hw5

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Import the necessary libraries

```
library(forecast)#Source of gold dataset
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
library(tryCatchLog)
```

```
## futile.logger not found. Using tryCatchLog-internal functions for logging...
```

```
library(attempt)
library(TTR)
```

Daily morning gold prices in US dollars. 1 January 1985 – 31 March 1989.

```
data("gold")
```

The classic Box & Jenkins airline data. Monthly totals of international airline passengers, 1949 to 1960 from base-R datasets. Monthly data.

```
data(AirPassengers)
```

A time series object containing average air temperatures at Nottingham Castle in degrees Fahrenheit for 20 years.(1920-1939)

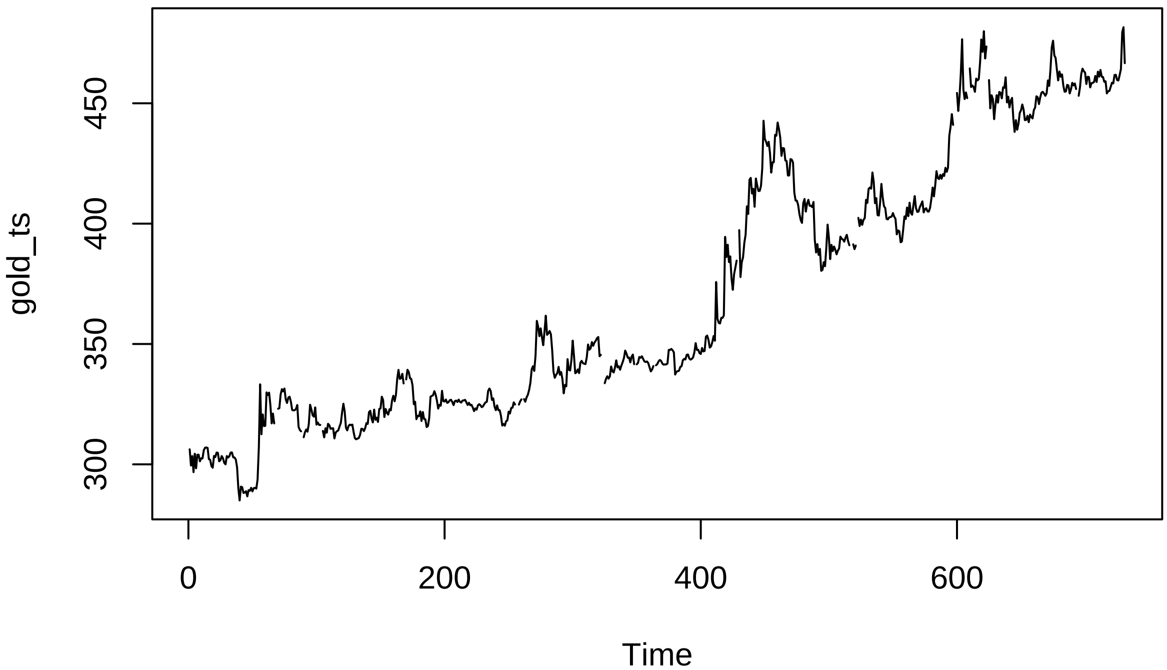
```
data(nottem)
```

Create time series objects from data

```
gold_ts <- ts(gold, start=1, end = 731) #January 1st 1985- Jan 1st 1987 subset
air_pass_ts <- AirPassengers
nottem_ts <- ts(nottem)
```

