## 9.7

1.(1) $(xdx+ydy)\wedge(zdz-zdx)=yz\,dx\wedge dy+yz\,dy\wedge dz-xz\,dz\wedge dx$

(2) $(dx+dy+dz)\wedge(xdx\wedge dy-zdy\wedge dz)=-z\,dx\wedge dy\wedge dz+x\,dz\wedge dx\wedge dy$

$=(x-z)\,dx\wedge dy\wedge dz$

2.(1) $d\omega=(y+z)dx+(z+x)dy+(x+y)dz$

(3) $d\omega=d(xy)\wedge dx+d(x^2)\wedge dy=x\,dy\wedge dx+2x\,dx\wedge dy$

$=x\,dx\wedge dy$

17. $dy_j=\sum_{i=1}^{n}\frac{\partial y_j}{\partial x_i}dx_i$, $j=1,2,\cdots,n$

故 $dy_1\wedge dy_2\wedge\cdots\wedge dy_n=(\sum_{i=1}^{n}\frac{\partial y_1}{\partial x_i}dx_i)\wedge\cdots\wedge(\sum_{i=1}^{n}\frac{\partial y_n}{\partial x_i}dx_i)$

$=\sum_{1\le i_1,\cdots,i_n\le n}((\prod_{k=1}^{n}\frac{\partial y_k}{\partial x_{i_k}})dx_{i_1}\wedge\cdots\wedge dx_{i_n})$

根据外积与行列式运算性质可知.

上式 $=\frac{\partial(y_1,\cdots,y_n)}{\partial(x_1,\cdots,x_n)}dx_1\wedge\cdots\wedge dx_n$

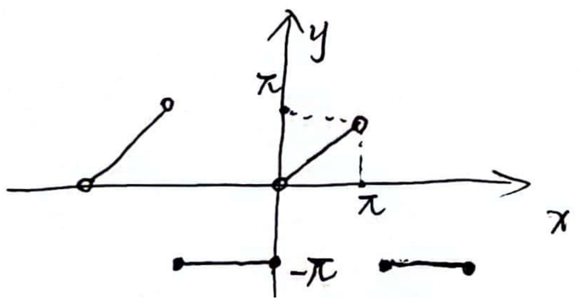
## 12.1

1.(1) $a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=-\pi+\frac{1}{\pi}\cdot\frac{\pi^2}{2}=-\frac{\pi}{2}$

$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx=\frac{(-1)^n-1}{n^2\pi}$ $(n\ge1)$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx=\frac{1-2(-1)^n}{n}$ $(n\ge1)$

$f(x)\sim-\frac{\pi}{4}+\sum_{n=1}^{\infty}(\frac{(-1)^n-1}{n^2\pi}\cos nx+\frac{1-2(-1)^n}{n}\sin nx)$




在 $x=2k\pi$ 处收敛到 $-\frac{\pi}{2}$

$x=(2k+1)\pi$ 处收敛到 0 $(k\in\mathbb{Z})$

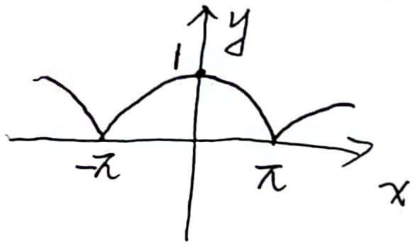
其他处收敛到 $f(x)$ 本身

1.(2) $a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=\frac{4}{\pi}$

$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx=\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(n-\frac{1}{2})x+\cos(n+\frac{1}{2})x\,dx=\frac{4(-1)^{n+1}}{(4n^2-1)\pi}$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\sin nx\cos\frac{x}{2}dx=0$

$f(x)\sim\frac{2}{\pi}+\sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{(4n^2-1)\pi}\cos nx$，处处收敛到 $f(x)$ 自身



