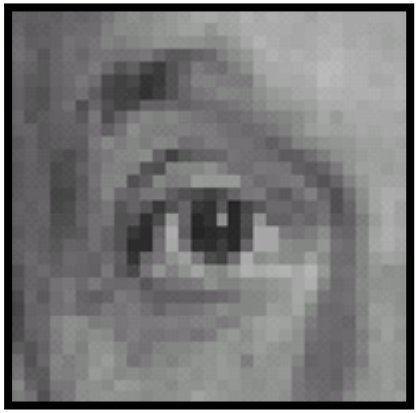
$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



Blur (with a box filter)

Slide credit: D. Lowe

Practice with linear filters



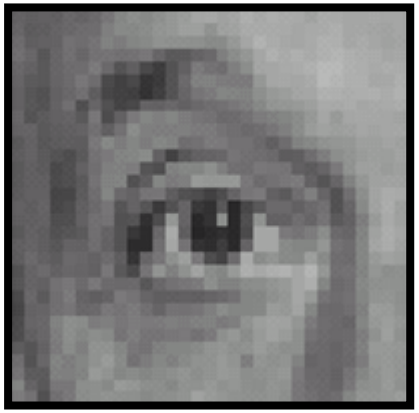
Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad ?$$

Slide credit: D. Lowe

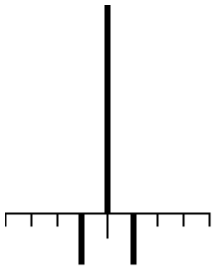
Practice with linear filters

original - smoothed = detail
original + detail = sharping
-> 2*original - smoothed



Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

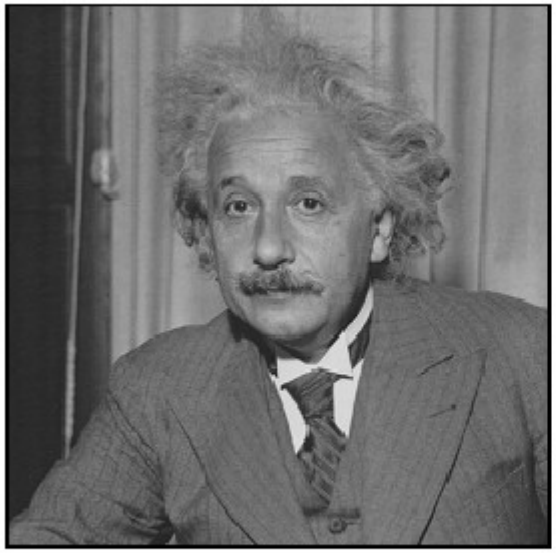


Sharpening filter:
accentuates differences with local average

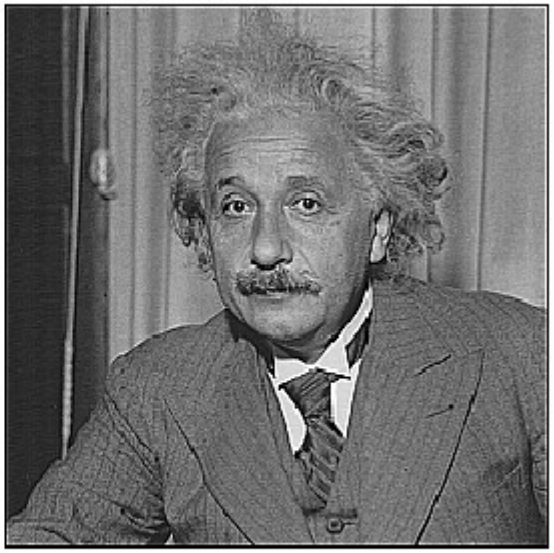
original + original -smoothed
= original + detail
= sharping

Slide credit: D. Lowe

Filtering examples: sharpening



before



after

Slide credit: K. Grauman

Sharpening

- What does blurring take away?

