CAN data smoothing

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1 Introduction

This document explains the problem formulation for CAN data smoothing. The goal is to smooth position, velocity and acceleration data from CAN bus while maintaining internal consistency.

Notations Below is a list of notations used in this work:

- p: vehicle position time series vector
- v: vehicle velocity vector
- a: vehicle acceleration vector
- j: vehicle jerk vector

Dynamics Consider the following 3rd order 1D motion model:

$$\begin{aligned}\dot{p} &= v\\ \dot{v} &= a\\ \dot{a} &= j,\end{aligned} \tag{1}$$

where p, v, a, j are position, velocity, acceleration and jerk, respectively. A discrete-time approximation is

$$\begin{aligned}p[t+1] &= p[t] + v[t]\Delta T\\ v[t+1] &= v[t] + a[t]\Delta T\\ a[t+1] &= a[t] + j[t]\Delta T,\end{aligned} \tag{2}$$

with ΔT as the timestep. Let $\mathbf{p} = [p[1], p[2], ..., p[N]]^T$ be the position vector for N timesteps, $\mathbf{v} = [v[1], v[2], ..., v[N-1]]^T$ be the velocity vector, $\mathbf{a} = [a[1], a[2], ..., a[N-2]]^T$ the acceleration vector and $\mathbf{j} = [j[1], j[2], ..., j[N-3]]^T$ jerk vector. Eq (2) can be written in matrix multiplication form:

$$\begin{aligned}\mathbf{v} &= D^{(1)}\mathbf{p}\\ \mathbf{a} &= D^{(2)}\mathbf{p}\\ \mathbf{j} &= D^{(3)}\mathbf{p},\end{aligned} \tag{3}$$

$D^{(k)} \in \mathbb{R}^{(N-k)\times N}$ represents the k^{th}-order differentiation operator, specifically

$$D^{(1)} = \frac{1}{\Delta T}\begin{bmatrix} -1 & 1 & 0 & ... & 0 & 0 & 0\\ 0 & -1 & 1 & ... & 0 & 0 & 0\\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & ... & -1 & 1 & 0\\ 0 & 0 & 0 & ... & 0 & -1 & 1 \end{bmatrix}$$

$$D^{(2)} = \frac{1}{\Delta T^2}\begin{bmatrix} 1 & -2 & 1 & 0 & ... & 0 & 0 & 0\\ 0 & 1 & -2 & 1 & ... & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & .... & 1 & -2 & 1 \end{bmatrix}$$

$$D^{(3)} = \frac{1}{\Delta T^3}\begin{bmatrix} -1 & 3 & -3 & 1 & ... & 0 & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & .... & -1 & 3 & 3 & 1 \end{bmatrix}$$

Consider the measurement vector $\mathbf{y} = [\mathbf{p}, \mathbf{v}, \mathbf{a}]^T \in \mathbb{R}^{3N-3}$. Let $\mathbf{x} \in \mathbb{R}^N$ denote the ground truth position vector. Model the noisy measurement as:

$$\mathbf{y} = \mathbf{x} + \mathbf{w}. \tag{4}$$

We want to find the reconstructed position $\hat{\mathbf{x}}$ that is smooth in k^{th}-order derivatives:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{p} - \mathbf{x}\|_2^2 + \lambda_1 \|\mathbf{v} - D^{(1)}\mathbf{x}\|_2^2 + \lambda_2 \|\mathbf{a} - D^{(2)}\mathbf{x}\|_2^2 + \lambda_3 \|D^{(3)}\mathbf{x}\|_2^2. \tag{5}$$

The formulation (5) minimizes a linear combination of position, velocity and acceleration errors, as well as a term for jerk smoothing. The optimal solution $\hat{\mathbf{x}}$ gives self-consistent signals $\hat{\mathbf{v}} = D^{(1)}\hat{\mathbf{x}}$, $\hat{\mathbf{a}} = D^{(2)}\hat{\mathbf{x}}$, and $\hat{\mathbf{j}} = D^{(3)}\hat{\mathbf{x}}$. All time series should be smooth with proper tuning of λ_1, λ_2 and λ_3.