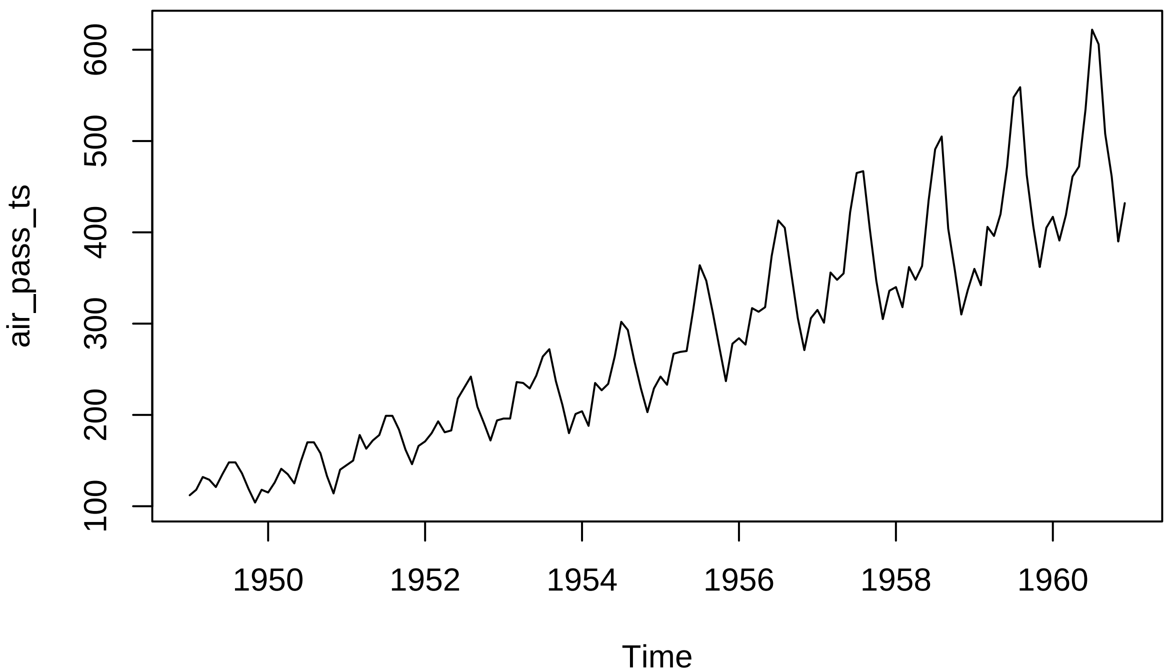
Plot gold time series

```
plot.ts(gold_ts)
```



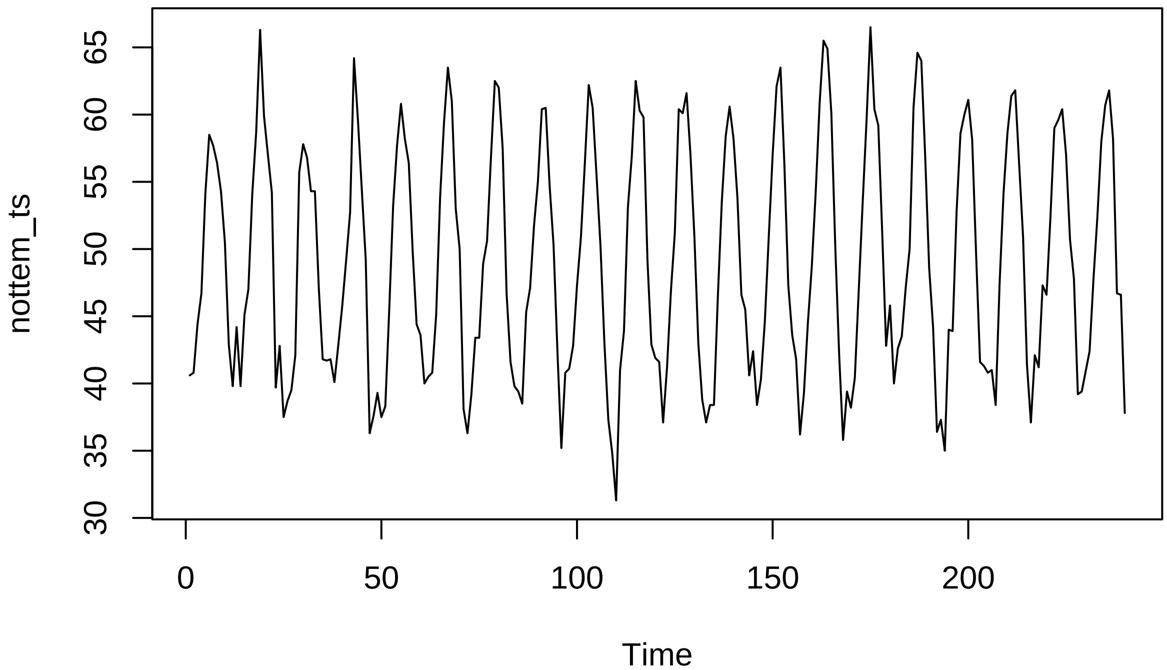
Plot Air Passengers time series

```
plot.ts(air_pass_ts)
```