2.(1) $a_0=\frac{2}{\pi}\int_0^{\pi}(1-\sin\frac{x}{2})dx=2-\frac{4}{\pi}$

$a_n=\frac{2}{\pi}\int_0^{\pi}f(x)\cos nx\,dx=\frac{4}{(16n^2-1)\pi}$

$b_n=\frac{2}{\pi}\int_0^{\pi}f(x)\sin nx\,dx=\frac{16n}{(16n^2-1)\pi}$

$f(x)\sim(1-\frac{2}{\pi})+\sum_{n=1}^{\infty}\left[\frac{4}{(16n^2-1)\pi}\cos 2nx+\frac{16n}{(16n^2-1)\pi}\sin 2nx\right]$

在 $x\neq k\pi$ 时，收敛到 $f(x)$，$x=k\pi$ 时，收敛到 $\frac{1}{2}$. $(k\in\mathbb{Z})$

(3) $a_0=\frac{1}{l}\int_{-l}^{l}e^{ax}dx=\frac{e^{al}-e^{-al}}{al}$

$a_n=\frac{1}{l}\int_{-l}^{l}e^{ax}\cos\frac{n\pi}{l}x\,dx=\frac{al(-1)^n(e^{al}-e^{-al})}{a^2l^2+n^2\pi^2}$

$b_n=\frac{1}{l}\int_{-l}^{l}e^{ax}\sin\frac{n\pi}{l}x\,dx=\frac{-n\pi(-1)^n(e^{al}-e^{-al})}{a^2l^2+n^2\pi^2}$

$f(x)\sim\frac{e^{al}-e^{-al}}{2al}+\sum_{n=1}^{\infty}\frac{(-1)^n(e^{al}-e^{-al})}{a^2l^2+n^2\pi^2}(al\cos\frac{n\pi x}{l}-n\pi\sin\frac{n\pi x}{l})$

在 $x=kl$，$k\in\mathbb{Z}$ 时收敛于 $\frac{e^{al}+e^{-al}}{2}$，其它点处收敛于 $f(x)$.

3.(1) 正弦级数：$f(x)=\begin{cases}-2x^2 & -\pi\le x\le 0\\ 2x^2. & 0\le x\le\pi\end{cases}$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx=\frac{8}{n^3\pi}[(-1)^n-1]-\frac{4\pi}{n}(-1)^n$

$f(x)\sim\sum_{n=1}^{\infty}\left[\frac{8[(-1)^n-1]}{n^3\pi}-\frac{4\pi(-1)^n}{n}\right]\sin nx.$

余弦级数：$f(x)=2x^2,\ -\pi\le x\le\pi$

$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}2x^2dx=\frac{4\pi^2}{3}$

$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}2x^2\cos nx=\frac{8(-1)^n}{n}.$

$f(x)\sim\frac{2\pi^2}{3}+\sum_{n=1}^{\infty}\frac{8(-1)^n}{n}\cos nx.$

5.(1) 注意到 $S(x)$ 是 $f(x)$ 偶延拓后展成的余弦级数.

$S(\frac{9}{4})=S(\frac{1}{4})$. $S(-\frac{5}{2})=S(\frac{5}{2})=S(\frac{1}{2})$

$f(x)$ 在 $(0,\frac{1}{2})$ 和 $(\frac{1}{2},1)$ 上分别连续可微 $\lim_{x\to\frac{1}{2}^-}f(x)=\frac{1}{2}$, $\lim_{x\to\frac{1}{2}^+}f(x)=1$.

故 $S(\frac{9}{4})=f(\frac{1}{4})=\frac{1}{4}$, $S(-\frac{5}{2})=\frac{\frac{1}{2}+1}{2}=\frac{3}{4}.$

5.(2). $S(3\pi)=S(\pi)=\frac{-1+(1+\pi^2)}{2}=\frac{\pi^2}{2}.$

$S(4\pi)=S(0)=\frac{-1+1}{2}=0$