Time Series HW8

Yaniv Bronshtein(yb262)

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Import the necessary libraries

```
library(sarima)
```

```
## Loading required package: stats4
```

```
library(forecast)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

III. For the following models, simulate 400 observations, estimate the model, obtain 1-step ahead to 12-step ahead predictions and their standard errors, and plot the last 24 observed observations with the predictions and their 95% confidence intervals.

1. $(x_t - 20) = -0.7(x_{t-1} - 20) + \epsilon_t, \epsilon_t \sim N(0, 5^2)$

```
set.seed(123)
sim1 <- arima.sim(model=list(order=c(1,0,0), ar=c(-0.7)), n=400, sd = 5)+20
model1 <- arima(sim1, order=c(1,0,0))
model1.pred <- predict(model1, n.ahead=12)
```

Predictions for model 1

```
model1.pred$pred
```

```
## Time Series:
## Start = 401
## End = 412
## Frequency = 1
##  [1] 22.61501 18.21987 21.20547 19.17736 20.55505 19.61919 20.25492 19.82307
##  [9] 20.11642 19.91715 20.05251 19.96056
```

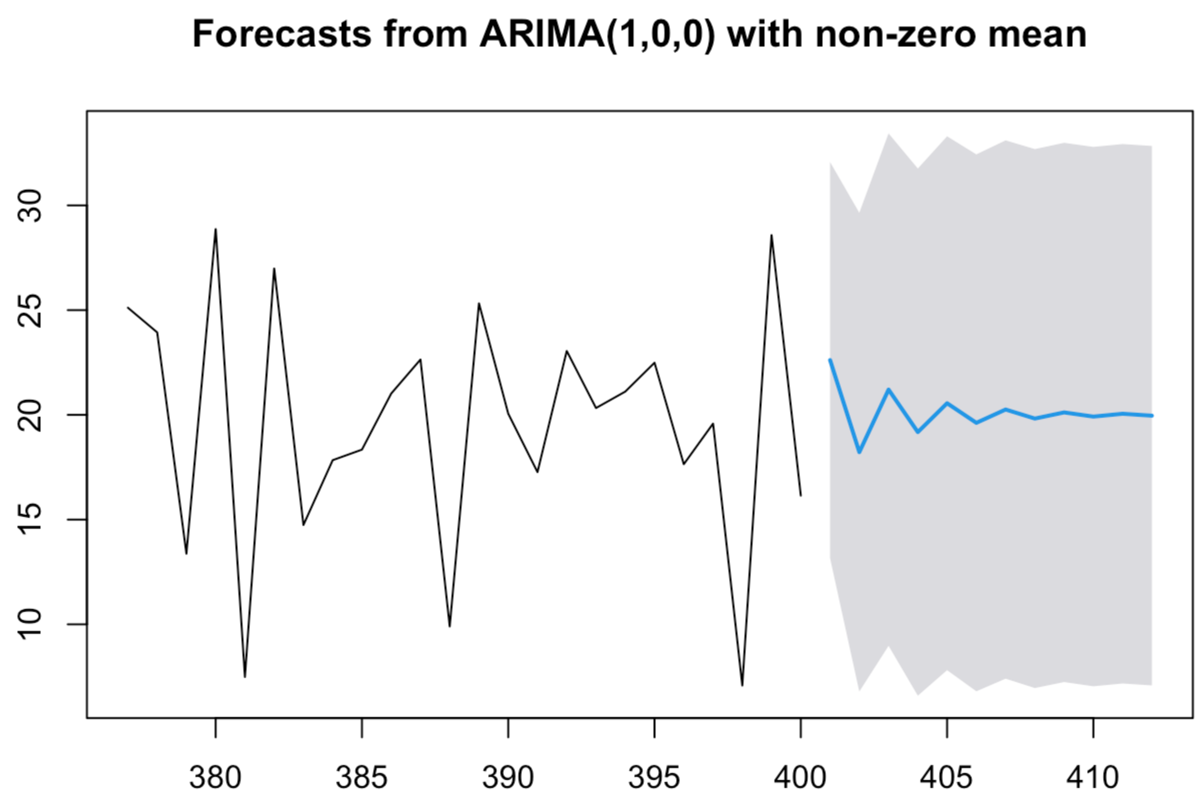
Standard errors for model 1 predictions

```
model1.pred$se
```

```
## Time Series:
## Start = 401
## End = 412
## Frequency = 1
##  [1] 4.821014 5.828133 6.238277 6.418704 6.500273 6.537569 6.554708 6.562601
##  [9] 6.566240 6.567919 6.568693 6.569051
```

Plot last 24 observations with 95% confidence intervals for predictions