Plot nottem time series

```
plot.ts(nottem_ts)
```



Decomposing Gold time series.Impossible, so there is no seasonal component

```
try_catch(decomposed_gold_additive <- decompose(gold_ts, type='additive'), .e=~print("Cannot decompose a
```

```
## [1] "Cannot decompose additive agold"
```

```
try_catch(decomposed_gold_mult <- decompose(gold_ts, type='multiplicative'), .e=~print("Cannot decompose
```

```
## [1] "Cannot decompose multiplicative gold"
```

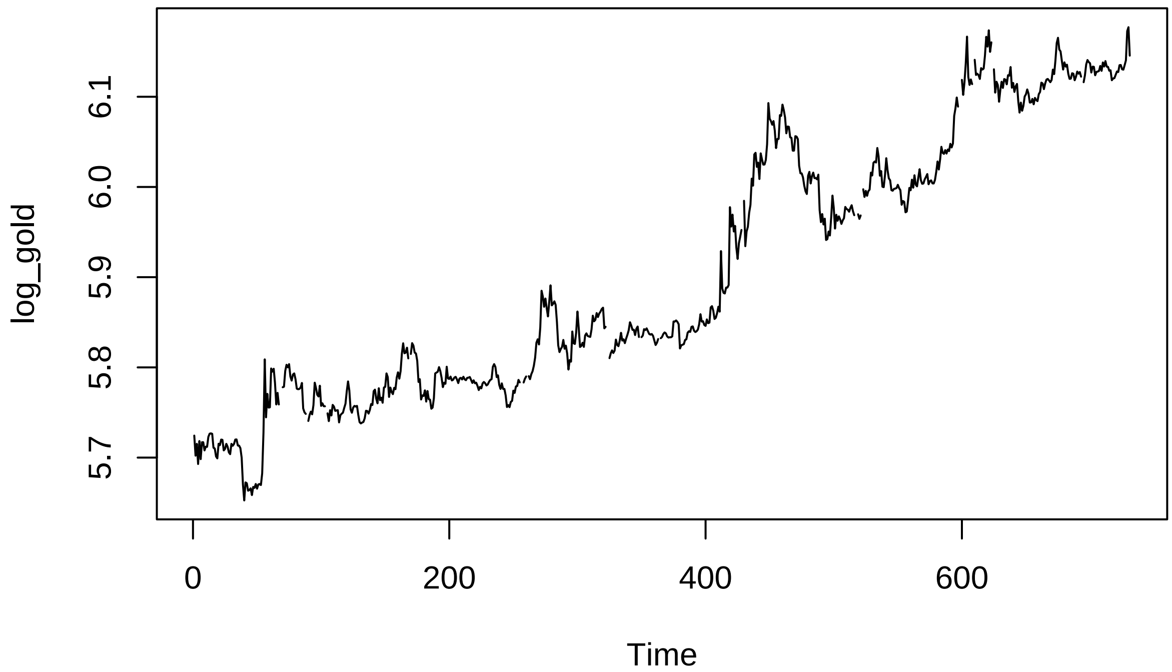
Instead, let us try to use SMA for Gold data.

```
try_catch(sma_gold <- SMA(gold_ts), .e=~print("Cannot get moving average gold"))
```

```
## [1] "Cannot get moving average gold"
```

