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**Introduction(p.3-5)**

The modern world faces increasing challenges related to armed conflicts. International, internal, and regional conflicts remain some of the greatest threats to global stability and security. Their analysis and understanding are critical, especially in the context of dynamic political, economic, and social changes. While the vision of entirely predicting wars and eliminating them seems utopian, every improvement in conflict analysis methods brings benefits in reducing their impact and potential escalation.

The goal of this thesis is to contribute to this process by adopting a mathematical approach to the analysis of armed conflicts. Studying such conflicts not only enhances the understanding of their causes and mechanisms but also facilitates the development of strategies to minimize them.

The development of theories and models of armed conflicts has led to many proposals. However, these do not always support the process of scientific inquiry. An excess of models can disrupt the science of conflicts, generating contradictory results and interpretations. For instance, models based on overly simplified assumptions about the rationality of conflict actors often fail in atypical conflicts where cultural and historical factors are decisive. Therefore, it is essential to select models that are not only theoretically sound but also suitable for mathematical representation and practical analysis.

This thesis attempts such an approach, beginning with a review of existing concepts and tools. The initial step involved studying the article “Natural Resources, Conflict, and Conflict Resolution: Uncovering the Mechanisms” by Macartan Humphreys. The author discusses six key mechanisms linking natural resources to armed conflicts, including:

* **Rebel Greed Mechanism:** Resources serve as a source of conflict financing.
* **Greedy Outsiders Mechanism:** The role of third-party states and corporations in exploiting resources.
* **Weak States Mechanism:** Resource dependence weakens state structures.
* **Sparse Networks Mechanism:** Lack of robust economic ties in resource-based economies.

Although these mechanisms are compelling, their application in mathematical models encounters many obstacles. The lack of uniform data and the difficulty of operationalizing these mechanisms make them less practical for creating quantitative models.

An alternative proved to be the **Correlates of War (COW) project**, a comprehensive database initiated by J. David Singer. Recognized as one of the most prominent peace and conflict researchers, Singer developed this project in the 1960s as a tool for analyzing international conflicts. The COW database arose in response to the need for organized knowledge about wars and their determinants, particularly in the context of President Richard Nixon’s administration policies. This project allows systematic analysis of international wars, providing data on state capabilities, armed conflicts, and their effects.

By utilizing this database and the in-depth analyses of its creator, it became possible to apply mathematical methods to the analysis of armed conflicts. This thesis adopts Singer’s approach as the foundation for developing an analytical model, which is further modified and adapted to contemporary research needs.

The objective of this study is not only to present Singer’s approach to armed conflict analysis but also to attempt its modification based on modern data. Such an approach aims to contribute to a better understanding of conflict dynamics and the development of more effective methods for analyzing and forecasting them.

**Chapter 1: Theoretical Foundations of Armed Conflict Analysis (p.5-13)**

**1.1 Introduction (p.5-6)**

**Definition of Armed Conflict in the Context of Mathematical Analysis (p.5)**  
Armed conflict can be defined as a social and political phenomenon in which at least two entities, such as states, social groups, or organizations, use military or armed means to pursue conflicting political, economic, territorial, or ideological interests. These conflicts are characterized by dynamism, complexity, and the influence of multiple political, social, economic, and technological factors.

In the context of mathematical analysis, armed conflict is viewed as a dynamic system where interactions between conflicting parties can be modeled using variables representing resources, strategies, goals, and potential outcomes. Key elements in the mathematical analysis of conflicts include:

1. **Conflict Parties** – Entities involved in the conflict that aim to achieve their goals using available means. These can include states, coalitions, armed groups, or other organizations.
2. **Resources** – Means at the disposal of the conflict parties, such as armed forces, economic resources, political support, or territory.
3. **Strategies** – Potential actions that conflict parties can undertake, such as attack, defense, negotiation, or withdrawal.
4. **Variables and Parameters** – Mathematical quantities representing key aspects of the conflict, such as the number of military units, intensity of actions, duration of the conflict, or the probability of a specific outcome.
5. **Interactions** – Mechanisms and dependencies determining how the actions of one party influence the actions and outcomes of the other party.

Mathematical analysis of armed conflicts involves creating models that enable quantitative representation of the relationships between these elements.

**Application of mathematical models in the study of international conflicts(p.6)**

In making decisions related to armed conflicts, relying solely on the psychology or intuition of one person is extremely risky. As J. David Singer notes in his works, policies based on "intuitive hunches" lead to erroneous predictions and bad decisions. The lack of solid criteria for selecting behaviors makes decision-makers prone to making decisions under the influence of the most persuasive supporters, regardless of whether their theories are sound or not.

This emphasizes the need for more objective and systematic approaches to conflict analysis. An alternative is mathematical modeling, which allows not only to predict the development of war, but also to analyze conflicts from a completely different perspective. Such models allow for the study of the structure of the conflict, the identification of its regularities, and the assessment of the effects of various political decisions, which in turn can help avoid catastrophic mistakes and minimize risk [6].

It is worth emphasizing that the purpose of mathematical analysis of armed conflicts is not only to predict the course of war. These models also serve as tools to better understand the mechanisms of conflict, identify potential critical points, and support peace processes by assessing the effectiveness of alternative de-escalation strategies. In this way, mathematical analysis of conflicts can contribute to more informed and rational decision-making in the field of international policy and security.

However, there are shortcomings of many models in the literature. They often fail to meet basic methodological requirements and lack a longitudinal perspective that would take into account the historical continuum of conflicts. Many models focus solely on one level of analysis, ignoring the links between them, namely the individual, national and international levels. Therefore, a more effective model must incorporate a multi-level approach that recognizes the complex interaction between human nature, national interests and the broader international system [3].

The mathematical model underlying this work allows for the complexity of conflicts to be accounted for and for the identification of key factors influencing their course. The aim of this approach is to understand how different variables, such as the distribution of national capabilities and their changes over time, can be incorporated into a model to predict the occurrence of war. In particular, the model can be adapted to assess the effects of these predictor variables either sequentially (where one variable leads to another) or simultaneously. This chronological aspect is critical because it allows researchers to capture how national capabilities affect the likelihood of war in subsequent periods, essentially linking past conditions to future events [10].

The distribution of capabilities can be measured at specific points in time, while changes in this distribution and movement between states are recorded in the intervals preceding the occurrence of war. Integrating these perspectives allows for a more robust understanding of international dynamics and increases the predictive accuracy of conflict trends [10].

**1.2 Mathematical description of the Singer model(p.7-13)**

The model presented here is designed to answer the question of how the distribution and redistribution of power among states affects the probability of war. There are two distinct and incompatible views on this issue: one predicts that there will be fewer wars when there is approximate parity (and change towards it) between states and a relatively fluid power hierarchy. The other predicts that there will be fewer wars when there is overwhelming power concentrated in the hands of a very few states and a relatively stable rank order among the major powers [10].

These two perspectives were consolidated in a single basic multivariate regression model including three predictor variables (power concentration, the rate and direction of change in concentration, and the movement of power between great powers) and a dependent variable (the intensity of international wars).

**1.2.1 Characteristics of model variables and methods of their measurement(p.7 - 11)**

**Dependent variable: war intensity(p.7)**

In the analysis, we distinguish three main types of armed conflicts: international wars - conflicts in which the main parties are sovereign states; non-state wars - conflicts between a recognized state and a non-state entity; civil wars - conflicts between organized groups within the same state [5]. In this work, we focus exclusively on international wars that meet certain criteria: at least one of the major powers must be an active participant in the conflict, and each side must have suffered at least 1,000 fatalities in battles.

The variable used in the model reflects the intensity of warfare (1.1), measured as the average annual duration of wars waged in that period [10]. Το means that for each country that participated in wars in a given period t (years in which we want to calculate the average number of months of war duration), the months of participation in the war are counted, then summed up and divided by t to average the result:

(1.1)

Where:

w - is the number of wars in the period t2-t1,

p - number of war participants,

m\_{w,p} - number of months of participation of the p participant in the w war.

**The power of the state in the system(Capacities of states)(p.7-9)**

David Singer's analysis of state power uses three basic elements of capacity: demographic, industrial and military. This decision was based on their universality, the possibility of comparison in different historical periods and the availability of data [8]. Below is a detailed justification for the selection of these elements and an explanation of why other potential indicators were rejected.

1. Demographic capacity. Demographic capacity reflects the population potential of a state, which affects its production and military capabilities. Singer proposed measuring this capacity using two key sub-elements:
   1. Total population of a state the number of people is the basis for mobilizing resources, both in the economy and in warfare. The larger the population, the greater the potential of the state.
   2. Urban population ("urban agglomerations") an indicator of the number of people living in urban areas, which are more productive and easier to mobilize in crisis situations.

These sub-items are particularly useful because total values ​​(e.g. total population) are more universal and easier to compare across eras and countries. These indicators were chosen over more specific ones, such as the urbanization index, which better describes economic development but is not crucial for measuring the material capacities of a state.

1. Industrial capacity. Industrial capacity measures a country's ability to produce material resources that are key to its economy and defense. Singer chose two elements:

* Industrial energy consumption, converted into tonnes of coal equivalents, includes various energy sources such as coal, hydropower and nuclear power. This element reflects the overall industrial power of the country.
* Iron and steel production – key indicators of material production, especially in the context of building military infrastructure such as weapons, tanks and ships.

These indicators were considered to be best suited for comparisons over a long historical period. Alternative indicators, such as electronics or chemicals production, were discarded due to difficulties in comparisons across countries and eras.

1. Military capability. Military capability describes the military capabilities of a state and is measured by:

* Military personnel numbers include the number of active soldiers in the armed forces. Although this indicator does not include reserves or paramilitary units, it reflects the direct defense potential of the state.
* Military expenditure measured as cumulative values ​​over the last five years. This indicator illustrates the state's commitment to maintaining and modernizing the armed forces.

Singer chose to exclude other potential elements of capability, such as geography, natural resources, political institutions, culture, or economic indicators.

Geography, while important in specific cases (e.g. access to the sea, mountainous terrain), is too contextual and variable depending on the era and technology. This prevents comparisons over time and between countries.

Natural resources, such as oil, have historically contextualized importance. In addition, their influence is partly reflected in industrial capacity.

Although political institutions affect resource efficiency, their measurement is subjective and difficult to compare. They are not a direct component of the material capabilities of the state.

The influence of culture or ideology is difficult to measure in an objective and uniform way. They are contextual rather than universal.

Gross domestic product, although often used in international analyses, does not directly reflect military or mobilization capacity. Singer found that energy consumption and steel production were better indicators of economic capacity.

Technology changes dynamically, making it difficult to compare historically. Its influence is, however, partly reflected in industrial and military capabilities.

Singer's choice of elements is based on their universality, measurability, and ability to be compared across time and space. Demographic, industrial, and military capabilities are central elements of a state's material power, making them crucial in comparative and long-term analyses. By rejecting alternative indicators, Singer focused on those aspects that best capture the essence of state power in the context of their material capabilities.

The process of calculating state power in the international system, based on six elements of capacity (total state population, urban population, industrial energy consumption, iron and steel production, number of military personnel, military expenditure), includes the following steps [10]:

1. Calculation of total system values ​​for each capability element. In each year, the values ​​of each country in the entire international system are summed for each capability element, and if there is no data for a country, it contributes nothing to the sum. This produces total system values ​​for each component for that year.
2. Determine each country's share in the system. For each country, its share in the system is calculated by dividing the value of the component by the total system value of that component in a given year. If a country does not have a value for a component, its share is coded as missing. This gives each country a share in the system for each of the six capability components.
3. Calculation of state power. The power index for each state in a given year is the arithmetic mean of the non-missing values ​​of that state's shares in all available elements.

The final power value for country i is denoted as S\_i and takes values ​​from 0 to 1.

**Predictor variable: distribution of capasities(**Concentration of Power**) (p.9-10)**

The CON (Concentration of Power Index) variable (1.2) is a key tool for analyzing the concentration of power in the international system. Its main purpose is to quantify the extent to which power in the system is concentrated in the hands of one or a few states or dispersed among many states [7].

The process of calculating the concentration index (CON) involves two key steps. First, the standard deviation of the power of the major powers in the system is calculated. This value is then normalized by dividing it by the maximum possible standard deviation for the system that would occur if one country had 100% power and the others had none:

(1.2)

Where:

S\_i - the power of the state and in time t\_0,

N\_{t\_0} - is the number of countries in the system at time t\_o.

The indicator takes values ​​between 0 and 1. A value of 0 indicates perfect equality in the system, indicating that all states have identical power. A value of 1 indicates complete concentration of power in the system, where one state controls all capabilities.

The concept of the index is based on the assumption that the distribution of power influences the likelihood of conflicts. The calculation of the CON index reflects the methodology of J. David Singer, who aimed for precision and comparability across different historical periods and geopolitical contexts. When analyzing the concentration of power in the international system, the CON index meets key requirements:

* Takes into account the availability of historical data.
* Focuses on objective indicators such as population, military strength and industrial capacity, avoiding subjective assessments.
* It enables tracking changes in the balance of power over time and their impact on the risk of armed conflict.
* Includes all units.
* It is simple to interpret and robust to problems resulting from changes in the number of countries in the system over different time periods.

This method of calculation makes it possible to precisely determine the level of power concentration in international systems and track their changes over time.

