Lecture 4:
Massive MIMO

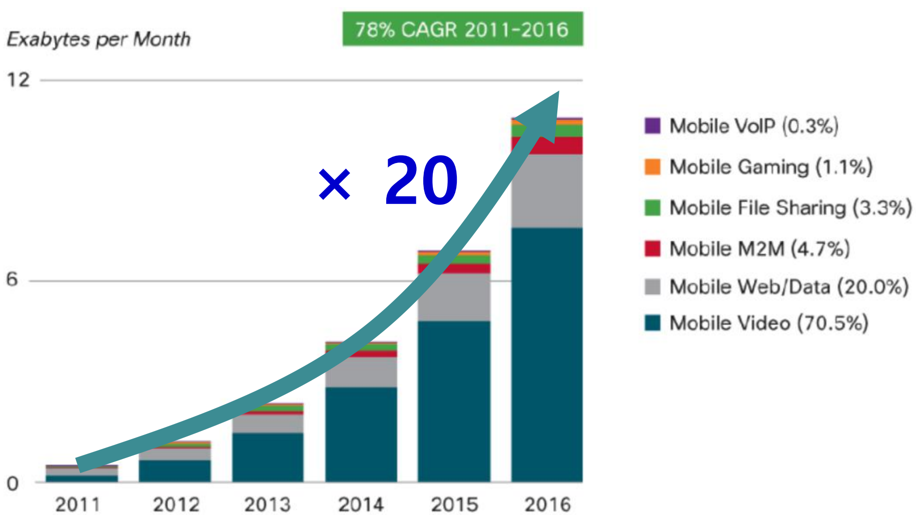
Contents

Introduction to Massive MIMO

Introduction to Massive MIMO
: Beyond 4G Network

MTC : Machine Type Communications
VoIP : Voice over Internet Protocol
M2M : Machine-to-machine

- Future mobile data traffic
  - Global exponential mobile data traffic increase
    - By a factor of ~ 20 from 2011 until 2016, and more expected in 2020.
  - More devices, higher bit rates, always active
  - Larger variety of traffic types e.g. Video, MTC



Figures in legend refer to traffic share in 2016.
Source: Cisco VNI Mobile, 2012

Further capacity enhancement is needed

[1] RWS-120002, “Technologies for Rel-12 and Onwards,” Samsung, 3GPP RAN WS on Rel-12 and onwards, Ljubljana, Slovenia, June 11–12, 2012.
[2] T. L. Marzetta et, “Beyond LTE: Hundreds of Base Station Antennas! “, 2010 IEEE Communication Theory Workshop

Introduction to Massive MIMO
: Candidate of Beyond 4G Network

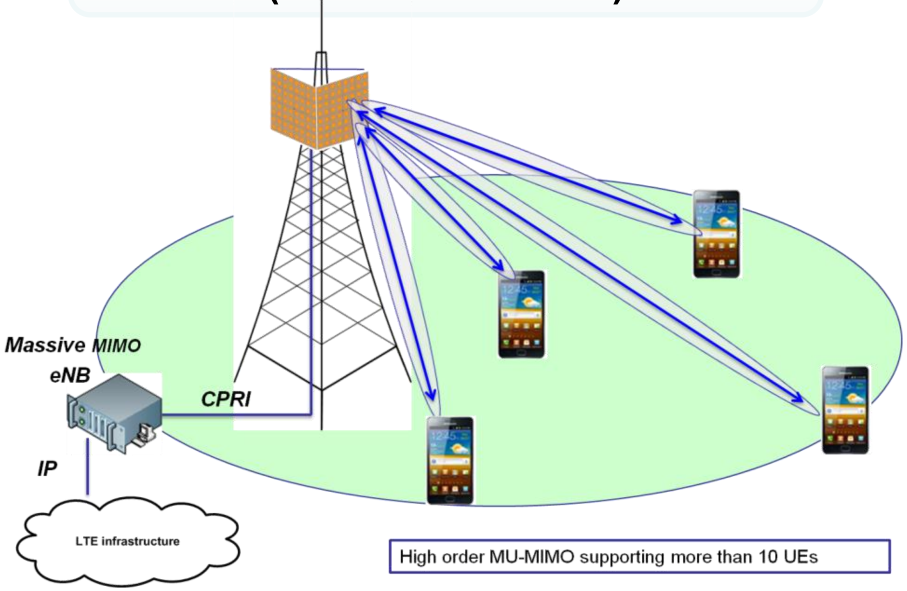
CSI : Channel State Information

Solution for Capacity Demand[1],[2]

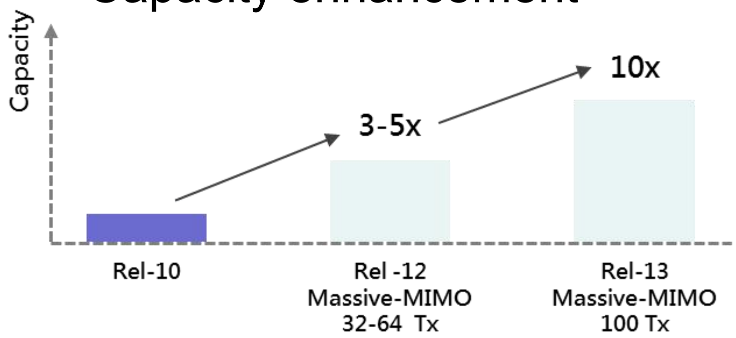
Massive MIMO

Using hundreds of antennas at BS

Support the dozens of UEs (Multi-user MIMO)

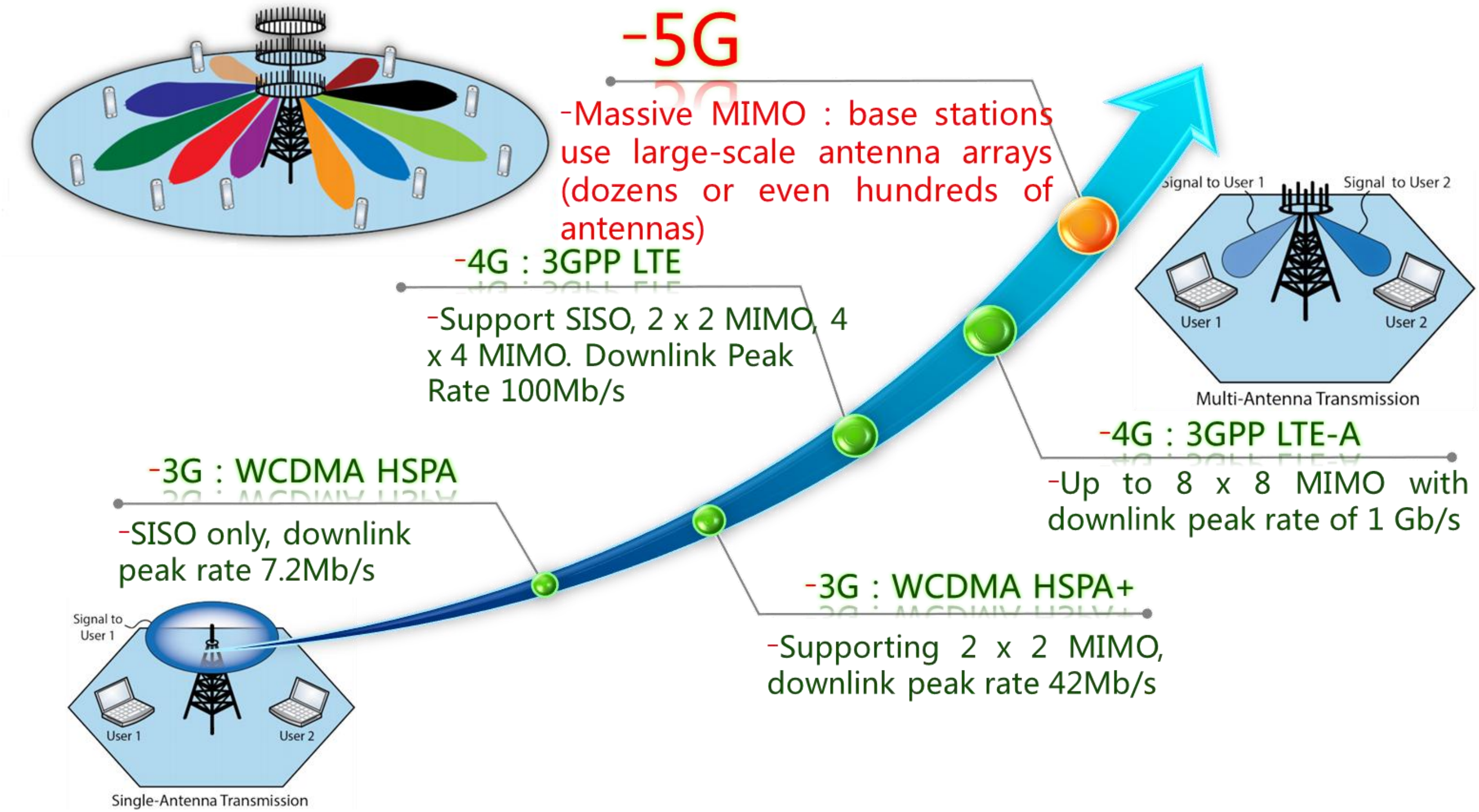


- **Benefit of Massive Antenna**
  - Capacity enhancement[1]



  - Mathematically Exact[2]
    - Required **Tx energy/bit** is arbitrarily **small**
    - **Eliminate** the effects of uncorrelated noise & fast fading
    - **Compensate** the poor-quality CSI

Introduction to Massive MIMO
: Evolution of MIMO Technology



[5] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," IEEE Trans. Wireless Commun., vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

Introduction to Massive MIMO

MAC : Multiple Access Channel

- Motivation of Massive MIMO[5]
  - Consider a $M \times K$ MIMO MAC (M : # of BSs antennas, K : # of user)

$$\underset{[M\times 1]}{\mathbf{y}} = \overset{[M\times K]}{\mathbf{H}}\underset{[K\times 1]}{\mathbf{x}} + \underset{[M\times 1]}{\mathbf{n}}$$