We can now try exponential smoothing on gold data

```
log_gold <- log(gold_ts)
#Now that we have taken the log, let's try to fit a linear model
plot.ts(log_gold)
```

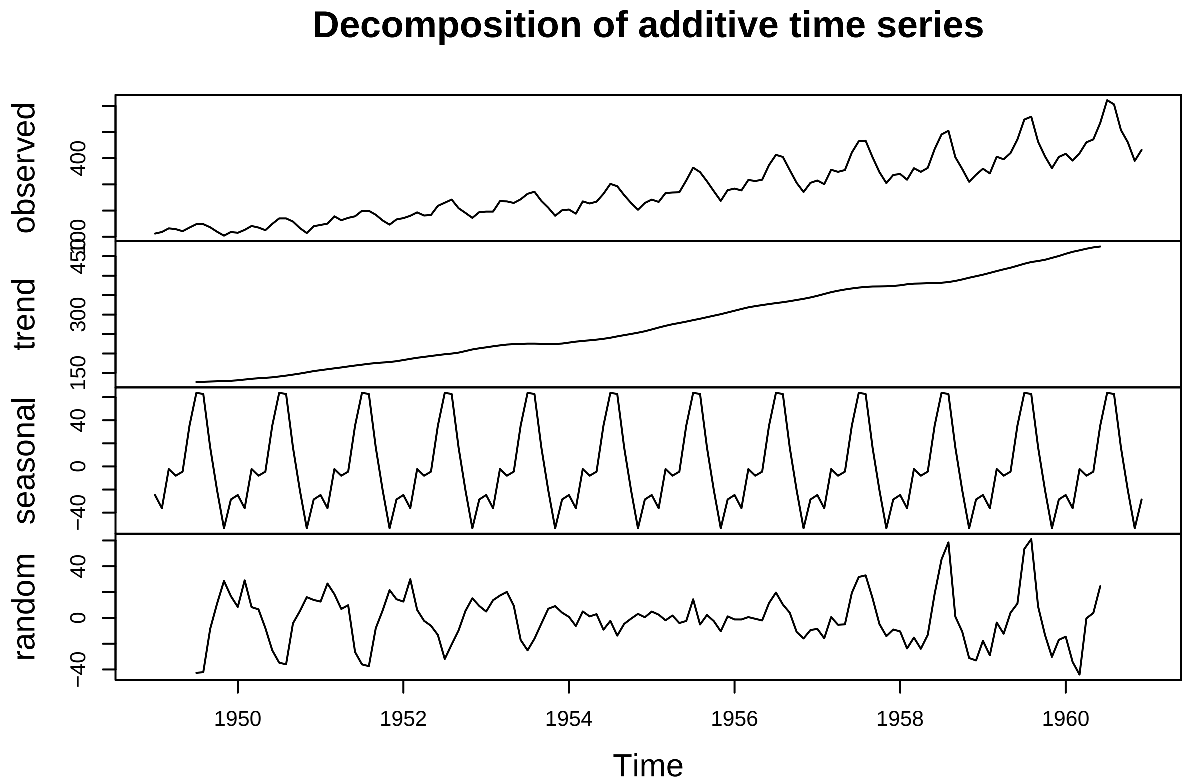


Decomposing Airpassenger time series.Successful

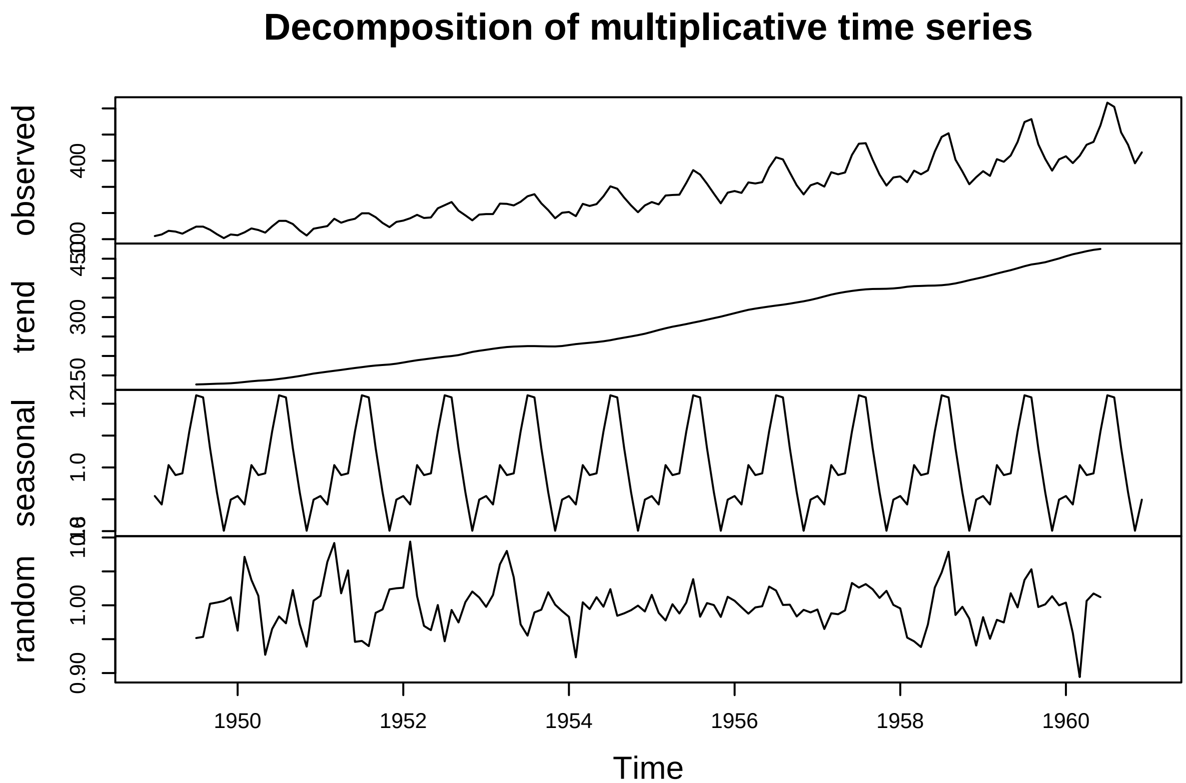
```
try_catch(decomposed_air_pass_additive <- decompose(air_pass_ts, type='additive'), .e=~print("Cannot dec
try_catch(decomposed_air_pass_mult <- decompose(air_pass_ts, type='multiplicative'), .e=~print("Cannot d
```

Since Airpassenger decomposition was successful, let us plot the decomposed version. We see that both plots are successful.

```
plot(decomposed_air_pass_additive)
```