```
plot(forecast(model1,h=12, level=0.95), include=24)
```



2. $(1 - 1.4B + 0.48B^2)(x_t - 20) = (1 + 1.2B + 0.35B^2)\epsilon_t, \epsilon_t \sim N(0, 5^2)$

```
set.seed(123)
sim2 <- arima.sim(model=list(order=c(2,0,2),ar=c(1.4,-0.48), ma=c(1.2,0.35)), n=400,
sd=5)+20
model2 <- arima(sim2, order=c(2,0,2))
model2.pred <- predict(model2, n.ahead=12)
```

Model 2 predictions

```
model2.pred$pred
```

```
## Time Series:
## Start = 401
## End = 412
## Frequency = 1
##  [1] 65.05035 54.46571 45.90735 39.25102 34.23000 30.53848 27.88528 26.01794
##  [9] 24.73006 23.85973 23.28400 22.91196
```

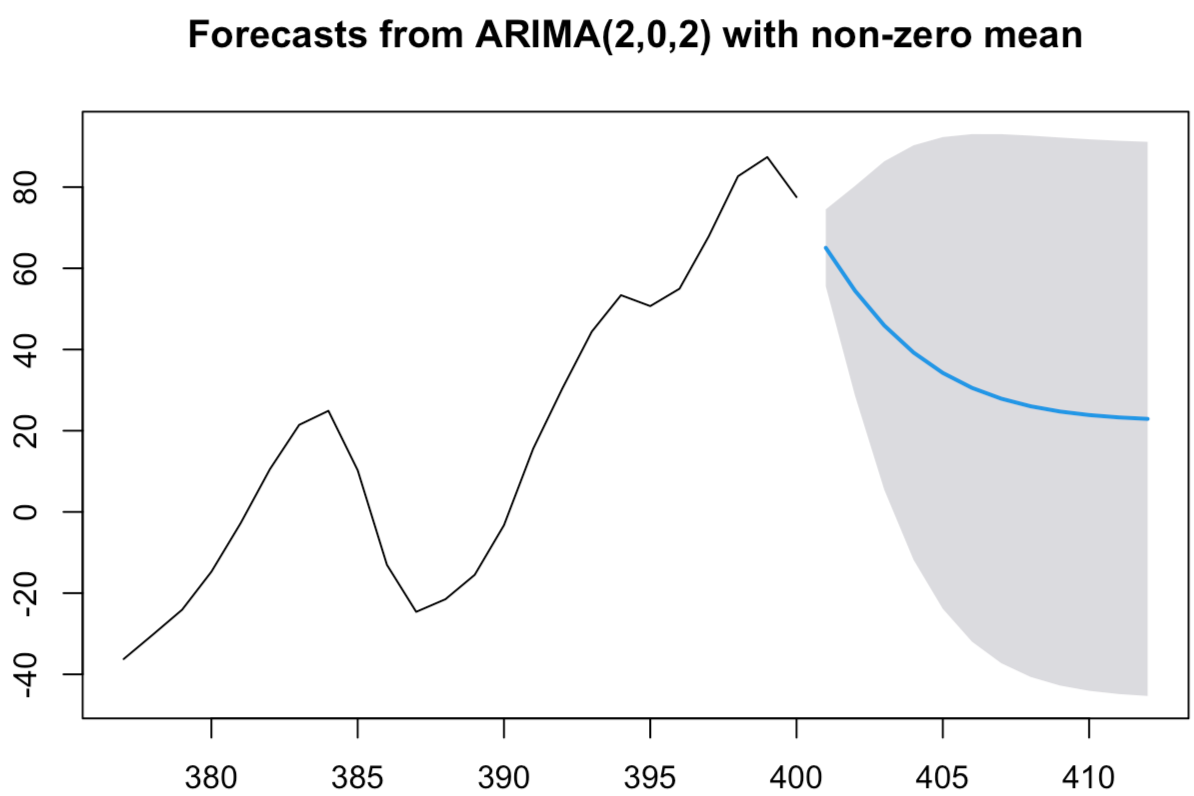
Standard errors for model 2 predictions

```
model2.pred$se
```

```
## Time Series:
## Start = 401
## End = 412
## Frequency = 1
##  [1]  4.82509 13.17455 20.63327 26.03732 29.63439 31.89041 33.23671 34.00483
##  [9] 34.42500 34.64576 34.75729 34.81150
```

Plot last 24 observations with 95% confidence intervals for predictions