$\mathbf{H}$, $\mathbf{n}$: i.i.d. with zero mean and unit variance

  - If the BS process its receive signal by matched filtering,

$$\mathbf{y} \Rightarrow \frac{1}{M}\mathbf{H}^H\mathbf{y} = \frac{1}{M}\mathbf{H}^H\mathbf{H}\mathbf{x} + \frac{1}{M}\mathbf{H}^H\mathbf{n}$$

  - By the strong law of large numbers (大数定律),

$$\boxed{\frac{1}{M}\mathbf{H}^H\mathbf{y} \xrightarrow[M\to\infty,\ K=\text{const.}]{a.s.} \mathbf{x}}$$

$$\frac{1}{M}\mathbf{H}^H\mathbf{H} = \begin{bmatrix} \frac{\|\mathbf{h}_1\|^2}{M} & \frac{\mathbf{h}_1^H\mathbf{h}_2}{M} & \cdots & \frac{\mathbf{h}_1^H\mathbf{h}_K}{M} \\ \frac{\mathbf{h}_2^H\mathbf{h}_1}{M} & \frac{\|\mathbf{h}_2\|^2}{M} & & \frac{\mathbf{h}_2^H\mathbf{h}_K}{M} \\ \vdots & & \ddots & \vdots \\ \frac{\mathbf{h}_K^H\mathbf{h}_1}{M} & \frac{\mathbf{h}_K^H\mathbf{h}_2}{M} & \cdots & \frac{\|\mathbf{h}_K\|^2}{M} \end{bmatrix}$$

as $M \to \infty$

$\xrightarrow[M\to\infty]{a.s.} 0$

$\xrightarrow[M\to\infty]{a.s.} 1$

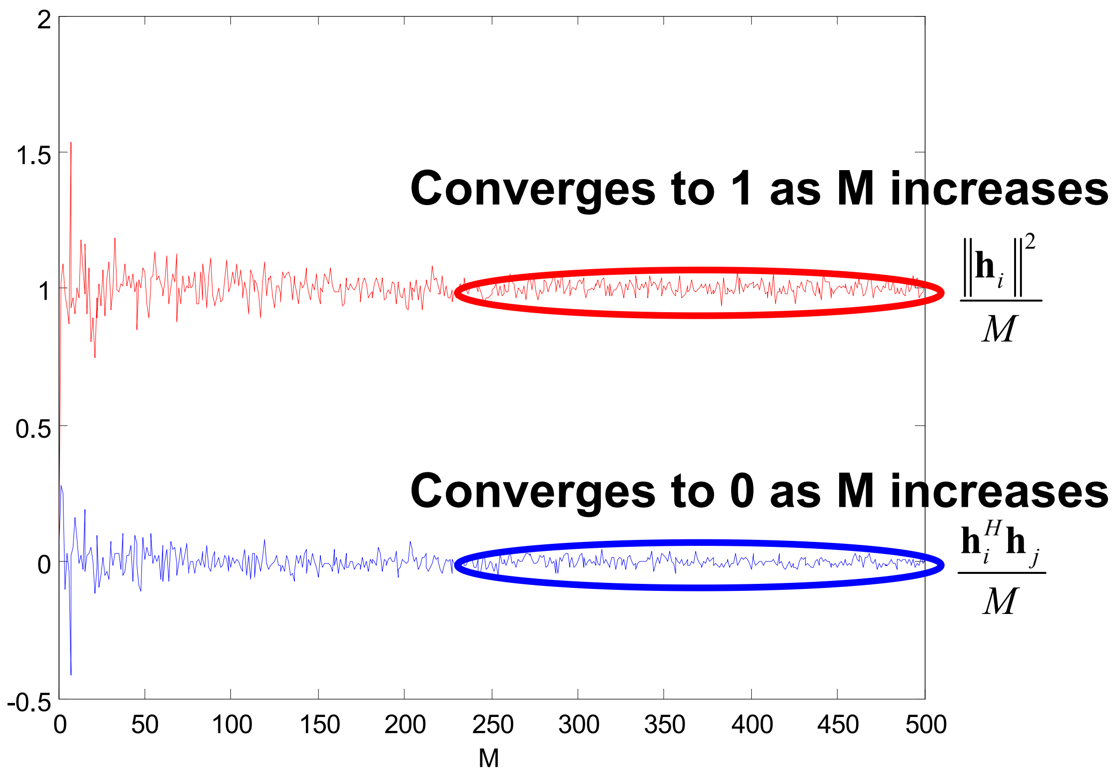
By strong law of large numbers

With an **unlimited number of antennas**

- Uncorrelated interference and noise vanish
- The matched filter is optimal
- The transmit power can be made arbitrarily small

Introduction to Massive MIMO

- Simulation result
  - M = 1 ~ 500, $\mathbf{h}_i$ = M X 1 Real Gaussian Vector



[6] H. Q. Ngo, E. G. Larsson and T. L. Marzetta, "Uplink power efficiency of multiuser MIMO with very large antenna arrays," in Proc. of Allerton Conference on Communication, Control, and Computing, Sept. 2011.

Introduction to Massive MIMO

- On channel estimation and pilot contamination[5]
  - The receiver estimates the channels based on pilot sequences.
  - The # of orthogonal sequences is limited by the coherence time
  - Thus, the pilot sequences must be reused
  - Assume that transmitter *m* & *j* **use** the **same pilot sequence**

$$\mathbf{y} = \mathbf{H}_m \mathbf{x}_m + \underbrace{\mathbf{H}_j \mathbf{x}_j}_{\text{pilot contamination}} + \mathbf{n} \qquad \widehat{\mathbf{H}}_m = \mathbf{H}_m + \underbrace{\mathbf{H}_j}_{\text{pilot contamination}} + \underbrace{\mathbf{n}_m}_{\text{estimation noise}}$$

  - Thus, the BS process its receive signal by matched filtering

$$\boxed{\frac{1}{M}\widehat{\mathbf{H}}_m^H \mathbf{y} \xrightarrow[M\to\infty,\ K=\text{const.}]{a.s.} \mathbf{x}_m + \mathbf{x}_j}$$

By strong law of large numbers

With an **unlimited number of antennas**

- Uncorrelated interference, noise and estimation errors vanish
- The matched filter is optimal
- The transmit power can be made arbitrarily small[6] $\left(\sim 1/\sqrt{M}\right)$
- The performance is limited by pilot contamination

Introduction to Massive MIMO

CSIT : Channel State Information at the Transmitter
CSIR : Channel State Information at the Receiver

- Analysis of Massive MIMO with sufficient large # of BS antennas
  - Single-cell massive MIMO scenario
    - Massive MIMO Downlink scenario
      - Analysis the performance of various linear precoding/beamforming technique for Massive MIMO based on channel information
      - Perfect CSIT/ imperfect CSIT cases
    - Massive MIMO Uplink scenario
      - Linear receiver technique for Massive MIMO Uplink
      - Perfect CSIR/ imperfect CSIR cases
  - Multi-cell massive MIMO scenario
    - Inter-cell interference problem
    - Pilot contamination problem

Fundamental Overview: Massive MIMO

Fundamental Overview: Massive MIMO
: Point-to-Point MIMO (1/4)

SNR : Signal to Noise Ratio

- Channel Model

$$\underset{[N\times 1]}{\mathbf{y}} = \sqrt{\frac{p_d}{M}} \underset{[N\times M]}{\mathbf{H}} \underset{[M\times 1]}{\mathbf{x}} + \underset{[N\times 1]}{\mathbf{n}}$$

  - \# of BS antennas *M*, # of UE antennas *N*
  - IID complex-Gaussian channel **H**, **x**, **n** with zero mean and variance 1
  - p_d is downlink transmission power
  - Receiver has perfect knowledge of **H**

- Received SNR/ Capacity at Receiver

$$\mathrm{SNR} = \frac{p_d \left\|\mathbf{H}\right\|^2}{N_0} = p_d \left\|\mathbf{H}\right\|^2$$

M>N

$$C = \log_2 \det(\mathbf{I}_N + \frac{p_d}{M}\mathbf{H}\mathbf{H}^H)_{M>N}$$

M<N

$$C = \log_2 \det(\mathbf{I}_M + \frac{p_d}{M}\mathbf{H}^H\mathbf{H})_{M<N}$$

Fundamental Overview: Massive MIMO
: Point-to-Point MIMO (2/4)

- Capacity at Receiver (*M*>*N*)

$$C = \log_2 \det(\mathbf{I}_N + \frac{p_d}{M}\mathbf{H}\mathbf{H}^H)$$

$$\frac{1}{M}\mathbf{H}\mathbf{H}^H = \frac{1}{M}\begin{bmatrix} -\mathbf{h}_1- \\ \vdots \\ -\mathbf{h}_N- \end{bmatrix}\begin{bmatrix} | & & | \\ \mathbf{h}_1^H & \cdots & \mathbf{h}_N^H \\ | & & | \end{bmatrix} = \frac{1}{M}\begin{bmatrix} \|\mathbf{h}_1\|^2 & \mathbf{h}_1\mathbf{h}_2^H & \cdots & \mathbf{h}_1\mathbf{h}_N^H \\ \mathbf{h}_2\mathbf{h}_1^H & \|\mathbf{h}_2\|^2 & & \\ \vdots & & \ddots & \\ \mathbf{h}_N\mathbf{h}_1^H & \mathbf{h}_N\mathbf{h}_2^H & \cdots & \|\mathbf{h}_N\|^2 \end{bmatrix}$$

$$\text{where } \underset{[1\times M]}{\mathbf{h}_i} = \begin{bmatrix} h_1^i & h_2^i & \cdots & h_M^i \end{bmatrix}$$

- For large *M* with IID complex-Gaussian channel **H**,

$$\frac{\|\mathbf{h}_i\|^2}{M} = \frac{|h_1^i|^2 + \cdots + |h_M^i|^2}{M} \approx \underbrace{\mathrm{Var}[h]}_{1} + \underbrace{(E[h])^2}_{0} = 1$$

