

Week 8: Generative Adversarial Networks

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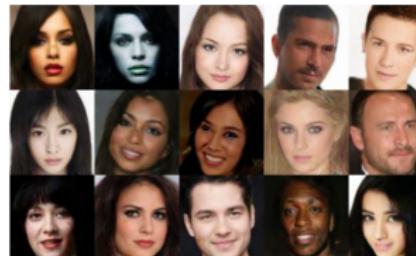
18 April, 2019

1 Generative adversarial network

2 Wasserstein GANs

Generative model

- Objective: generate new data by learning from training data
- Various applications: realistic new designs, super-resolution, colorization, etc.



- Data augmentation: more realistic data for model training
- Data mining: exploring latent structure/representation of data

Figures here and in the next 7 slides from Stanford CS231n 2017 Lecture 13;
also see Goodfellow et al., "Generative adversarial nets", NIPS, 2014

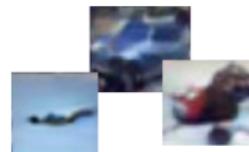
Generative model (cont')

- What to learn from training data?

One way is to learn data's density distribution from training data, then generate new data from the learned distribution.



Training data $\sim p_{\text{data}}(x)$



Generated samples $\sim p_{\text{model}}(x)$

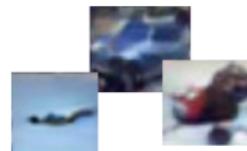
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Generated samples $\sim p_{\text{model}}(x)$

- Difficulty: never know real data complex distribution $P_{data}(x)$, therefore difficult to learn its approximation $P_{model}(x)$
 - Solution: learn to generate new data directly, without learning the data distribution!

Generative model (cont')

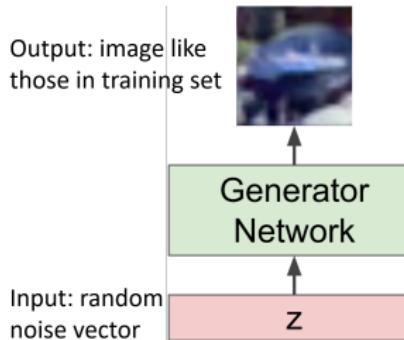
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- Idea: sample from simple distribution (e.g., of random noise), and learn a complex transformation from simple distribution to the (unknown) complex distribution of image data.

Generative model (cont')

- Problem: How to generate data without learning complex high-dim (image) data distribution?
 - Idea: sample from simple distribution (e.g., of random noise), and learn a complex transformation from simple distribution to the (unknown) complex distribution of image data.
 - If using a neural network to represent the transformation, the objective is to train the network to generate realistic images whose distribution is similar to that of training dataset.

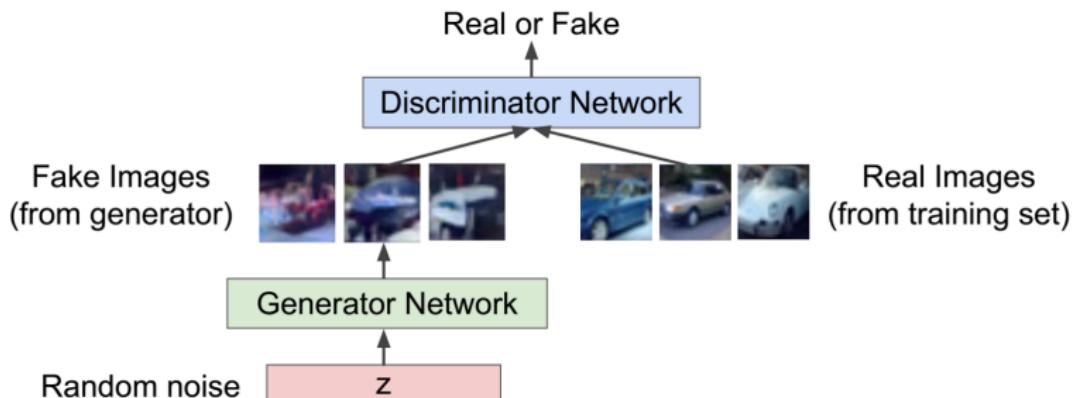


Generative adversarial network (GAN): a two-player game

- How to evaluate whether generated images have a similar distribution to the training set?

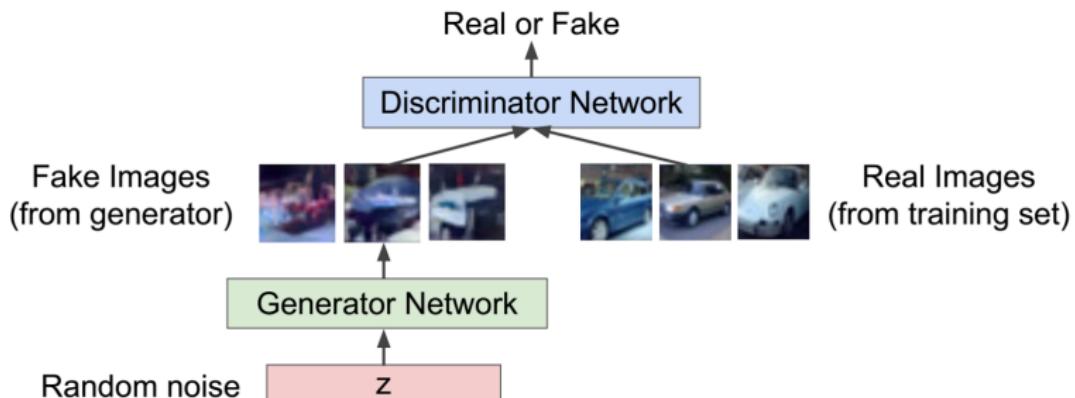
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 - Key insight: Use another network to help evaluate it!