−

=

Let's add it back:

+

=

Slide credit: S. Lazebnik

Unsharp mask filter

滤波器核为一个冲激I减去一个平滑滤波器g，例如高斯，效果是一个Laplacian of Gaussian

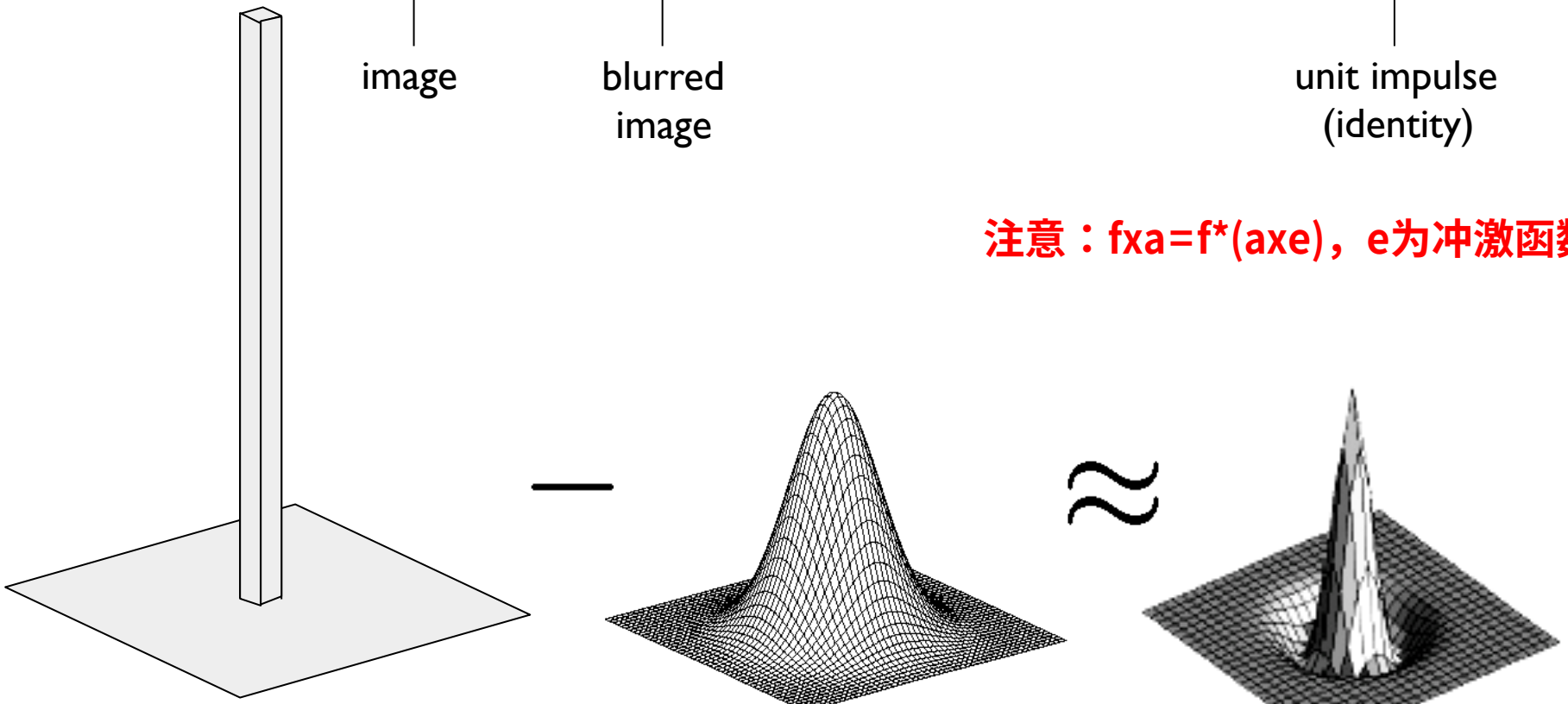
$$f + \alpha(f - f * g) = (1+\alpha)f - \alpha f * g = f * ((1+\alpha)e - g)$$

image

blurred image

unit impulse (identity)