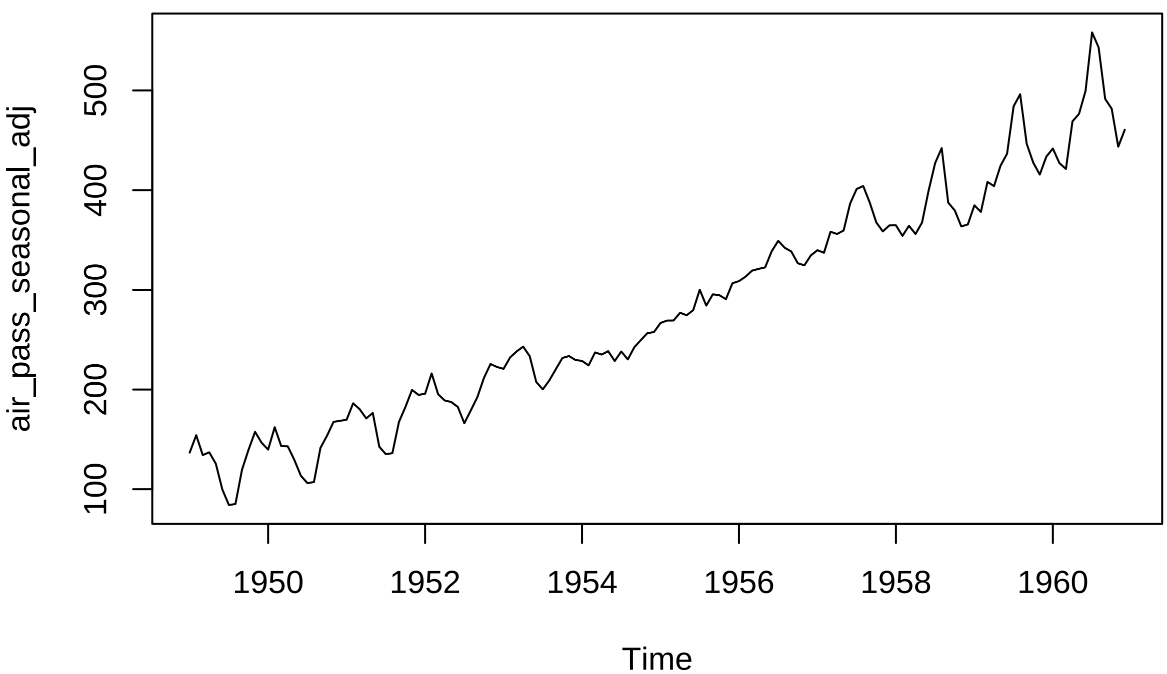


```
plot(decomposed_air_pass_mult)
```



Let us now try seasonal adjustment for AirPassengers

```
air_pass_seasonal_adj <- air_pass_ts - decomposed_air_pass_additive$seasonal
plot(air_pass_seasonal_adj)
```



Decomposing Nottem time series.Impossible

```
try_catch(decomposed_nottem_additive <- decompose(nottem_ts, type='additive'), .e=~print("Cannot decompo
```

```
## [1] "Cannot decompose additive nottem"
```

```
try_catch(decomposed_nottem_mult <- decompose(nottem_ts, type='multiplicative'), .e=~print("Cannot decom
```

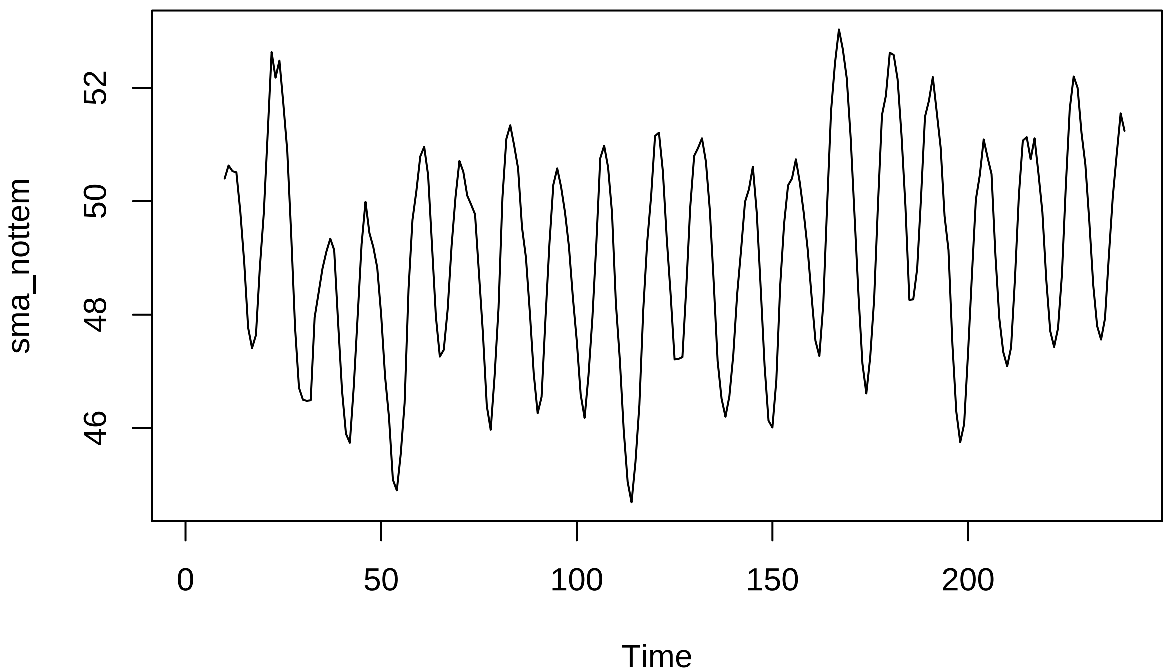
```
## [1] "Cannot decompose multiplicative nottem"
```

