32: $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial x}=\frac{\partial z}{\partial u}+\frac{\partial z}{\partial v}$

$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial y}=-2\frac{\partial z}{\partial u}+a\frac{\partial z}{\partial v}$

$\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}(\frac{\partial z}{\partial u}+\frac{\partial z}{\partial v})=\frac{\partial^2 z}{\partial u^2}+2\frac{\partial^2 z}{\partial u\partial v}+\frac{\partial^2 z}{\partial v^2}$

$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial x}(-2\frac{\partial z}{\partial u}+a\frac{\partial z}{\partial v})=-2\frac{\partial^2 z}{\partial u^2}+(a-2)\frac{\partial^2 z}{\partial u\partial v}+a\frac{\partial^2 z}{\partial v^2}$

$\frac{\partial^2 z}{\partial y^2}=\frac{\partial}{\partial y}(-2\frac{\partial z}{\partial u}+a\frac{\partial z}{\partial v})=4\frac{\partial^2 z}{\partial u^2}-4a\frac{\partial^2 z}{\partial u\partial v}+a^2\frac{\partial^2 z}{\partial v^2}$

原方程化为：$(5a+10)\frac{\partial^2 z}{\partial u\partial v}+(6+a-a^2)\frac{\partial^2 z}{\partial v^2}=0$

$\begin{cases}5a+10\neq 0\\ 6+a-a^2=0\end{cases}\Rightarrow a=3$

35: $u(x,2x)=x$ 两端对 $x$ 求导. 得. $u_x+2u_y=1$.

再求一次，得 $u_{xx}+2u_{xy}+2u_{yx}+4u_{yy}=0$

同理，在 $u_x(x,2x)=x^2$ 两端对 $x$ 求导，得.

$u_{xx}+2u_{xy}=2x$. 又因为 $u_{xx}=u_{yy}$, $u_{xy}=u_{yx}$.

代入可得 $u_{xx}(x,2x)=-\frac{4}{3}x$, $u_{xy}=\frac{5}{3}x$; $u_{yy}=-\frac{4}{3}x$

36.(3) $du=(f_x+2tf_y+3t^2f_z)dt$

(5) $\frac{\partial u}{\partial x}=\frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x}+\frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial x}+\frac{\partial u}{\partial \zeta}\frac{\partial \zeta}{\partial x}=2xf_\xi+2xf_\eta+2yf_\zeta$

$\frac{\partial u}{\partial y}=\frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial y}+\frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y}+\frac{\partial u}{\partial \zeta}\frac{\partial \zeta}{\partial y}=2yf_\xi-2yf_\eta+2xf_\zeta$

故 $du=(2xf_\xi+2xf_\eta+2yf_\zeta)dx+(2yf_\xi-2yf_\eta+2xf_\zeta)dy$

37: $\frac{\partial}{\partial x}=\frac{\partial r}{\partial x}\frac{\partial}{\partial r}+\frac{\partial \theta}{\partial x}\frac{\partial}{\partial \theta}+\frac{\partial \varphi}{\partial x}\frac{\partial}{\partial \varphi}$

$\frac{\partial}{\partial y}=\frac{\partial r}{\partial y}\frac{\partial}{\partial r}+\frac{\partial \theta}{\partial y}\frac{\partial}{\partial \theta}+\frac{\partial \varphi}{\partial y}\frac{\partial}{\partial \varphi}$

$\frac{\partial}{\partial z}=\frac{\partial r}{\partial z}\frac{\partial}{\partial r}+\frac{\partial \theta}{\partial z}\frac{\partial}{\partial \theta}+\frac{\partial \varphi}{\partial z}\frac{\partial}{\partial \varphi}$

$$\frac{\partial^2}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial r}{\partial x}\frac{\partial}{\partial r}+\frac{\partial \theta}{\partial x}\frac{\partial}{\partial \theta}+\frac{\partial \varphi}{\partial x}\frac{\partial}{\partial \varphi}\right)$$

$$\frac{\partial^2}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial r}{\partial y}\frac{\partial}{\partial r}+\frac{\partial \theta}{\partial y}\frac{\partial}{\partial \theta}+\frac{\partial \varphi}{\partial y}\frac{\partial}{\partial \varphi}\right)$$

$$\frac{\partial^2}{\partial z^2}=\frac{\partial}{\partial z}\left(\frac{\partial r}{\partial z}\frac{\partial}{\partial r}+\frac{\partial \theta}{\partial z}\frac{\partial}{\partial \theta}+\frac{\partial \varphi}{\partial z}\frac{\partial}{\partial \varphi}\right)$$

$$\Delta=\nabla^2=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \varphi^2}$$