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- G network: try to fool D by generating realistic images
 - D network: try to distinguish between real and fake images

GAN training

- The two objectives can be achieved by

$$\min_{G_\theta} \max_{D_w} \{ \mathbb{E}_{x \sim P_r} [\log D_w(x)] + \mathbb{E}_{z \sim P(z)} [\log(1 - D_w(G_\theta(z)))] \}$$

where discriminator network D_w is a binary classifier.

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Train the two networks alternatively:

- 1: Training D network, i.e. $\max_{D_w}\{\cdot\}$, makes $D_w(x)$ close to 1 for real images and $D_w(G(z))$ close to 0 for fake images $G(z)$

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- 2: Training G network, i.e. $\min_{G_\theta}\{\cdot\}$, makes $D_w(G(z))$ close to 1, fooling D into thinking $G(z)$ is real.

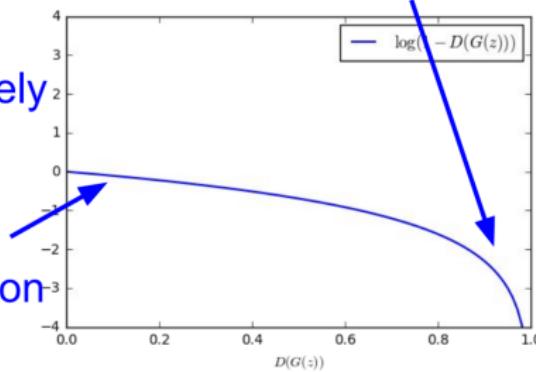
$$\min_{G_\theta} \{ \mathbb{E}_{z \sim P(z)} [\log(1 - D_w(G_\theta(z)))] \}$$

GAN training (cont')

- However, optimizing generator's objective does not work well!
- Figure: x-axis for $D(G(z))$ and y-axis for $\log(1 - D(G(z)))$

Gradient signal
dominated by region
where sample is
already good

When sample is likely fake, want to learn from it to improve generator. But gradient in this region is relatively flat!

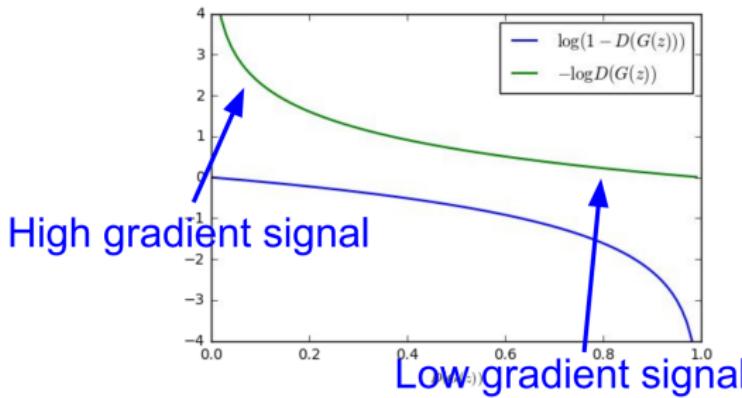


GAN training (cont')

- Instead, optimizing generator by

$$\min_{G_\theta} \{ \mathbb{E}_{z \sim P(z)} [-\log(D_w(G_\theta(z)))] \}$$

which also tries to fool discriminator, but now with higher gradient signal for poor generator.



GAN implementation

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(m)}\}$ from noise prior $p_g(\mathbf{z})$.
- Sample minibatch of m examples $\{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\mathbf{x})$.
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D_{\theta_d}(x^{(i)}) + \log(1 - D_{\theta_d}(G_{\theta_g}(z^{(i)}))) \right]$$

Update Discriminator

end for

- Sample minibatch of m noise samples $\{\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(m)}\}$ from noise prior $p_g(\mathbf{z})$.
- Update the generator by ascending its stochastic gradient (improved objective):

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log(D_{\theta_d}(G_{\theta_g}(z^{(i)})))$$

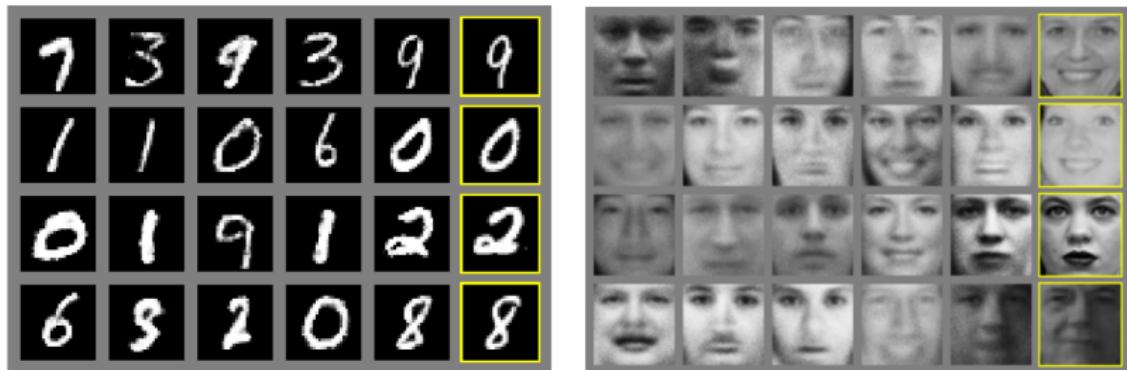
Update Generator

end for

- Note: $D_{\theta_d} = D_w$ and $G_{\theta_g} = G_{\theta}$

GAN result

- Generator (consisting of FC layers) generated realistic images
- Generator does not simply remember training images



- Yellow boxes: nearest training example of neighboring generated sample

Figure from Goodfellow et al., "Generative adversarial nets", NIPS, 2014

DCGAN: deep convolutional GAN

- Generator is a ‘deconvolutional’ network
- Discriminator is a convolutional network
- It is not that easy to make DCGAN work!