注意：fxa=f*(axe)，e为冲激函数



unit impulse

Gaussian

Laplacian of Gaussian

Slide credit: S. Lazebnik

Sharpening using Unsharp Mask Filter

Original

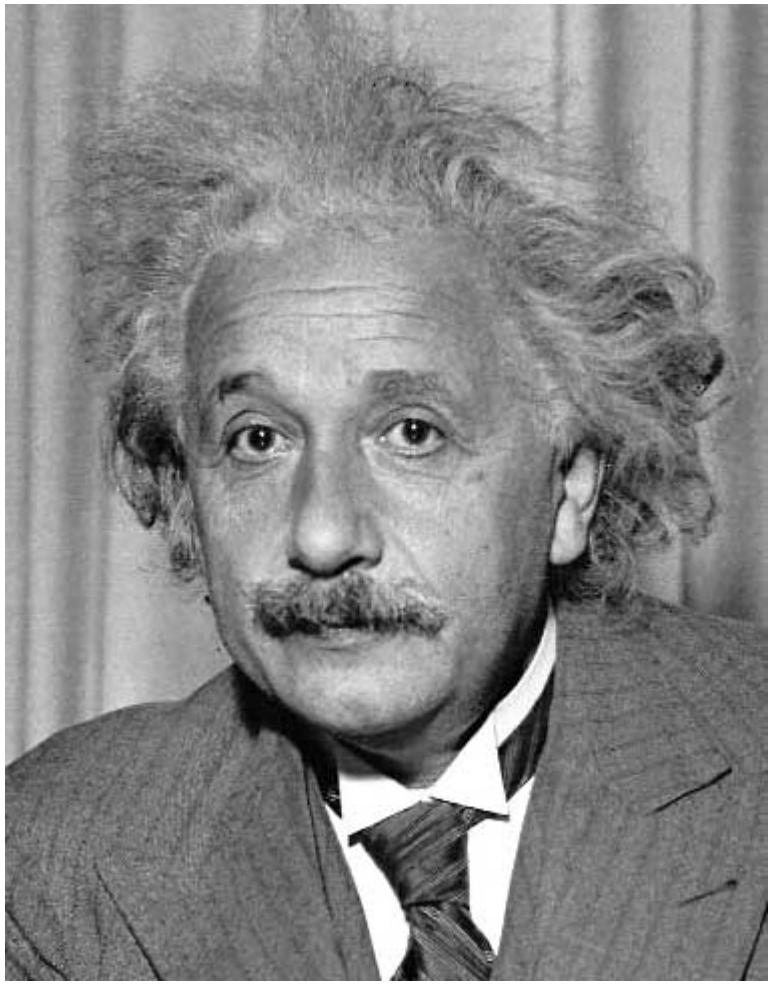
Filtered result