Conclusion

$$\boxed{\frac{1}{M}\mathbf{H}\mathbf{H}^H \approx \mathbf{I}_N}$$

$$\frac{\mathbf{h}_i\mathbf{h}_j^H}{M}_{(i\neq j)} = \frac{1}{M}\left(\underbrace{h_1^i h_1^{j*}}_{\text{Gaussian}} + \underbrace{h_2^i h_2^{j*}}_{\text{Gaussian}} + \cdots + \underbrace{h_M^i h_M^{j*}}_{\text{Gaussian}}\right) = \frac{g_1 + g_2 + \cdots + g_M}{M} \approx E[g] = E[h] = 0$$

Fundamental Overview: Massive MIMO
: Point-to-Point MIMO (3/4)

- Capacity at Receiver (*N*>*M*)

$$C = \log_2 \det(\mathbf{I}_M + \frac{p_d}{M}\mathbf{H}^H\mathbf{H})$$

$$\frac{1}{M}\mathbf{H}^H\mathbf{H} = \frac{N}{M}\frac{1}{N}\begin{bmatrix} -\mathbf{h}_1^H - \\ \vdots \\ -\mathbf{h}_M^H - \end{bmatrix}\begin{bmatrix} | & & | \\ \mathbf{h}_1 & \cdots & \mathbf{h}_M \\ | & & | \end{bmatrix} = \frac{N}{M}\frac{1}{N}\begin{bmatrix} \|\mathbf{h}_1\|^2 & \mathbf{h}_1^H\mathbf{h}_2 & \cdots & \mathbf{h}_1^H\mathbf{h}_M \\ \mathbf{h}_2^H\mathbf{h}_1 & \|\mathbf{h}_2\|^2 & & \\ \vdots & & \ddots & \\ \mathbf{h}_M^H\mathbf{h}_1 & \mathbf{h}_M^H\mathbf{h}_2 & \cdots & \|\mathbf{h}_M\|^2 \end{bmatrix}$$

where $\underset{[N\times 1]}{\mathbf{h}_i} = \begin{bmatrix} h_1^i & h_2^i & \cdots & h_N^i \end{bmatrix}^T$

- For large *N* with IID complex-Gaussian channel **H**,

$$\frac{\|\mathbf{h}_i\|^2}{N} = \frac{|h_1^i|^2 + \cdots + |h_N^i|^2}{N} \approx \underbrace{\mathrm{Var}[h]}_{1} + \underbrace{(E[h])^2}_{0} = 1$$

$$\left.\frac{\mathbf{h}_i\mathbf{h}_j^H}{N}\right|_{(i\neq j)} = \frac{1}{N}\left(\underbrace{h_1^i h_1^{j*}}_{\text{Gaussian}} + \underbrace{h_2^i h_2^{j*}}_{\text{Gaussian}} + \cdots + \underbrace{h_N^i h_N^{j*}}_{\text{Gaussian}}\right) = \frac{g_1 + g_2 + \cdots + g_N}{N} \approx E[g] = E[h] = 0$$

Conclusion

$$\frac{N}{M}\frac{1}{N}\mathbf{H}^H\mathbf{H} \approx \frac{N}{M}\mathbf{I}_M$$

Fundamental Overview: Massive MIMO
: Point-to-Point MIMO (4/4)

- Point-to-point MIMO
  - Large number of transmit antennas

$$\frac{1}{M}\mathbf{H}\mathbf{H}^H \approx \mathbf{I}_N$$

$$C_{M>>N} = \log_2 \det(\mathbf{I}_N + \frac{p_d}{M}\mathbf{H}\mathbf{H}^H) \approx \log_2 \det(\mathbf{I}_N + p_d \mathbf{I}_N)$$

$$= \log_2 \det\left(\begin{bmatrix} 1+p_d & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1+p_d \end{bmatrix}_{N\times N}\right) = N \log_2(1+p_d)$$

Independent with *M*
Linearly increase as *N*

  - Large number of receive antennas

$$\frac{N}{M}\frac{1}{N}\mathbf{H}^H\mathbf{H} \approx \frac{N}{M}\mathbf{I}_M$$

$$C_{N>>M} = \log_2 \det\left(\mathbf{I}_M + \frac{p_d}{M}\mathbf{H}^H\mathbf{H}\right) \approx \log_2 \det(\mathbf{I}_M + \frac{Np_d}{M}\mathbf{I}_M)$$

$$= \log_2 \det\left(\begin{bmatrix} 1+\frac{Np_d}{M} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1+\frac{Np_d}{M} \end{bmatrix}_{M\times M}\right) = M \log_2(1+\frac{Np_d}{M})$$

Increase as N with log shape

Fundamental Overview: Massive MIMO
: Multi-User MIMO (1/4)

- Multi-user MIMO Uplink
  - *M* antennas simultaneously serves *K* users
  - Each user has single antenna, *N* = 1
  - Channel modeling (large scale + small scale)
    - Propagation matrix **G**

$$\underset{[M\times K]}{\mathbf{G}_{UL}} = \underset{[M\times K]}{\mathbf{H}}\,\underset{[K\times K]}{\mathbf{D}^{1/2}}$$

H: small scale fading
D: large scale fading (path loss, shadow fading)

- Massive MU-MIMO Uplink

$$\left(\frac{\mathbf{G}^H\mathbf{G}}{M}\right)_{M>>K} = \mathbf{D}^{1/2}\left(\frac{\underset{[K\times M]}{\mathbf{H}^H}\,\underset{[M\times K]}{\mathbf{H}}}{M}\right)\mathbf{D}^{1/2} \approx \mathbf{D}$$

$$\boxed{\frac{1}{M}\underset{[K\times M]}{\mathbf{H}}\,\underset{[M\times K]}{\mathbf{H}^H} \approx \mathbf{I}_K}$$

  - Only large scale fading coefficients remain

Fundamental Overview: Massive MIMO
: Multi-User MIMO (2/4)

MRC: Maximal Ratio Combining

- Massive MU-MIMO Uplink
  - UL Received vector for *K* users

$$\mathbf{y} = \sqrt{p_u} \underset{[M\times K][K\times 1]}{\mathbf{G}\mathbf{x}} + \underset{[K\times 1]}{\mathbf{n}}$$

> - Assume Perfect CSIT
> - Symbol power: normalized to 1

  - Total capacity of UL MU-MIMO

$$\left(\frac{\mathbf{G}^H\mathbf{G}}{M}\right)_{M>>K} = \mathbf{D}^{1/2}\left(\frac{\mathbf{H}^H\mathbf{H}}{M}\right)\mathbf{D}^{1/2} \approx \mathbf{D}$$

$$C_{\text{sum\_UL}} = \log_2 \det(\mathbf{I}_K + p_u \mathbf{G}^H \mathbf{G})$$

$$C_{\text{sum\_UL}\ M>>K} \approx \log_2 \det(\mathbf{I}_K + M p_u \mathbf{D})$$

  - Detection using MRC

$$\mathbf{G}^H\mathbf{y} = \sqrt{p_u}\mathbf{G}^H\mathbf{G}\mathbf{x} + \mathbf{G}^H\mathbf{n} \approx M\sqrt{p_u}\mathbf{D}\mathbf{x} + \mathbf{G}^H\mathbf{n}$$

Achievable by detection using MRC

    - SNR vector for *K* users

$$\boldsymbol{\rho} = \frac{M^2 p_u \mathbf{D}^2}{\mathbf{G}^H\mathbf{G}} \approx \frac{M^2 p_u \mathbf{D}^2}{M\mathbf{D}} = M p_u \mathbf{D}$$

Fundamental Overview: Massive MIMO
: Multi-User MIMO (3/4)

- Multi-user MIMO Downlink
  - *M* antennas simultaneously serves *K* users
  - Each user has single antenna, *N* = 1
  - Channel modeling (large scale + small scale)
    - Propagation matrix **G**

$$\underset{[K\times M]}{\mathbf{G}_{DL}} = \underset{[K\times K]}{\mathbf{D}^{1/2}} \underset{[K\times M]}{\mathbf{H}}$$

H: small scale fading
D: large scale fading (path loss, shadow fading)

- Massive MU-MIMO Downlink

$$\left(\frac{\mathbf{G}\mathbf{G}^H}{M}\right)_{M>>K} = \mathbf{D}^{1/2}\left(\frac{\underset{[K\times M]}{\mathbf{H}}\,\underset{[M\times K]}{\mathbf{H}^H}}{M}\right)\mathbf{D}^{1/2} \approx \mathbf{D} \qquad \boxed{\frac{1}{M}\underset{[K\times M]}{\mathbf{H}}\,\underset{[M\times K]}{\mathbf{H}^H} \approx \mathbf{I}_N}$$

  - Only large scale fading coefficients remain

Fundamental Overview: Massive MIMO
: Multi-User MIMO (4/4)

- Massive MU-MIMO Downlink
  - DL Received vector for BS (equal power allocation assumption)

$$\mathbf{y} = \sqrt{p_{\mathrm{d}}} \underset{[K\times M][M\times 1]}{\mathbf{Gx}} + \underset{[K\times 1]}{\mathbf{n}}$$

$$\left(\frac{\mathbf{GG}^H}{M}\right)_{M>>K} = \mathbf{D}^{1/2}\left(\frac{\mathbf{HH}^H}{M}\right)\mathbf{D}^{1/2} \approx \mathbf{D}$$

  - Total capacity of DL MU-MIMO

$$C_{\mathrm{sum\_DL}} = \log_2 \det(\mathbf{I}_M + p_{\mathrm{d}}\mathbf{G}^H\mathbf{G}) = \log_2 \det(\mathbf{I}_M + p_{\mathrm{d}}\mathbf{H}^H\mathbf{D}\mathbf{H})$$

$$= \log_2 \det(\mathbf{I}_K + p_{\mathrm{d}}\mathbf{D}\mathbf{H}\mathbf{H}^H) \approx \log_2 \det(\mathbf{I}_K + Mp_{\mathrm{d}}\mathbf{D})$$

  - Maximal Ratio Transmission (MRT) precoding based received signal

$$\mathbf{x} = \frac{1}{\sqrt{\gamma}}\mathbf{H}^H\tilde{\mathbf{x}} \approx \frac{1}{\sqrt{M}}\mathbf{H}^H\tilde{\mathbf{x}}$$