```
plot(forecast(model2,h=12, level=0.95), include=24)
```



3. $(1-0.8B)\Delta x_t = (1+0.6B)\epsilon_t, \epsilon_t \sim N(0, 5^2)$

```
set.seed(123)
sim3 <- arima.sim(model=list(order=c(1,1,1),ar=c(0.8), ma=c(0.6)), n=400, sd=5)
model3 <- arima(sim3, order=c(1,1,1))
model3.pred <- predict(model3, n.ahead=12)
```

Model 3 predictions

```
model3.pred$pred
```

```
## Time Series:
## Start = 402
## End = 413
## Frequency = 1
##  [1] 283.0256 299.3452 311.1959 319.8014 326.0503 330.5881 333.8833 336.2761
##  [9] 338.0136 339.2754 340.1916 340.8570
```

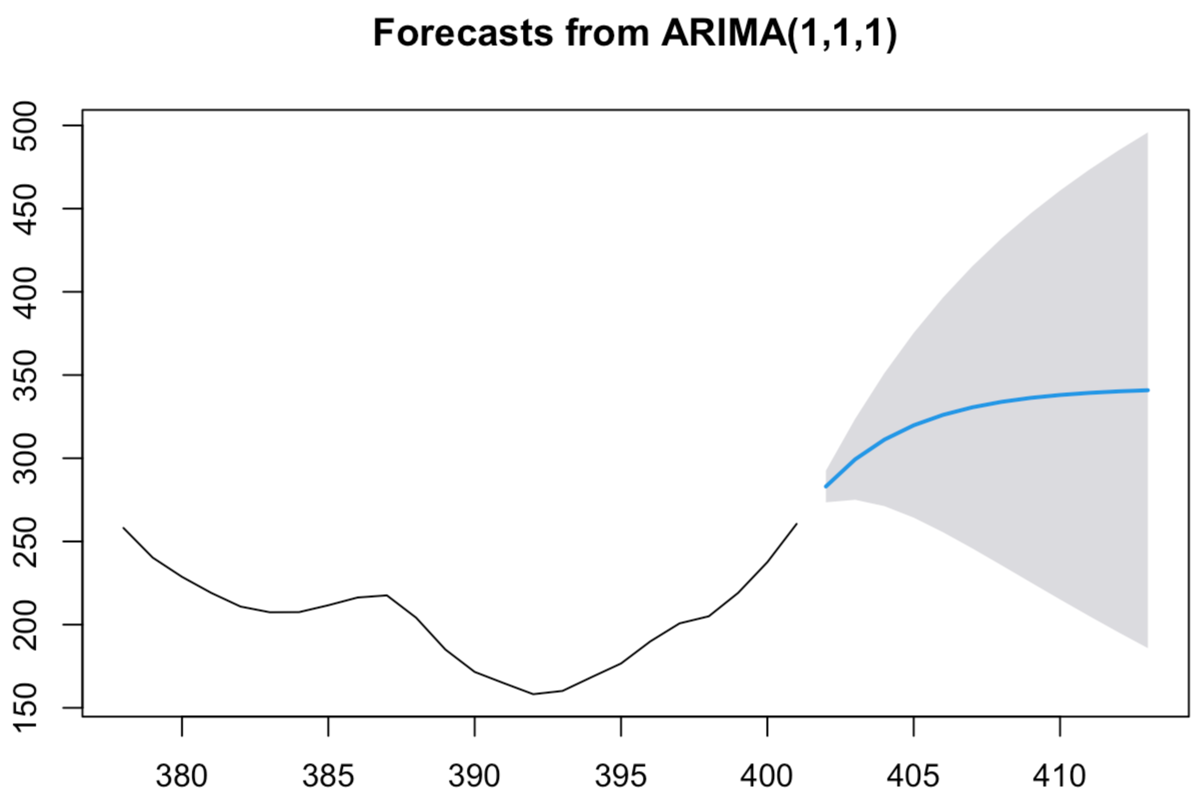
Standard errors for model 3 predictions

```
model3.pred$se
```

```
## Time Series:
## Start = 402
## End = 413
## Frequency = 1
##  [1]  4.837976 12.392390 20.380972 28.307836 35.951017 43.219673 50.088131
##  [8] 56.563335 62.667865 68.430860 73.883155 79.054761
```

Plot last 24 observations with 95% confidence intervals for predictions

```
plot(forecast(model3,h=12, level=0.95), include=24)
```



4. $(1 - B^{12})x_t = (1 + 0.8B)(1 + 0.8B^{12})\epsilon_t, \epsilon_t \sim N(0, 5^2)$.

$ARIMA(0,0,1)(1,0,1)_{12}$