**Predictor variable: change in concentration of power (change of capabilities distributions)(p.10-11)**

*(DCON – Delta CON, it can be written like ACON too, mean change of CON)*

The DCON (1.3) variable is a quantitative measure used to analyze changes in the concentration of power in the international system over a specified time period. Its main purpose is to reflect the extent to which the value of the concentration (CON) has increased or decreased over the analyzed period t: (1.3)

Where:

CONto, CONt1 are the power concentration at times to, t₁ respectively, and t₁ - to = t.

The DCON value allows us to precisely determine the direction and intensity of these changes, which makes it an important tool in researching the dynamics of the balance of power in the international system.

The change in concentration can take both positive and negative values. A positive value indicates an increase in the concentration of power in the system. A negative value, on the other hand, indicates a decrease in concentration, which reflects a more balanced distribution of power among states.

The ACON index enables the analysis of long-term trends in the international system, which supports a better understanding of the mechanisms of the changing balance of power and their impact on the stability and dynamics of armed conflicts.

**Predictor variable: capacity(power) movement between great powers(p.11)**

The MOVE index (1.4) is a more complex measure of power redistribution in the international system, reflecting the amount of power that has shifted among major powers in a given period, regardless of whether this leads to a change in their total power. MOVE provides information on the intensity of power redistribution, enabling the analysis of the dynamics of the balance of power in particular periods [10].

The construction of the indicator starts with a comparison of the shares held by each country at the beginning and end of the analyzed period t. In order to make the indicator comparable across periods, taking into account the changing number of members of the international system, normalization is necessary. This process consists in dividing the MOVE value by the maximum possible level of power redistribution. This maximum occurs in the hypothetical case where the country with the lowest initial power would take over all the power in the system, ending up with 100%. In such a scenario, the denominator is calculated as the difference between 100% and the power of the lowest ranked country, multiplied by 2, since any power gained must be lost by other countries:

(1.4)

Where:

S\_{it\_0} - the power of the state and in time t\_0,

S\_{it\_1} - power of state i at time t\_1,

N\_t - number of countries in the system in period t.

S\_{mt} - state with the lowest power at time t = t₁ - to.

When calculating the MOVE indicator, it is necessary to take into account a potential problem. The anomaly is that the international system changes its composition over time, gaining and losing members. Such variability requires normalizing the size of the system to eliminate the inconvenience of differences in the number of members at different points of observation. For consistency, the MOVE indicator only considers the movement of power between states that were members of the system at both the beginning and the end of the analyzed period t. This excludes the distortions resulting from differences in the number of members of the system at different points of observation.

Thanks to the power movement index, it is possible not only to study the dynamics of the balance of power, but also to identify the intensity of power redistribution in the international system, which is an important contribution to the analysis of stability and changes in the global political order.

**1.2.2 Design and operation of the Singer model(p.11-13)**

The basic mathematical model (1.5) that describes the relationship between the configuration of capabilities in the international system and the occurrence of wars can be presented as in the form of a multivariate regression equation:

(1.5)

Where:

WARt1→t2 dependent variable representing the occurrence of war in the time interval between t₁ and t2;

CON is the level of concentration of power over time t\_0;

ACONto→t1 change in power concentration between to and t₁;

MOVEto→t1 amount of shifted power between to and t₁;

a - constant;

B; - regression coefficient, reflecting the influence of individual independent variables on the occurrence of war;

e - random component.

This model allows for the examination of the influence of three key independent variables: power concentration (CON), its change (ACON) and redistribution (MOVE), on the probability of war (WAR).

**Interpretation of time parameters (p.12)**

The selected analysis period is divided into fixed-length intervals:

(1.6)

For example, in the analysis covering the years 1900–1950 zt 5, the first five-year period includes data from January 1, 1900, to December 31, 1904. In this period:

* CONto is calculated based on data from January 1, 1900,
* ACONto→t1, MOVEto→t1 are calculated based on data from January 1, 1900 to December 31, 1904,
* WARt1→t2 represents the occurrence of war in the subsequent period, from January 1, 1905 to December 31, 1909.

This model design allows for the assessment of the impact of power concentration indicators on the probability of armed conflict in the future.

It is worth noting that the length of the time intervals t in the model can be adjusted depending on the needs of the analysis. For shorter time intervals (t = 1), the analysis will be more detailed, while for longer periods (t> 5) it allows for capturing more general trends. In Singer's original model, the value t = 5 was assumed, which means dividing the analysis into five-year periods.

In this paper, it is assumed that the values ​​of t can be adjusted to perform the analysis on different time scales. This approach allows for the precise determination of how the power variables in the international system affect the risk of armed conflict in the short and long term.

**Singer Model Versions (p.13)**

In addition to the basic model, there are three different versions of it, which take into account both the nature of the variable influences (additive or multiplicative) and the chronological order of combining the predictor variables [10].

(1.7)

(1.8)

(1.9)

(1.10)

The distinction between the additive (1.7) (1.8) and multiplicative (1.9) (1.10) versions concerns the way in which the predictor variables affect the probability of war. In the additive version, the effects of the individual variables are additive, meaning that a high value of any variable can lead to an increase in the probability of conflict. In contrast, in the multiplicative version, all variables must be high to signal war, because a low value of any one variable completely negates the effect of the others.

In addition to the nature of the influences of the variables, the chronological sequence in which the variables are measured and combined to explain the occurrence of wars is also important. In this context, two main versions of the model are distinguished. In the first version, CON LEADS (1.7) (1.9), the level of power concentration (CON) is measured at the beginning of period t. This sequence assumes that the level of power concentration signals the subsequent changes and redistribution of power in that period. In contrast, in the CON LAGS version (1.8) (1.10), the level of power concentration (CON) is measured at the end of period t. This sequence assumes that the change and redistribution of power throughout period t indicates the subsequent level of concentration, which in turn determines the probability of war.

Taking both versions into account enables the analysis of the interdependencies between the predictor variables and allows for a better understanding of the dynamics of the relationship between the concentration of power, its changes and redistribution, and the occurrence of wars.

**Chapter 2**

**Empirical analysis of Singer's model(p.15 – 88)**

The analysis of armed conflicts is an important element of research on international relations and the mechanisms of the emergence of wars. In this chapter, we will focus on the application of the Singer model, which allows for the systematic study of conflicts based on specific data and indicators. The inspiration for this research is the approach of J. D. Singer, who developed one of the most influential methods of conflict analysis, the model based on the Correlates of War project. This model allows for the identification of key factors determining the likelihood of the outbreak of conflicts.

The aim of this chapter is to conduct an empirical analysis of armed conflict models in accordance with the methodological assumptions of their author and to extend his approach to new research aspects. We will begin by discussing the data that constitute the basis of the analysis, then move on to the application of key methods and the interpretation of the results in the context of Singer's theory. Then, we will propose our own analysis taking into account different time periods t, and an attempt to extend Singer's study to predict wars for periods later than those originally analyzed.

The conclusions resulting from this research will serve as a basis for assessing the effectiveness of quantitative methods in conflict analysis and their potential application in political and research practice.

It is worth noting at the outset that in the social sciences the interpretation of the results of dependency modeling differs from the approach typical of the exact sciences. This is due to the high variability of parameters and the lower precision in social research. The interpretation of modeling results in this field is more patronizing, because models rarely achieve a high degree of fit (e.g. in the case of the R² index), but their goal is to capture trends, not precise predictions. As the later graphs will show, although the coefficients may be insignificant, the model can capture general trends, even over a period of several years. The key goal of these models is not to accurately predict the date of conflict, but to understand the impact of various indicators on the probability of wars.

**Data (p.15-17)**

The analysis was carried out using data from the Correlates of War (COW) project, which was initiated by J. D. Singer in the 1960s [1]. This project collects, analyzes, and systematizes data on armed conflicts. It contains a variety of data sets covering aspects such as territories, states, religions, and other variables relevant to the analysis of international conflicts. Due to its broad scope and structure, the COW project has become one of the most important data sources in the study of armed conflicts.

The first dataset used is National Material Capabilities (v6.0), which contains information on state capabilities in the years 1816-2016 [10, 8]. This data concerns sovereign states and refers to aspects such as demographic, economic and military potential, which is discussed in detail in Chapter 1.2.1 "The power of the state in the system". This dataset is particularly important in the analysis because it allows for the assessment of the relative power of states in a given period, which is crucial for understanding the dynamics of international conflicts.

It is also important to highlight the specific taxonomy regarding the recognition of states within the COW project. The criteria according to which entities are considered as states in the international system depend on the historical period [1]:

* Before 1920: the entity had to have a population of over 500,000 and maintain diplomatic missions at least at the level of chargé d'affaires to Great Britain and France.
* After 1920: an entity had to be a member of the League of Nations or the United Nations, or meet the population criteria (over 500,000 inhabitants) and be recognized by two major powers.

Such standardization allows for a more objective picture of the phenomena being studied.

The second key dataset used in this analysis is the Inter-State War Data (v4.0), which contains information on various types of wars [5]. The data cover both domestic and international conflicts. However, as discussed in Section 1.2.1 “The Dependent Variable WAR”, this analysis focuses solely on international wars. The dataset covers the period from 1816 to 2003 and provides detailed information on conflicts, their intensity, and the parties involved in the warfare.

The data used in this analysis differ from those available to J. D. Singer in the 1970s, when the Correlates of War (COW) project was in its infancy. Singer worked with early versions of the data sets. Since then, the project has undergone numerous modifications and updates, significantly improving its precision, comprehensiveness, and temporal coverage.

An example of this is the National Material Capabilities dataset, which is used in this paper in its sixth version (NMC v6). This dataset contains data from 1816 to 2016, covering a range of indicators such as demographic, economic and military potential. However, at the time of Singer, when the COW project was still in its infancy, the first version of this dataset was used, much less extensive in terms of both time coverage and variable detail. In the first version, some countries and variables were not yet included, and the data collection methodology was not as precise as it is today.

It is worth emphasizing that the differences between the data versions do not mean that Singer's analyses were less valuable. On the contrary, his work provided the foundation for the project's development and allowed subsequent generations of researchers to refine the methodology and expand the data sets. Modern versions, such as NMC v6, are evidence of the evolution of the COW project and its ability to adapt to new research challenges and available data resources. The use of these latest versions in this analysis allows for more detailed and up-to-date results that are also consistent with the assumptions of the Singer model.

**2.1. Conducting the analysis according to the assumptions of the Singer model(p.17-33)**

In this section, we will present the analytical process according to the assumptions of Singer's model. The aim of this section is to recreate Singer's approach based on his model and apply it to newer data. This will allow us to see whether Singer's conclusions remain valid and how the model performs in a changing international context.

Singer's analysis was conducted for the years 1820-1965 and included two sub-periods: the years 1820-1890 (identifies the 19th century) and the years 1890-1965 (identifies the 20th century). This division allowed for the examination of how variables in the model influenced the occurrence of conflicts in different historical eras, taking into account dynamic changes in the international system, such as industrialization, technological development, or changing alliance structures.

In his analysis, Singer calculated all indicators every five years (t = 5). This means that all predictor variables were calculated for the period 1820–1960, while the dependent variable WAR was calculated for the period 1825–1965. As indicated in Section 1.2.2 “Interpretation of time parameters”, the dependent variable WAR refers to the period t after the period in which the independent variables were calculated. Therefore, the division of periods for the independent variables is 1820–1890 and 1890–1960, and for the dependent variable 1825–1895 and 1895–1965.

When presenting the results in this paper, the periods of calculation of independent variables will be noted to simplify the description, which means that the dependent variable WAR is calculated for the period t = 5 years later.

**Descriptive statistics of model variables(p.17-19)**

Tables 2.1, 2.2 and 2.3 present descriptive statistics for model variables (WAR, CON, ACON, MOVE) in different time periods: 1820-1960, 1820-1890 and 1890-1960. Analysis of these tables allows us to identify differences in variables depending on the period.

(Tables 2.1, 2.2 and 2.3)

The WAR variable shows a high expected value and a large standard deviation in the table for the period 1820-1960 (Table 2.1), which indicates significant fluctuations in the intensity of conflicts throughout the period. At the same time, the positive skewness of this variable suggests the presence of rare but significant high-intensity conflicts. In comparison, the analysis of sub-periods indicates significant differences between the 19th and 20th centuries. In the years 1820-1890 (Table 2.2), the average value of WAR was relatively low, which may indicate a relative stability of the international system at that time. In the period 1890-1960 (Table 2.3), on the other hand, a sharp increase in the intensity of conflicts was observed, probably related to the two world wars and the general increase in geopolitical tensions.

The CON variable, describing the concentration of power, shows a clear difference between the sub-periods. In the years 1820-1890 its expected value was higher, which indicates a greater centralization of power in the 19th century. This can be associated with the dominance of a few major powers and a limited number of state actors. In the 20th century (1890-1960) we observe a decrease in the average CON value, which may be the result of decolonization processes and a greater dispersion of power in the international political system. The increase in the variability of this variable in this period may also indicate greater dynamics in the flow and use of resources.

An interesting element is the analysis of the MOVE variable, representing the movement of power. In the 19th century, the movement of power was even lower, which can be explained by technological limitations and less international integration. In the 20th century, an increase in the MOVE value was observed, which can be attributed to the development of transport and technology, although these values ​​still remain moderate, indicating the existence of structural barriers to the flow of resources.