Achievable by MRT

$$\Rightarrow \mathbf{y} = \sqrt{\frac{p_d}{M}}\mathbf{G}\mathbf{H}^H\tilde{\mathbf{x}} + \mathbf{n} = \sqrt{\frac{p_d}{M}}\mathbf{D}^{1/2}\mathbf{H}\mathbf{H}^H\tilde{\mathbf{x}} + \mathbf{n} = \sqrt{Mp_d}\,\mathbf{D}^{1/2}\underbrace{\frac{\mathbf{HH}^H}{M}}_{\rightarrow \mathbf{I}_K}\tilde{\mathbf{x}} + \mathbf{n} \approx \sqrt{Mp_d}\,\mathbf{D}^{1/2}\tilde{\mathbf{x}} + \mathbf{n}$$

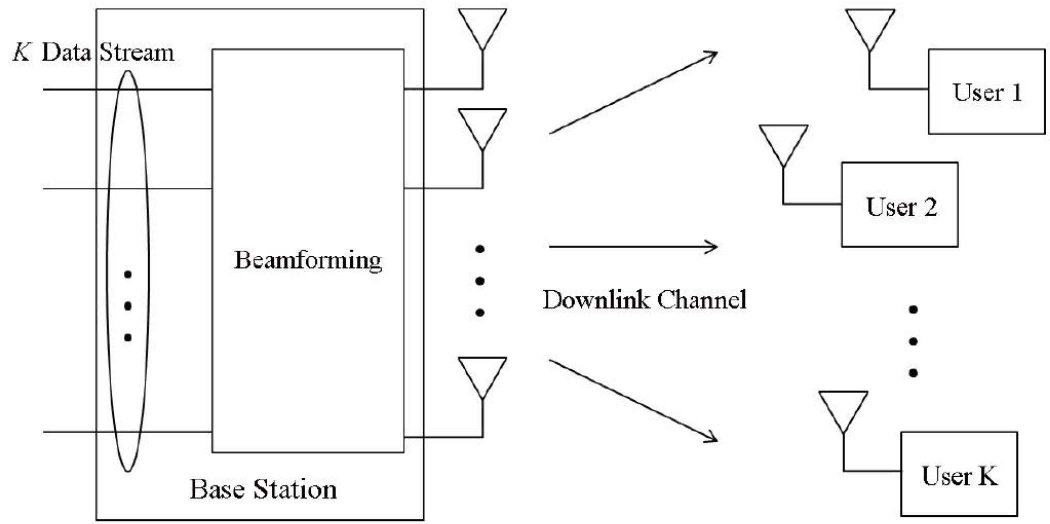
    - SNR vector for K users: $\boldsymbol{\rho} = Mp_d\mathbf{D}$

Massive MIMO Downlink Channel

Deterministic Equivalent for the Achievable Sum Rate

Massive MIMO Downlink Channel
: System Model

- Parameters
  - $\mathbf{h}_k$: small scale fading
  - $\mathbf{w}_k$: beamforming vector
  - M : # of BS antenna
  - K: # of users
  - Single antenna at user



- Received vector

$$\underset{[K\times 1]}{\mathbf{y}} = \sqrt{p_d}\underset{[K\times M]}{\mathbf{H}}\underset{[M\times 1]}{\mathbf{x}} + \underset{[K\times 1]}{\mathbf{n}} = \sqrt{p_d}\,\mathbf{H}\underset{[M\times K]}{\mathbf{W}}\underset{[K\times 1]}{\mathbf{s}} + \mathbf{n}$$

$$E\left[\|\mathbf{x}\|^2\right] = \mathrm{tr}\left(\mathbf{W}^H\mathbf{W}\right) \le 1,\ n_i \sim \mathcal{CN}\left(0,1\right)$$

$$\mathbf{H} \triangleq \left[\mathbf{h}_1,\ldots,\mathbf{h}_K\right],\ \mathbf{W} \triangleq \left[\mathbf{w}_1,\ldots,\mathbf{w}_K\right]$$

Massive MIMO Downlink Channel
: Linear Precoding

MRT: Maximal Ratio Transmission
ZFBF: Zero-Forcing Beamforming

- Conventional linear precoding

MRT	ZFBF
$\mathbf{W} = \mathbf{H}^H$	$\mathbf{W} = \mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H \right)^{-1}$

- Received signal after using linear precoding

$$y_k = \underbrace{\sqrt{p_d}\,\mathbf{h}_k \mathbf{w}_k s_k}_{\text{desired signal}} + \underbrace{\sqrt{p_d} \sum_{i=1, i\neq k}^{K} \mathbf{h}_k \mathbf{w}_i s_i}_{\text{interference}} + \underbrace{\mathbf{n}}_{\text{noise}}$$

- SINR of the *k*th user

$$\mathrm{SINR}_k = \frac{p_d \left| \mathbf{h}_k \mathbf{w}_k \right|^2}{p_d \sum_{i=1, i\neq k}^{K} \left| \mathbf{h}_k \mathbf{w}_i \right|^2 + 1}$$

- Rate of user k

$$R_k = \log_2 \left(1 + \mathrm{SINR}_k \right)$$

- Ergodic sum rate

$$R_{\text{sum}} = \sum_{k=1}^{K} E\left\{ R_k \right\}$$

Massive MIMO Downlink Channel
: Linear Precoding – Perfect CSI

SINR: Signal-to-Interference-plus-Noise Ratio
CSI: Channel State Infromation

- Deterministic form of the $\mathrm{SINR}_k/R_{\mathrm{sum}}$ as M, $K \to \infty$, $M/K=\alpha$
  - MRT $\boxed{\frac{1}{K}\sum_{i=1,i\neq k}^{K}\left|\mathbf{h}_k\mathbf{h}_i^H\right|^2 \approx E\left\{\left|\mathbf{h}_k\mathbf{h}_i^H\right|^2\right\} = M \left(\because \left|\mathbf{h}_k\mathbf{h}_i^H\right|^2 \sim \chi_M^2\right),\ \gamma = \left\|\mathbf{H}^H\right\|_F^2 \approx KM}$

$$\mathrm{SINR}_k^{mrt} = \frac{\frac{p_d}{\gamma}\left|\mathbf{h}_k\mathbf{h}_k^H\right|^2}{\frac{p_d}{\gamma}\sum_{i=1,i\neq k}^{K}\left|\mathbf{h}_k\mathbf{h}_i^H\right|^2+1} \xrightarrow{a.s.} \frac{p_d\alpha}{p_d+1} \text{ as } M, K \to \infty$$

$$R_{\mathrm{sum}}^{mrt} = K\cdot\log_2\left(1+\frac{p_d\alpha}{p_d+1}\right)$$

  - ZFBF $\boxed{1/\mathrm{tr}\left(\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right) \approx \text{Diversity order of ZF-BF} = \frac{M-K}{K}}$

$$\mathrm{SINR}_k^{zf} = \frac{p_d}{\mathrm{tr}\left(\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right)} \xrightarrow{a.s.} p_d(\alpha-1) \text{ as } M, K \to \infty$$

$$R_{\mathrm{sum}}^{zf} = K\cdot\log_2\left(1+p_d(\alpha-1)\right)$$

Massive MIMO Downlink Channel
: Linear Precoding – Imperfect CSI

- Estimated CSI ($\hat{\mathbf{H}}$) using MMSE channel estimation

$$\hat{\mathbf{H}} = \xi \mathbf{H} + \sqrt{1-\xi^2}\mathbf{E}$$

  - $\mathbf{E}$: Error matrix where $\mathbf{e}_i \sim \mathcal{CN}(0,1)$
  - ξ : Reliability of the estimation

- Deterministic form of R_{sum} with imperfect CSI

MRT

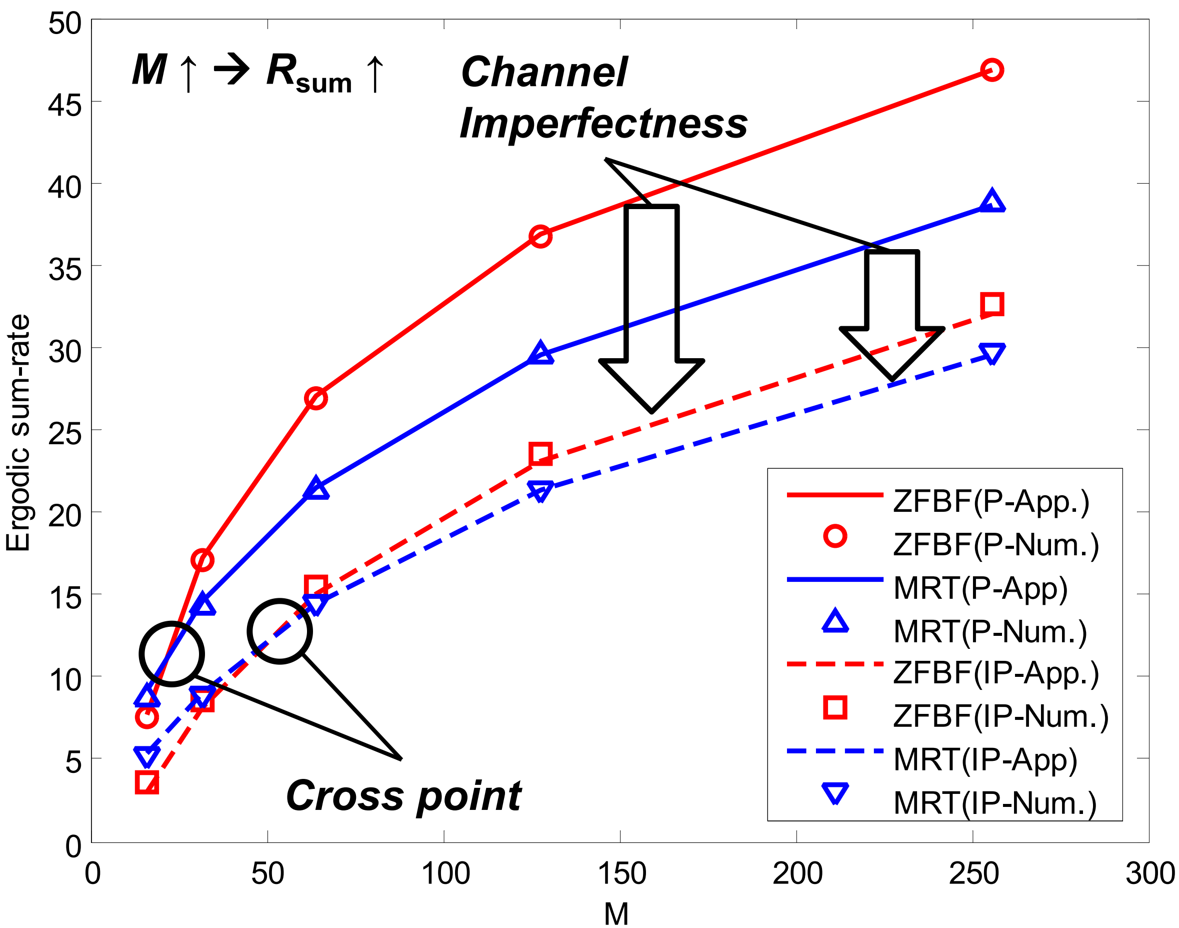
$$R_{\text{sum}}^{mf} \approx K \cdot \log_2\left(1+\frac{\xi^2 p_d \alpha}{p_d+1}\right)$$