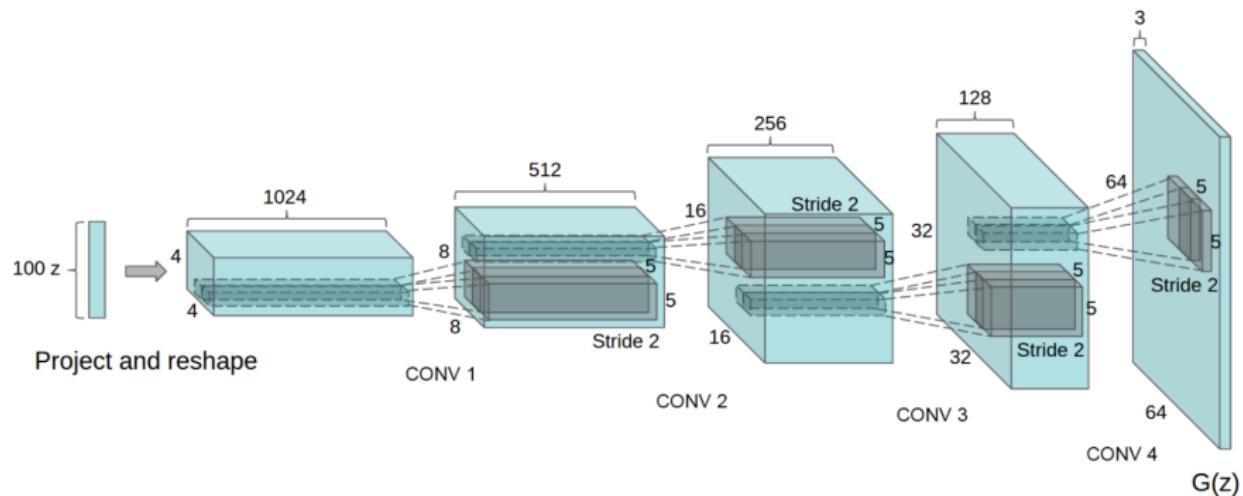
Architecture guidelines for stable Deep Convolutional GANs

- Replace any pooling layers with strided convolutions (discriminator) and fractional-strided convolutions (generator).
- Use batchnorm in both the generator and the discriminator.
- Remove fully connected hidden layers for deeper architectures.
- Use ReLU activation in generator for all layers except for the output, which uses Tanh.
- Use LeakyReLU activation in the discriminator for all layers.

Table here and figures in next 5 slides from Radford, Metz, Chintala, "Unsupervised representation learning with deep convolutional generative adversarial networks", ICLR, 2016

DCGAN (cont')

- The generator's structure



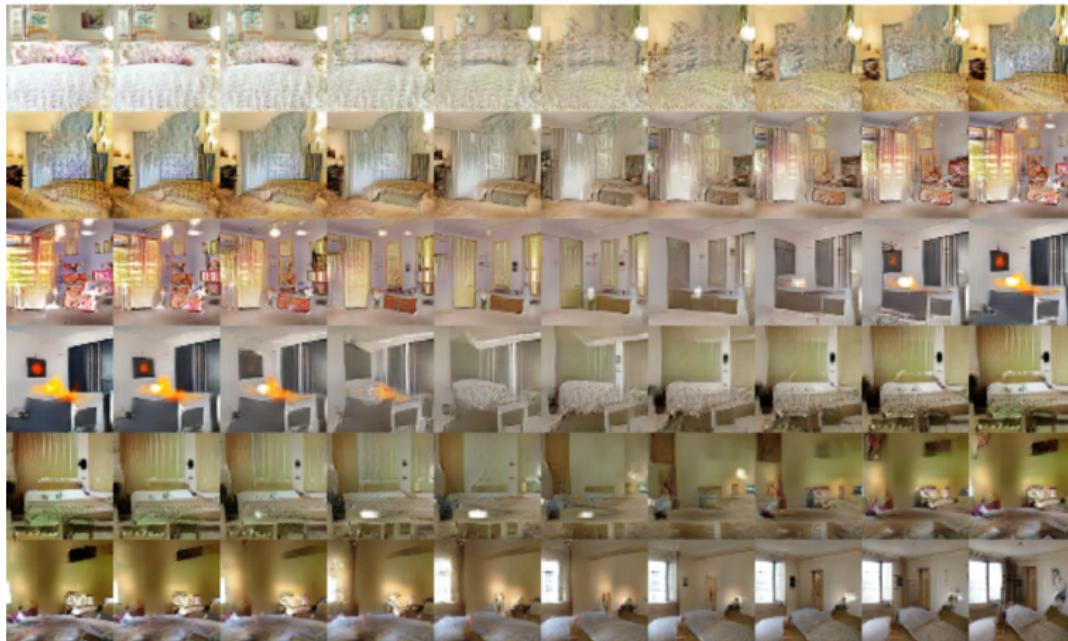
DCGAN result

- Generated images are already realistic after one epoch training



DCGAN result (cont')