When analyzing the descriptive statistics in the context of their distribution, it is worth noting the high kurtosis for the ACON and MOVE variables, especially over the entire period 1820-1960. This indicates a concentration of most values ​​around the mean, with the simultaneous occurrence of outliers. These values ​​emphasize the importance of a thorough analysis of the distributions in order to understand the mechanisms driving changes in the international system.

Comparing these results with the analyses by J. D. Singer (Appendix: Table 2.25), one can see agreement in general trends, such as the increase in the intensity of conflicts and the decrease in the concentration of resources in the 20th century. However, differences in the average values ​​may be due to differences in methodology, e.g. in the way of classifying conflicts.

In summary, the results of the analysis indicate significant changes in the nature of the international system, especially in the context of the increase in the number of conflicts and transformations in the structure of the concentration and movement of power. The observed changes reflect the evolution of international political and economic relations and adaptation to dynamically changing technological and social conditions. These conclusions are crucial for understanding historical processes in the international system and provide a basis for further mathematical analyses and modeling of these processes.

**Bivariate analysis(p.19-20)**

J. D. Singer believed that the influence of indicators (independent variables) on the probability of war should be analyzed both separately and jointly in order to understand their individual and joint influence on the studied phenomenon. For this reason, Table 2.4 is presented below with the results of correlation of each indicator with the probability of war, taking into account three different time periods. The value of the correlation coefficient R allows for assessing to what extent the model is able to capture the general nature of the relationship between variables. Then, the coefficient of determination R² was determined, which specifies the proportion of the variance of the dependent variable explained by the independent variables. Higher values ​​of R² indicate a better fit [4].

[Table 2.4: Correlation of independent variables with the probability of war occurrence]

For the whole period (1820-1960) there is a significant negative correlation (R = -0.42) for the CON indicator, which suggests that a greater concentration of power in the international system reduces the probability of conflict. The DCON (R = 0.33) and MOVE (R = 0.40) indicators show a positive correlation, which may mean that the change in the concentration of power and the movement of power promote an increased risk of conflict.

The R² values ​​for the whole period indicate that these indicators moderately predict the occurrence of conflict. The best fit is observed for CON (R^2 = 0.17), suggesting that concentration of power is a relatively good predictor of conflict in the long run. The MOVE indicator (R^2 = 0.16) also shows some influence, while DCON has a lower value (R^2 = 0.11).

In the sub-period 1820-1890 the correlations change their characteristics. The MOVE indicator records the highest positive correlation (R = 0.54, R^2 = 0.29), which means that the movement of power was a key factor promoting conflicts in the 19th century. The DCON indicator also shows a positive correlation (R = 0.29), but its strength is moderate. It is worth noting that the CON indicator loses its significance in this period (R = 0.11, R^2 = 0.01), indicating that power concentration had minimal impact on the occurrence of conflicts in the 19th century.

In the period 1890-1960 the situation changes. The correlation of the CON indicator is more negative (R = -0.38, R2 = 0.14), which again confirms that greater concentration of power reduced the probability of conflicts in the 20th century. The DCON (R = 0.31, R2 = 0.10) and MOVE (R = 0.27, R2 = 0.07) indicators maintain a positive correlation, although their strength is smaller compared to the 19th century. This means that in the 20th century, movement and changes in power had a less direct impact on the occurrence of conflicts than in the previous century.

Comparing these results with Singer’s analyses (Appendix: Table 2.26), one can see similarities in the general trends. However, the DCON index in Singer’s model is negative for the entire period and for the 20th century, indicating differences in methodology or interpretation of the results. In contrast, the MOVE index in both analyses shows variability by period but retains a positive correlation.

In summary, the correlation analysis indicates that these indicators contribute to explaining conflict dynamics to a different extent in different historical periods. Movement of power (MOVE) was a key predictor of conflict in the 19th century, while power concentration (CON) was more important in the 20th century, while changes in concentration (DCON) had a smaller impact. These results underline the complexity of the dynamics of the international system and the need to consider different factors depending on the historical period.

**Multivariate analysis(p. 20-33)**

This chapter is devoted to the mathematical analysis of models of armed conflicts developed by J. David Singer, taking into account the specificity of their application in different historical periods. In particular, four models are examined: ADD/CON LEADS (1.7), ADD/CON LAGS (1.8), MULT/CON LEADS (1.9) and MULT/CON LAGS (1.10), which are tools for describing and analyzing the dynamics of international conflicts. The aim of the chapter is to assess the fit of these models to empirical data and to identify periods in which individual approaches show the greatest adequacy.

The chapter includes: estimation of model parameters, evaluation of models, analysis of residuals. The chapter focuses on answering the question: which of Singer's models best describe conflicts in specific time periods? These results are of great importance for understanding the dynamic nature of international conflicts and the possibility of using multivariate analysis methods in social sciences.

**Parameter estimation (p. 20-23)**

The tables below present the results of parameter estimation for four linear regression models analyzing the impact of power concentration (CON), changes in concentration (DCON) and power movement (MOVE) on the dependent variable. The estimations were performed for three different periods: the entire analyzed range of 1820-1960 (Table 2.5), the subperiod 1820-1890 (Table 2.6) and the subperiod 1890-1960 (Table 2.7). The models take into account both the additive (ADD) and multiplicative (MULT) approaches, as well as different variants of calculating the CON variable: at the beginning of the period t (LEADS) and at the end (LAGS).

The aim of the analysis is to examine to what extent the above indicators affect the dependent variable in different periods and which of them are statistically significant. p-values ​​allow us to determine whether a given coefficient is significant at the assumed significance level (a = 0.05), and the coefficients themselves indicate the direction and strength of the relationship.

[Table 2.5: Estimated parameters for the years 1820 – 1960]

For the full range of years 1820-1960 the value of the constant (a) is statistically significant only in multiplicative models (MULT), where p = 0.022. In additive models (ADD) the constant is higher (a = 43.52), but is not significant (p = 0.266).

The coefficient for resource concentration (B1) is negative in all models, suggesting that greater resource concentration reduces the likelihood of conflict. However, in none of the models does the p-value indicate statistical significance (p > 0.05).

Change in concentration (DCON) has a positive coefficient (B2) in additive models, which may suggest that larger changes in concentration increase the likelihood of conflicts. In multiplicative models, this effect is not significant (B2 = -0.874, p = 0.968).

Суть в том, что мы естемируем параметры для модели, определяя сколько «частей» независимых переменных (CON, DCON i MOVE) нужно для прогноза зависимой переменных WAR.

Movement power (MOVE) shows the highest coefficients (ẞ3 = 759.93 in ADD and β3 = 35.40 in MULT). Despite relatively high values, MOVE is not statistically significant in any of the models (p > 0.05).

[Table 2.6: Estimated parameters for the years 1820 - 1890]

In the period 1820-1890 the value of the constant (a) is not significant in all models, where p >> 0.05, although the value of the coefficient itself (a = 0.576) is relatively low.

For the power concentration index (CON), the coefficient is negative in all models, confirming the observation from the full period that higher concentration reduces the risk of conflict. However, the coefficient is insignificant (p >> 0.05).

The change in concentration (DCON) shows negative values ​​in multiplicative models (B2 = -39.52, p = 0.707), which may suggest an inverse relationship, but the lack of statistical significance prevents unambiguous conclusions. Additive models indicate a moderately positive effect, also without statistical significance (β2 = 295.08, p = 0.705).

Power mobility (MOVE) has a positive coefficient in the ADD (B3 = 2874.89, p = 0.119) and MULT (β3 = 379.59, p = 0.061) models. Although it does not reach significance, the relatively high values ​​of the coefficient may indicate a potential effect of power mobility on conflicts.

[Table 2.7: Estimated parameters for the years 1890-1960]

In the analysis of the years 1890-1960, the value of the constant (a) is clearly higher in the ADD models (a = 64.92), but statistically insignificant (p = 0.339). In the MULT models, the value of a is significant (p = 0.012) and equal to 5.232, which indicates a significant basis for the model.

Power concentration (CON) has negative coefficients in all models and approaches significance in the MULT models (β₁ = -11.43, p = 0.072). This suggests that greater power concentration may have reduced the likelihood of conflict in the 20th century.

Change in concentration (DCON) shows positive coefficients in the MULT/CON LAGS model (B2 = 55.37, p = 0.080), suggesting that abrupt changes in concentration may have increased the risk of conflict in the 20th century, although this effect is not statistically significant.

Power movement (MOVE) in the ADD and MULT models has negative coefficients, which differs from the previous periods. The coefficient (β3 = -6.579, p = 0.885) does not show significance, but suggests a reduction in the effect of mobility on conflicts in this period.

**Comparison of periods.** The coefficient on CON is negative in all periods and models, which may indicate a general trend that greater resource concentration reduces the likelihood of conflict. In the period 1890-1960, this effect is more pronounced, although still insignificant.

The DCON variable shows different directions in the different periods. In the years 1820-1890 the effect is more volatile and moderately positive, while in the years 1890-1960 it gains importance in the MULT models, which may result from the dynamic changes in the international system in the 20th century.

The power movement (MOVE) shows the highest coefficients in the period 1820-1890, which may indicate its importance in the 19th century, especially in ADD models. In the 20th century, the movement effect weakens.

The MULT LAGS model is the best for the whole period due to the stability and statistical significance of the constant. The ADD LEADS model for the period 1820-1890 is the best because it highlights the important potential of movement of power(MOVE) as a key factor of conflicts in the 19th century. The MULT LAGS model best describes the period 1890-1960, taking into account both changes in the change in concentration of power (DCON) and their overall level (CON).

Analysis of the results for different periods shows that the dynamics of the CON, DCON and MOVE indicators change over time, and their impact on the dependent variable varies depending on the model and period. None of the indicators is unequivocally statistically significant.

**Model evaluation (p. 23-25)**

In linear regression analysis, the next step was model evaluation, which involves both data preparation and assessment of the quality of the model fit to the explanatory and dependent variables. This process began with standardization of variables, which is an essential step in multivariate analysis. Standardization involves transforming variables so that they have a mean of zero and a standard deviation of one. In addition, standardization improves the stability of calculations and eliminates problems related to differences in the scales of variables.

The next step was to calculate the multiple correlation coefficient R, and R². However, to account for the number of variables and observations in the model, the adjusted coefficient of determination R2 was also calculated. This is a more precise measure that penalizes the model for an excessive number of predictors, protecting against the problem of overfitting.

An important element of the analysis were the standardized regression coefficients (b). They allow us to determine which independent variables have the greatest impact on the dependent variable, assuming that the other variables are constant.

To further examine the effect of each independent variable on the dependent variable, partial correlation coefficients (r²) were calculated. Partial correlations allow us to assess which variables are most important in explaining the dependent variable after taking into account the relationships between the independent variables.

The results presented in Tables 2.8, 2.9 and 2.10 show the fit of standardized linear regression models for three different periods: 1820–1960, 1820–1890 and 1890–1960.

[Table 2.8: Fit of standardized models for 1820-1960]

In Table 2.8, the highest value of the correlation coefficient R was achieved by the MULT models (R = 0.520), which indicates a better fit of these models compared to the ADD models. Similarly, the coefficient of determination R² and its adjusted R2 value for these models are the highest (R2 = 0.271, R2 = 0.180). The ADD models are characterized by lower values ​​of R and R², which suggests a poorer fit.

In the analysis of independent variables, the MOVE variable has the highest values ​​of the regression coefficient (b = 0.211) and the squared partial correlation coefficient (r2 = 0.033), which means that it is the most important predictor in explaining the dependent variable in this period. In turn, the CON and DCON variables have lower b I r² values, indicating a smaller impact on the variability of the dependent variable.

[Table 2.9: Fit of standardized models for the years 1820 – 1890]

The results for the period 1820-1890 are presented in Table 2.9. The values ​​of the correlation coefficient R and the determination coefficient R² are higher compared to the results for the entire period 1820-1960. The best fit is observed for the MULT models (R = 0.562, R2 = 0.316, R2 = 0.111), which means that in this subperiod these models better explain the variability of the dependent variable.

Among the independent variables, the MOVE variable again shows the highest values ​​of b (b = 0.611) and r² (r² = 0.309), confirming its key role in the model. The CON variable has a negative regression coefficient in the MULT models (b= -0.039 and b = -0.052), which suggests that its effect on the dependent variable is inverse, but not very significant (r2 = 0.002). The DCON variable is also less important (b=0.015 and r² = 0.035).

[Table 2.10: Fit of standardized models for 1890-1960]

Table 2.10 presents the results for the period 1890-1960. Compared to the earlier period 1820-1890, the correlation coefficient R and the determination coefficient R² are significantly higher for the MULT models (R = 0.646, R2 = 0.417, R2 = 0.242), indicating a clear improvement in model fit in this period. The ADD models have significantly lower values ​​of Ri R2 (R = 0.395, R2 = 0.156), suggesting their weaker ability to explain the variation in the dependent variable.