ZFBF

$$R_{\text{sum}}^{zf} \approx K \cdot \log_2\left(1+\frac{\xi^2 p_d (\alpha-1)}{(1-\xi^2) p_d+1}\right)$$

Massive MIMO Downlink Channel
: Linear Precoding – Simulation Results (2/2)

- Ergodic sum-rate vs. *M*



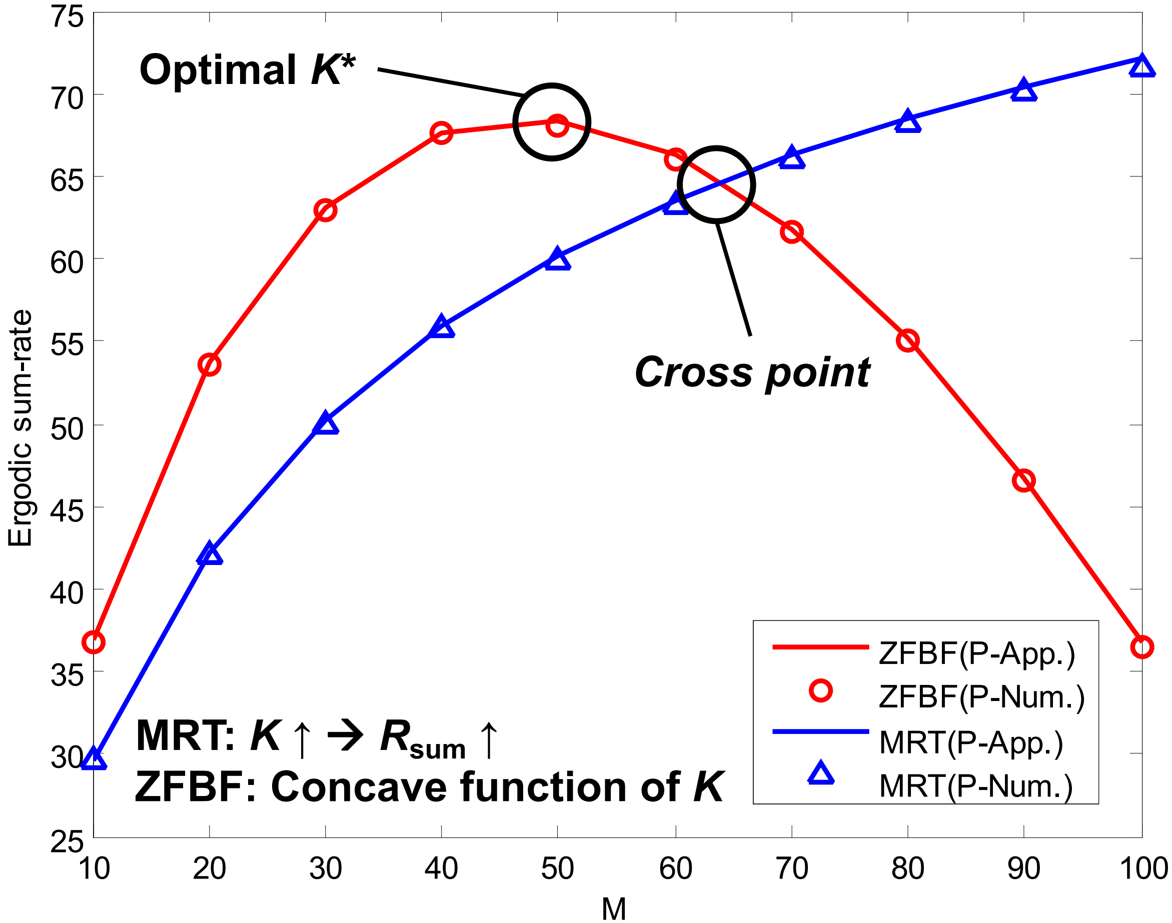
Simulation parameters
- $M = \{2^4, 2^5, 2^6, 2^7, 2^8\}$
- $K = 10$
- $p_d = 0\text{dB}$
- $\xi^2 = 0.5$

Deterministic form of R_{sum}
- Perfect CSI, Imperfect CSI
- The approximations is very cl ose to the numerical result
- It is also accurate even for s mall *M*, *K*.

Performance comparison
- As α>>1, ZFBF > MRT
- As α=1, MRT > ZFBF

Massive MIMO Downlink Channel
: Linear Precoding – Simulation Results (2/2)

- Ergodic sum-rate vs. *K*



- **Simulation parameters**
  - $M = 128$
  - $K = \{10,\ 20,\ \ldots,\ 100\}$
  - $p_d = 0\text{dB}$
  - $\xi^2 = 0.5$
- **Performance comparison**
  - As α>>1, ZFBF > MRT
  - As α=1, MRT > ZFBF
- **Optimal *K** for ZFBF**
  - $K^* = \left\{ K \,\middle|\, \frac{\partial R_{\text{sum}}^{zf}}{\partial K} = 0 \right\}$
  - ex) $K^* = \frac{M+1}{e}$ where $p_d = 0\text{dB}$

Massive MIMO Uplink Channel

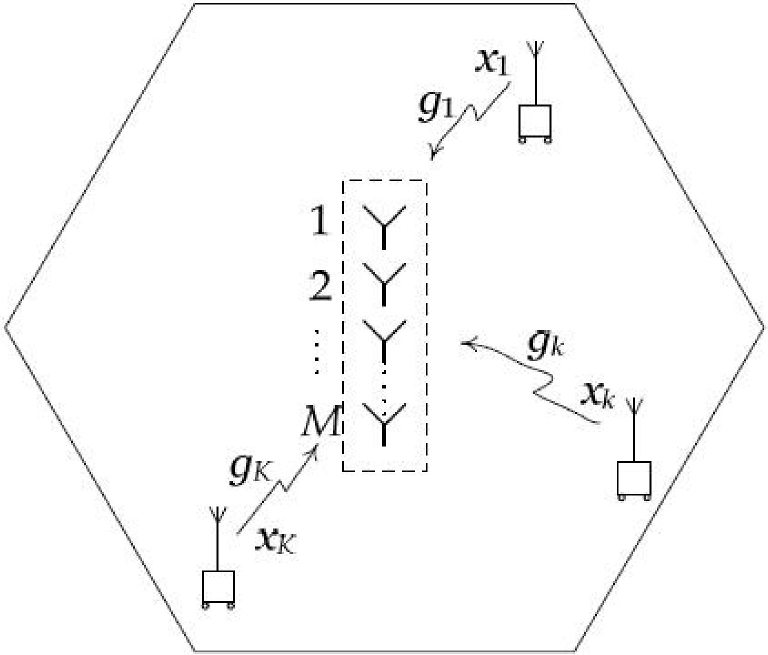
Energy and Spectral Efficiency with Linear Receivers

Massive MIMO Uplink Channel
: Contents

- Massive MIMO Uplink Channel
  - System model
  - Uplink power efficiency – Perfect CSI case
  - Uplink power efficiency – Imperfect CSI case
  - Energy efficiency and spectral efficiency tradeoff

Massive MIMO Uplink Channel
: System Model

- Parameters
  - $\mathbf{g}_k = \sqrt{\beta_k}\mathbf{h}_k$
  - $\mathbf{h}_k$: small scale fading
  - β_k : path loss + shadowing
  - SNR for the *k*th UE: $p_u \beta_k$



- Received signal

$$\underset{[M\times 1]}{\mathbf{y}} = \sqrt{p_u}\underset{[M\times K]}{\mathbf{G}}\underset{[K\times 1]}{\mathbf{x}} + \underset{[M\times 1]}{\mathbf{n}}$$

$$E\left[\left|x_k\right|^2\right] = 1,\ n_i \sim \mathcal{CN}\left(0,1\right)$$

$$\mathbf{G} = \mathbf{H}\underset{[K\times K]}{\mathbf{D}}^{1/2},\ \ \mathbf{H} \triangleq \left[\mathbf{h}_1,\ldots,\mathbf{h}_K\right],\ \ \mathbf{D} \triangleq \mathrm{diag}\left(\beta_1,\ldots,\beta_K\right)$$

Massive MIMO Uplink Channel
: Linear Detector

MRC: Maximal Ratio Combining
ZF: Zero-Forcing
MMSE: Minimum Mean Square Error

- Conventional linear detector

MRC	ZF	MMSE
$\mathbf{A}=\mathbf{G}$	$\mathbf{A}=\mathbf{G}\left(\mathbf{G}^H\mathbf{G}\right)^{-1}$	$\mathbf{A}=\mathbf{G}\left(\mathbf{G}^H\mathbf{G}+\frac{1}{p_u}\mathbf{I}_K\right)^{-1}$