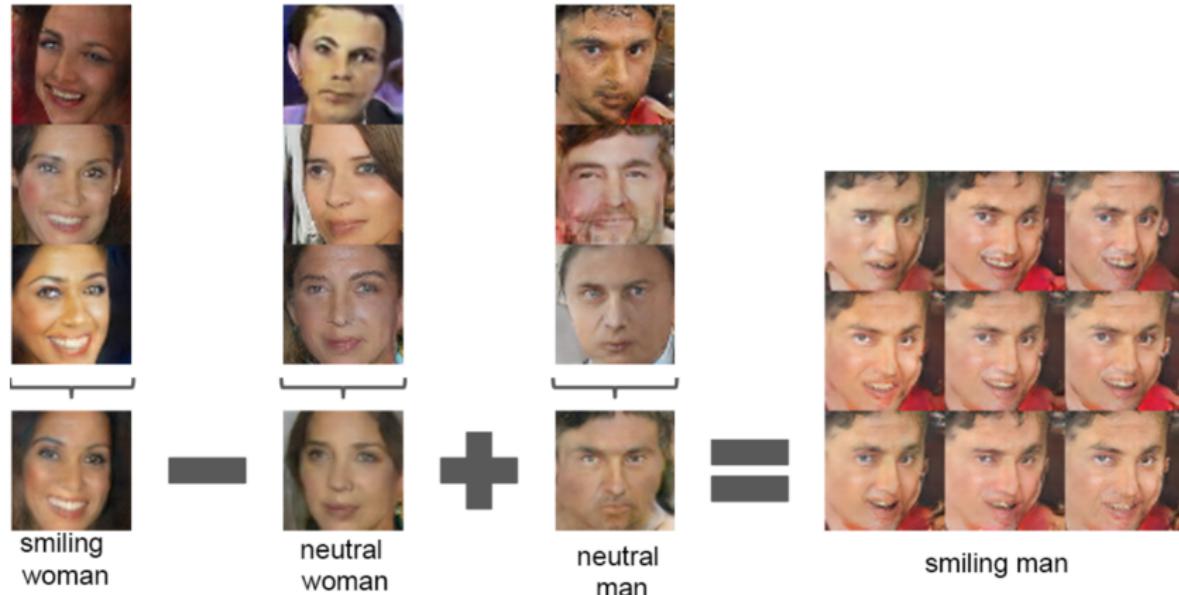
- Interpolations between random points in the latent (input) space result in semantic changes in generated images



DCGAN result (cont'): interpretable vector arithmetic

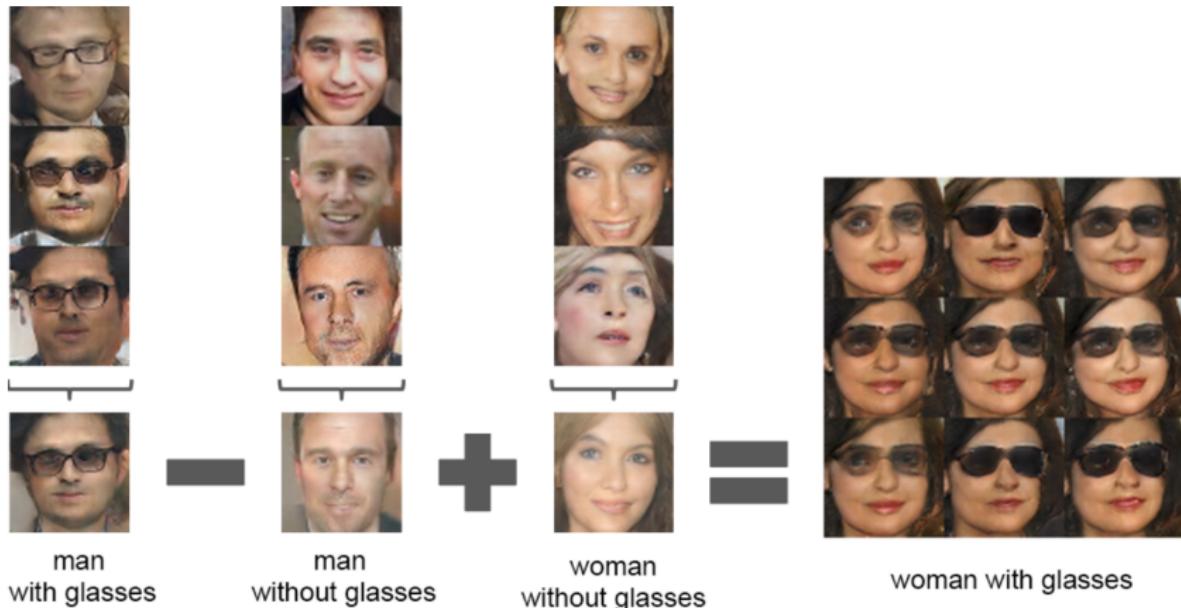
In latent space:

mean smiling woman vector - mean neural woman vector + mean neural man vector = mean smiling man vector!



DCGAN result (cont'): interpretable vector arithmetic

- Similarly, women-with-glass images can be generated by vector arithmetic in the latent space.



LSGAN: Least Square Generative Adversarial Networks

- Uses least-square loss to relieve gradient vanishing issue.
Original GAN used (sigmoid) binary cross-entropy loss.
- Tries to achieve the same objective as original GAN

Original GAN

$$\min_G \max_D V_{\text{GAN}}(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

LS-GAN

$$\begin{aligned} \min_D V_{\text{LSGAN}}(D) &= \frac{1}{2} \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [(D(\mathbf{x}) - 1)^2] + \frac{1}{2} \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [(D(G(\mathbf{z})))^2] \\ \min_G V_{\text{LSGAN}}(G) &= \frac{1}{2} \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [(D(G(\mathbf{z}))) - 1]^2. \end{aligned}$$

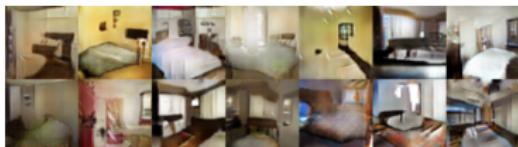
Formulae here and figures in next slide from Mao, Li, Xie, Lau, Wang, Smolley, "Least squares generative adversarial networks", ICCV, 2017

LSGAN result

- LSGAN generated good-quality (e.g., outdoor and indoor) images



- LSGAN is more stable than regular GAN, when batch normalization is removed in generator.



(a) LSGANs.



(b) Regular GANs.

Issues in GAN training

- Unstable training: when Discriminator becomes perfect, loss becomes zero, so gradient for Generator vanishes!

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- Unstable training: when Discriminator becomes perfect, loss becomes zero, so gradient for Generator vanishes!
On the other hand, if Discriminator behave badly, Generator would have bad feedback and cannot update well.
- Mode collapse: generator generates realistic images, but with low varieties



- Lack of proper evaluation metric: do not know when to stop training.

See theoretical proof behind the issues from Arjovsky, Bottou, "Towards principled methods for training generative adversarial networks", arXiv, 2017

Essentially,...