In this period, the CON variable in the MULT models reaches relatively high absolute values ​​of b (b = -0.670 and b = -0.688) and r² (r2 = 0.288), which indicates its impact on the variability of the dependent variable. The MOVE variable also remains a significant predictor (b = -0.047, r2 = 0.002), but its significance is smaller compared to the previously analyzed periods. The DCON variable in the MULT / CON LAGS models records the highest values ​​of the regression coefficient (b = 0.740) and the squared partial correlation coefficient (r2 = 0.275), which suggests that its role in explaining the variability of the dependent variable increases in the later period.

When comparing the results of the analysis with the results of J. D. Singer (Appendix: Table 2.27), there are significant differences in the assessment of the fit of the models. For the entire period 1820-1960 Singer finds ADD models to have better fit (R = 0.56, R2 = 0.31) compared to MULT models which obtain lower values ​​(R = 0.43, R2 = 0.19). In the present analyses the results suggest an advantage of MULT models (R = 0.520, R2 = 0.271) over ADD models which may be due to differences in the version of the dataset used.

In summary, the results indicate that MULT models explain the variability of the dependent variable better than ADD models in each of the analyzed periods. The role of the MOVE variable is significant in the entire analyzed time period, especially in the period 1820-1890. In the later period (1890-1960), the importance of the CON and DCON variables increases, which may reflect the changing dynamics of the international system and the growing importance of the concentration and change of concentration of power in the context of armed conflicts. The results suggest that MULT models are more adequate for the analysis of dependencies in the international system, especially in the 20th century.

**The charts illustrating the model fitting (p. 25-29)**

This chapter presents the modeling results for different versions of the ADD and MULT models, illustrating the fit of the predicted values ​​of the number of conflicts (WAR) to the actual historical data. The graphs allow for the assessment of the quality of the predictions of each model in the analyzed periods (1820-1960, 1820-1890, 1890-1960) and the comparison of their efficiency. Additionally, the analysis includes the mean square errors (MSE) and absolute errors (MAE). These indicators are key tools in assessing the accuracy of the models, allowing for an objective comparison of their efficiency in predicting the number of conflicts.

The mean square error (MSE) measures the average difference between the forecasted and actual values ​​by squaring the difference, which accentuates larger deviations and penalizes significant errors. The mean absolute error (MAE) measures the average of the absolute deviations, which better captures the typical level of errors without squaring them [4]. In this way, MSE provides information about extremes, while MAE better reflects the overall accuracy of the forecasts. The results of these calculations are presented below in Table 2.11.

[Table 2.11: Mean Absolute Error (MAE) and Mean Squared Error (MSE) for the models in three time periods]

Figure 2.1 shows the fit graphs of the four models considered for the entire period (1820-1960). The ADD models show noticeable fluctuations in their predictions compared to the actual values. The results indicate that the values ​​predicted by these models often deviate from the actual number of conflicts, leading to a moderate fit

[Figure 2.1: Model Fit for 1820–1960]

The mean square error (MSE) for the ADD models is 988.23 (LEADS) and 1060.59 (LAGS), respectively, and the mean absolute error (MAE) is 19.93 and 20.48. In turn, the MULT models are characterized by a slightly higher MSE (1330.60 and 1383.13), which indicates their greater variability, but also better representation of high WAR values. The MAE for the MULT models is 18.35 (LEADS) and 19.94 (LAGS), respectively, which indicates their greater accuracy in forecasting values ​​with greater dynamics.

[Figure 2.2: Model Fit for 1820-1890]

Figure 2.2 presents the results for the 19th century (1820-1890). In this period, the MULT models, both in the LEADS and LAGS versions, show a better fit to the actual data than in the previous period considered. The mean square error (MSE) for these models is 106.83 and 106.87, and the mean absolute error (MAE) is 8.52 and 8.56, respectively. The MULT models in this period have slightly higher MSE values: 108.02 (LEADS) and 102.14 (LAGS). Nevertheless, the MAE for the MULT models, which are 6.36 and 6.20, indicate their higher accuracy in forecasting. The analysis of the graphs suggests that the MULT/CON LAGS models respond best to dynamic changes in the data, which makes them more effective in capturing local fluctuations.

[Figure 2.3: Model Fit for 1890–1960]

Figure 2.3 shows the model fit for the 20th century (1890–1960). During this period, the MULT models perform better than the ADD models, which is reflected in the lower MAE values. The mean square error (MSE) for the MULT models is 2382.69 (LEADS) and 2220.72 (LAGS), which is higher than that of the ADD models. However, the MAE values ​​of 28.19 and 28.35, respectively, indicate their better ability to capture the variability in real data. The ADD models have significantly higher MSE values ​​(1756.20 and 1984.84) ​​and MAE values ​​(31.26 and 32.88), which indicates their difficulty in capturing the more dynamic changes in the international system of the 20th century.

The presented results show that ADD models perform better in the 19th century, where geopolitical dynamics were relatively lower, which resulted in smaller prediction errors. In the 20th century, characterized by greater changes and higher conflict dynamics, MULT models, especially MULT/CON LAGS, achieve better fit. This indicates their greater ability to respond to data variability and capture significant changes. It is worth emphasizing that LAGS models in both groups (ADD and MULT) are more sensitive to sudden jumps in data, which makes them more effective in the analysis of dynamic historical periods.

**Analysis of residuals (p. 29 - 33)**

Residuals – это разница между настоящим значением и значением которое посчитала модель, то есть errors.

This chapter presents a graphical analysis of residuals, i.e. differences between actual values ​​(WAR) and values ​​predicted by regression models. Residuals are a key element in assessing the quality of a model, as they allow for identifying potential problems with model fit to data. The presented graphs allow for visualization of the distribution of residuals depending on the predicted values ​​for different models (ADD/CON LEADS, ADD/CON LAGS, MULT/CON LEADS, MULT/CON LAGS) and time periods (1820-1960, 1820-1890, 1890-1960). The aim of the analysis is to assess whether the residuals are randomly scattered around the horizontal axis (predicted values), which would indicate the correctness of the model assumptions, including linearity, homoscedasticity, and normality of errors.

In Figure 2.4, showing the results for the period 1820–1960, the ADD models show a more concentrated dispersion of residuals around the predicted values. However, some outliers are visible, indicating a poor fit of the models for extreme values ​​of WAR. The MULT models have a larger range of residuals, indicating that they have difficulty predicting actual values ​​over such a wide period. The MULT/CON LAGS model responds better to jumps in the data, making it more useful for analyses involving dynamic changes in the number of conflicts.

In Figure 2.5 for the 19th century (1820–1890), the ADD models, and especially the ADD/CON LAGS models, are characterized by a more even distribution of residuals and a smaller range of errors. This indicates that they fit the data better in a historical period with relatively low dynamics. The MULT models, although characterized by a larger variability of residuals, are good at predicting outliers, which makes them more suitable for the analysis of sudden events.

In Figure 2.6, for the period 1890–1960, the MULT models show a better fit than the ADD models. The residuals for the MULT/CON LAGS model are more concentrated around zero, indicating smaller forecast errors in this dynamic period. The ADD models, especially the ADD/CON LEADS, show more scatter in the residuals, suggesting difficulties in predicting actual values ​​in times of intense geopolitical change.

[Figure 2.4: Residuals (e) for 1820 – 1960]

The Shapiro-Wilk tests performed for all models and periods showed that the residuals were not normally distributed. This may indicate a discrepancy with the model assumptions or a need for further optimization.

Тест Shapiro-Wilk проверяет имеют ли residuals нормальное распределение(normal distribution), что означает что они рандомные

By analyzing all three figures, it can be seen that the ADD/CON LAGS models perform best in stable periods such as 1820 1890, where the residuals are the least dispersed.

[Figure 2.5: Residuals (e) for the years 1820 – 1890]

In contrast, in dynamic periods such as 1890–1960, the MULT/CON LAGS models achieve a better fit, as evidenced by smaller residual errors. The LEADS models in both groups (ADD and MULT) have difficulty predicting extreme WAR values, which indicates their limitations in responding to sudden changes.

[Figure 2.6: Residuals (e) for the years 1890 – 1960]

The analysis of residuals shows that LAGS are more effective in predicting dynamic changes, especially in periods of intense historical transformations, such as the 20th century. ADD models show a better fit in periods of less dynamics, such as the 19th century. These results emphasize the importance of selecting an appropriate model depending on the specificity of the analyzed historical period. The lack of normality of residuals is an indication for further analyses to improve the prediction quality of the models.

**Summary (p. 33)**

The analysis of the Singer model allowed for a detailed evaluation of four versions of the Singer model (CON, ACON, MOVE) in three different periods: 1820-1960, 1820-1890 and 1890-1960. The results indicate that the choice of the best model depends on the dynamics of the period under study. In the 19th century, characterized by greater international stability, the ADD models, especially ADD/CON LEADS, achieved better fit, achieving lower values ​​of prediction errors (MSE and MAE) and higher R2. The values ​​predicted by these models were closer to the actual data, which indicates their accuracy in less dynamic conditions.

In the 20th century, which was characterized by greater volatility and geopolitical dynamics, MULT models, and especially MULT/CON LAGS, showed better performance. These models better reproduced sudden changes in the data, as confirmed by both the prediction error results and the analysis of residuals. MULT models proved to be more flexible and able to capture the complexity of the international system in that period.

In summary, ADD models perform better in stable periods, while MULT models, especially with lags (LAGS), perform better in more dynamic conditions. The final choice of the model should be tailored to the specific period under consideration and the goals of the analysis, taking into account differences in the dynamics of the international system. MULT/CON LAGS models can be considered more versatile in studies of the 20th century, while ADD/CON LEADS remain effective in the analysis of more stable historical periods.

**2.2 Own analysis of Singer's model with the change of parameter t (p.33-56)**

In this part of the work, Singer's own analysis of the model will be carried out, taking into account the modification of the key time parameter t. Singer's research so far was based on the assumption that the predictor variables are calculated for five-year periods (t = 5), which allowed for taking into account some changes in the dynamics of the international system. The aim of this chapter is to examine how changing the value of the parameter t affects the model's effectiveness in predicting wars.

**Descriptive statistics**

Tables 2.12 and 2.13 present descriptive statistics for variables calculated from t = 1 and t = 10 in different time periods: 1820–1960, 1820–1890, and 1890–1960. Analysis of these tables allows us to identify differences in the model variables depending on the period.

[Table 2.12: Descriptive statistics for variables calculated for t = 1 in all periods]

[Table 2.13: Descriptive statistics for variables calculated for t = 10 in all periods]

In the statistical analysis of variables describing armed conflicts, such as WAR, CON, DCON and MOVE, the results for time divisions when t = 1 and t = 10 (Tables 2.12, 2.13) were compared with the results for t = 5 in section 2.1 "Descriptive statistics of model variables". The key observations and conclusions resulting from the analysis of the tables are presented below.

The expected value for the WAR variable clearly increases with the length of the time interval t. The results for t = 10 show a higher expected value of the number of armed conflicts compared to t = 5 and t = 1, which results from the accumulation of conflicts in the longer period. However, the trends in the distribution of the number of conflicts remain consistent – ​​in the 19th century the number of wars was noticeably smaller than in the 20th century, which reflects changes in geopolitical dynamics and the intensification of conflicts in later periods.

For the variables CON and MOVE, the trends are stable regardless of the value of t. The expected values ​​for these variables indicate similar relationships in different time periods. At the same time, for MOVE it can be seen that this value is larger for t = 1 than for t = 5 and t = 10, which suggests larger fluctuations of movements in shorter time intervals.

The DCON analysis shows similar trends in different time periods for t = 1 and t = 10. These values ​​reflect the stability of changes in the CON variable, but for t = 5 the behavior of this variable differs, which may indicate different dynamics in the average time intervals. Nevertheless, the average values ​​of CON and DCON are similar for all analyzed t, which indicates their stability as indicators in the study of conflict dynamics.

The distributions of the variables WAR, CON, DCON, and MOVE in the analyzed periods indicate asymmetry, especially for WAR and MOVE, where positive values ​​dominate. The results for CON are more symmetric, and the kurtosis of variables such as DCON indicates the presence of outliers, which requires consideration in further interpretation of the results.

In summary, the analysis of statistical results for different t shows consistency of key trends, while revealing differences in the behavior of variables such as MOVE and DCON depending on the length of the analyzed time interval. These differences and consistencies confirm that the models are sensitive to the length of time periods.

**Bivariate analysis (p. 35 - 37)**

The aim of this chapter is to examine the relationship between selected independent variables and the probability of armed conflicts in different time periods. The analysis is based on the calculation of the correlation coefficient (R) and the coefficient of determination (R2) for the variables CON, DCON, MOVE. The results are presented for two different values ​​of t = 1 and t = 10 in Table 2.14 and compared with the results of correlation calculations for variables calculated every 5 years presented in section 2.1 "Bivariate analysis". The study covers both the entire analyzed period 1820-1960 and its subperiods: the 19th century (1820-1890) and the 20th century (1890-1960). This analysis allows for the identification of variables that show stable relationships with the occurrence of conflicts in different historical contexts, and for the assessment of the optimal length of the time period t for modeling these relationships.

[Table 2.14: Correlation of independent variables with the probability of war for different time periods with divisions t = 1 and t = 10]

The results of the correlation analysis for different values ​​of t indicate significant differences in the effectiveness of the models depending on the adopted time interval. For t = 10 and t = 5, a relatively good fit was obtained for different variables, while the results for t = 1 show the worst values ​​of the coefficient of determination (R²). For t = 1, only the variables CON and MOVE are stable, especially in the analysis of the entire period (1820-1960) and the 20th century (1890 - 1960).

