Scrap

Monday, October 19, 2020 3:54 PM

## Normal distribution

1) First moment about origin is

$$E(x_i) = \mu$$

2) Second moment about Mean is

$$Var(x_i) = E((x_i - \mu)^2) = \sigma^2$$

Equating

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i = E(x)$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

So $\hat{\mu} = \bar{x}$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

↳ verience of data

$$sd = \sqrt{\sigma^2}$$

So we directly return the Mean and Veriance of data as the estimated Mean and verience Perameters

## Exponential distribution

It is already given in the question,

Mean of data $= \beta$

But R uses $1/\beta$ as the input.

So dividing Mean by one

Estimate $\hat{\beta} = \frac{1}{\bar{x}}$

## Gamma distribution

As enplaind in Mini lecture,

Mean is given as $\alpha\beta$

Variance $= \alpha\beta^2$

So $\beta = \frac{\sigma^2}{\bar{x}}$

$\alpha = \frac{\bar{x}}{\beta}$ or $\frac{(\bar{x})^2}{\sigma^2}$ $\left[\because \frac{\alpha^2\beta^2}{\alpha\beta^2} = \alpha\right]$

## Beta distribution

For this,

Mean $= \alpha/(\alpha+\beta) = \bar{x}$

Var $= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = s^2$

Solving,

$$\frac{\alpha}{(\alpha+\beta)} = \bar{x}$$

$$\alpha+\beta = \frac{\alpha}{\phantom{x}}$$

$$x$$

$$\beta = \frac{\alpha}{\bar{x}} - \alpha$$

$$= \frac{\alpha(1-\bar{x})}{\bar{x}}$$

Substituting this in $s^2$ formula which is second moment.

$$s^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$= \frac{\alpha\left(\frac{\alpha(1-\bar{x})}{\bar{x}}\right)}{\left(\alpha+\frac{\alpha(1-\bar{x})}{\bar{x}}\right)^2\left(\alpha+\frac{\alpha(1-\bar{x})}{\bar{x}}+1\right)}$$

$$= \frac{(\alpha^2(1-\bar{x}))/\cancel{\bar{x}}}{\frac{(\cancel{\alpha\bar{x}}+\alpha-\cancel{\alpha\bar{x}})^2}{\bar{x}^2}\left(\frac{\cancel{\alpha\bar{x}}+\alpha-\cancel{\alpha\bar{x}}+\bar{x}}{\cancel{\bar{x}}}\right)}$$

$$= \frac{\cancel{\alpha^2}(1-\bar{x})\,\bar{x}^2}{(\cancel{\alpha^2})(\alpha+x)}$$

$$(1-\bar{x})\ \bar{x}^2$$

$$s^2 = \frac{(1-\bar{x})\,\bar{x}^2}{(\alpha + n)}$$

$$(\alpha + n)s^2 = (1-\bar{x})\,\bar{x}^2$$

$$\alpha s^2 = (1-x)\,\bar{x}^2 - n s^2$$

$$\alpha = \frac{(1-x)\,\bar{x}^2 - n s^2}{s^2}$$

$$= \left( \left(\frac{\bar{x}^2}{s^2}\right)(1-\bar{x}) - \bar{x} \right)$$

$$\boxed{\alpha = \bar{x}\left( \frac{x}{s^2}(1-\bar{x}) - 1 \right)}$$

using this we get $\beta$

$$\boxed{\beta = \frac{\alpha(1-\bar{x})}{\bar{x}}}$$

## T - distribution

for this we've in book

$$s^2 = \frac{v}{v-2}$$

$$(v-2)s^2 = v$$

$$\frac{v-2}{v} = \frac{1}{s^2}$$

$$1-\frac{2}{v} = \frac{1}{s^2}$$

$$\frac{2}{v} = 1-\frac{1}{s^2} = \frac{s^2-1}{s^2}$$

$$v = \left(\frac{2s^2}{s^2-1}\right)$$

Chi-Square distribution

For Chi-Square, it is directly given

then Mean of data = P

So we directly return the Mean value

Multinomial distribution

Here, we calculate the values for each row

For each row, Mean is given by $np$

$$s^2 = \underbrace{np}_{\text{mean}}(1-p)$$

$$\frac{s^2}{\bar{x}} = 1-p$$

$$p_i = \left(1 - \frac{s_i^2}{\bar{x}_i}\right)$$

$n$ can be deduled from this

$$n = \Sigma\left(\frac{\bar{x}}{p}\right)$$

## Multi veriete normal

for this we just find Mean and Veriance of each row by using colMeans and summation