- Generator is used to generate fake data whose distribution P_θ approximates real but unknown data distribution P_r
 - Discriminator helps measures difference between P_r and P_θ (using loss function of GAN based on discriminator's output)

Any better way to measure difference between distributions?

Distance measurements between two distributions

- Kullback-Leibler (KL) divergence

$$KL(P_r \parallel P_\theta) = \int P_r(x) \log \left(\frac{P_r(x)}{P_\theta(x)} \right) dx$$

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- Jenson-Shannon (JS) divergence, with $P_m = (P_r + P_\theta)/2$:

$$JS(P_r, P_\theta) = \frac{1}{2}KL(P_r \parallel P_m) + \frac{1}{2}KL(P_\theta \parallel P_m)$$

(GAN's loss function with optimal D) = $2JS(P_r, P_\theta) - 2\log 2$

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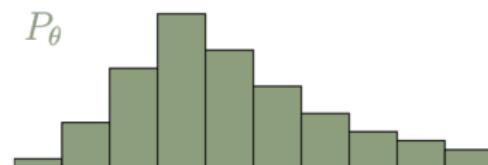
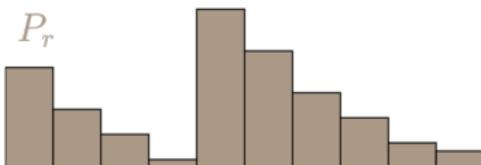
- Earth Mover's distance or Wasserstein distance:

$$W(P_r, P_\theta) = \inf_{\gamma \in \Pi(P_r, P_\theta)} \mathbb{E}_{(x,y) \sim \gamma} [\|x - y\|].$$

where $\Pi(P_r, P_\theta)$ is the set of all joint distributions whose marginal distributions are P_r and P_θ .

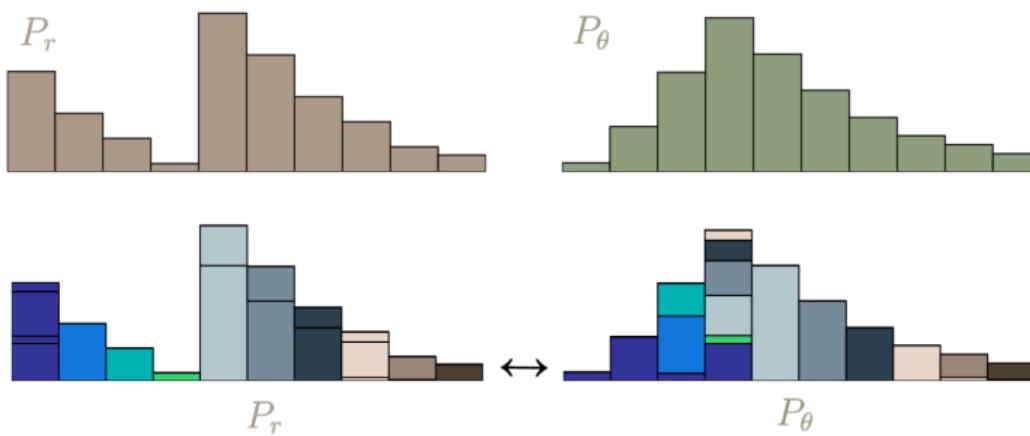
Earth Mover's Distance (EMD)

- EMD is the minimal total amount of work it takes to transform one heap into the other
- 'work': amount of moved earth in a chunk times the distance moved



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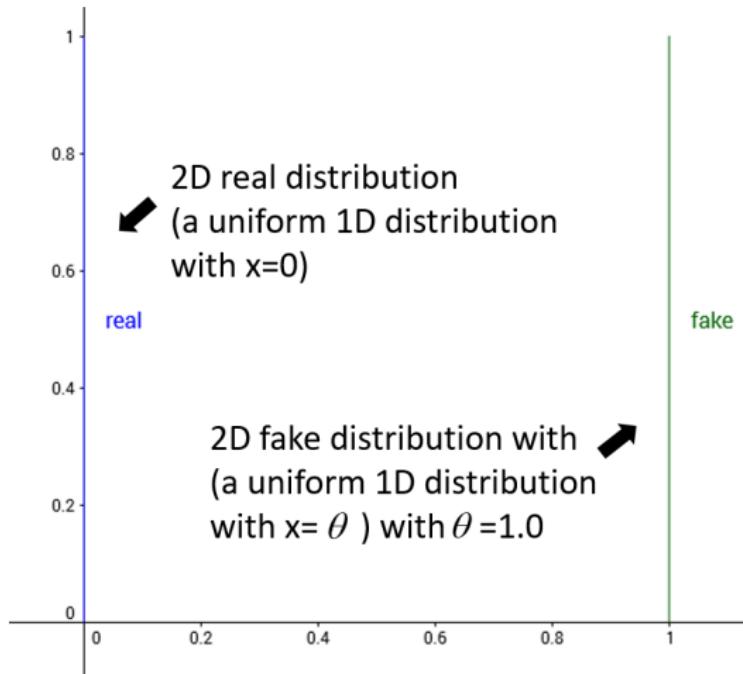
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Figures from <https://vincentherrmann.github.io/blog/wasserstein/>

EMD or Wasserstein distance

- Example of measuring the distance between real distribution P_0 (i.e., P_r) and fake distribution P_θ



EMD or Wasserstein distance (cont')

- $W(\mathbb{P}_0, \mathbb{P}_\theta) = |\theta|,$
- $JS(\mathbb{P}_0, \mathbb{P}_\theta) = \begin{cases} \log 2 & \text{if } \theta \neq 0 , \\ 0 & \text{if } \theta = 0 , \end{cases}$
- $KL(\mathbb{P}_\theta \| \mathbb{P}_0) = KL(\mathbb{P}_0 \| \mathbb{P}_\theta) = \begin{cases} +\infty & \text{if } \theta \neq 0 , \\ 0 & \text{if } \theta = 0 , \end{cases}$

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- The gradient of JS and KL divergence (over θ) is always 0, therefore failing to update θ .
- Wasserstein distance is continuous and has non-zero gradient almost everywhere
- Therefore, JS and KL divergences are not good choices to measure the distance between two distributions.

