Solving Buckets' Partitioning Problem

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Problem Formulation

Consider problem of splitting set of buckets into smaller almost equivalent(equivalent if possible) in sense of coordinatewise sums subsets. Each bucket represents vector of route's and vehicle's features which are integer numbers themselves.

$$Buckets = \{v_0, v_1, \ldots v_n\}$$
$$v_i = (v_{i0}, v_{i1}, \ldots v_{im})$$

$$Buckets = \bigcup_{i=1}^{k} B_i$$

Mathematical View

We can reformulate the vectors' partitioning problem like a Integer Linear Programming Problem:

$$t = max_{j \in \{0 \ldots m\}} \left\{ max_{l \in \{1 \ldots k\}} \sum_{i \in B_l} v_{ij} - min_{l \in \{1 \ldots k\}} \sum_{i \in B_l} v_{ij} \right\}$$

$$\begin{cases} t \to min, \\ \texttt{subject to} \sum_{j=1}^{k} x_{ij} = 1, i \in \{0 \ldots n\}, \\ |\sum_{i=0}^{n} v_{ij} x_{ik_1} - \sum_{i=0}^{n} v_{ij} x_{ik_2}| \leq t, \\ x_{ij} \in \{0, 1\}, \end{cases}$$

where k_1, k_2 - all couples from $\{1 \ldots k\}$

Proposed Approach

The basic idea of the proposed approach is to fix a number of groups and variables in order to obtain a reduced critical subproblem which is then solved by the ILP solver. The key issue is how to choose the groups and variables to fix. The main idea use LP-relaxation for considered ILP problem and fix groups G_i which have $x_{ij_i} = 1$ in LP-relaxation solution.

Implementation and Results

Consider as metric

$$\Delta t = max_{j \in \{0 \ldots m\}} \frac{max_{l \in \{1 \ldots k\}} \sum_{i \in B_l} v_{ij} - min_{l \in \{1 \ldots k\}} \sum_{i \in B_l} v_{ij}}{max_{l \in \{1 \ldots k\}} \sum_{i \in B_l} v_{ij}}$$

nmbr of buckets	m	k	Δt	time, sec
500	3	6	0,08	4,69
500	4	6	0,1	7,01
500	5	6	0,099	13,98
500	6	6	0,11	24,65
1000	3	6	0,07	9,64
1000	4	6	0,11	12,03
1000	5	6	0,1	15,43
1000	6	6	0,11	26,29