It is worth noting that the CON variable is always negatively correlated with the probability of war, except for t = 1 in the 19th century (1820-1890), where it takes a positive value. This is consistent with theoretical assumptions that suggest that higher values ​​of CON (concentration of power) should reduce the risk of conflict. In turn, ACON and MOVE are always positively correlated, regardless of the value of t, which indicates their constant relationship with the probability of conflict, especially in shorter time periods.

For the whole analyzed period (1820-1960) the best performance is achieved by variables calculated for t = 5, which can be attributed to the balanced approach between short-term fluctuations and long-term trends. The R² values ​​for them are higher compared to t = 1 and comparable to t = 10, which indicates that these variables best capture the dynamics of the studied processes.

The coefficient of determination (R2) for t = 1 is generally low, suggesting that the models are not able to capture the variability of the data over short time periods sufficiently well. This is probably due to the greater randomness and less stability of the data over shorter time periods.

Optimal time interval. The results suggest that t = 5 is an optimal time interval for correlation analysis, combining model stability (R2) with the ability to capture long-term trends and changes. A value of t = 10 is also useful, especially for long-term analysis, but may lead to loss of detail regarding short-term changes.

The results of the correlation analysis emphasize the importance of choosing the right time interval t in studying the dynamics of armed conflicts. While t = 5 provides the best overall fit for the different variables, t = 10 allows for the analysis of long-term relationships. The results for t = 1 indicate the limited usefulness of this interval, especially especially in the context of low R² values. However, the stability of the CON and MOVE variables over short periods suggests that they may be useful in analyzes specific to the 20th century.

**Multivariate analysis (p. 37-55)**

**Parameter estimation (p. 37-39)**

The tables below present parameter estimation results for various linear regression models analyzing the impact of power concentration (CON), concentration changes (DCON ), and power movement (MOVE) on the dependent variable. Estimations were carried out for three different periods: the entire studied range of years 1820-1960, the subperiod 1820-1890 and the subperiod 1890-1960. The models take into account both the additive approach (ADD) and the multiplicative approach (MULT), as well as various variants of calculating the CON variable: at the beginning of period t (LEADS) and at the end (LAGS). The results for different divisions t = 1 (Table 2.15) and t = 10 (Table 2.16) are presented below.

[Table 2.15: Estimated parameters for t = 1 for all periods]

Based on the results presented for t = 1 in Table 2.15, it can be seen that most of the regression coefficients in the analyzed models reach a good level of significance compared to other divisions, which suggests that the independent variables explain the variability of the dependent variable WAR well.

The ADD models show a particularly good fit in the periods 1820–1960 and 1890–1960. The values ​​of the coefficients for MOVE (e.g. β3 = 277.88, p < 0.001 in the period 1820–1960) indicate a significant positive effect of power movement on WAR. A similar situation occurs in the period 1890–1960 (β3 265.22, p = 0.002). This may be due to the dynamic growth of the importance of power flows in the context of industrialization and the growth of political tensions in the 19th century, where the growth of military and economic capabilities could directly translate into the probability of armed conflicts.

The DCON value in all periods is characterized by a high p-value (p ≥ 0.05), which suggests that the variability of CON concentration is not sufficiently captured by models for t = 1. This may mean that the adopted delay period does not reflect the full dynamics of changes in concentration in the international system. In turn, the CON variable in the 19th century (β₁ = 57.67, p = 0.016) has a positive effect on WAR, which may result from the fact that a higher concentration of power favored the dominance of a few major states, which more often initiated conflicts in order to strengthen their position. The DCON and MOVE variables show a positive effect in all periods.

The models for the 19th century (1820-1890) are characterized by poorer fit results compared to other periods, this is visible in higher p-values ​​(e.g. for MOVE, p = 0.833). The 19th century was a period of rapid technological and social changes, which could distort the relationships between variables.

The MULT models also show significant coefficient significance, especially for MOVE. For example, over the period 1890–1960, the coefficient ẞ3 = 25.19 (p = 0.000) indicates a significant effect of this variable. For the period 1890–1960, MULT/CON LEADS is a better choice because it offers more balanced coefficients and variable significance (β₁ = -3.77, p = 0.060; β3 = 25.19, p = 0.000), which may better capture the complexity of the international system at that time.

[Table 2.16: Estimated parameters for t = 10 for all periods]

Comparison of the results for t = 1 (Table 2.15) with the results for t = 10 (Table 2.16) indicates significant differences both in the values ​​of the regression coefficients and in the level of statistical significance (p-values).

Based on the results of parameter estimation presented in Table 2.16 for t = 10, it can be seen that the level of statistical significance is generally low. P-values ​​for most variables are high, which suggests that these variables do not have a significant effect on the prediction of armed conflicts. The exception are, however, some coefficients in the MULT models, which reach moderate significance. In the period 1890 - 1960 in the model, the coefficients ẞ2 corresponding to the DCON variable have moderate significance (p = 0.24, p = 0.29, p = 0.08, p = 0.10). This indicates a possible, although weak, influence of changes in the concentration of power on the prediction of conflicts in this period.

The MOVE variables in both sub-periods show a negative effect on the occurrence of conflicts, which is unusual compared to past results. It is also worth noting that the CON variable does not show a significant effect on the prediction of conflicts in any of the periods studied. The high p-values ​​for the ẞ₁ coefficient for this variable suggest no clear relationship between power concentration and conflicts.

The best model for the period 1820 - 1960 is ADD/CON LAGS because the variables reach moderate significance. Similarly, for the period 1820-1890 the best model is MULT/CON LEADS, due to the significance of the ACON variable. In the period 1890-1960 the best model can be considered MULT/CON LAGS, because the CON variable shows the highest statistical significance, which suggests its role in predicting conflicts in this period. MULT models generally have better results in terms of the level of significance compared to ADD models.

Comparison of both t values ​​clearly shows that variables in the models are more significant at a smaller time interval (t = 1), which may suggest greater dynamics of changes in the international system in short periods. On the other hand, for a larger time interval (t = 10), the models lose significance and precision in reflecting real conflicts, which may be due to the small number of observations (14 for the period 1820-1960, 7 for sub-periods).

**Model evaluation (p.39-42)**

The results of the analysis of multivariate regression models for variables in different historical periods provide interesting observations regarding the dynamics of armed conflicts and the significance of individual variables in explaining the dependent variable. In this chapter, the following were calculated: standardized multiple correlation coefficients (R) and (R2), the corrected coefficient of determination (R2), and for each variable, the partial correlation coefficients (r²) and standardized regression coefficients (b) were calculated. The calculations were carried out for all time periods and for different t (t1, t = 10) and are presented in the following Tables (2.17, 2.18)

Analyzing the results of t = 1 partitions (Table 2.17) for the full range of years 1820–1960, we find a moderate strength of the relationship between the explanatory variables and conflict (R = 0.473 for the ADD models and R = 0.409 for the MULT). Of particular interest is the dominant role of the MOVE variable, whose standardized regression coefficient (b = 0.356 and b = 0.328) and partial coefficient of determination (r2 = 0.122 and r2 = 0.099) indicate its contribution to explaining the variability of conflicts. The results suggest that power movement was the main factor in the escalation of conflicts in the period under study.

In turn, the variables CON and DCON had a relatively low impact on explaining conflicts. The negative coefficients for CON (b = -0.213) may mean that stability in the international system acted as a brake on conflict escalation, but was not a strong enough determining factor. The change in concentration (DCON) showed an even smaller impact (b = -0.053 and b = 0.069), which indicates that this variable did not play a significant role in the long run.

[Table 2.17: Fit of the standardized models for t = 1 for all periods]

The results for the period 1820–1890 show that the models had limited ability to explain conflicts (R = 0.172, R = 0.205), and negative R2 values ​​indicate a possible underfit of the models to the data. These values ​​suggest that the dynamics of conflicts in the first half of the 19th century were more complex and could depend on other factors not included in the models, such as colonial structures or local geopolitical tensions.

It is worth noting that the CON variable still showed some positive influence, suggesting that power concentration was an important factor, although its significance was less pronounced than in later periods. DCON MOVE showed little or no contribution to explaining conflict, which may be due to the relative stability of the international system in this era and the limited number of large international conflicts.

The highest values ​​were obtained for the period 1890-1960 with the ADD model (R = 0.531, R = 0.250). This indicates a stronger relationship between the explanatory variables and armed conflicts in this era. This can be interpreted as a result of the increase in the intensity of global conflicts (two world wars) and better data availability and greater stability of international geopolitical structures.

MOVE emerged as the dominant variable during this period. High values ​​of b = 0.393 and r2 = 0.159 suggest that power movement was a key predictor of conflict escalation, which is consistent with historical realities of increasing armament and aggression. In contrast, the CON variable had a moderate effect, with negative coefficients (b = -0.268), confirming that the stability of power concentration could reduce the risk of conflict, but its effect was limited by other factors, such as the expansionist goals of some states.

Change in concentration (DCON) showed mixed effects (b = -0.087 for ADD/CON LEADS and b = 0.080 for ADD/CON LAGS), which may reflect the heterogeneity of states' responses to changing international conditions. These results underscore the need for deeper analysis of political contexts and their interactions with conflict dynamics.

The analysis showed that the MOVE variable was consistently the most important predictor of armed conflict, regardless of the period. This suggests that the intensity of power movement plays a key role in the escalation of conflicts, especially in more complex international systems.

Analyzing the results for t = 10 (Table 2.18), it can be seen that for the whole period 1820-1960 the models show a moderate ability to explain the variability of conflicts, with R values ​​ranging from 0.486 (MULT/CON LEADS) to 0.509 (ADD/CON LEADS). The coefficient of determination R² suggests that the models explain about 23-26% of the total variance. The results for the CON variable indicate its negative effect on conflict escalation in the ADD models. On the other hand, the DCON variable played a significant role, especially in the ADD/CON LAGS model, where the regression coefficient was 0.356, suggesting that slow systemic changes could lead to conflicts. However, the most important variable in this period was MOVE (movements of power), which showed the highest values ​​of regression coefficients (from 0.341 to 0.441) and partial coefficient of determination (from 0.098 to 0.150), which emphasizes the key role of military actions in the escalation of conflicts.

[Table 2.18: Fit of the standardized models for t = 10]

In the period 1820-1890 the results show much greater variability. The MULT/CON models achieve exceptionally high values ​​(R = 0.855 and R2 = 0.731), which means that they explain more than 73% of the variance in the data, indicating that these models are exceptionally effective in explaining the dynamics of conflict in this era. The CON variable had a small effect here, suggesting that stability played a marginal role in deterring conflict in this period. In contrast, the DCON variable turned out to be crucial, with very high coefficients (b = 1.391 in the MULT/CON LAGS model) and significant partial coefficients of determination (r2 = 0.687). These results indicate that sudden systemic changes were a major factor leading to conflict. The MOVE variable also had a significant effect, although smaller than DCON, suggesting that power movement interacted with systemic changes in predicting conflict in this period.

The period 1890–1960 is characterized by a poorer fit of the models to the data, as seen in the negative values ​​of the adjusted coefficient of determination (R). The results indicate a potential under-fitting of the models to the more complex conflicts of that time, which may have required the inclusion of additional variables. The CON variable shows a negative effect on conflicts, which may indicate that the disruption of concentration had a destabilizing effect. In contrast, DCON was of some significance, especially in the MULT/CON LAGS model, where the coefficient was 0.886, suggesting that systemic changes continued to play an important role, although their impact was less consistent than in previous periods.

In summary, the analysis shows that the MOVE variable was the most important predictor of conflicts throughout the analyzed period, while DCON had a particularly strong impact in the 19th century. In contrast, the period 1890-1960 suggests more complex conflict dynamics, which requires further consideration of additional factors in the models. These results confirm the evolution of the dynamics of armed conflicts and emphasize the need for a flexible approach to modeling these phenomena.

Comparison of the results for t = 1 and t = 10 shows that the variables have a different role depending on the time horizon. For t = 1, the largest influence is exerted by MOVE, which consistently dominates as a predictor of conflicts in all periods. In contrast, the variables CON and DCON play a smaller role, suggesting that in the shorter term, system stability and changes in the status quo have a limited impact on conflict escalation.

For t = 10, higher R and R2 values ​​are visible in the 19th century, especially for the DCON variable, which plays a key role in explaining conflicts in this period. The regression coefficients indicate that changes in the international system had a significant impact on the dynamics of conflicts. In the 20th century, the fit of the models decreases, which may be due to more complex determinants of conflicts, such as technology or economy. In the longer term, the importance of MOVE is still significant, but it gives way to DCON, emphasizing the key role of systemic changes in generating conflicts.

**Graphs illustrating model fit (p. 42-46)**

This chapter presents the modeling results for different versions of the ADD and MULT models, illustrating the fit of the predicted values ​​of the number of conflicts (WAR) to the actual historical data. The graphs allow for the assessment of the quality of the predictions of each model in the analyzed periods (1820-1960, 1820-1890, 1890-1960) and the comparison of their efficiency in the case of counting variables with divisions t = 1 and t = 10. Additionally, the analysis includes the mean square errors (MSE) and absolute errors (MAE) presented in Table 2.19.