- Received signal after using the linear detector

$$\mathbf{r}=\sqrt{p_u}\mathbf{A}^H\mathbf{G}\mathbf{x}+\mathbf{A}^H\mathbf{n}\rightarrow r_k=\sqrt{p_u}\mathbf{a}_k^H\mathbf{G}\mathbf{x}+\mathbf{a}_k^H\mathbf{n}=\underbrace{\sqrt{p_u}\mathbf{a}_k^H\mathbf{g}_k x_k}_{\text{desired signal}}+\underbrace{\sqrt{p_u}\sum_{i=1,i\neq k}^{K}\mathbf{a}_k^H\mathbf{g}_i x_i}_{\text{interference}}+\underbrace{\mathbf{a}_k^H\mathbf{n}}_{\text{noise}}$$

- SINR of the *k*th user

$$\mathrm{SINR}_{\mathrm{k}}=\frac{p_u\left|\mathbf{a}_k^H\mathbf{g}_k\right|^2}{p_u\sum_{i=1,i\neq k}^{K}\left|\mathbf{a}_k^H\mathbf{g}_i\right|^2+\left\|\mathbf{a}_k\right\|^2}$$

Massive MIMO Uplink Channel
: Uplink Power Efficiency – Perfect CSI (1/2)

- Ergodic achievable uplink rate of the *k*th user

$$R_{P,k} = E\left\{\log_2\left(\mathrm{SINR}_k\right)\right\} = E\left\{\log_2\left(1 + \frac{p_u \left|\mathbf{a}_k^H \mathbf{g}_k\right|^2}{p_u \sum_{i=1, i\neq k}^{K} \left|\mathbf{a}_k^H \mathbf{g}_i\right|^2 + \left\|\mathbf{a}_k\right\|^2}\right)\right\}$$

- ***Proposition 1***: Assume that the BS has perfect CSI and the transmit power of each user is scaled with M_t according to $p_u = \frac{E_u}{M}$, where E_u is fixed. The n,

$$R_{P,k} \to \log_2\left(1 + \beta_k E_u\right), \quad M \to \infty$$

  - Massive MIMO effect
    - Small-scale fading/ Inter-user interference goes away in the limit

Massive MIMO Uplink Channel
: Uplink Power Efficiency – Perfect CSI (2/2)

- Uplink performance with MRC – Perfect CSI

Capacity lower bound

$$R_{P,k}^{\mathrm{mrc}} = E\left\{\log_2\left(1+\frac{p_u\left\|g_k\right\|^4}{p_u\sum_{i=1,i\neq k}^{K}\left|g_k^H g_i\right|^2+\left\|g_k\right\|^2}\right)\right\}$$

$$\geq \log_2\left(1+\left(E\left\{\frac{p_u\sum_{i=1,i\neq k}^{K}\left|g_k^H g_i\right|^2+\left\|g_k\right\|^2}{p_u\left\|g_k\right\|^4}\right\}\right)^{-1}\right)$$

$$=\log_2\left(1+\frac{p_u\left(M_t-1\right)\beta_k}{p_u\sum_{i=1,i\neq k}^{K}\beta_i+1}\right)\triangleq \tilde{R}_{P,k}^{\mathrm{mrc}}$$

$p_u=E_u/M$

Limit case

$$\tilde{R}_{P,k}^{\mathrm{mrc}}=\log_2\left(1+\frac{\frac{E_u}{M}\left(M-1\right)\beta_k}{\cancel{\frac{E_u}{M}\sum_{i=1,i\neq k}^{K}\beta_i}+1}\right)$$

$$\rightarrow \log_2\left(1+\beta_k E_u\right)\ \text{as}\ M\rightarrow\infty$$

- **Small-scale fading/ Inter-user interference goes away in the limit !**
- **Tx power can be scaled as** $\propto 1/M$ **!!**

Massive MIMO Uplink Channel
: Uplink Power Efficiency – Imperfect CSI (1/3)

- Estimated CSI ($\hat{\mathbf{G}}$) using MMSE channel estimation

$$\hat{\mathbf{G}} = \mathbf{G} + \mathbf{E}$$

  - $\mathbf{E}$: Error matrix where $\mathbf{e}_i \sim \mathcal{CN}\left(0, \frac{\beta_i}{p_p \beta_i + 1} \mathbf{I}_M\right)$
  - $p_p = \tau p_u$: Uplink pilot power
  - τ : # of pilots

- Received signal vector after using the linear detector

$$\hat{\mathbf{r}} = \hat{\mathbf{A}}^H \left(\sqrt{p_u} \hat{\mathbf{G}} \mathbf{x} - \sqrt{p_u} \mathbf{E} \mathbf{x} + \mathbf{n} \right)$$

$$r_k = \sqrt{p_u} \hat{\mathbf{a}}_k^H \hat{\mathbf{G}} \mathbf{x} - \sqrt{p_u} \hat{\mathbf{a}}_k^H \mathbf{E} \mathbf{x} + \hat{\mathbf{a}}_k^H \mathbf{n}$$

$$= \underbrace{\sqrt{p_u} \hat{\mathbf{a}}_k^H \hat{\mathbf{g}}_k x_k}_{\text{desired signal}} + \underbrace{\sqrt{p_u} \sum_{i=1, i \neq k}^{K} \hat{\mathbf{a}}_k^H \hat{\mathbf{g}}_i x_i}_{\text{inter-user interference}} - \underbrace{\sqrt{p_u} \sum_{i=1}^{K} \hat{\mathbf{a}}_k^H \mathbf{e}_i x_i}_{\substack{\text{interference from} \\ \text{channel estimation error}}} + \underbrace{\hat{\mathbf{a}}_k^H \mathbf{n}}_{\text{noise}}$$

Massive MIMO Uplink Channel
: Uplink Power Efficiency – Imperfect CSI (2/3)

- Ergodic achievable uplink rate of the *k*th user

$$R_{IP,k} = E\left\{\log_2\left(1+\frac{p_u\left|\hat{\mathbf{a}}_k^H\hat{\mathbf{g}}_k\right|^2}{p_u\sum_{i=1,i\neq k}^{K}\left|\hat{\mathbf{a}}_k^H\hat{\mathbf{g}}_i\right|^2+p_u\left\|\hat{\mathbf{a}}_k\right\|^2\sum_{i=1}^{K}\frac{\beta_i}{\boxed{\tau p_u}\beta_i+1}+\left\|\hat{\mathbf{a}}_k\right\|^2}\right)\right\}$$

Pilot power (p_p)

  - If we cut the Tx power

    → Both data and pilot signal suffer from the reduction in power.

    → We cannot reduce power proportionally to 1/*M*.

- ***Proposition 2:*** Assume that the BS has imperfect CSI, obtained by MMSE estimation from uplink pilots, and that the transmit power of each user is $p_u = \frac{E_u}{\sqrt{M}}$, where E_u is fixed. Then,

$$R_{IP,k} \to \log_2\left(1+\tau\beta_k^2E_u^2\right),\ \text{as } M\to\infty$$

Massive MIMO Uplink Channel
: Uplink Power Efficiency – Imperfect CSI (3/3)

- Uplink performance with MRC – Imperfect CSI

Capacity lower bound

Squaring effect

$$\tilde{R}_{IP,k}^{\text{mrc}} = \log_2\left(1 + \frac{\boxed{\tau p_u^2}\left(M-1\right)\beta_k^2}{p_u\left(\tau p_u \beta_k + 1\right)\sum_{i=1,i\neq k}^{K}\beta_i + \left(\tau+1\right)p_u\beta_k + 1}\right)$$

Limit case ($M \to \infty$)

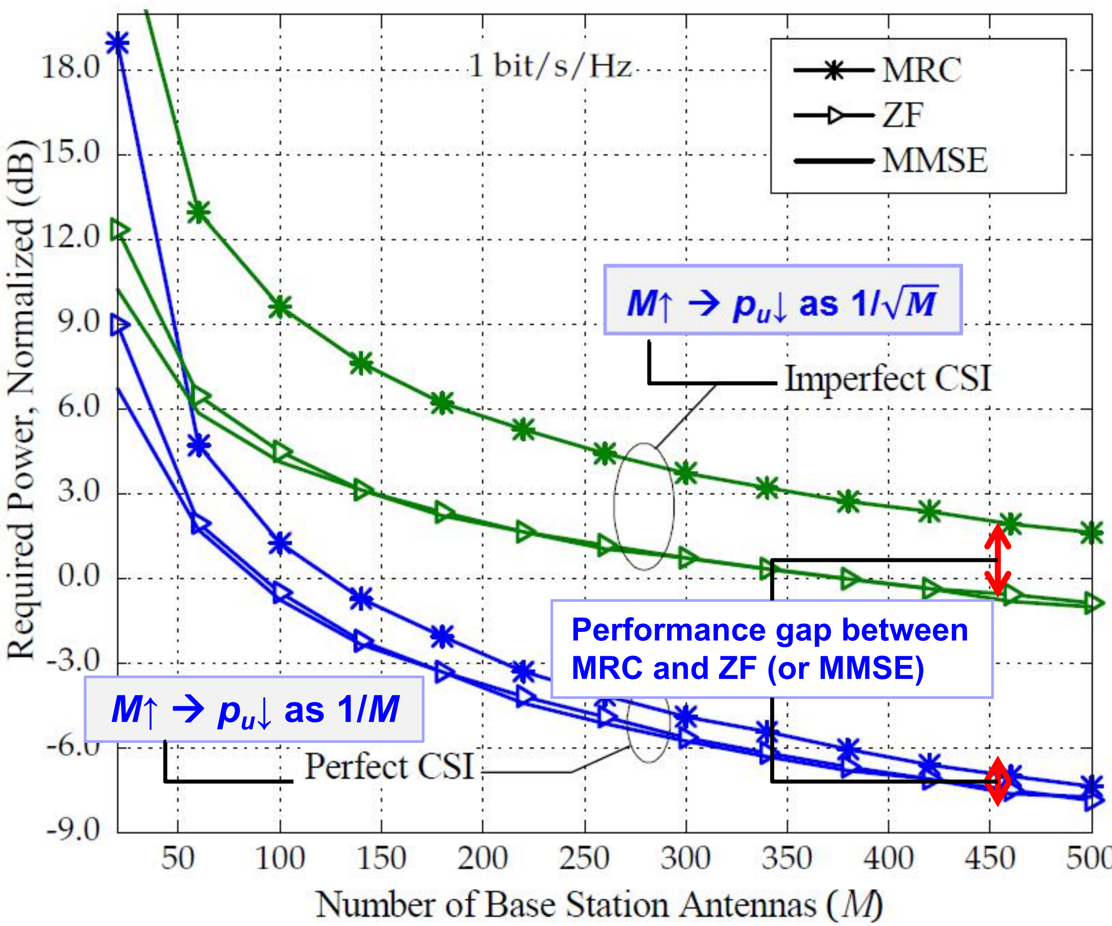
$$\tilde{R}_{IP,k}^{\text{mrc}} \to \begin{cases} \infty & p_u = E_u \\ 0 & p_u = E_u / M \\ \log_2\left(1 + \tau\beta_k^2 E_u^2\right) & p_u = E_u / \sqrt{M} \end{cases}$$

- **Case 1. Without reduction of p_u**
- **Case 2. With scaled p_u as 1/M**
- **Case 3.With scaled p_u as 1/$\sqrt{M}$**

□ **Tx power can be scaled as** $\propto 1/\sqrt{M}$!!

