## Appendix A
## The derivation of QBER

Quantum Bit Error Rate (QBER) refers to the bit-flip error caused by the collapse decoherence here. First, we provide a theorem:

**Theorem 1.** *Given the collapse decoherence model with the entanglement fidelity* $F(|\psi\rangle_d^f)$*, its QBER is deduced as* $p'' = F^{-1}(F^d_{p''}, d)$.

*Proof.* The quality of each link-level entanglement is compromised to varying degrees due to the bit-flip errors caused by the collapse decoherence noise model. Therefore, the QBER of each link-level entanglement transmission is determined by the entanglement quality. In our works, the link-level entanglement fidelity is used to evaluate the entanglement quality, and denoted as

$$F(|\psi\rangle_d^f) = \sqrt{fd}p(1-p)^{f\cdot d-1}e^{-\tau(\Delta x)^2}, \qquad (1)$$

It is determined by the entanglement dimension $d$, measured probability $p$, and the entanglement link length $\Delta x$ with the assumption of $f = 1$.

For two-qubit entanglement, Zhao *et al.* [1] introduce an equation to describe the relationship between the QBER and the fidelity:

$$F = p''^2 + (1-p'')^2. \qquad (2)$$

It is derived from the flipping and no-flipping simultaneously occurring on both two qubits, which maintains identical two-qubit entanglement state, i.e., the fidelity indicates the degree to which quantum states remain stable.

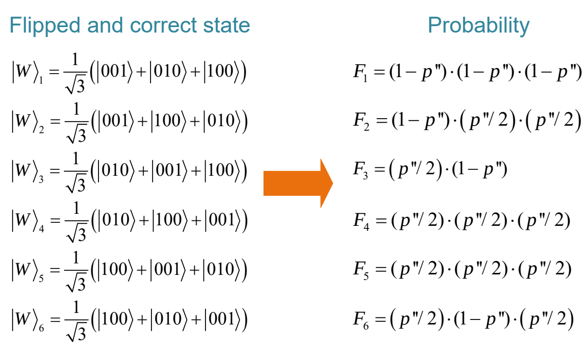


Fig. 1. A three-qubit W entanglement state.

It is completely different under the scheme of the multi-qubit entanglement. As is shown in Fig. 1, a three-qubit entanglement is written as $W$ state as $|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$. If the first qubit in the 1-dimension state $|001\rangle$ flips from $|0\rangle$ to $|1\rangle$, the state in 1-dimension changes from $|001\rangle$ to $|100\rangle$ according to the rules of $W$ state which indicates only one qubit can be at state $|1\rangle$ in a dimension. For a 3-dimensional entanglement state, there are totally $3! = 6$ bit-flip results that can maintain the original results, i.e., $|W\rangle = |W_i\rangle, i = 1, 2, ..., 6$. Fig. 1 provides the fidelity of each bit-flip result. Hence, the total fidelity of $|W\rangle$ is denoted as $F^3 = \sum_{i=1}^{3!} F_i^3$. It is a misalignment problem in the square matrix. And so forth, the fidelity of $d$-dimensional entanglement is derived as:

$$F^d_{p''} = C_d^d(1-p'')^d + \sum_{k=2}^{d}\left[C_d^{d-k}\left(k!\sum_{kk=0}^{k}\frac{(-1)^{kk}}{kk!}\right)(1-p'')^{d-k}(p''/2)^k\right]. \qquad (3)$$

Given a dimension number $d$ and a link-level fidelity $F^d$, we can obtain the QBER $p''$ of this entanglement link by the inverse function $F^{-1}$ of $F^d$. Further, we can obtain the average QBER of an E2E entanglement route for a request. □

## Appendix B
## The derivation of QBER

The phase flip is a specific error in the quantum state, which has no equivalent in classical computing like the bit-flip error. Whereas a bit flip swaps the probabilities of a qubit being $|0\rangle$ or $|1\rangle$, a phase flip swaps the probabilities of the qubit being $+$ or $-$, i.e., the direction of the particle rotation. First, we provide a theorem:

**Theorem 2.** *Given the collapse decoherence model with the entanglement fidelity* $F(|\psi\rangle_d^f)$*, its QPER is deduced as* $p' = F^{-1}(F^d_{p'}, d)$.

*Proof.* Assume a QPER for a two-qubit entanglement is denoted by $p'$, its fidelity can be deduced by $F^2 = p'^2 + (1-p')^2$ because $|W\rangle = \frac{1}{\sqrt{2}}(+|01\rangle + |10\rangle)$ is oppositely same as $|W\rangle = \frac{1}{\sqrt{2}}(-|01\rangle - |10\rangle)$. That two phases both flip is equal in that there is no flipping for entanglement state $|W\rangle$. Likewise, for the three-qubit link-level entanglement, it fidelity is $F^3 = p'^3 + (1-p')^3$. Further, the fidelity of $d$-dimensional link-level entanglement is

$$F^d = p'^d + (1-p')^d. \qquad (4)$$

From our introduced collapse decoherence model $F(|\psi\rangle_d^f) = \sqrt{fd}p(1-p)^{f\cdot d-1}e^{-\tau(\Delta x)^2}$, we can obtain a QPER $p'$ of a link-level $d$-dimensional entanglement solved by the inverse function $F^{-1}(F^d_{p'}, d)$. □

We can see that, QBER is the same as QPER for a link-level entanglement in the two-qubit transmission scheme due to the same fidelity formulation $F^2 = p^2 + (1-p)^2$.

## References


[1] Zhao Y, Zhao G, Qiao C. E2E fidelity aware routing and purification for throughput maximization in quantum networks[C]//IEEE INFOCOM 2022-IEEE Conference on Computer Communications. IEEE, 2022: 480-489.