Instead, let us try to use SMA for Nottem data

```
try_catch(sma_nottem <- SMA(nottem_ts), .e=~print("Cannot get moving average SMA"))
```

Since no error was generated, we can plot sma_nottem

```
plot.ts(sma_nottem)
```



Observations and Conclusions

In the subset of data taken from the gold prices dataset, the price of Gold is increasing. That is not the case for the entire dataset. Neither Gold nor nottem data are additive models which is why decompose() does not work for them.

However, Air passengers is additive and multiplicative as both decompositions are possible There is a clear upward trend; As time increases, the number of passengers increases For nottem data, we were able to compute the moving average using SMA().

2. $X_t = W_t(1-W_{t-1})$ $\quad$ $\{W_t\}$ and $\{Z_t\}$ are i.i.d

$P(W_t=0)=P(W_t=1)=\frac{1}{2}$

$P(Z_t=-1)=P(Z_t=1)=\frac{1}{2}$

A process is white if the following hold:

$\mu_t=0 \quad \gamma_{t,t}=\sigma_w^2$ for all $t\in T$ and $\gamma_{s,t}=0$ for all $s\neq t$

(1) $E[X_t]=E[W_t(1-W_{t-1})Z_t]=E[W_t]*E[(1-W_{t-1})]*E[Z_t]$

$=(0*\frac{1}{2}+1*\frac{1}{2})\,E[1-W_t]*(-1*\frac{1}{2}+1*\frac{1}{2})=$

$=\frac{1}{2}*(1-(0*\frac{1}{2}+1*\frac{1}{2}))*(0)=\frac{1}{2}*\frac{1}{2}*0=\boxed{0}$

(2) $Var[X_t]=E[X_t^2]-(E[X_t])^2=\frac{1}{4}-0=\boxed{\frac{1}{4}}$

$E[X_t^2]=E[(W_t(1-W_{t-1})Z_t)^2]=E[W_t^2(1-W_{t-1})^2Z_t^2]=$

$=E[W_t^2]*E[(1-W_{t-1})^2]*E[Z_t^2]$

$=(0^2*\frac{1}{2}+1^2*\frac{1}{2})(1-(0^2*\frac{1}{2}+1^2*\frac{1}{2}))*((-1)^2*\frac{1}{2}+(1)^2*\frac{1}{2})$

$=(\frac{1}{2})(\frac{1}{2})(1)=\frac{1}{4} \qquad \gamma_{t,t}=\sigma_w^2\neq 0$

0 b/c $E[X_t]=0$

(3) $Cov(X_s,X_t)=E[X_sX_t]-E[X_s]*\cancel{E[X_t]}$

$=E[W_s(1-W_{s-1})Z_s*W_t(1-W_{t-1})Z_t]$

$=E[\cancel{Z_t}^{\,0}]*E[W_s(1-W_{s-1})Z_s*W_t(1-W_{t-1})]$

$=\boxed{0}$

To show X_t is not I.I.D:

$P(X_{t-1}=c, X_t=c)\neq P(X_{t-1}=c)*P(X_t=c)$

Suppose $X_{t-1}=1$

$X_{t-1}=W_{t-1}(1-W_{t-2})Z_{t-1}$

$1=W_{t-1}(1-W_{t-2})Z_{t-1}$

$W_{t-2}=0$ and $W_{t-1}=1$ and $Z_{t-1}=1$

$P(W_{t-2}=0)=\frac{1}{2}$, $P(W_{t-1}=1)=\frac{1}{2}$, $P(Z_{t-1}=1)=\frac{1}{2}$

Thus $P(X_{t-1}=1)=\frac{1}{8}$

Let us now examine X_t

$X_t=W_t(1-W_{t-1})*Z_t$

From before, $W_{t-1}=1$ so $X_t=0$

$P(X_t=1, X_{t-1}=1)=0$

Suppose that $\{X_t\}$ is i.i.d.

