IMPORTANT NOTES. Please upload your homework to Canvas or email your homework to our TA: ys688 at stat.rutgers.edu. For the simulation and data analysis problems, please put the code you developed at the end of the homework report (no separated files).

1. Generate $n = 100$ observations of the time series by $x_t = w_{t-1} + 2w_t + w_{t+1}$, where $\{w_t\} \sim N(0, 1)$. Plot the sample autocorrelation function.

2. Rewrite the following time series models using the backward shift operator B and the difference operator Δ and in terms of the time series x_t and a_t (that is, no x_{t-1}, y_t or a_{t-1} should show up in the final expression).

(1) $x_t = \phi_1 x_{t-1} + a_t + \theta_1 a_{t-1}$
(2) $y_t = \phi_1 y_{t-1} + a_t + \theta_1 a_{t-1}$ and $y_t = x_t - x_{t-1}$.
(3) $x_t = \Phi_1 x_{t-4} + a_t$
(4) $y_t = \phi_1 y_{t-1} + a_t + \theta_1 a_{t-1}$ and $y_t = x_t - x_{t-4}$.

3. Define the operator Δ_4 as $\Delta_4 x_t := x_t - x_{t-4}$. Rewrite the following time series models in the original forms **WITHOUT** the operator B, the difference operator Δ and Δ_4.

(1) $(1 - \phi_1 B)x_t = (1 + \theta B)a_t$
(2) $(1 - \phi_1 B)(1 - \Phi_1 B^4)x_t = a_t$
(3) $\Delta\Delta_4 x_t = (1 + \theta_1 B)a_t$
(4) $(1 - \phi_1 B)\Delta_4 x_t = (1 + \theta_1 B)a_t$