

2. Rewrite the following time series models using the backward shift operator B and the difference operator Δ and in terms of the time series X_t and a_t (that is, no X_{t-1} , y_t or a_{t-1} should show up in the final expression).

$$(1) X_t = \phi_1 X_{t-1} + a_t + \theta_1 a_{t-1} \quad BX_t = X_{t-1}$$

$$X_t = \phi_1 BX_t + a_t + \theta_1 Ba_t \quad (\text{My solution})$$

$$X_t - \phi_1 BX_t = a_t + \theta_1 Ba_t$$

$$X_t(1 - \phi_1 B) = a_t(1 + \theta_1 B)$$

$$(2) y_t = \phi_1 y_{t-1} + a_t + \theta_1 a_{t-1} \quad \text{and} \quad y_t = X_t - X_{t-1}$$

$$y_t = \phi_1 By_t + a_t + \theta_1 Ba_t$$

plug in y_t

$$X_t - X_{t-1} = \phi_1 B(X_t - X_{t-1}) + a_t + \theta_1 Ba_t$$

$$\Delta X_t = \phi_1 B(\Delta X_t) + a_t + \theta_1 Ba_t$$

$$\Delta X_t - \phi_1 B(\Delta X_t) = a_t(1 + \theta_1 B)$$

$$\Delta X_t(1 - \phi_1 B) = a_t(1 + \theta_1 B)$$

$$(3) \cdot X_t = \Phi_1 X_{t-4} + a_t$$

$$X_t = \Phi_1 B^4 X_t + a_t$$

$$X_t - \Phi_1 B^4 X_t = a_t$$

$$X_t (1 - \Phi_1 B^4) = a_t$$

$$(4) \cdot y_t = \phi_1 y_{t-1} + a_t + \theta_1 a_{t-1} \text{ and } y_t = X_t - X_{t-4}$$

$$y_t = \phi_1 B y_t + a_t + \theta_1 B a_t \text{ and } y_t = X_t - X_{t-4}$$

$$y_t (1 - \phi_1 B) = a_t (1 + \theta_1 B)$$

$$(X_t - X_{t-4}) (1 - \phi_1 B) = a_t (1 + \theta_1 B)$$

$$\Delta^4(X_t) (1 - \phi_1 B) = a_t (1 + \theta_1 B)$$

3. Define the operator Δ_4 as $\Delta_4 X_t = X_t - X_{t-4}$
 Rewrite the following time series models in the original forms WITHOUT the operator B , the difference operator Δ and Δ_4 .

$$(1) (1 - \phi_1 B) X_t = (1 + \theta B) a_t$$

$$X_t - \phi_1 B X_t = a_t + \theta B a_t$$

$$X_t - \phi_1 X_{t-1} = a_t + \theta a_{t-1}$$

$$X_t = \phi_1 X_{t-1} + a_t + \theta a_{t-1}$$

$$(2) (1 - \phi_1 B)(1 - \phi_1 B^4) X_t = a_t$$

$$(1 - \phi_1 B^4 - \phi_1 B + \phi_1^2 B^5) X_t = a_t$$

$$X_t - \phi_1 B^4 X_t - \phi_1 B X_t + \phi_1^2 B^5 X_t = a_t$$

$$X_t = \phi_1 X_{t-4} + \phi_1 X_{t-1} - \phi_1^2 X_{t-5} + a_t$$

$$(3) \Delta \Delta_4 X_t = (1 + \theta_1 B) a_t$$

$$(1 - B)(X_t - X_{t-4}) = a_t + \theta_1 B a_t$$

$$X_t - X_{t-4} - B X_t + B X_{t-4} = a_t + \theta_1 B a_t$$

$$X_t - X_{t-4} - X_{t-1} + X_{t-5} = a_t + \theta_1 a_{t-1}$$

$$X_t = X_{t-4} + X_{t-1} - X_{t-5} + a_t + \theta_1 a_{t-1}$$

$$(4) (1 - \phi_1 B) \Delta_4 X_t = (1 + \theta_1 B) a_t$$

$$(1 - \phi_1 B)(X_t - X_{t-4}) = a_t + \theta_1 B a_t$$

$$X_t - X_{t-4} - \phi_1 B X_t + \phi_1 B X_{t-4} = a_t + \theta_1 a_{t-1}$$

$$X_t - X_{t-4} - \phi_1 X_{t-1} + \phi_1 X_{t-5} = a_t + \theta_1 a_{t-1}$$

$$X_t = X_{t-4} + \phi_1 X_{t-1} - \phi_1 X_{t-5} + a_t + \theta_1 a_{t-1}$$