Slide credit: C. Dyer

Unsharp Masking

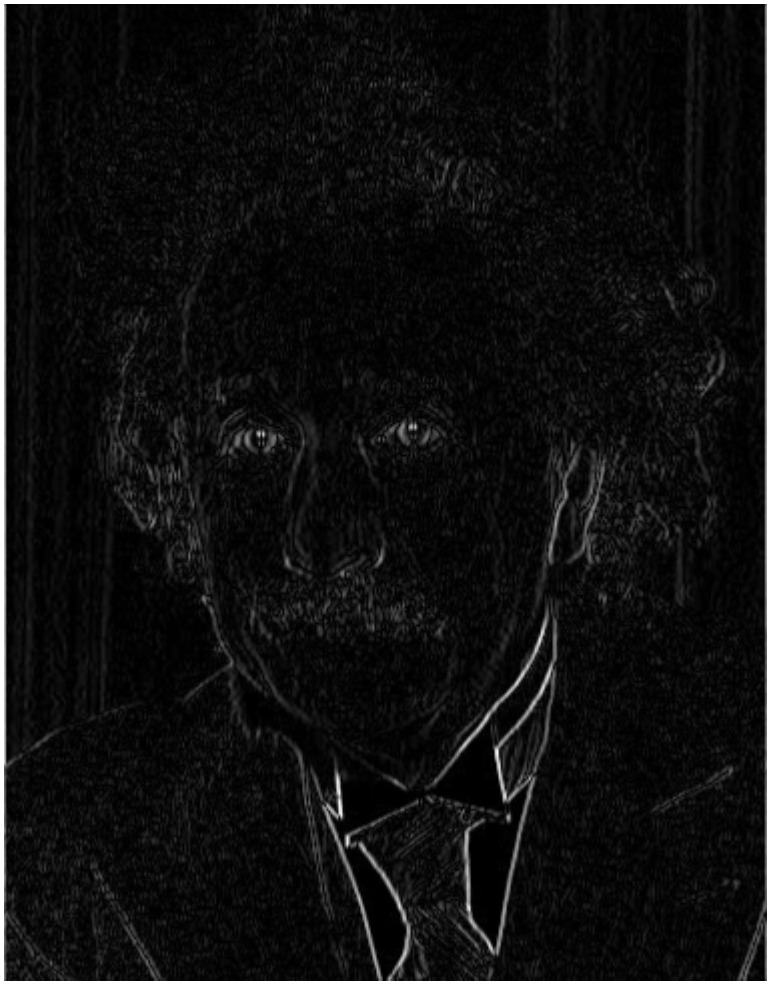
Slide credit: C. Dyer

Other filters



1	0	-1
2	0	-2
1	0	-1

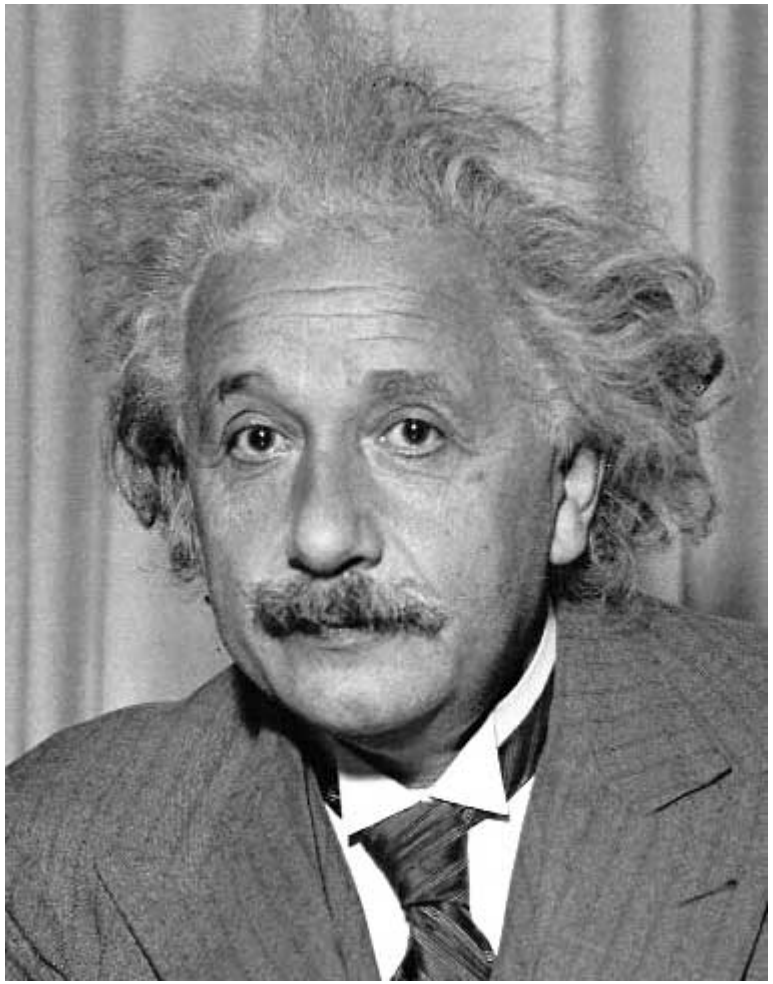
Sobel



Vertical Edge
(absolute value)