Wasserstein distance

- It is intractable to compute Wasserstein distance exactly

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- But it shows W is equivalent to

$$W(P_r, P_\theta) = \sup_{\|f\|_L \leq 1} \mathbb{E}_{x \sim P_r} [f(x)] - \mathbb{E}_{x \sim P_\theta} [f(x)]$$

where $\|f\|_L \leq 1$ means f is a 1-Lipschitz function.

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- Let d_X and d_Y be distance functions on spaces X and Y . A function $f : X \rightarrow Y$ is K-Lipschitz if for all $x_1, x_2 \in X$,

$$d_Y(f(x_1), f(x_2)) \leq K d_X(x_1, x_2)$$

Intuitively: if x_1, x_2 are close, $f(x_1)$ and $f(x_2)$ are also close.

Wasserstein distance approximation

- The supremum over 1-Lipschitz functions is still intractable

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- Suppose there are a parameterized function family $\{f_w\}_{w \in \mathcal{W}}$ with parameter w and function f_w being K-Lipschitz, then

$$\begin{aligned} & \max_{w \in \mathcal{W}} \mathbb{E}_{x \sim P_r} [f_w(x)] - \mathbb{E}_{x \sim P_\theta} [f_w(x)] \\ & \leq \sup_{\|f\|_L \leq K} \mathbb{E}_{x \sim P_r} [f(x)] - \mathbb{E}_{x \sim P_\theta} [f(x)] \end{aligned}$$

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- Then $W(P_r, P_\theta)$ can be approximated by

$$\max_{w \in \mathcal{W}} \mathbb{E}_{x \sim P_r} [f_w(x)] - \mathbb{E}_{x \sim P_\theta} [f_w(x)]$$

Note: K can be absorbed into learning rate during max.

Wasserstein GAN

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- Answer: weight clamping! Clipping w to be within $[-c, c]$ after every update of w .

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- Go back to original objective: In order to train Generator g_θ such that the fake $P_\theta = g_\theta(Z)$ matches real P_r , we need a distance measurement to estimate the difference between P_θ and P_r . Here the distance measurement is Wasserstein distance which can be approximately computed based on the optimal f_w , i.e., by optimizing a network work f_w . We call such a network f_w 'critic function', corresponding to Discriminator in original GAN.

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- Wasserstein GAN: generator + critic

Wasserstein GAN (cont')

- Once getting optimal f_w (by training critic network with maximization of $\mathbb{E}_{x \sim P_r} [f_w(x)] - \mathbb{E}_{x \sim P_\theta} [f_w(x)]$), then

$$W(P_r, P_\theta) = \mathbb{E}_{x \sim P_r} [f_w(x)] - \mathbb{E}_{x \sim P_\theta} [f_w(x)]$$

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- Then update generator g_θ (with fixed f_w) by minimizing W

$$\begin{aligned}\nabla_\theta W(P_r, P_\theta) &= \nabla_\theta (\mathbb{E}_{x \sim P_r} [f_w(x)] - \mathbb{E}_{z \sim Z} [f_w(g_\theta(z))]) \\ &= -\mathbb{E}_{z \sim Z} [\nabla_\theta f_w(g_\theta(z))]\end{aligned}$$

Wasserstein GAN (cont')

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$$W(P_r, P_\theta) = \mathbb{E}_{x \sim P_r} [f_w(x)] - \mathbb{E}_{x \sim P_\theta} [f_w(x)]$$

- Then update generator g_θ (with fixed f_w) by minimizing W

$$\begin{aligned}\nabla_{\theta} W(P_r, P_\theta) &= \nabla_{\theta} (\mathbb{E}_{x \sim P_r} [f_w(x)] - \mathbb{E}_{z \sim Z} [f_w(g_\theta(z))]) \\ &= -\mathbb{E}_{z \sim Z} [\nabla_{\theta} f_w(g_\theta(z))]\end{aligned}$$

In summary, repeat the following steps to train WGAN:

- Step 1: Fix generator g_θ , compute approximation of $W(P_r, P_\theta)$ by training f_w , i.e., by $\max_{w \in \mathcal{W}} \mathbb{E}_{x \sim P_r} [f_w(x)] - \mathbb{E}_{x \sim P_\theta} [f_w(x)]$
- Step 2: Fix 'critic' f_w , update g_θ with gradient $-\mathbb{E}_{z \sim Z} [\nabla_{\theta} f_w(g_\theta(z))]$ by sampling several z from uniform distribution Z .

WGAN algorithm

Algorithm 1 WGAN, our proposed algorithm. All experiments in the paper used the default values $\alpha = 0.00005$, $c = 0.01$, $m = 64$, $n_{\text{critic}} = 5$.

Require: : α , the learning rate. c , the clipping parameter. m , the batch size.
 n_{critic} , the number of iterations of the critic per generator iteration.

