11.2

1.(6) 交线：$x^2+y^2=2(\frac{x}{2}+1)^2$，即 $\frac{(x-2)^2}{8}+\frac{y^2}{4}=1$.

于是 $S=\iint_{\frac{(x-2)^2}{8}+\frac{y^2}{4}\le 1}\sqrt{1+z_x^2+z_y^2}\,dxdy=\sqrt{2}\iint_{\frac{(x-2)^2}{8}+\frac{y^2}{4}\le 1}dxdy=\sqrt{2}\times 4\sqrt{2}\pi=8\pi$

1.(8) 考虑该曲面在第一卦限中的面积 S，在球坐标下方程为 $r=a\sin\varphi\sqrt{\sin 2\theta}$

面积元素 $dS=\sqrt{(r^2+(\frac{\partial r}{\partial\varphi})^2)\sin^2\varphi+(\frac{\partial r}{\partial\theta})^2}\cdot r\,d\varphi d\theta$.

于是 $S=\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\frac{a\sin\varphi}{\sqrt{\sin 2\theta}}\cdot a\sin\varphi\sqrt{\sin 2\theta}\,d\varphi d\theta=a^2\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\sin^2\varphi\,d\varphi d\theta=\frac{\pi^2a^2}{8}$

根据对称性，曲面面积为 $4S=\frac{\pi^2a^2}{2}$

2.(2) $\iint_S xyz\,dS=\iint_\Omega xy(1-x-y)\sqrt{1+z_x^2+z_y^2}\,dxdy$

$=\sqrt{3}\int_0^1dy\int_0^{1-y}xy-x^2y-xy^2\,dx=\sqrt{3}\int_0^1\frac{y(1-y)^3}{6}dy=\frac{\sqrt{3}}{120}$

2.(7) $\iint_S|xyz|\,dS=\iint_{x^2+y^2\le 1}|xy(x^2+y^2)|\sqrt{1+4x^2+4y^2}\,dxdy$

$=4\int_0^{\frac{\pi}{2}}d\theta\int_0^1 r\cos\theta\cdot r\sin\theta\cdot r^2\cdot\sqrt{1+4r^2}\cdot r\,dr$

$=2\int_0^{\frac{\pi}{2}}\sin 2\theta\,d\theta\int_0^1 r^5\sqrt{1+4r^2}\,dr$

$=\frac{125\sqrt{5}-1}{420}$

3.(1) $\iint_S(x^2+y^2)dS=\frac{2}{3}\iint_S(x^2+y^2+z^2)dS=\frac{2}{3}R^2\cdot 4\pi R^2=\frac{8}{3}\pi R^4$

3.(2) $\iint_S x\,dS=\iint_S y\,dS=0$.

故 $I=\iint_S z\,dS=\iint_\Omega\sqrt{a^2-x^2-y^2}\cdot\sqrt{1+z_x^2+z_y^2}\,dxdy=\iint_\Omega a\,dxdy=\pi a^3$

4: $z=-\frac{A}{C}x-\frac{B}{C}y-\frac{D}{C}$. $S_G=\iint_{G_1}\sqrt{1+\frac{A^2}{C^2}+\frac{B^2}{C^2}}\,dxdy=\sqrt{\frac{A^2+B^2+C^2}{C^2}}\iint_{G_1}dxdy$

$=\sqrt{\frac{A^2+B^2+C^2}{C^2}}\cdot S_{G_1}$

5. $M=\iint\rho\,dS=\iint_{x^2+y^2\le 2}\frac{x^2+y^2}{2}\sqrt{1+x^2+y^2}\,dxdy=\int_0^{2\pi}d\theta\int_0^{\sqrt{2}}\frac{r^2}{2}\sqrt{1+r^2}\cdot r\,dr$

$= 2\pi \times \frac{1+6\sqrt{3}}{15} = \frac{2\pi}{15}(1+6\sqrt{3})$

11.3

1.(1) $\int_L (x^2+y^2)dx+(x^2-y^2)dy = \int_0^1 2x^2dx + 0dx + \int_1^2 x^2+(2-x)^2dx - [x^2-(2-x)^2]dx$

$= \frac{2}{3}+\frac{2}{3} = \frac{4}{3}$

1.(4) $\int_L y^2dx+xydy+xzdz = \int_0^1 0dx + \int_0^1 ydy + \int_0^1 zdz = 1$

1.(6) 记Σ是平面$x+y=2$被L所截的部分，方向朝不含原点的一侧

由Stokes公式：$\int_L ydx+zdy+xdz = \iint_\Sigma (-1,-1,-1)\cdot\vec{n}dS$

$= \iint_\Sigma (-1,-1,-1)\cdot(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0)dS = -\sqrt{2}S_\Sigma = -2\sqrt{2}\pi$

2. $\int_L (y+z,z+x,x+y)\cdot d\vec{r} = \int_0^\pi (2a\sin t\cos t + a\cos^2 t, a\cos^2 t + a\sin^2 t, a\sin^2 t + 2a\sin t\cos t)$

$\cdot(2a\sin t\cos t, 2a\cos^2 t - 2a\sin^2 t, -2a\cos t\sin t)\,dt = 0$

3. $W = \int_L F\cdot d\vec{r} = \int_L k|\vec{r}|\cdot\frac{-\vec{r}}{|\vec{r}|}\cdot d\vec{r} = k\int_L(-x,-y)\cdot d\vec{r}$

$= k(a^2-b^2)\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta\,d\theta = \frac{1}{2}k(a^2-b^2)$

4.(2) $I = -\iint_{\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1}(y+1)-(x+1)dxdy = \iint_{\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1}(-y+x)dxdy = 0$

4.(3) $I = \iint_D (y^3+e^y-x^3-e^y)dxdy = \iint_D (y^3-x^3)dxdy$

其中D是由L所围成的区域，故D关于坐标轴对称.

于是 $\iint_D y^3dxdy = \iint_D x^3dxdy = 0 \Longrightarrow I = 0$

4.(6) 记L为从O到A的直线段.

则 $\int_{AMO+L}(e^x\sin y - my)dx + (e^x\cos y - m)dy = \iint_D m\,dxdy = \frac{\pi}{8}ma^2$

而 $\int_L (e^x \sin y - my)dx + (e^x \cos y - m)dy = 0$

故 $I = \frac{\pi}{8} m a^2$

5.(1) $S = \int_L -y\,dx = \int_0^{2\pi} 3a^2 \sin^4 t \cos^2 t\, dt = 4\int_0^{\frac{\pi}{2}} 3a^2(\sin^4 t - \sin^6 t)dt$

$= \frac{3\pi}{8} a^2$

(2) 记 L 为摆线从 (0,0) 到 (2π,0) 再加上从 (2π,0) 到 (0,0) 的直线段

方向是 顺时针

则 $S = -\int_L x\,dy = \int_0^{2\pi} -a(t-\sin t)\cdot a\sin t\, dt = 3\pi a^2$