Figures 2.7, 2.8, 2.9 present graphs for t = 1 for the periods 1820–1960, 1820–1890, 1890–1960, respectively. For shorter time horizon divisions (t1), the graphs indicate a generally good ability of the models to reproduce trends of real conflicts in the historical periods studied. ADD/CON models are characterized by smoother predictions, which results in a smaller number of extreme values ​​compared to actual values. The results for MULT models, although also relatively accurate, show greater variability and stronger differences in fit for the highest actual values.

[Table 2.19: Mean Absolute Error (MAE) and Mean Squared Error (MSE) for Models with t = 1 and t = 10 in Three Time Periods]

For the years 1820-1890, the differences between actual and predicted values ​​are particularly noticeable in cases of extreme conflicts, where the models are unable to capture the full dynamics of the sudden spikes. At the same time, for the years 1820-1960 and 1890-1960 the models correctly capture the long-term changes in the number of conflicts, although they still have difficulty accurately matching periods of rapid escalation, such as the years of the world wars.

The analysis of errors for (t = 1) shows that the ADD/CON LEADS model achieves lower errors than MULT, which may indicate a greater stability of predictions within the first group of models. The MSE values ​​range from 84 to 226 (for different periods), while the MAE remains at a level from about 5.7 to 10.5.

Figures 2.10, 2.11, 2.12 show graphs for t = 10 for the periods 1820-1960, 1820-1890, 1890-1960, respectively. For a longer time horizon, the models show larger differences between the predicted and actual values, especially in the case of years with clear changes in the number of conflicts. The predictions for ADD and MULT tend to underestimate the actual values ​​in periods of rapid increases (e.g. conflicts in the years 1860-1870 or during the world wars). At the same time, the predictions for MULT/CON, despite greater variability, better reflect the dynamics of periods with an average number of conflicts.

In the period 1820-1890 the models are more stable than in the years 1890-1960, which is reflected in lower MSE and MAE values. MULT/CON LAGS reaches the lowest error values ​​(MSE: 52.19, MAE: 5.50) in this period, which indicates a better fit for the analyzed time interval. However, for the period 1890-1960 the errors increase significantly, reaching MSE above 800 and MAE at the level of 19-28, which indicates the difficulties of the models in predicting conflicts in the more complex conditions of the 20th century.

Comparison of results for different values ​​of t reveals clear differences in the ability of the models to reproduce conflict dynamics. For t = 1, predictions are more accurate, especially in shorter time periods. ADD/CON models are characterized by smaller errors and more stable predictions, which indicates their advantage in shorter time horizons. MULT/CON, although more volatile, are better at capturing long-term trends.

[Figure 2.7: Model fitting for t = 1 for the years 1820-1960]

For (t = 10) the models show more difficulty in fitting, especially in years with rapid changes in the number of conflicts. MULT/CON LAGS performs best in the period 1820-1890, but in the 20th century all models struggle with high errors, suggesting that conflicts in this period require the inclusion of more complex variables. Increasing t also results in a significant increase in MSE and MAE errors, which highlights the difficulties of long-term conflict forecasting.

[Figure 2.8: Model fitting for t 1 for the years 1820 – 1890]

In summary, t = 1 is better suited for short-term conflict analyses, where models achieve lower errors and better reflect real values. On the other hand, t = 10 requires a more advanced approach, especially in the 20th century, where the complexity of conflicts exceeds the current predictive capabilities of these models.

[Figure 2.9: Model fitting for t = 1 for the years 1890-1960]

**Analysis of residuals(p. 46 - 55)**

As part of the analysis of the quality of the model fit, the residuals for different time horizons (t = 1 and t = 10) and historical periods (1820-1960, 1820 1890, 1890-1960) were examined. Figures 2.13, 2.14, 2.15 present the residuals for t = 1, and Figures 2.16, 2.17, 2.18 present the residuals for t = 10. These graphs show the relationship between the predicted and actual values ​​of the number of armed conflicts. Additionally, the Shapiro-Wilk test was conducted to verify the normality of the residuals for each of the models.

[Figure 2.10: Model fitting for t = 10 for the years 1820 – 1960]

For a short time horizon (t = 1), the residual plots show that the models have difficulty accurately predicting extreme values, especially in periods 1820 - 1960 and 1890-1960. The residuals for the ADD models show some systematicity - there is a tendency to underpredict the higher actual values. In the case of the MULT models, the residuals are more scattered, which indicates lower stability of these models.

[Figure 2.11: Model fitting for t = 10 for the years 1820 – 1890]

Shapiro's test showed that the residuals for all models at t = 1 are not distributed normal.

[Figure 2.12: Model fitting for t = 10 for the years 1890–1960]

These results indicate potential problems with the linear model assumptions, which can reduce the quality of prediction. This is particularly visible in the residual plots, where there is no random distribution of points around the horizontal axis; patterns appear that indicate model misfit across different ranges of values.

For the longer time horizon (t = 10) the residual plots show a better fit models in most cases.

[Figure 2.13: Analysis of residuals for t = 1 for the years 1820 – 1960]

The residuals for ADD are more evenly distributed around the horizontal line (with value 0), suggesting that these models are better at predicting over longer time intervals. However, the MULT models in the period 1820–1890 have clear fit problems – the residuals show significant deviations and are not evenly distributed.

[Figure 2.14: Analysis of residuals for t = 1 for the years 1820 – 1890]

The Shapiro test confirmed that the residuals were normally distributed for most models at t = 10, except for MULT/CON LEADS and MULT/CON LAGS in the period 1820–1890. This indicates better fulfillment of the model assumptions in the longer time horizon, which may explain the better fit of these models compared to t = 1.

Comparing the results for t = 1 and t = 10, it can be seen that the longer time horizon (t = 10) leads to more stable residuals, especially in the ADD models.

[Figure 2.15: Analysis of residuals for t = 1 for the years 1890 – 1960]

The MULT models have difficulties at both t = 1 and t = 10, which may be due to the greater complexity of their assumptions. The residuals at t = 10 are less dispersed and more even, suggesting that the models cope better with historical data at a longer time interval.

However, the Shapiro test reveals a difference in the fulfillment of the assumptions of the normal distribution of residuals for t = 10, while for t = 1 none of the models satisfy it.

[Figure 2.16: Analysis of residuals for t = 10 for the years 1820 – 1960]

This indicates the potential benefits of using a longer time horizon in analyses, especially for predicting conflicts in more complex historical systems.

In summary, the models at t = 10 seem to be more reliable in the longer term.

[Figure 2.17: Analysis of residuals for t = 10 for the years 1820 – 1890]

However, for short-term forecasts (t = 1) it may be necessary to include additional variables or modify model assumptions to improve the quality of the fit.

[Figure 2.18: Analysis of residuals for t = 10 for the years 1890-1960]

**Summary(p.56)**

In this chapter, a detailed analysis of the Singer model was performed by modifying one of its key parameters, designated as parameter t. This parameter, as indicated, plays an important role in the model dynamics, directly influencing the solution trajectories and their stability over time.

Bivariate analysis indicated that for both t = 1 and t = 10, the CON variable has a negative effect on the dependent variable WAR, while the MOVE variable almost always shows the greatest effect on WAR.

In the model evaluation, the models for t = 10 showed a better ability to reflect real data compared to the models for t = 1. Moreover, for t = 1 the ADD models turned out to be the leader, not the MULT models, in contrast to the models for t = 5, although none of the models achieved high values ​​of the coefficient of determination R2.

The analysis also showed that in the case of t = 1 for the period 1820-1890 all models performed worse than in the other periods. The MULT/CON LEADS model achieved exceptional results for t = 10 in the same period, obtaining R2 = 0.731, with the DCON variable showing the greatest impact on WAR.

The analysis shows that modifying the parameter t in the Singer model can significantly change its results, which emphasizes the importance of selecting appropriate parameters in conflict modeling. The conclusions drawn provide a solid basis for further research on the use of mathematical models in this field.

**2.3 War forecasting for the time period up to the year 2000(p.57-88)**

Due to the fact that preparing the appropriate data and publishing the results is a time-consuming process, a significant challenge is to assess the possibility of using existing models to predict future phenomena. In the context of analyzing armed conflicts, it is particularly interesting to check whether a model based on historical data from previous years can be useful in predicting future conflicts.

In this chapter, an attempt was made to forecast the dependent variable WAR for the period up to the year 2000, using models prepared on the basis of data from different historical periods: 1820–1960, 1820–1890 and 1890–1960, and also using different values ​​of the parameter t (t = 1, t = 5, t = 10). Additionally, the results of these models will be compared with the model trained directly on data from the years 1950–2000. This analysis will allow us to answer the question of whether models based on historical data are able to effectively forecast future phenomena, or whether it is more effective to use models trained on current, contemporary data.

The first half of this chapter will present the forecast results of models with different values ​​of t, trained on the time periods proposed by J.D. Singer: 1820-1960, 1820-1890, 1890-1960. The analysis process included assessing the quality of forecasts by comparing the predicted WAR values ​​with the actual data, presenting graphs of the predicted values ​​with the actual values, and calculating the mean absolute error (MAE) and mean square error (MSE). In addition, the analysis of residuals was performed, including their visualization and testing for normality using the Shapiro-Wilk test.

The second half of this chapter will present results for models trained with different values ​​of t over the postwar period 1950–2000. These results include model evaluations, plotting predicted versus actual values, calculating MAE and MSE, and analyzing residuals.

**Forecasts of models with t = 1 based on historical data from the period 1820-1960 and its divisions (p.57-62)**

Figures 2.19, 2.20, 2.21 present the forecasts of the WAR variable until the year 2000 for the ADD/CON and MULT/CON (LEADS and LAGS) models trained on three different time periods: 1820–1890, 1820–1960 and 1890–1960. Table 2.20 also presents the values ​​of the mean absolute error (MAE) and the mean square error (MSE) for each of the models.

[Table 2.20: Mean Absolute Error (MAE) and Mean Squared Error (MSE) for t 1 models trained on three time periods]

Models trained on the period 1820-1890 are characterized by noticeable differences between actual and predicted values ​​for the years after 1890. It is particularly visible that the values ​​predicted by ADD models tend to underestimate actual values ​​in periods of increased conflict. MULT models, although they reproduce the dynamics of the WAR variable somewhat better, also have difficulty capturing extreme values.

The MAE and MSE values ​​for this period are lower compared to longer training periods, suggesting that the models are better suited to historical data, but their ability to predict future values ​​is limited. For example, the MAE for MULT/CON LAGS is only 12.24 and the MSE is 398.85, which is the lowest compared to other periods.

Period 1820-1960 Models trained on the full historical period have a better fit to actual values ​​in the post-war years (1945-1960), which is especially visible in ADD models. However, the values ​​predicted up to the year 2000 show significant deviations from actual values.

The MAE and MSE for this period are higher than for the 1820–1890 period, indicating the difficulty of the models in reproducing the more complex conflict dynamics spanning the 20th century. For example, MULT/CON LEADS achieves an MSE of 422.82, which is higher than for shorter periods.

Models trained on the period 1890–1960 show the weakest fit to historical data and forecasted values ​​to the year 2000. This is particularly visible in ADD models, which fail to reproduce actual values ​​during periods of intense conflict, such as world wars. MULT, although they reproduce values ​​better in the training period, also have difficulty predicting future values, and their forecasts for the years after 1960 show clear differences in relation to actual values.

[Figure 2.19: Model prediction for t = 1 trained on 1820 – 1960]

The MAE and MSE values ​​for this period are the highest among all analyzed. For example, ADD/CON LAGS reaches an MSE of 509.08, which is a clear indicator of a mismatch. The predicted values ​​in ADD models, which are often negative, may be due to the linear nature of the models and the lack of lower bounds on the predicted variable WAR.

[Figure 2.20: Model prediction for t = 1 trained on 1820 – 1890]

These models do not take into account the fact that the WAR variable represents the number of armed conflicts, which in reality cannot take negative values. A solution to this problem could be to use a logarithmic transformation (MULT models) or other methods that restrict the forecasts to non-negative values.

Models trained on longer periods of time tend to reproduce better trends in historical data, but their ability to predict values ​​outside the training range is limited. Negative values ​​predicted by the models indicate the need to modify the methodology, for example by including additional constraints or using more advanced modeling techniques. MULT models seem to be better at reproducing conflict dynamics, but their results also require further optimization.

**Analysis of residuals (p.61)**

Residual plots for the ADD/CON LEADS, ADD/CON LAGS, and MULT/CON LE-ADS, MULT/CON LAGS models trained on three time periods (1820–1890, 1820–1960, and 1890–1960) are presented in Figures 2.22, 2.23, 2.24. The residual plots allow us to assess the quality of the model fit and to check whether the assumptions about the normal distribution of the residuals are met. The results of the Shapiro test indicate that in each case the residuals are not normally distributed, suggesting potential problems with the accuracy of the models or their assumptions.

For the models trained on the period 1820-1890, the residuals are very diverse and show a clear asymmetry. The ADD/CON LEADS and LAGS models generate significant residuals for the actual values ​​of the WAR variable, especially in the regions of high predicted values. The residuals for the MULT/CON LEADS and LAGS models are somewhat more scattered, which indicates difficulties in representing the WAR variable at higher values.