38: $$\frac{\partial(x,y)}{\partial(r,\theta)}=\begin{vmatrix}\cos\theta & -r\sin\theta\\ \sin\theta & r\cos\theta\end{vmatrix}=r$$

3: $\frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{2x+y}{2y+x}$. 令 $\frac{dy}{dx}=0$，联立 $\begin{cases}x^2+xy+y^2-27=0\\ 2x+y=0\end{cases}$

可得 $(x,y)=(3,-6)$ 或 $(-3,6)$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=-6\frac{x^2+xy+y^2}{(x+2y)^3}$$

故 $\left.\frac{d^2y}{dx^2}\right|_{(3,-6)}=\frac{2}{9}$. $\left.\frac{d^2y}{dx^2}\right|_{(-3,6)}=-\frac{2}{9}$

于是 $y=y(x)$ 在 $x=3$ 时取得极小值 $-6$，在 $x=-3$ 时取得极大值 $6$.

4.(4) $F_1(dx-dy)+F_2(dy-dz)+F_3(dz-dx)=0$

于是 $dz=\frac{(F_1-F_3)dx+(F_2-F_1)dy}{F_2-F_3}$

6: 令 $F(x,y,z)=2\sin(x+2y-3z)-x-2y+3z$.

$F_x=2\cos(x+2y-3z)-1$, $F_y=4\cos(x+2y-3z)-2$, $F_z=-6\cos(x+2y-3z)+3$

故 $\frac{\partial z}{\partial y}+\frac{\partial z}{\partial x}=-\frac{F_y}{F_z}-\frac{F_x}{F_z}=\frac{2}{3}+\frac{1}{3}=1$

9: 令 $F(x,y)=y-f(x+t(x,y))$. 则 $\frac{dy}{dx}=-\frac{F_x}{F_y}=\frac{f'(1+\frac{\partial t}{\partial x})}{1-f'\frac{\partial t}{\partial y}}$

而 $\frac{\partial t}{\partial x}=-\frac{g_x}{g_t}$, $\frac{\partial t}{\partial y}=-\frac{1}{g_t}$. 代入可得 $\frac{dy}{dx}=\frac{f'(g_t-g_x)}{f'+g_t}$

11: 不难求得 $\frac{\partial u}{\partial x}=-\frac{xu+yv}{x^2+y^2}$, $\frac{\partial u}{\partial y}=\frac{xv-yu}{x^2+y^2}$

$$\frac{\partial v}{\partial x}=\frac{yu-xv}{x^2+y^2}.\quad \frac{\partial v}{\partial y}=-\frac{xu+yv}{x^2+y^2}$$

$$\left|\frac{\partial(u,v)}{\partial(x,y)}\right|=\frac{x^2u^2+y^2v^2+y^2u^2+x^2v^2}{(x^2+y^2)^2}=\frac{u^2+v^2}{x^2+y^2}$$

12: $\frac{\partial u}{\partial x}=\frac{\sin v}{e^u(\sin v-\cos v)+1}$, $\frac{\partial u}{\partial y}=\frac{-\cos v}{e^u(\sin v-\cos v)+1}$

$$\frac{\partial v}{\partial x}=\frac{-e^u+\cos v}{ue^u(\sin v-\cos v)+u},\quad \frac{\partial v}{\partial y}=\frac{e^u+\sin v}{ue^u(\sin v-\cos v)+u}$$

13: $\frac{du}{dx}=f_1+f_2\cos x-\frac{f_3}{\varphi_3}(\varphi_1 2x+\varphi_2 e^{\sin x}\cos x)$

2: 设 $\vec{r}(t)=(r_1(t),r_2(t),\cdots,r_n(t))$.

在 $r_1^2(t)+r_2^2(t)+\cdots+r_n^2(t)=1$ 两端对 $t$ 求导即得

$\langle \vec{r}(t),\frac{d\vec{r}}{dt}\rangle=0$.

意义: 距离原点恒为1的向量值函数在每一点处的切向量与径向垂直

3: $\frac{d\vec{r}}{dt}=(-a\sin t,\ a\cos t,\ b)$. $\vec{k}=(0,0,1)$

$\cos\theta=\frac{b}{\sqrt{(-a\sin t)^2+(a\cos t)^2+b^2}\cdot 1}=\frac{b}{\sqrt{a^2+b^2}}$, 故切线与z轴成定角

5.(1) $\vec{r}'(t)=(2a\sin t\cos t,-b\sin^2 t+b\cos^2 t,-2c\sin t\cos t)$

在 $t=\frac{\pi}{4}$ 处切向量为 $(a,0,c)$

切线方程: $\frac{x-\frac{a}{2}}{a}=\frac{y-\frac{b}{2}}{0}=\frac{z-\frac{c}{2}}{c}$

法平面方程: $a(x-\frac{a}{2})-c(z-\frac{c}{2})=0$