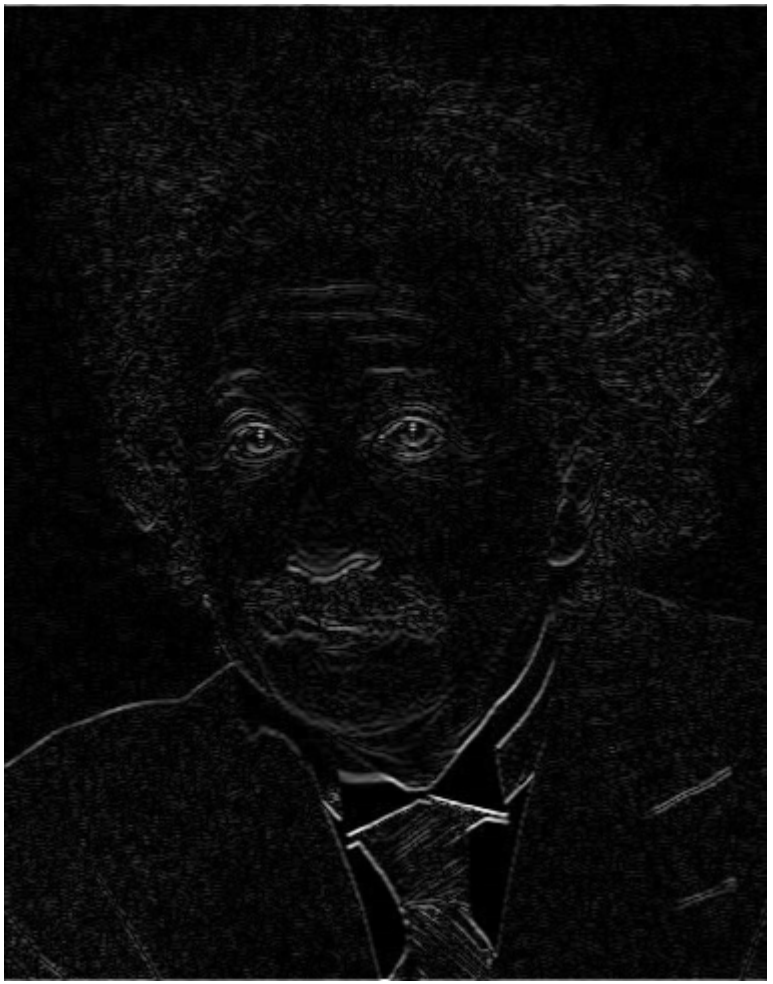
Slide credit: J. Hays

Other filters



1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

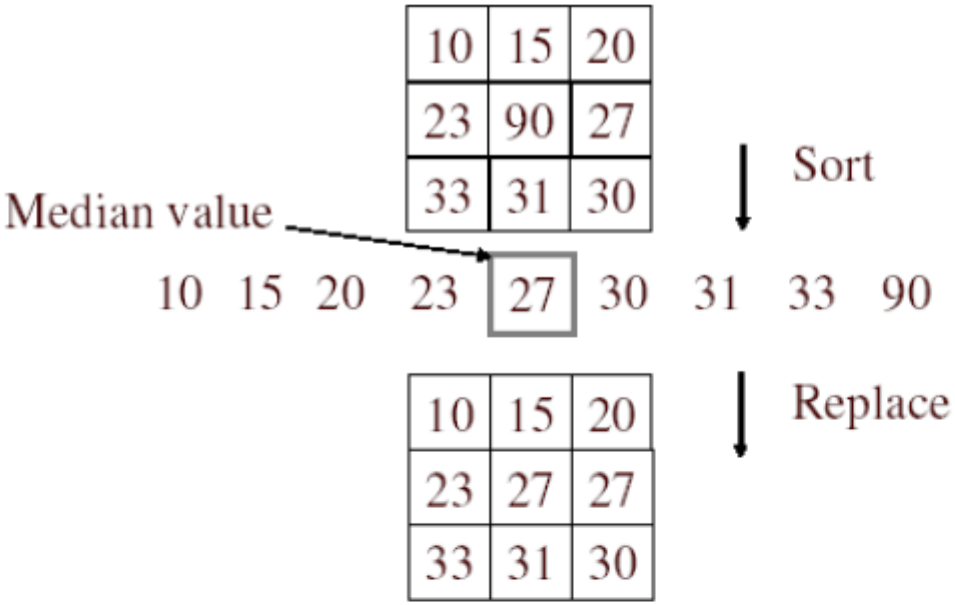
Slide credit: J. Hays

Median filters

- A **Median Filter** operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

adapted from: S. Seitz

Median filter

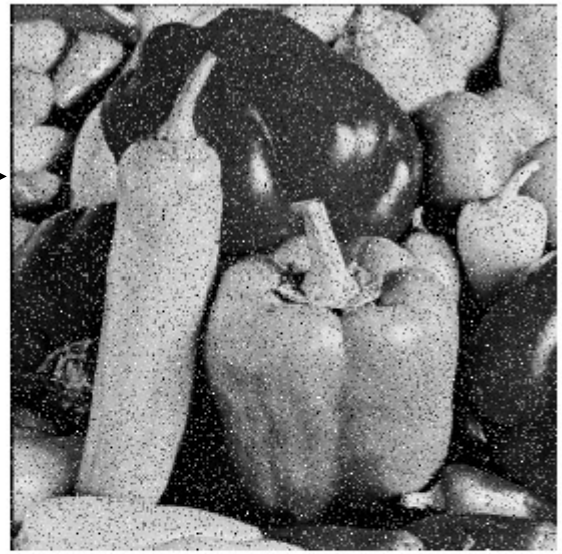


- No new pixel values introduced
- Removes spikes: good for impulse, salt & pepper noise
- Non-linear filter