```
set.seed(123)
sim4 <- sim_sarima(model=list(siorder=1, ma=c(0.8), sma=c(0.8), nseasons=12), n=400,
sd=5)

model4 <- arima(sim4, order=c(0,0,1), seasonal=list(order=c(1,0,1), period=12))

model4.pred <- predict(model4, n.ahead=12)
```

Model 4 predictions

```
model4.pred$pred
```

```
## Time Series:
## Start = 401
## End = 412
## Frequency = 1
##  [1] -3.3739258  5.7240993 18.3386909  4.5111090  7.2202443 10.6474639
##  [7] -1.0901893 -7.5589013 -9.5697695 -3.6018103  0.5235469 -4.4363520
```

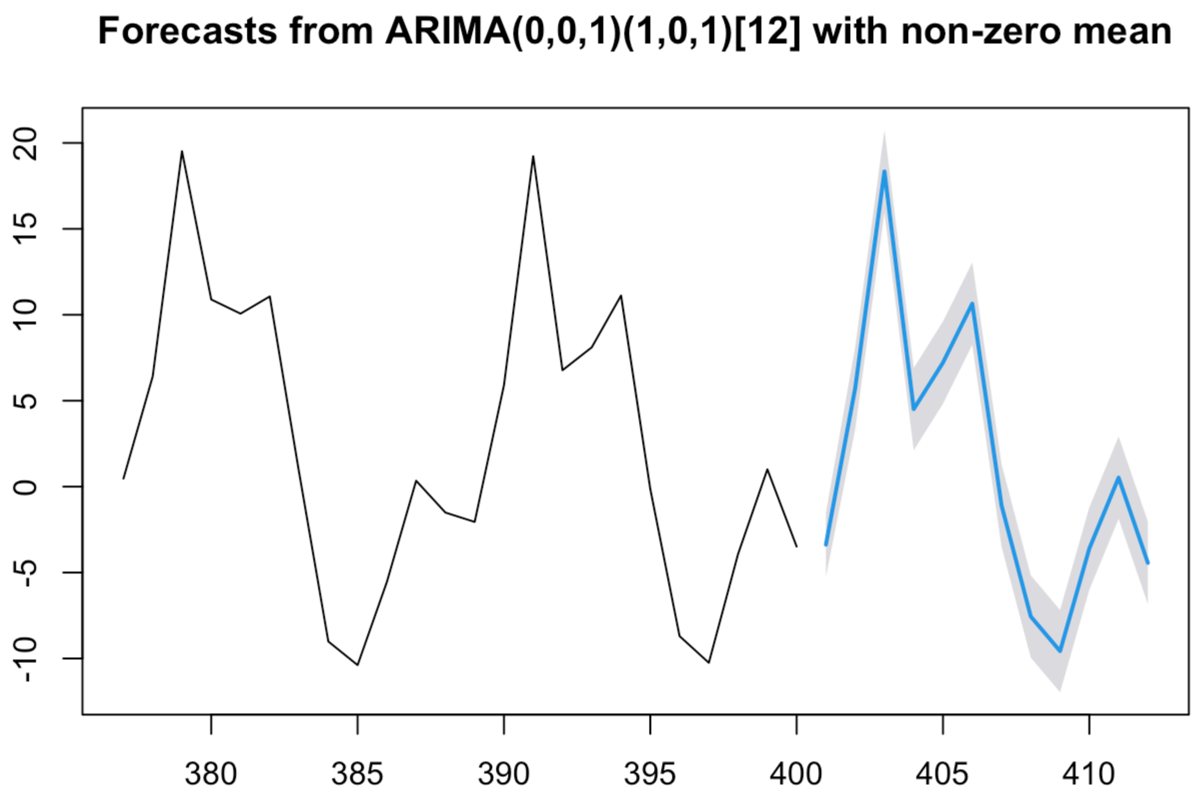
Standard errors for model 4 predictions

```
model4.pred$se
```

```
## Time Series:
## Start = 401
## End = 412
## Frequency = 1
##  [1] 0.944145 1.219792 1.219792 1.219792 1.219792 1.219792 1.219792 1.219792
##  [9] 1.219792 1.219792 1.219792 1.219792
```

Plot last 24 observations with 95% confidence intervals for predictions