Require: : w_0 , initial critic parameters. θ_0 , initial generator's parameters.

```

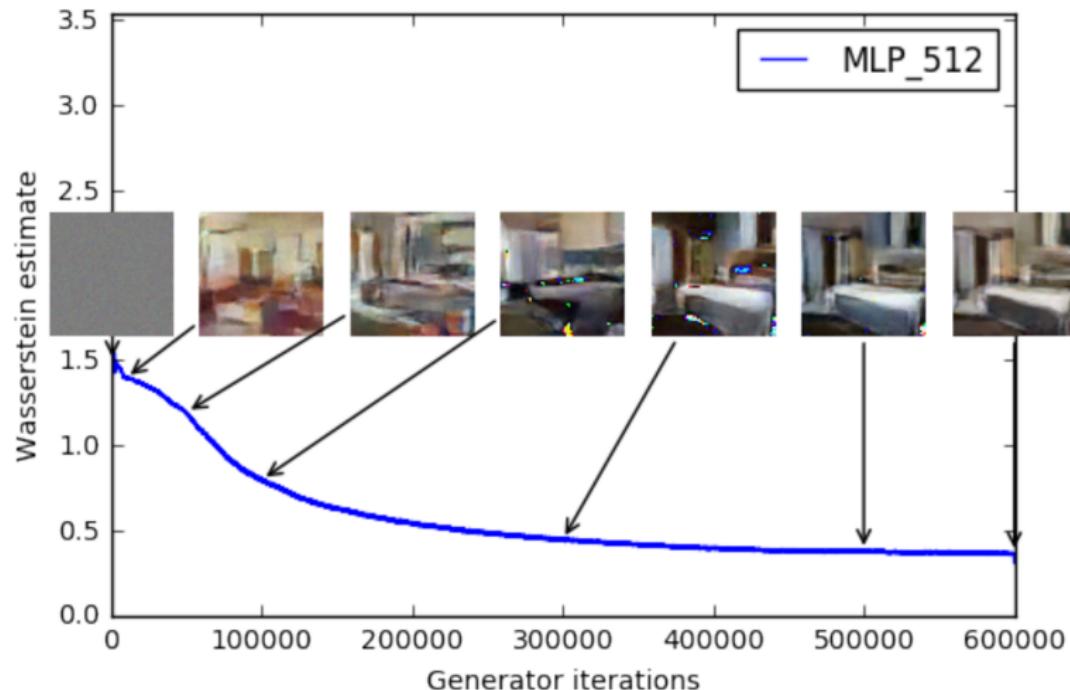
1: while  $\theta$  has not converged do
2:   for  $t = 0, \dots, n_{\text{critic}}$  do
3:     Sample  $\{x^{(i)}\}_{i=1}^m \sim \mathbb{P}_r$  a batch from the real data.
4:     Sample  $\{z^{(i)}\}_{i=1}^m \sim p(z)$  a batch of prior samples.
5:      $g_w \leftarrow \nabla_w [\frac{1}{m} \sum_{i=1}^m f_w(x^{(i)}) - \frac{1}{m} \sum_{i=1}^m f_w(g_\theta(z^{(i)}))]$ 
6:      $w \leftarrow w + \alpha \cdot \text{RMSProp}(w, g_w)$ 
7:      $w \leftarrow \text{clip}(w, -c, c)$ 
8:   end for
9:   Sample  $\{z^{(i)}\}_{i=1}^m \sim p(z)$  a batch of prior samples.
10:   $g_\theta \leftarrow -\nabla_\theta \frac{1}{m} \sum_{i=1}^m f_w(g_\theta(z^{(i)}))$ 
11:   $\theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, g_\theta)$ 
12: end while
```

Step 1

Step 2

WGAN results

- Wasserstein loss correlates well with image quality



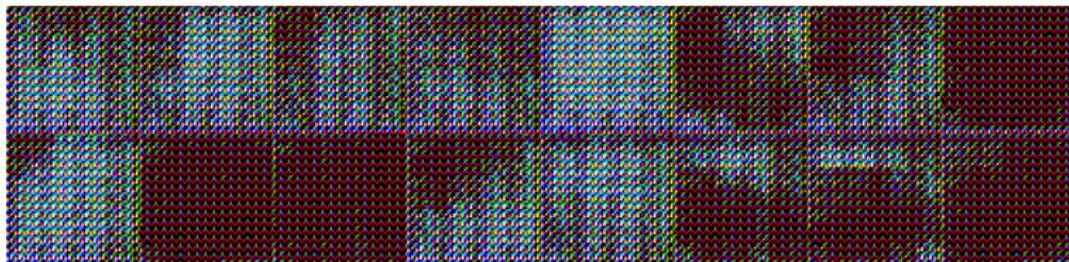
WGAN results (cont')

- WGAN (top 2 rows; with same DCGAN structure) performs well (if not better) compared to DCGAN (bottom 2 rows)



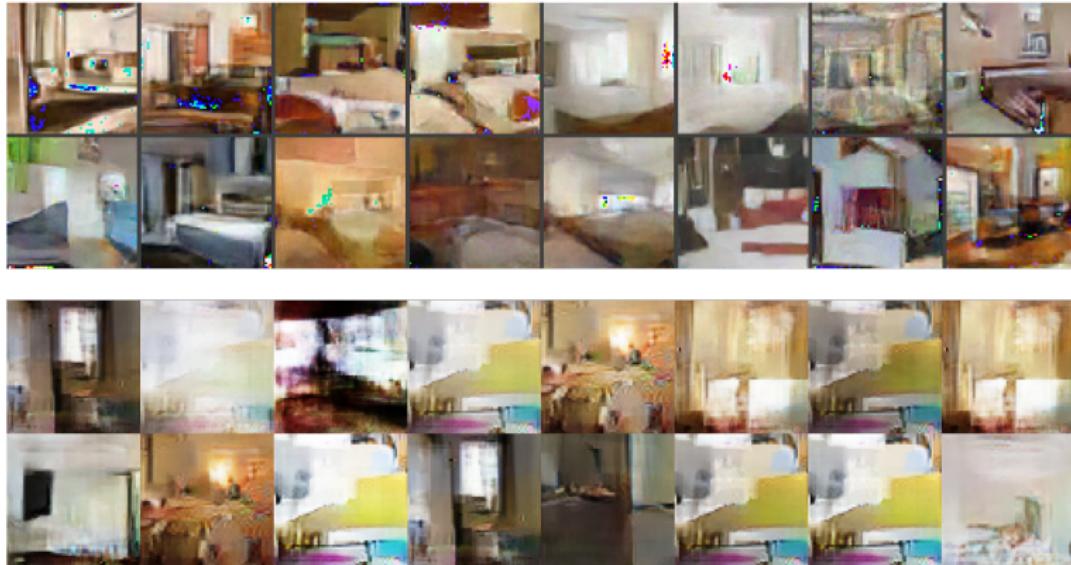
WGAN results (cont')