The distribution of residuals for this period shows a lack of randomness, which may suggest poor model fit or missing key variables in the analysis. This confirms the result of the Shapiro test, which indicates that the distribution of residuals significantly deviates from normality.

Models trained on a longer period (1820-1960) generate residuals with a larger range of values, which is especially visible in ADD models. For these models, it can be seen that the residuals systematically increase with the predicted value, which indicates the possible presence of heteroscedasticity (the variability of the residuals is not constant).

In the MULT/CON LEADS and LAGS models, the residuals are more compactly distributed, but there are still significant deviations for high WAR values. It is worth noting that a longer training period introduces more variability in the residuals, which may be due to more complex trends in the historical data.

For the models trained on the period 1890-1960 the residuals are the least random, which is especially visible in the ADD/CON LEADS and LAGS models. These models generate clearly systematic errors, which indicates their poor fit to the data.

For the MULT/CON LEADS and LAGS models, the residuals are also not normally distributed, but are more evenly distributed than for the ADD models. These results indicate that the MULT models may be better suited to the data from this period, but they still have difficulty representing the true WAR variable.

[Figure 2.22: Plots of residuals for t = 1 trained on the years 1820 – 1960]

**Forecasts of models with t = 5 based on historical data from the period 1820-1960 and its divisions (p.62-71)**

The analysis of forecasts of the WAR variable for the model parameters (t=5) and for three time periods (1820-1960, 1820-1890, 1890-1960) is presented in Table 2.21.

(MAE and MSE) and in Figures 2.25, 2.26, 2.27 indicate significant differences in the prediction quality between the ADD models and the MULT models, as well as a significant influence of the training period on the model results.

[Figure 2.23: Plots of residuals for t = 1 trained on the years 1820-1890]

The ADD/CON LEADS and ADD/CON LAGS models for all three time periods show relatively stable predictions of the WAR variable. The forecast graphs values, have small deviations from the actual values, especially in more recent years.

[Figure 2.24: Plots of residuals of models for t = 1 trained on the years 1890-1960]

The MSE and MAE error rates in these models are comparable, with the ADD/CON LEADS model having MSE = 2432.10 and MAE = 36.31 for the period 1820-1960, and MSE = 2247.09 and MAE = 31.77 for the period 1820-1890. These values ​​indicate that the ADD models generally fit well, although their prediction quality declines with distance from the training period.

[Figure 2.25: Forecast of models for t = 5 trained on the years 1820-1960]

For the MULT/CON LEADS and MULT/CON LAGS models the situation is diametrically different. In the case of these models, very large forecast values ​​of the WAR variable were observed, especially in the period 1820-1890. For example, in the MULT/CON LEADS model in the years 1920 and 1940 the forecasts reach values ​​exceeding 500,000, which is definitely unrealistic and indicates significant overfitting of the model.

[Figure 2.26: Model prediction for t = 5 trained on 1820-1890]

Such results are the result of improper scaling of values ​​in the MULT models, which include products between variables, leading to escalation of predicted values ​​in the case of large ranges of predicted values. The error scores for these models are also unacceptably high, especially for the period 1820-1890.

[Figure 2.27: Model prediction for t = 5 trained on 1890 – 1960]

Analyzing the period 1890-1960, MULT models show smaller deviations of predicted values, but still their MSE and MAE are larger than in the case of ADD models. This is due to the fact that the training period includes more similar values ​​of variables to those occurring in the forecasted period.

[Table 2.21: Mean Absolute Error (MAE) and Mean Square Error (MSE) for Models with t = 5 trained in three time periods]

In summary, the MULT models in the analysis with t = 5 show significantly worse fit than the ADD models. Their excessively high predicted values ​​and very large MSE and MAE values ​​indicate that they cannot be used in practical forecasts. The ADD models, on the other hand, despite some limitations in accuracy, are more stable and offer better fit in different time periods. These results emphasize the importance of proper choice of model structure and precise scaling of variables in quantitative analysis.

**Analysis of residuals(p.68)**

For models with t = 5, analysis of residuals indicates significant differences in fit for different time periods and models. The analysis is presented in Figures 2.28, 2.29, 2.30.

Based on the residual plots and the results of the Shapiro-Wilk test, the following conclusions can be drawn.

In the period 1820-1960 in the ADD/CON LEADS and ADD/CON LAGS models the residuals show moderate deviations and their distribution is asymmetric, which indicates the non-normality of the residuals in these cases. The values ​​of the residuals are significantly smaller compared to the MULT/CON models, which confirms the better fit of the ADD models in this period. The Shapiro test confirms the non-normality of the residuals in both the ADD and MULT models.

The MULT/CON LEADS and MULT/CON LAGS models show very large residuals, especially in periods where WAR values ​​were significantly overestimated, which indicates a problem with prediction at large values ​​of input variables. The residuals in the MULT models fall outside the range of expectations, which makes these models unreliable.

For the ADD/CON LEADS and LAGS models in the period 1820-1890 the residuals have a smaller range compared to the other periods. Additionally, the Shapiro test shows normality of the residuals in both ADD models, which indicates a better quality of fit of these models in this period.

The MULT models produce unusually high residuals in this period, which is due to errors in predicting extreme WAR values. These models are clearly inappropriate for this period, as evidenced by both the residuals and the MSE results (which in the case of MULT/CON LEADS reach a value of 16224055348.28).

The ADD/CON LEADS and ADD/CON LAGS models show a relatively better fit in the period 1890-1960, with smaller residuals and relatively lower MSE values ​​compared to the period 1820-1960. The Shapiro test confirms the normality of the residuals in the ADD/CON LAGS model, confirming its stability in this period.

[Figure 2.28: Plots of residuals for t = 5 trained on the years 1820 – 1960]

The MULT models continue to show very high residual values ​​that are disproportionate to the actual WAR values. These results indicate that the MULT models are inappropriate for the analysis of this period.

The analysis of residuals and Shapiro test results indicates that the ADD models are significantly more stable and predictable compared to the MULT models, especially for the 19th century period.

[Figure 2.28: Plots of residuals for t = 5 trained on the years 1820 – 1960]

The MULT models continue to show very high residual values ​​that are disproportionate to the actual WAR values. These results indicate that the MULT models are inappropriate for the analysis of this period.

The analysis of residuals and Shapiro test results indicates that the ADD models are significantly more stable and predictable compared to the MULT models, especially for the age 19 period.

[Figure 2.29: Plots of residuals for t = 5 trained on the years 1820-1890]

MULT models generate disproportionately high residual values, which makes them inadequate for the analysis of armed conflicts. It is worth noting that ADD models, despite their better fit, also require further optimization, especially for the period 1820-1960, where the normality of residuals was not confirmed.

[Figure 2.30: Plots of residuals of models for t = 5 trained on the years 1890-1960]

**Forecasts of models with t = 10 based on historical data from the period 1820-1960 and its divisions(p.71-79)**

In the analysis of the model forecast results for t = 10, the presented graphs (Figures 2.31, 2.32, 2.33) show the predicted values ​​of the WAR variable for the trained models on the time periods: 1820–1960, 1820–1890, and 1890–1960. Table 2.22 presents the results of calculating the mean errors MAE and MSE.

[Table 2.22: Mean Absolute Error (MAE) and Mean Square Error (MSE) for Models with t = 10 trained in three time periods]

Let's take a closer look at the results for different periods.

For the period 1820-1960 the results indicate a relatively good fit of the ADD models, especially in the case of the ADD/CON LEADS model, where the predicted values ​​gradually decrease, reflecting the general downward trend of the actual values. Nevertheless, the differences between the predictions and the reality are still visible, which is reflected in the relatively high values ​​of the MSE and MAE. The MULT models, both LEADS and LAGS, perform worse, which is especially visible in the larger deviations of the predicted values ​​from the actual ones.

In the period 1820-1890, MULT models show extreme predicted values ​​that significantly deviate from the actual data. The graphs show predicted values ​​in the millions, which is a result of the excessive sensitivity of these models to the variability of the training data. This result can be explained by the small number of observations in this period, which limits the model's ability to generalize. Such a problem often leads to unstable results in more complex models such as MULT. Moreover, the MSE and MAE values ​​are huge, which further indicates the inadequacy of these models in this context.

For the period 1890-1960, the ADD models show a better fit, and the predicted values ​​of the WAR variable show a downward trend similar to the actual data. However, there are still deviations, especially in the LAGS model, which shows larger errors for this variant. The MULT models, although less extreme than for the period 1820-1890, still have greater prediction problems, as evidenced by the increased MSE and MAE values.

In general, the results of the analysis for t = 10 show that the ADD models are more stable and fit better in each analyzed period. The MULT models, especially in the period 1820-1890, are not suitable for prediction, which is due to the small number of observations and the greater complexity of these models, which makes them susceptible to overfitting. In the research context, these results suggest that the simpler ADD models are more suitable for analyzing historical trends in the WAR variable, especially in the case of limited data.

**Analysis of residuals (p.72-79)**

The results of the residual analysis for the t = 10 models are presented in Figures 2.34, 2.35, 2.36 and indicate significant differences in the quality of forecasts depending on the time period and model used. Models were analyzed for three periods: 1820-1960, 1820-1890 and 1890-1960.

[Figure 2.31: Model prediction for t = 10 trained on 1820 – 1960]

The models trained on the period 1820-1960 show normality of residuals. In the period 1820-1890, the ADD/CON LEADS and ADD/CON LAGS models trained on 19th century have normal distribution of residuals, while the remaining models, in particular MULT, show significant deviations from normality.

[Figure 2.32: Model prediction for t = 10 trained on the period 1820-1890]

In ADD models trained on the period 1820-1960, the residual values ​​fluctuate around zero, which indicates a relatively good agreement with the actual data. The values ​​predicted by ADD models are moderately accurate, as confirmed by the lower MAE and MSE values ​​compared to MULT models.

[Figure 2.33: Model prediction for t = 10 trained on 1890–1960]

MULT models are characterized by a larger scatter of residuals and predictions that are slightly far from reality. However, for this period, all models meet the criteria for normality of the residual distribution. In this period 1820-1890, it is clearly visible that the ADD/CON LEADS and ADD/CON LAGS models achieve the best results in terms of forecast agreement with actual data.

[Figure 2.34: Plots of residuals for t = 10 trained on the period 1820 – 1960]

MULT models produce extremely high predicted values, leading to very large residuals and reduced forecast quality.

The ADD models for the period 1890-1960 continue to perform moderately well, with residual values ​​concentrated around zero. The MULT models continue to exhibit larger deviations and produce residual values ​​that are clearly different from reality. The values Of MSE and MAE for the MULT models are much larger compared to the ADD models, which clearly indicates the advantage of the ADD models in this period.

[Figure 2.35: Plots of residuals for t = 10 trained on the period 1820 – 1890]

It should be noted that in the MULT models the predictions for the period 1820-1890 reach extremely high values ​​(in the order of millions in some cases), which makes these models completely useless in a research context. Such high values result from the fact that the small number of observations and the specificity of the data in the period 1820-1890 lead to an overestimation of the parameters of the MULT models.

[Figure 2.36: Plots of residuals of models for t = 10 trained on the period 1890-1960]

As a result, the MULT models become unstable and unreliable in forecasting.

The analysis of the residuals for t = 10 confirms that the ADD models show significantly better results compared to the MULT models, both in terms of the agreement of the predictions with the data

MULT models, especially in the period 1820-1890, are prone to instabilities resulting from the small number of observations, which leads to the generation of useless forecasts. These results confirm the need for a cautious approach to MULT models in the case of limited historical data.

**Forecasts of models with t = {1,5, 10} based on historical data from the period 1950 – 2000 (p.79 - 87)**

**Model evaluation (p. 79-80)**

The analysis of the model evaluations for different values ​​of t covers the period 1950–2000 and is presented in Table 2.23. This table presents the estimates of the coefficients R, R², R², as well as the values ​​of b and r² for the dependent variables CON, DCON and MOVE.

[Table 2.23: Fit of standardized models for all t for the period 1950- 2000]

For t = 1, the best results are achieved by the ADD/CON LEADS model, for which R2 = 0.194, and R2 = 0.142. All predictors in this model show negative coefficient values ​​with respect to the WAR variable. This is an atypical result, compared to previous analyses, it indicates a general tendency to reduce the probability of conflict with the growth of the analyzed variables, which may result from the dynamics of the post-war period and the role of the balance of power in the international system. The DCON value has the greatest influence in the LEADS models (b = -0.420, r2 = 0.153), while CON plays a key role in the LAGS models (b = -0.341, r2 = 0.101). The MOVE variable shows a negligible influence, which suggests its limited significance in forecasts with smaller t divisions.

For t = 5, the MULT/CON LEADS model is characterized by a significant increase in fit, where R2 = 0.618, and R2 = 0.426. The greatest influence on the model result has the MOVE variable, whose coefficient b = -0.736, and r² = 0.477. The negative value of the MOVE coefficient in this case is surprising and may indicate a specific role of power movement in generating tensions during the Cold War. The DCON variable in the MULT/CON LAGS model also achieves a high level of fit (r² = 0.435).