Massive MIMO Uplink Channel
: Uplink Power Efficiency – Simulation Results

- Required Power vs. *M*



- **Simulation parameters**
  - ***K*=10**
  - **Target rate: 1bit/s/Hz**
- **Power scaling low**
  - **Perfect CSI** $\propto 1/M$
  - **Imperfect CSI** $\propto 1/\sqrt{M}$
- **As *M* increases, the difference in performance between MRC and ZF (or MMSE) decreases.**
  - **Perfect CSI: less than 1dB**
  - **Imperfect CSI: less than 3dB**

Multi-Cell Massive MIMO

Pilot Contamination & Inter-Cell Interference Problem

Multi-Cell Massive MIMO
: Inter-Cell Interference Problem (1/3)

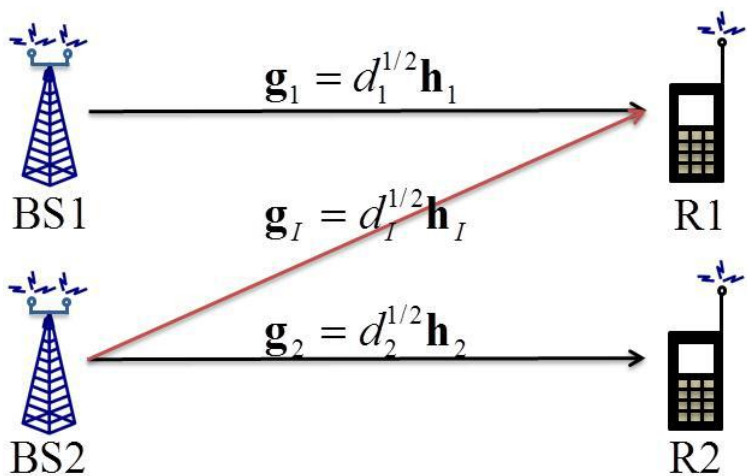
- Non-cooperative multi-cell environment
  - Assumption: MRT precoder / 2 cell assumption
  - MRT based transmit signal
    - Transmit signal at BS 1: $x_1 = \frac{1}{\sqrt{\gamma_1}}\mathbf{h}_1^H \tilde{x}_1 \approx \frac{1}{\sqrt{M}}\mathbf{h}_1^H \tilde{x}_1$
    - Transmit signal at BS 2: $x_2 = \frac{1}{\sqrt{\gamma_2}}\mathbf{h}_2^H \tilde{x}_2 \approx \frac{1}{\sqrt{M}}\mathbf{h}_2^H \tilde{x}_2$

For large M

$$\left(x \approx \frac{1}{\sqrt{M}}\mathbf{h}^H \tilde{x} \right)$$

  - Received signal for R1

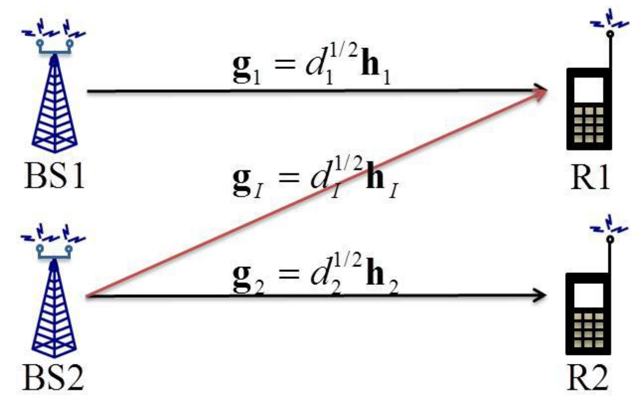
$$y_1 \approx \sqrt{\frac{p_d}{M}} d_1^{1/2}\mathbf{h}_1\mathbf{h}_1^H \tilde{x}_1 + \sqrt{\frac{p_d}{M}} d_I^{1/2}\mathbf{h}_I\mathbf{h}_2^H \tilde{x}_2 + \mathbf{n}$$



Multi-Cell Massive MIMO
: Inter-Cell Interference Problem (2/3)

- Non-cooperative multi-cell environment
  - Received signal for R1

$$y_1 \approx \sqrt{\frac{p_d}{M}} d_1^{1/2} \mathbf{h}_1 \mathbf{h}_1^H \tilde{x}_1 + \sqrt{\frac{p_d}{M}} d_I^{1/2} \mathbf{h}_I \mathbf{h}_2^H \tilde{x}_2 + \mathbf{n}$$



  - Scaling by $1/\sqrt{M}$ at received signal y_1

$$\frac{1}{\sqrt{M}} y_1 \approx \sqrt{p_d} d_1^{1/2} \frac{\mathbf{h}_1 \mathbf{h}_1^H}{M} \tilde{x}_1 + \sqrt{p_d} d_I^{1/2} \frac{\mathbf{h}_I \mathbf{h}_2^H}{M} \tilde{x}_2 + \frac{1}{\sqrt{M}} \mathbf{n} \rightarrow \mathbf{0}$$

  - For large MIMO (large M),

$$\frac{\|\mathbf{h}_i\|^2}{M} = \frac{|h_1^i|^2 + \cdots + |h_M^i|^2}{M} \approx \underbrace{\mathrm{Var}[h]}_{1} + \underbrace{\left(E[h]\right)^2}_{0} = 1$$

$$\left.\frac{\mathbf{h}_i \mathbf{h}_j^H}{M}\right|_{(i \neq j)} = \frac{1}{M}\left(\underbrace{h_1^i h_1^{j*}}_{\text{Gaussian}} + \underbrace{h_2^i h_2^{j*}}_{\text{Gaussian}} + \cdots + \underbrace{h_M^i h_M^{j*}}_{\text{Gaussian}}\right) = \frac{g_1 + g_2 + \cdots + g_M}{M} \approx E[g] = E[h] = 0$$

Multi-Cell Massive MIMO
: Inter-Cell Interference Problem (3/3)

- Non-cooperative multi-cell environment
  - Scaling by $1/\sqrt{M}$ at received signal y_1 with large M,

$$\frac{1}{\sqrt{M}}y_1 \approx \sqrt{p_d}d_1^{1/2}\underbrace{\frac{\mathbf{h}_1\mathbf{h}_1^H}{M}}_{\to 1}\tilde{x}_1 + \sqrt{p_d}d_I^{1/2}\underbrace{\frac{\mathbf{h}_I\mathbf{h}_2^H}{M}}_{\to 0}\tilde{x}_2 + \underbrace{\frac{1}{\sqrt{M}}\mathbf{n}}_{\to 0}$$

$$\approx \sqrt{p_d}d_1^{1/2}\tilde{x}_1$$

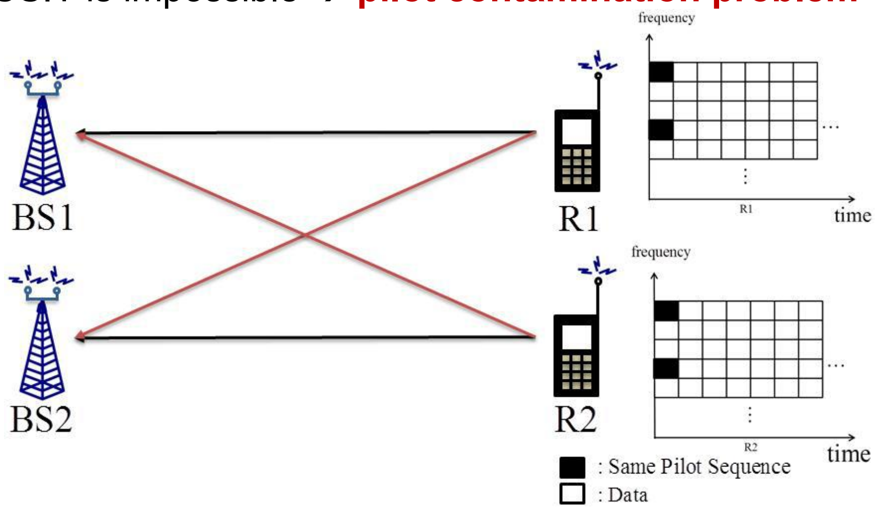
  - Received signal y_1

$$y_1 \approx \sqrt{p_d M}d_1^{1/2}\tilde{x}_1$$

    - No inter-cell interference + noise component
    - Increase as M and DL transmit power

Multi-Cell Massive MIMO
: Pilot Contamination Problem (1/5)