- WGAN (top 2 rows; with same DCGAN structure) still performs well when removing batch normalization, but DCGAN not (bottom 2 rows)



WGAN results (cont')

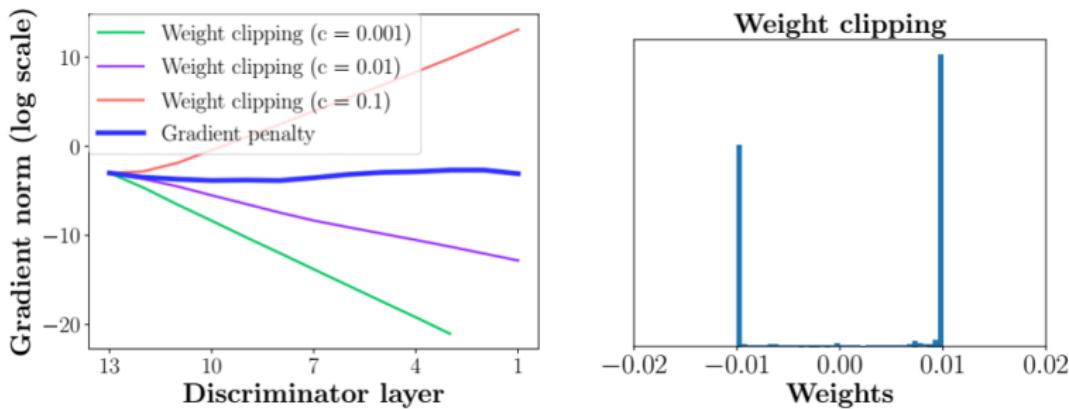
- WGAN (top 2 rows; with same MLP structure) reduce issue of mode collapse, compared to original GAN (bottom 2 rows).



However, ...

Issues of weight clipping in WGAN:

- makes critic f_w within a small subset of K-Lipschitz functions.
- causes gradient vanishing or exploding (Fig. left).
- pushes weights to extremes of clipping range (Fig. right), in turn causing gradient exploding and slow training.



WGAN-GP: GAN with gradient penalty

- Alternative way to enforce Lipschitz constraint: a function is 1-Lipschitz if and only if it has gradients (over input variable) with norm less or equal to 1.0 everywhere.

WGAN-GP: GAN with gradient penalty

- Alternative way to enforce Lipschitz constraint: a function is 1-Lipschitz if and only if it has gradients (over input variable) with norm less or equal to 1.0 everywhere.
- Critic loss (to be minimized)

$$\begin{aligned} L = & -\mathbb{E}_{x \sim P_r} [f_w(x)] + \mathbb{E}_{\tilde{x} \sim P_\theta} [f_w(\tilde{x})] + \\ & \lambda \mathbb{E}_{\hat{x} \sim P_{\hat{x}}} [(\|\nabla_{\hat{x}} f_w(\hat{x})\|_2 - 1)^2] \end{aligned}$$

where the last term is ‘gradient penalty’ term, and

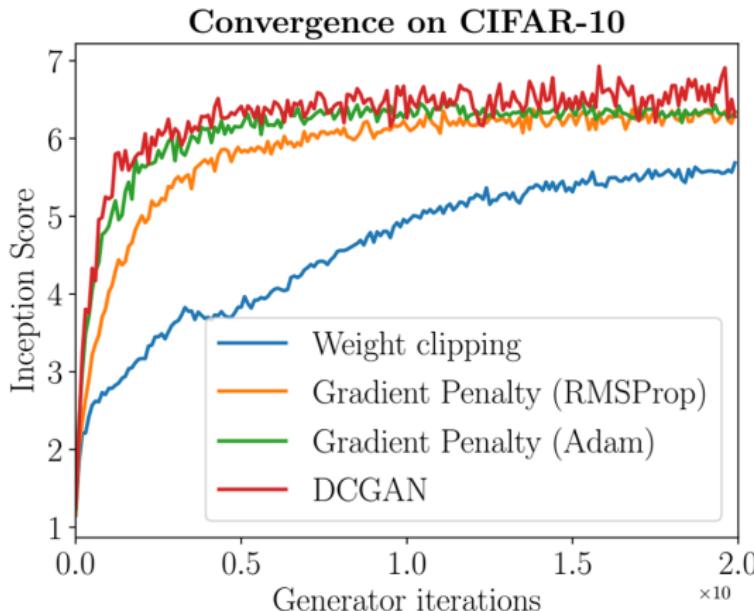
$$\begin{aligned} \epsilon &\sim U[0, 1], x \sim P_r, \tilde{x} \in P_\theta \\ \hat{x} &= \epsilon x + (1 - \epsilon) \tilde{x} \end{aligned}$$

considering optimal critic has gradient norm 1.0 on straight lines connecting points from P_r to P_θ .

- The improved WGAN is called WGAN-GP

WGAN-GP result

- WGAN-GP improves training speed compared to WGAN



- 'Inception score' is used to measure image quality

WGAN-GP result

- WGAN-GP successfully train difficult GAN architectures

DCGAN



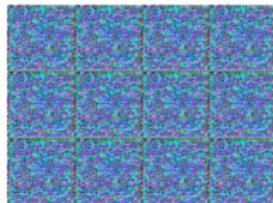
LSGAN



WGAN (clipping)



WGAN-GP

tanh nonlinearities everywhere in G and D 

Summary

- GANs provide an effective way to generate data
- GANs do not explicitly model data distribution
- GAN training: unstable and mode collapse
- WGAN: 1st attempt to improve GAN with theoretical proof
- More GANs already proposed, more GANs to be proposed!

Further reading:

- Miyato et al., Spectral normalization for generative adversarial networks, ICLR, 2018
- Dai et al., Good semi-supervised learning that requires a bad GAN, arXiv, 2017