Mathematics 202B — Spring 2024 — Introduction to Topology and Analysis

Instructor: Michael Christ, `mchrist@berkeley.edu`, 809 Evans Hall.
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Course control number: 25751

Lectures: MWF 1:10–2:00 PM in 219 Dwinelle Hall

Office hours (Christ) Thursdays 2:00-3:30 PM in 809 Evans Hall.
(Sanchez) Th 9:30-10:30 and F 10-11 in 1061 Evans.

Text: *Real Analysis*[1] (2nd edition) by G. B. Folland. (ISBN-10: 0471317160; ISBN-13: 978-0471317166) Detailed lecture notes will be provided, but are not a replacement for the more carefully written text.

bCourses: Course resources will be available on bCourses, including archived announcements, problem set assignments and solutions, and lecture notes.

Prerequisites: Successful completion of Math 202A plus the equivalent of Math 110, or equivalent with permission of instructor.

Required Work: Final exam, and one midterm exam. Weekly problem sets. Assigned readings from the text.

Exam schedule: Midterm exam: Monday, February 26, tentatively in class.
Final exam: Tuesday, May 7, 8-11 AM

Course content: This course is very much a continuation of 202A. It focuses on Lebesgue integration in spaces more general than $\mathbb{R}^1$, and on an introduction to functional analysis, with a bit of Fourier analysis to illustrate the more abstract topics. Topology is closely intertwined with these topics but — the course title notwithstanding — is not the primary object of study. Specific topics include:

○ Products of measure spaces, and Fubini's theorem.
○ Lebesgue measure on $\mathbb{R}^d$ for $d > 1$.
○ Banach spaces; dual spaces. Subspaces. Hahn-Banach theorem.
○ Baire category theorem, and its applications to functional analysis: Uniform Boundedness Principle, Closed Graph theorem, Open Mapping theorem.
○ Weak and weak-star topologies. Banach-Alaoglu theorem and a variant.
○ Hilbert spaces. Cauchy-Schwarz inequality. Riesz-Fischer theorem.
○ L^p spaces. Inequalities of Hölder and Minkowski. Duality. Essential boundedness.
○ Riesz representation theorems for $C_0(X)$ and $C_c(X)$.
○ Introduction to Fourier series and Fourier integrals, including some applications of material from earlier in the course.
○ Supplementary topics as/if time permits.

Grading: Midterm exam 15%, problem sets 50%, final exam 35%. Participation, e.g. discussion in office hours or questions in class, may be taken into account as a grade booster.

Graduate course expectation: Reading of the text is expected. Not all details will be treated in class. Comprehensiveness may sometimes be sacrificed for the sake of clarity in lectures.

[1] Subtitle *Modern Techniques and Their Applications*

Problem sets:

○ Problem sets will ordinarily be due on Friday evenings at 11:59 PM. Submission via Gradescope.

○ Only selected problems will be graded each week. Partial credit will be awarded for any significant attempt. Solutions will be provided for most problems.

○ Typewritten work is welcome, but is not expected. (It is your responsibility to ensure that submissions are legible. Please be kind to our reader.)

○ The course teaching assistant's office hours are a resource for advice concerning problem sets.

○ You may freely discuss problem sets with other enrolled students, but should submit your own writeups. No points will be deducted for collaboration on problem sets. Credit must be given, in writing, to anyone who has contributed.[2]

○ Solutions to many problems can be found on the internet and elsewhere. Use of any such solutions in written work is strongly discouraged, must be acknowledged, and will not earn full credit. One learns mathematics largely by struggling with problems.

Copyright: Materials produced for this course by Professor Christ, including lecture videos, lecture notes, problem set solutions, exams, and all related materials, are supplied for the personal use of enrolled students and authorized auditors. Personal use includes consultation of study partners or tutors. To share these documents for other purposes, to post them on the internet, or to otherwise distribute them is not authorized by their author, and is a violation of copyright law.

Students with disabilities: Course staff will work with the Disabled Students Program (DSP) to meet the needs of students with disabilities. If you may have a disability, please consult DSP (dsp.berkeley.edu) promptly at the beginning of the semester.

Course materials are available in larger font size by request.

Incomplete grades: I grades are awarded only in exceptional circumstances beyond a student's control, such as serious illness. Consult ls.berkeley.edu for details of university policy.

Alternative texts and supplementary reading[3]:

Real analysis and Lebesgue integration:
Real and Complex Analysis by W. Rudin; *Real Analysis* by E. M. Stein and R. Shakarchi; *Real Analysis* by A. Knapp; *Real Analysis for Graduate Students* by R. F. Bass; *Probability and Measure* by P. Billingsley; *Real and Functional Analysis* by S. Lang; *Real Analysis* by H. Royden.

Functional analysis:
Functional Analysis by W. Rudin; *Functional Analysis* by P. D. Lax.

Point set topology:
Topology by J. Munkres; *General Topology* by J. Kelley.

Undergraduate real analysis:
Real Mathematical Analysis by C. Pugh; *Principles of Mathematical Analysis* by W. Rudin; *The Way of Analysis* by R. Strichartz.

[2]To fail to give appropriate credit to others, is academic misconduct and a violation of the UCB honor code.

[3]I do not generally find alternative treatments of essentially identical material to be helpful, but your experience may be different. These references supply some material outside the scope of this course.