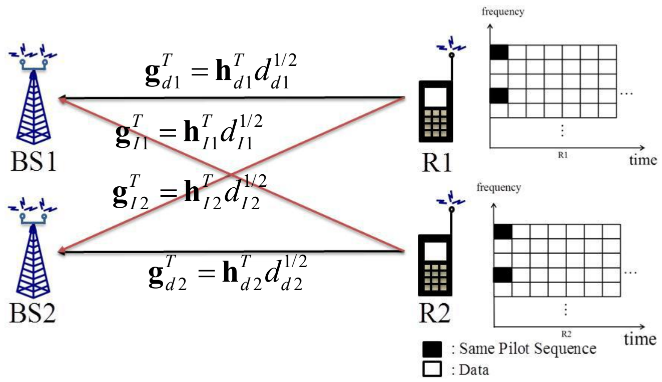
- Practical problems for non-cooperative multi-cell
  - Assumption [7]
    - Non-cooperative multi-cell with TDD mode
    - MIMO-OFDM system with MU-MIMO
  - To perfectly mitigate interference at large *M*, exact MRT scheme is needed
  - Perfect CSIT is necessary to design exact MRT precoder
    - UL pilots are allocated in same time-frequency elements to obtain perfect CSIT
    - UL pilots can be separated by orthogonal sequences → Can obtain perfect CSIT
  - Problem: Exact same time-frequency elements with same pilot sequence
    - Perfect CSIT is impossible → **pilot contamination problem**



Multi-Cell Massive MIMO
: Pilot Contamination Problem (2/5)

- Practical problems for non-cooperative multi-cell
  - From pilot contamination, exact MRT precoder cannot be obtained
    - Inter-cell interference cannot be perfectly mitigated
    - Also, intra-cell interference cannot be perfectly mitigated
  - Interference cannot be vanished even $M \rightarrow \infty$
    - → Inter-cell interference + intra-cell interference remains even $M \rightarrow \infty$

- SIR for 2-cell MU-MIMO with pilot contamination
  - Assume 2 cell environment



$$\hat{\mathbf{g}}_{d1} = \mathbf{g}_{d1} + \mathbf{g}_{I1} + \underbrace{\mathbf{n}_1}_{\text{estimation noise}}, \quad \hat{\mathbf{g}}_{d2} = \mathbf{g}_{d2} + \mathbf{g}_{I2} + \underbrace{\mathbf{n}_2}_{\text{estimation noise}}$$

  - By using MRT

$$y_1 \approx \sqrt{\frac{p_d}{M}} \mathbf{g}_{d1} \hat{\mathbf{g}}_{d1}^H \tilde{x}_1 + \sqrt{\frac{p_d}{M}} \mathbf{g}_{I2} \hat{\mathbf{g}}_{d2}^H \tilde{x}_2 + \mathbf{n}$$

$$= \sqrt{\frac{p_d}{M}} \mathbf{g}_{d1} \left(\mathbf{g}_{d1}^H + \mathbf{g}_{I1}^H + \mathbf{n}_1^H \right) \tilde{x}_1 + \sqrt{\frac{p_d}{M}} \mathbf{g}_{I2} \left(\mathbf{g}_{d2}^H + \mathbf{g}_{I2}^H + \mathbf{n}_2^H \right) \tilde{x}_2 + \mathbf{n}$$

Multi-Cell Massive MIMO
: Pilot Contamination Problem (3/5)

- SIR for 2-cell MU-MIMO with pilot contamination
  - Received signal at R1

$$y_1 \approx \sqrt{\frac{p_d}{M}}\mathbf{g}_{d1}\left(\mathbf{g}_{d1}^H + \mathbf{g}_{I1}^H + \mathbf{n}_1^H\right)\tilde{x}_1 + \sqrt{\frac{p_d}{M}}\mathbf{g}_{I2}\left(\mathbf{g}_{d2}^H + \mathbf{g}_{I2}^H + \mathbf{n}_2^H\right)\tilde{x}_2 + \mathbf{n}$$

  - Desired signal

$$\sqrt{\frac{p_d}{M}} d_{d1}^{1/2}\mathbf{h}_{d1}\left(\mathbf{h}_{d1}^H d_{d1}^{1/2} + \mathbf{h}_{I1}^H d_{I1}^{1/2} + \mathbf{n}_1^H\right)\tilde{x}_1 = \sqrt{Mp_d} d_{d1}^{1/2}\left(\frac{\mathbf{h}_{d1}\mathbf{h}_{d1}^H d_{d1}^{1/2} + \mathbf{h}_{d1}\mathbf{h}_{I1}^H d_{I1}^{1/2} + \mathbf{h}_{d1}\mathbf{n}_1^H}{M}\right)\tilde{x}_1 \approx \sqrt{Mp_d} d_{d1}\tilde{x}_1$$

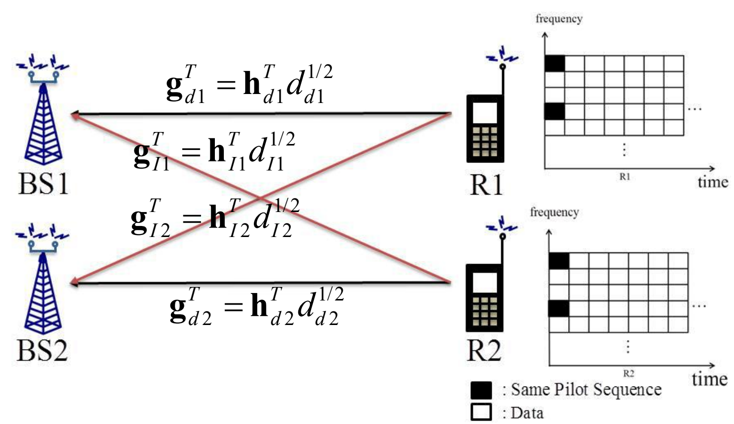
  - Interference signal

$$\sqrt{\frac{p_d}{M}} d_{I2}^{1/2}\mathbf{h}_{I2}\left(\mathbf{h}_{d2}^H d_{d2}^{1/2} + \mathbf{h}_{I2}^H d_{I2}^{1/2} + \mathbf{n}_2^H\right)\tilde{x}_2 = \sqrt{Mp_d} d_{I2}^{1/2}\left(\frac{\mathbf{h}_{I2}\mathbf{h}_{d2}^H d_{d2}^{1/2} + \mathbf{h}_{I2}\mathbf{h}_{I2}^H d_{I2}^{1/2} + \mathbf{h}_{I2}\mathbf{n}_1^H}{M}\right)\tilde{x}_2 \approx \sqrt{Mp_d} d_{I2}\tilde{x}_2$$

  - Received SIR

$$\frac{Mp_d d_{d1}^2}{Mp_d d_{I2}^2} = \frac{d_{d1}^2}{d_{I2}^2}$$

No small scale fading effects
Pilot contamination is independent from pilot power



Multi-Cell Massive MIMO
: Pilot Contamination Problem (4/5)

- SIR for multi-cell MU-MIMO with pilot contamination [7]
  - MRT based SINR for *l*-th UE

$$\mathrm{SIR}_l^{\mathrm{MRT}} = \frac{d_{jjl}^2}{\sum_{n \neq j} d_{jnl}^2}$$

No small scale fading effects
Pilot contamination is independent from pilot power

  - ZF-BF based SINR for *l*-th UE [7]

$$\mathrm{SIR}_l^{\text{ZF-BF}} = \frac{d_{jjl}^2 / \left(\sum_i d_{ijl} + 1/p_p\right)^2}{\sum_{n \neq j} d_{jnl}^2 / \left(\sum_i d_{inl} + 1/p_p\right)^2}$$

No small scale fading effects
Pilot contamination is dependent from pilot power

d_{kjl} : Large scale fading between BS j and UE l in cell k

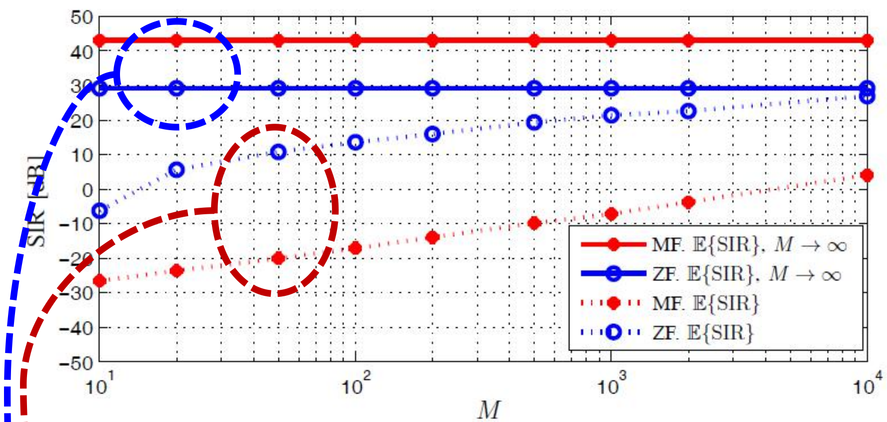
p_p : Pilot Transmission Power

    - If p_p → 0, MRT = ZF-BF

$$\mathrm{SIR}_l^{\text{ZF-BF}} = \frac{d_{jjl}^2 / \left(\sum_i d_{ijl} + 1/p_p\right)^2}{\sum_{n \neq j} d_{jnl}^2 / \left(\sum_i d_{inl} + 1/p_p\right)^2} \approx \frac{d_{jjl}^2 / \left(1/p_p\right)^2}{\sum_{n \neq j} d_{jnl}^2 / \left(1/p_p\right)^2} = \frac{d_{jjl}^2}{\sum_{n \neq j} d_{jnl}^2}$$

Multi-Cell Massive MIMO
: Pilot Contamination Problem (5/5)

- Simulation Results [7]



SIR for MRT and ZF as function of M and K=10

Finite M: ZF > MRT

Infinite M: ZF < MRT

- Simulation Parameters
  - MRT / ZF-BF is considered
  - Number of users (K): 10
  - Cell diameter: 1600m
  - Large scale + small scale considered
  - Performance is limited by pilot contamination
- Conclusion from simulation results
  - Finite *M* → MRT < ZF-BF
  - Infinite *M* → MRT > ZF-BF

Homework

- 1. What are the advantages of Massive MIMO technology？
- 2. For a 100 x 4 Massive MIMO channel, when the transmit power Pd=10 dB, what is the capacity?

- Tips: For Q1, see P5; For Q2, see P15