2. Rewrite the following time series models using the backward shift operator B and the difference operator Δ and in terms of the time series x_t and a_t (that is, no x_{t-1}, y_t or a_{t-1} should show up in the final expression).

(1) $x_t = \phi_1 x_{t-1} + a_t + \theta_1 a_{t-1}$ $\quad Bx_t = x_{t-1}$

$x_t = \phi_1 B x_t + a_t + \theta_1 B a_t$ (My Solution)

$x_t - \phi_1 B x_t = a_t + \theta_1 B a_t$

$$\boxed{x_t(1-\phi_1 B) = a_t(1+\theta_1 B)}$$

(2) $y_t = \phi_1 y_{t-1} + a_t + \theta_1 a_{t-1}$ and $y_t = x_t - x_{t-1}$

$y_t = \phi_1 B y_t + a_t + \theta_1 B a_t$

Plug in y_t

$x_t - x_{t-1} = \phi_1 B(x_t - x_{t-1}) + a_t + \theta_1 B a_t$

$\Delta x_t = \phi_1 B(\Delta x_t) + a_t + \theta_1 B a_t$

$\Delta x_t - \phi_1 B(\Delta x_t) = a_t(1+\theta_1 B)$

$\Delta x_t(1-\phi_1 B) = a_t(1+\theta_1 B)$

(3). $x_t = \Phi_1 x_{t-4} + a_t$

$$x_t = \Phi_1 B^4 x_t + a_t$$

$$x_t - \Phi_1 B^4 x_t = a_t$$

$$\boxed{x_t(1 - \Phi_1 B^4) = a_t}$$

(4) $y_t = \phi_1 y_{t-1} + a_t + \theta_1 a_{t-1}$ and $y_t = x_t - x_{t-4}$

$$y_t = \phi_1 B y_t + a_t + \theta_1 B a_t \quad \text{and} \quad y_t = x_t - x_{t-4}$$

$$y_t(1 - \phi_1 B) = a_t(1 + \theta_1 B)$$

$$(x_t - x_{t-4})(1 - \phi_1 B) = a_t(1 + \phi_1 B)$$

$$\boxed{\Delta^4(x_t)(1 - \phi_1 B) = a_t(1 + \phi_1 B)}$$

3. Define the operator Δ_4 as $\Delta_4 x_t := x_t - x_{t-4}$
Rewrite the following time series models in the original forms WITHOUT the operator B, the difference operator Δ and Δ_4.

(1) $(1-\phi_1 B)x_t = (1+\theta B)a_t$

$x_t - \phi_1 B x_t = a_t + \theta B a_t$

$x_t - \phi_1 x_{t-1} = a_t + \theta a_{t-1}$

$\boxed{x_t = \phi_1 x_{t-1} + a_t + \theta a_{t-1}}$

(2) $(1-\phi_1 B)(1-\phi_1 B^4)x_t = a_t$

$(1 - \phi_1 B^4 - \phi_1 B + \phi_1^2 B^5) x_t = a_t$

$x_t - \phi_1 B^4 x_t - \phi_1 B x_t + \phi_1^2 B^5 x_t = a_t$

$\boxed{x_t = \phi_1 x_{t-4} + \phi_1 x_{t-1} - \phi_1^2 x_{t-5} + a_t}$

(3) $\Delta\Delta_4 x_t = (1+\theta_1 B)a_t$

$(1-B)(x_t - x_{t-4}) = a_t + \theta_1 B a_t$

$x_t - x_{t-4} - Bx_t + Bx_{t-4} = a_t + \theta_1 B a_t$

$x_t - x_{t-4} - x_{t-1} + x_{t-5} = a_t + \theta_1 a_{t-1}$

$\boxed{x_t = x_{t-4} + x_{t-1} - x_{t-5} + a_t + \theta_1 a_{t-1}}$

(4) $(1-\phi_1 B)\Delta_4 x_t = (1+\theta_1 B)a_t$

$(1-\phi_1 B)(x_t - x_{t-4}) = a_t + \theta_1 B a_t$

$x_t - x_{t-4} - \phi_1 B x_t + \phi_1 B x_{t-4} = a_t + \theta_1 a_{t-1}$

$x_t - x_{t-4} - \phi_1 x_{t-1} + \phi_1 x_{t-5} = a_t + \theta_1 a_{t-1}$

$\boxed{x_t = x_{t-4} + \phi_1 x_{t-1} - \phi_1 x_{t-5} + a_t + \theta_1 a_{t-1}}$

Regression & Time Series HW 6

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1. Generate n=100 observations of the time series by xt=wt-1 + 2wt + wt+1 where {wt} ~ N(0,1). Plot the sample autocorrelation

```
set.seed(1)
w <- rnorm(n=102, mean=0, sd=1)
wtm1 <- w[1]
wt <- w[2:101]
wtp1 <- w[102]
xt <- wtm1 + 2 * wt + wtp1
```

Create the plot

```
acf(xt, main='Autocorrelation of xt')
```