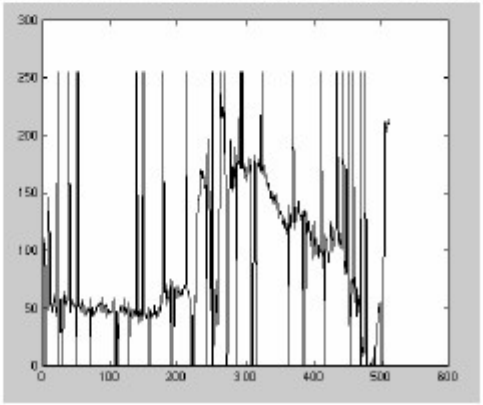
Slide credit: K. Grauman

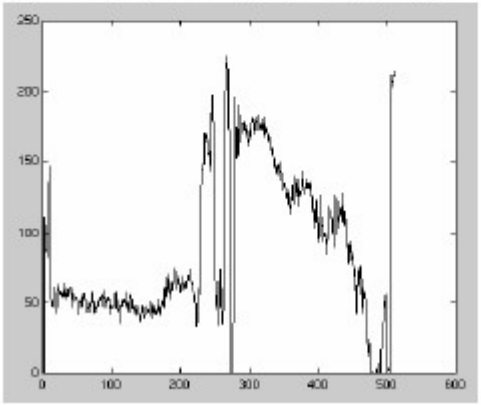
Median filter

Salt and pepper noise



Median filtered





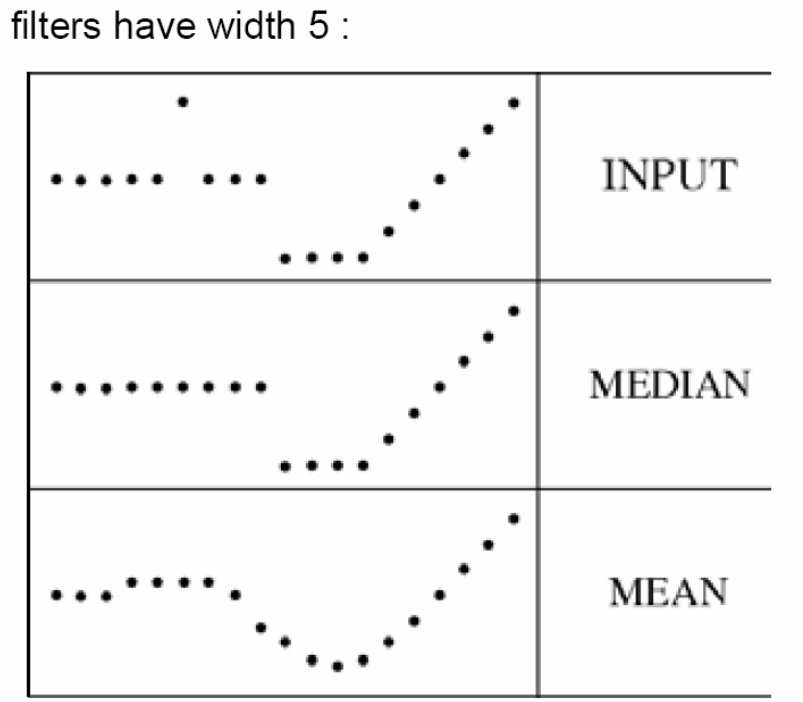
Plots of a row of the image

Matlab: `output im = medfilt2(im, [h w]);`

Slide credit: M. Hebert

Median filter

- What advantage does median filtering have over Gaussian filtering?
  - Robustness to outliers
  - Median filter is edge preserving



Slide credit: K. Grauman