Then we should have

$$P(X_t=1, X_{t-1}=1) = P(X_{t-1}=1)*P(X_t=1)=\left(\frac{1}{8}\right)^2.$$

But instead, $0 \neq \frac{1}{64}$. Hence we prove by contradiction that X_t is not I.I.D ■

3.

A process is weakly stationary if:

(1) u_t is constant over time

(2) $\gamma_{s,t}=\gamma_{k,l}$, $\forall$ $s,t,k,l \in T$ s.t $|s-t|=|k-l|$

(a). $X_t=W_t-W_{t-3}$

$E[X_t]=E[W_t-W_{t-3}]=E[W_t]-E[W_{t-3}]=0$

since all W_t are i.i.d

$\mathrm{Cov}(X_s, X_t) = E[X_s X_t] - E[X_s]E[X_t] =$ 0 b/c $E[X_t]=0$

$= E[(W_s - W_{s-3}) * (W_t - W_{t-3})] - E[X_s]\cancel{E[X_t]}$

$= E[W_s W_t - W_{s-3} W_t - W_s W_{t-3} + W_{s-3} W_{t-3}]$

$= E[W_s W_t] - E[W_{s-3} W_t] - E[W_s W_{t-3}] + E[W_{s-3} W_{t-3}]$

We will now show that this is a function of $|s-t|$ and is consequently a stationary process.

Using indicator functions we can rewrite as

$\mathbb{1}_{\{s=t\}} - \mathbb{1}_{\{s=t-3\}} - \mathbb{1}_{\{s-3=t\}} + \mathbb{1}_{\{s-3=t-3\}} =$

$= 2\mathbb{1}_{\{s-t|0\}} - 2*\mathbb{1}_{\{|s-t|=3\}}$

This is in fact a function of $|s-t|$. and stationary

(b) $X_t = W_3$

$E[X_t] = E[W_3] = 0$

$\mathrm{Cov}(X_s, X_t) = E[W_3 * W_3] - E[W_3] * \cancel{E[W_3]} = 0$

$= \sigma^2 + \mu^2 = 1 + 0^2 = \boxed{1}$ We have a constant variance and 0 mean

so the process is stationary

(c). $X_t = t + W_3$

(1) $E[X_t] = E[t + W_3] = E[t] + \cancel{E[W_3]} = t$ 0

Since the expectation is not constant, this process is not stationary.

(d). $X_t = W_t^2$

$E[X_t] = E[W_t^2] = 1$

$\mathrm{Cov}(X_s, X_t) = E[X_s X_t] - E[X_s] * E[X_t]$

$= E[X_s X_t] - E[X_s]$

$= E[W_s^2 W_t^2] - E[W_s^2]$

If $s=t$:

$Cov(X_s, X_t) = E[W_t^4] - E[W_t^2] = 3-1=2$

If $s \neq t$:

$Cov(X_s, X_t) = E[W_s^2] * E[W_t^2] - E[W_s^2] = 0$

Thus, we have a stationary process

(e). $X_t = W_t W_{t-2}$

$E[X_t] = E[W_t W_{t-2}] = E[W_t] E[W_{t-2}] = 0$

$$\begin{aligned} Cov(X_s, X_t) &= E[X_s X_t] - E[X_s] * E[X_t] \\ &= E[W_s W_{s-2} W_t W_{t-2}] - \cancelto{0}{E[W_s W_{s-2}] E[W_t W_{t-2}]} \\ &= \mathbb{1}_{\{s=t\}} \end{aligned}$$

Thus, we have a stationary process.