```
plot(forecast(model4,h=12, level=0.95), include=24)
```



1. Without using any computer software, make 1 step to 3 step ahead forecast for the following models.
Assume $\hat{\phi}_1=0.7$, $\hat{\phi}_2=0.4$, $\hat{\theta}_1=0.6$, $\hat{\theta}_2=0.4$, $\hat{\mu}=40$, $x_{n-2}=30$, $x_{n-1}=20$
$x_n=25$, $\hat{\varepsilon}_{n-2}=2$, $\hat{\varepsilon}_{n-1}=1$, $\hat{\varepsilon}_n=3$

(AR(1))

(1) $(x_t-\mu)=\phi_1(x_{t-1}-\mu)+\varepsilon_t$ AR(1)

$$\hat{x}_t(1)=\mu+\hat{\phi}_1(x_t-\mu)=40+0.7(25-40)=29.5$$
$$\hat{x}_t(2)=\mu+\phi_1(\hat{x}_t(1)-\mu)=40+0.7(29.5-40)=32.65$$
$$\hat{x}_t(3)=\mu+\hat{\phi}_1(\hat{x}_t(2)-\mu)=40+0.7(32.65-40)=34.855$$

(2) $x_t-\mu=\varepsilon_t+\theta_1\varepsilon_{t-1}$ MA(1)

$$\hat{x}_t(1)=\mu+\hat{\theta}_1\hat{\varepsilon}_t=40+0.6(3)=41.8$$
$$\hat{x}_t(2)=\hat{x}_t(3)=\mu=40$$

(3) $(1-\phi_1B)\Delta x_t=(1+\theta_1B)\varepsilon_t$ ARIMA(1,1,1)

$$\Delta x_t-\phi_1B\Delta x_t=\theta_1\varepsilon_{t-1}+\varepsilon_t$$
$$x_t-x_{t-1}-\phi_1B(x_t-x_{t-1})=\theta_1\varepsilon_{t-1}+\varepsilon_t$$
$$x_t-x_{t-1}-\phi_1x_{t-1}+\phi_1x_{t-2}=\theta_1\varepsilon_{t-1}+\varepsilon_t$$
$$x_t=(1+\phi_1)x_{t-1}-\phi_1x_{t-2}+\theta_1\varepsilon_{t-1}+\varepsilon_t$$

$$\hat{x}_t(1)=(1+\hat{\phi}_1)x_n-\hat{\phi}_1x_{n-1}+\hat{\theta}_1\hat{\varepsilon}_n$$
$$=(1+0.7)25-0.7*20+0.6*3$$
$$=30.3$$

$$\hat{X}_t(2) = (1+\hat{\phi}_1)\hat{X}_t(1) - \hat{\phi}_1 X_n$$

$$= (1+0.7)30.3 - 0.7 * 25 = 34.01$$

$$\hat{X}_t(3) = (1+\hat{\phi}_1)\hat{X}_t(2) - \hat{\phi}_1 \hat{X}_t(1)$$

$$= (1+0.7)34.01 - 0.7(30.3) = 36.607$$

(III) Simulate 400 observations, estimate the model, obtain 1-step ahead to 12 step ahead predictions and their standard errors an

(2) $(1-1.4B+0.48B^2)(x_t-20)=(1+1.2B+0.35B^2)\varepsilon_t, \quad \varepsilon_t \sim N(0,5^2)$

$$(x_t-20)=\underbrace{1.4}_{\phi_1}B(x_t-20)-\underbrace{0.48}_{\phi_2}B^2(x_t-20)+\underbrace{1.2}_{\theta_1}B\varepsilon_t+\underbrace{0.35}_{\theta_2}B^2\varepsilon_t+\varepsilon_t$$

(3) $(1-0.8B)\Delta x_t=(1+0.6B)\varepsilon_t, \quad \varepsilon_t \sim N(0,5^2)$

$$\Delta x_t = \underbrace{0.8}_{\phi_1}B\Delta x_t + \underbrace{0.6}_{\theta_1}B\varepsilon_t+\varepsilon_t$$

(4) $(1-B^{12})x_t=(1+0.8B)(1+0.8B^{12})\varepsilon_t, \quad \varepsilon_t \sim N(0,5^2)$

$$x_t = B^{12}x_t + (1+0.8B)(1+0.8B^{12})\varepsilon_t$$

$$x_t = \underbrace{B^{12}}_{\Phi_1}x_t + (1+\underbrace{0.8}_{\theta_1}B+\underbrace{0.8}_{\Theta_1}B^{12}+\underbrace{(0.8)^2}_{\theta_1\Theta_1}B^{13})\varepsilon_t$$

$$\underbrace{(0,0,1)}_{\text{Non-seasonal}}\times\underbrace{(1,0,1)_{12}}_{\text{Seasonal}}$$