The results for t = 10 indicate an outstanding model fit, reaching almost perfect values ​​of R2 = 0.998 and R2 = 0.993 for the ADD/CON LEADS model, and R2 = 1.000 for the MULT models. The predictors in these models have negative coefficients, which may result from the systematic interaction of variables in the long term t. The exception is DCON in the ADD/CON LAGS model, where r² = 0.687, which indicates possible differences in the dynamics of variables in the long-term modeling.

Comparing the results for different values ​​of t, there is a clear increase in the predictive ability of the models as the analysis period t is extended. The results for t = 10 predict almost 100% of the variability, which indicates the stability of long-term relations in the international system. The negative values ​​of the coefficients suggest that these variables may be indicators of adaptation or balancing mechanisms in the international system that reduce the likelihood of conflict.

The results highlight the importance of model conceptualization in the analysis of international relations and the need to consider both short and long divisions of variable counting in order to better understand the mechanisms in the system.

**Model fitting (p. 80-84)**

The analysis of the models for different values ​​of t (t = 1, t = 5, t = 10) in the period 1950 - 2000 was performed on the basis of mean absolute errors (MAE), mean squared errors (MSE) (Table 2.24) and graphs comparing the actual values ​​of the WAR variable with the values ​​predicted by the models (Figures 2.37, 2.38, 2.39).

[Table 2.24: Mean Absolute Error (MAE) and Mean Square Error (MSE) for models with different t]

In the case of t = 1, the models show relatively low errors, which, however, does not translate into high quality of predictions. The MULT/CON LAGS model achieved the lowest error values ​​(MAE = 11.35, MSE = 310.77), indicating its relatively better forecasting ability. The ADD models have similar results (MAE ≈ 12.35–12.86), but their MSEs are higher, suggesting larger discrepancies between the actual and predicted values ​​at some points. MULT/CON LEADS achieved the highest error (MSE = 902.26), which is the result of overestimating the WAR variable in key years such as the 1970s.

The graphs show that the models have difficulty capturing the variability of the actual WAR values. The ADD models are close to the actual values ​​in most years, but there are noticeable discrepancies in periods of large fluctuations, such as the 1970s and 1980.

[Figure 2.37: Model fitting for t = 1 for the years 1950-2000]

In turn, the MULT models, especially MULT/CON LEADS, significantly overestimate the WAR values ​​in key years, which leads to a high MSE error. MULT/CON LAGS performs slightly better, but still does not fully capture the variability of the data. The graphs for t = 1 indicate the difficulties of the models in capturing large fluctuations in the actual values ​​of the WAR variable. The models are able to predict general trends, but they have problems with dynamic changes that characterize short-term events in the international system.

[Figure 2.38: Model fitting for t = 5 for the years 1950-2000]

For t = 5 the model errors are clearly higher compared to t = 1, but the predictions become more stable. MULT/CON LAGS shows again the lowest error values ​​(MAE = 21.71, MSE = 1116.90), which indicates its ability to better forecast of the actual values ​​of the WAR variable in the medium term.

[Figure 2.39: Model fitting for t = 10 for the years 1950 – 2000]

The ADD models achieve similar results (MAE ≈ 22.24-22.96), but their MSE values ​​are higher, suggesting a less precise fit in years with large changes. MULT/CON LEADS obtains the highest squared error (MSE = 1115.23), indicating its limited performance in predicting dynamic changes.

In the graphs, the models show an improvement in reproducing the general trends, but still have difficulty accurately reproducing the actual values ​​in years of high volatility. ADD models are better at predicting values ​​in calmer periods, but their fit is weaker during extreme values, such as the 1960s and 1970s. MULT shows a clearly better reproduction of the actual trends, with MULT/CON LAGS achieving greater stability and accuracy in key years. For t = 10, the models show the greatest precision in predicting the value of the WAR variable. MULT/CON LAGS achieves an almost perfect fit (MAE = 0.05, MSE = 0.00), which means that this model perfectly reproduces the actual values ​​of the variable. Similarly, ADD/CON LEADS shows very low errors (MAE = 0.64, MSE = 0.81), which indicates its performance in long-term forecasting. In turn, ADD/CON LAGS achieves high error values ​​(MAE = 44.37, MSE = 3287.96), which indicates problems in long-term matching.

The fit graphs for t = 10 show that the MULT/CON LAGS and ADD/CON LEADS models almost perfectly reproduce the actual values ​​of the WAR variable. The stability of these models may be due to their ability to take into account recurring patterns and systemic dependencies in international relations. While ADD/CON LAGS and MULT/CON LEADS show clear divergences in the final period of analysis, although they also detect trends in general.

The fit graphs for t = 10 show that the MULT/CON LAGS and ADD/CON LEADS models almost perfectly reproduce the actual values ​​of the WAR variable. The stability of these models may be due to their ability to take into account recurring patterns and systemic dependencies in international relations. While ADD/CON LAGS and MULT/CON LEADS show clear divergences in the final period of analysis, although they also detect trends in general.

Comparison of the plots for different values ​​of t shows that the ability of the models to reproduce the actual values ​​of the WAR variable improves with the extension of the period t. For t = 1, the models have difficulty capturing dynamic changes and large fluctuations, leading to larger discrepancies between actual and predicted values. For t = 5, there is an improvement in capturing the overall trends, but the models still have problems with accuracy in key years. For t = 10, the models indicate their ability to integrate long-term dependencies.

**Analysis of residuals(p.84-87)**

Analysis of the model residuals for different values ​​of t allows for the assessment of the quality of fit and the identification of potential systematic errors. Residual plots show the differences between the actual and predicted values ​​of the WAR variable, which allows for the assessment of the distribution of prediction errors (Figures 2.40, 2.41, 2.42). In addition, Shapiro-Wilk tests were performed to check the normality of the distribution of residuals.

At t = 1, the residuals for the ADD models show a large dispersion and no clear symmetry around the 0-axis. In particular, these models tend to generate both high positive and negative residuals in years with extreme values ​​of the WAR variable. Even larger discrepancies are visible for the MULT models, especially for MULT/CON LEADS, where the residuals reach values ​​exceeding -150, indicating a strong overestimation in some cases. It is worth noting that the Shapiro-Wilk test did not confirm the normality of the distribution of residuals for any of the models at t = 1, indicating systematic deviations in the predictions.

For t = 5, the MULT models show a significant improvement in the distribution of residuals. The residuals for these models are more concentrated around the 0-axis, suggesting smaller biases. The Shapiro-Wilk test shows that the distribution of residuals in the MULT/CON models is normal, confirming their better fit in the medium term. In contrast, the ADD models still show large deviations and the residuals are more scattered, especially in years with large values ​​of the WAR variable. The lack of normality in the distribution of residuals in these models suggests that they do not fit the data well enough.

[Figure 2.40: Plots of residuals for t = 1 over the period 1950-2000]

For t = 10, the residual plots show an almost perfect fit for the MULT models. The residuals are close to 0 and well distributed along the axis of the predicted WAR values, indicating no systematic errors. The ADD models also show good quality of fit, although the ADD/CON LAGS model shows some deviations in the final analysis period. The Shapiro-Wilk test confirmed the normality of the residual distribution for all models at t = 10, which indicates the high quality of their predictions.

[Figure 2.41: Plots of residuals for t 5 over the period 1950-2000]

These results emphasize that extending t improves the quality of model forecasts over the period 1950–2000, especially for MULT models. These models are more effective in taking into account the long-term dependencies in the international system, while ADD models show greater problems with prediction accuracy over shorter periods.

[Figure 2.42: Plots of residuals for t = 10 over the period 1950-2000]

The normality of the distribution of residuals for t = 10 indicates that these models are well calibrated and can be a reliable tool in the analysis of armed conflicts.

**Summary (p. 88)**

**Summary of results for models trained on historical data (1820-1960, 1820-1890, 1890-1960).** Analysis of models trained on historical data showed significant differences in forecasting performance depending on the value of t and the period on which the models were trained.

ADD models generally performed better than MULT models. This was evident both in lower mean absolute error (MAE) and mean square error (MSE) values, and in more stable forecasts that were closer to the actual values ​​of the WAR variable. MULT models, especially for t = 5 and t = 10, showed significant deviations from the actual values, which was due to their susceptibility to instabilities with limited data. In the period 1820–1890, MULT models generated predicted values ​​that were many times higher than the actual WAR levels, making them unsuitable for practical use.

The analysis of residuals and Shapiro-Wilk tests showed that the normality of residuals was mainly satisfied in ADD models, which underlines their better prediction quality. MULT models, in most cases, did not meet this criterion, which indicates the presence of systematic errors. These results suggest that ADD models are more suitable for long-term forecasting of armed conflicts based on historical data, especially in situations where data are limited.

**Summary of results for models trained on data from 1950–2000**. Analysis of models trained on data from the period 1950–2000 confirmed the higher efficiency of models trained on contemporary data compared to models based on historical data.

The MULT models, especially for t = 10, achieved an almost perfect fit, which was visible both in the plots of forecasts and actual values, as well as in very low MAE and MSE values. MULT/CON LAGS at t = 10 showed almost zero errors (MAE = 0.05, MSE = 0.00), indicating a perfect representation of the actual WAR values. ADD models also achieved good results, although for some cases (e.g. ADD/CON LAGS) the errors were larger, especially in the long run.

Analysis of the residuals indicated that for t = 10 the residuals were normally distributed in all models, which was confirmed by Shapiro-Wilk tests. For t = 5, the normality of the residual distribution was present only in the MULT models, while for t = 1 none of the models met this criterion. These results suggest that extending the time horizon t allows for more stable and predictable results.

**Summary (p.89-90)**

In this paper, we conducted a detailed analysis of J. David Singer's model, including both a replication of the author's research and our own modifications and extensions of the model. The aim was to assess its effectiveness in the context of contemporary data and changes in methodology.

The first section of the empirical analysis (2.1) focused on the application of the classical Singer model with the assumption of t = 5, covering three historical periods: 1820–1960, 1820–1890, and 1890–1960. The results of this analysis showed differences compared to Singer’s original results, which may be related to changes in the quality and scope of data over the last fifty years. For example, in Singer’s analyses, the ADD models were dominant, while in this study the MULT/CON LAGS model proved more effective. These differences may be due to the development of data collection methods, the increased availability of information for a larger number of countries, and the subjectivity in assessing historical conflicts. This fact highlights a key challenge in historical research: the time perspective affects the objectivity of interpretation. Therefore, in the War Data Set, the data do not cover years after 2003, because at least two decades are needed for the assessments to become more objective.

The second section (2.2) analysed the impact of the change in parameter t on the model results. At t = 1 and t = 10, different patterns of the impact of the variables CON, ΔCΟΝ and MOVE on the dependent variable WAR were identified. For example, the bivariate analysis for the period 1820-1960 showed that the CON variable had a significant negative impact on the probability of conflict, which may reflect the historical dynamics of international power, in which political stability was more predictable in times of high concentration of power. In the same period, the best performance in the multivariate analysis is achieved by the MULT/CON LAGS model at t = 5, the CON and DCON variables played a key role, which can be associated with periods of intensive change in the balance of international power.

In the period 1820-1890 the highest efficiency was achieved by the MULT/CON LEADS model at t = 10, the DCON variable played a key role, which may result from relatively slow changes in international systems in the 19th century. In contrast, for the years 1890-1960 the MULT/CON LEADS model dominated at t = 5, indicating a more dynamic redistribution of power (CON) in this period, associated with violent world conflicts and the emergence of new international orders.

The ADD models were characterized by clear responses to changes in the predictor variables, which meant that they reflected short-term trends well. In turn, the MULT models, ignoring intense conflicts, were better at predicting stable periods. Similarly, the LAGS and LEADS models differed in their ability to predict depending on the dynamics of past and future changes in the balance of power.

In Chapter 2.3, the comparison of models trained on historical and contemporary data shows significant differences in their forecasting performance. Models trained on historical data were characterized by larger errors and lower stability, which was particularly true for MULT models. In the case of models trained on data from 1950–2000, their higher prediction quality was clearly noticeable, which resulted from better representativeness of the data for the analyzed period.

These results underscore the importance of using contemporary data in forecasting armed conflicts. ADD models are more stable and perform better in data-limited situations. In contrast, MULT models, although more prone to instability in data-limited conditions, were highly effective when trained on the full range of available data for the period 1950-2000.

The results of this work emphasize the importance of further developing Singer's methodology, as well as the need to continue the Correlates of War project, which is a fundamental source of knowledge about the dynamics of armed conflicts. Systematic data collection and their continuous analysis will allow for the creation of models that will not only reflect the dynamics of international conflicts, but will also contribute to their better understanding and prevention.

These studies also pointed out some limitations and challenges. At short time intervals (t = 1), the variables did not fully capture the changes in concentration (DCON) in the international system, which limited their effectiveness. At the same time, intervals t = 10, although promising, would require more data to be fully exploited. However, these challenges do not weaken the value of Singer's model, but show that it must evolve to meet new research requirements and challenges of the modern world. Continuing this research is also a key step towards more effective forecasting and reducing armed conflicts in the future.

**Appendix**

[Table 2.25: Results of calculations of descriptive statistics of model variables in the J. D. Singer’s analysis.]

[Table 2.26: Results of the calculations of the correlation of independent variables with the probability of war in the analysis by J. D. Singer]

[Table 2.27: Evaluation results of standardized models in the J. D. Singer analysis]

**Bibliography**

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