1. Without using any computer software, make 1 step to 3 step ahead forecast for the following models.
Assume $\hat{\phi}_1=0.7$, $\hat{\phi}_2=0.4$, $\hat{\theta}_1=0.6$, $\hat{\theta}_2=0.4$, $\hat{\mu}=40$, $x_{n-2}=30$, $x_{n-1}=20$
$x_n=25$, $\hat{\varepsilon}_{n-2}=2$, $\hat{\varepsilon}_{n-1}=1$, $\hat{\varepsilon}_n=3$

(AR(1))

(1) $(x_t-\mu)=\phi_1(x_{t-1}-\mu)+\varepsilon_t$ AR(1)

$\hat{x}_t(1)=\mu+\hat{\phi}_1(x_t-\mu)=40+0.7(25-40)=29.5$

$\hat{x}_t(2)=\mu+\phi_1(\hat{x}_t(1)-\mu)=40+0.7(29.5-40)=32.65$

$\hat{x}_t(3)=\mu+\hat{\phi}_1(\hat{x}_t(2)-\mu)=40+0.7(32.65-40)=34.855$

(2) $x_t-\mu=\varepsilon_t+\theta_1\varepsilon_{t-1}$ MA(1)

$\hat{x}_t(1)=\mu+\hat{\theta}_1\hat{\varepsilon}_t=40+0.6(3)=41.8$

$\hat{x}_t(2)=\hat{x}_t(3)=\mu=40$

(3) $(1-\phi_1B)\Delta x_t=(1+\theta_1B)\varepsilon_t$ ARIMA(1,1,1)

$\Delta x_t-\phi_1B\Delta x_t=\theta_1\varepsilon_{t-1}+\varepsilon_t$

$x_t-x_{t-1}-\phi_1B(x_t-x_{t-1})=\theta_1\varepsilon_{t-1}+\varepsilon_t$

$x_t-x_{t-1}-\phi_1x_{t-1}+\phi_1x_{t-2}=\theta_1\varepsilon_{t-1}+\varepsilon_t$

$x_t=(1+\phi_1)x_{t-1}-\phi_1x_{t-2}+\theta_1\varepsilon_{t-1}+\varepsilon_t$

$\hat{x}_t(1)=(1+\hat{\phi}_1)x_n-\hat{\phi}_1x_{n-1}+\hat{\theta}_1\hat{\varepsilon}_n$

$=(1+0.7)25-0.7*20+0.6*3$

$=30.3$

$$\hat{X_t}(2) = (1+\hat{\phi}_1)\hat{X_t}(1) - \hat{\phi}_1 X_n$$
$$= (1+0.7)30.3 - 0.7 * 25 = 34.01$$

$$\hat{X_t}(3) = (1+\hat{\phi}_1)\hat{X_t}(2) - \hat{\phi}_1 \hat{X_t}(1)$$
$$= (1+0.7)34.01 - 0.7(30.3) = 36.607$$

(III) Simulate 400 observations, estimate the model, obtain 1-step ahead to 12 step ahead predictions and their standard errors an

(2) $(1-1.4B+0.48B^2)(x_t-20)=(1+1.2B+0.35B^2)\varepsilon_t$, $\varepsilon_t \sim N(0,5^2)$

$$(x_t-20)=\underbrace{1.4}_{\phi_1}B(x_t-20)-\underbrace{0.48}_{\phi_2}B^2(x_t-20)+\underbrace{1.2}_{\theta_1}B\varepsilon_t+\underbrace{0.35}_{\theta_2}B^2\varepsilon_t+\varepsilon_t$$

(3) $(1-0.8B)\Delta x_t=(1+0.6B)\varepsilon_t$, $\varepsilon_t \sim N(0,5^2)$

$$\Delta x_t=\underbrace{0.8}_{\phi_1}B\Delta x_t+\underbrace{0.6}_{\theta_1}B\varepsilon_t+\varepsilon_t$$

(4) $(1-B^{12})x_t=(1+0.8B)(1+0.8B^{12})\varepsilon_t$, $\varepsilon_t \sim N(0,5^2)$

$$x_t=B^{12}x_t+(1+0.8B)(1+0.8B^{12})\varepsilon_t$$

$$x_t=\underbrace{B^{12}}_{\Phi_1}x_t+(1+\underbrace{0.8}_{\theta_1}B+\underbrace{0.8}_{\Theta_1}B^{12}+\underbrace{(0.8)^2}_{\theta_1\Theta_1}B^{13})\varepsilon_t$$

$$\underbrace{(0,0,1)}_{\text{Non-seasonal}}\times\underbrace{(1,0,1)_{12}}_{\text{Seasonal}}$$