Regression and Time Series HW7

Yaniv Bronshtein

11/18/2021

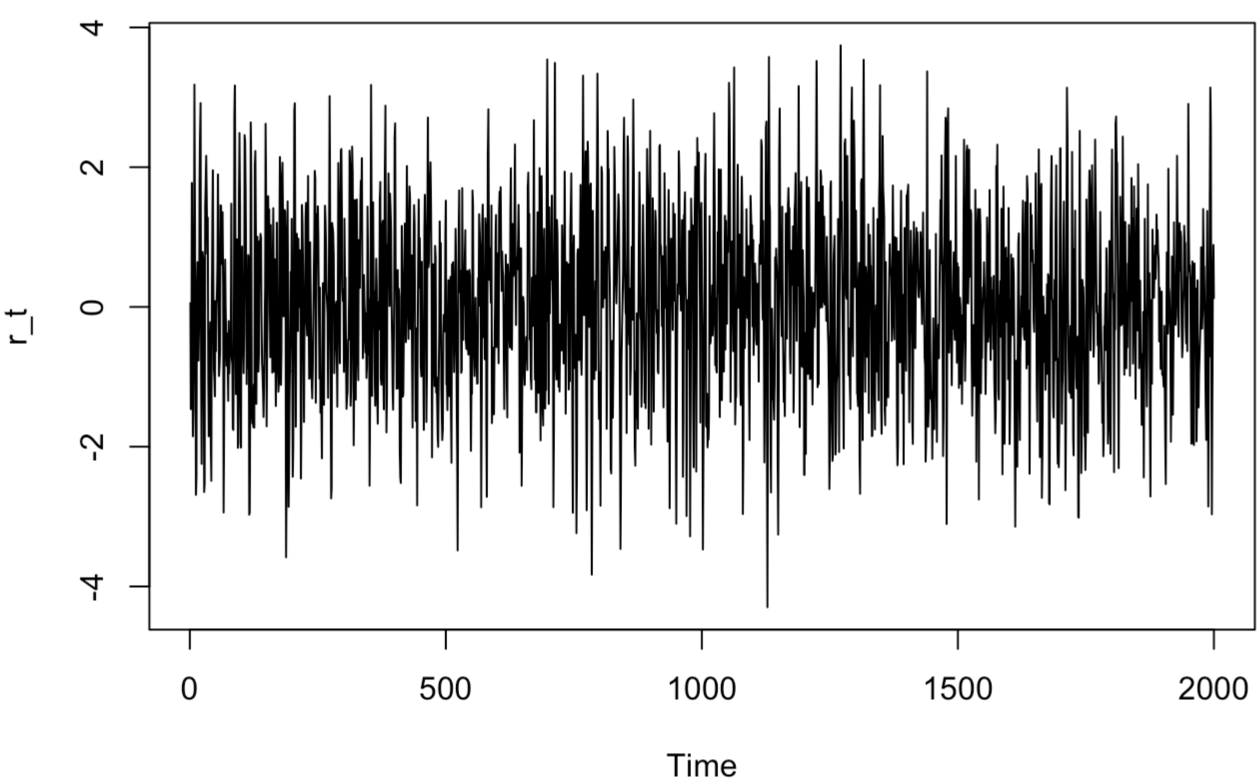
c. Simulate a time series of length T=2000 from this model. Create a time series plot. Compute the lag-1, lag-2, and lag-3 sample autocorrelations

```
set.seed(1)
T <- 2000
#rt = 0.01 + 0.6rt-1 - 0.4rt-2 + at
a_t <- rnorm(n=T, mean=0, sd=sqrt(0.02))

r_t <- 0.01 + arima.sim(model=list(order=c(2, 0, 0), ar=c(0.6, -.4)),n =T) + a_t
```

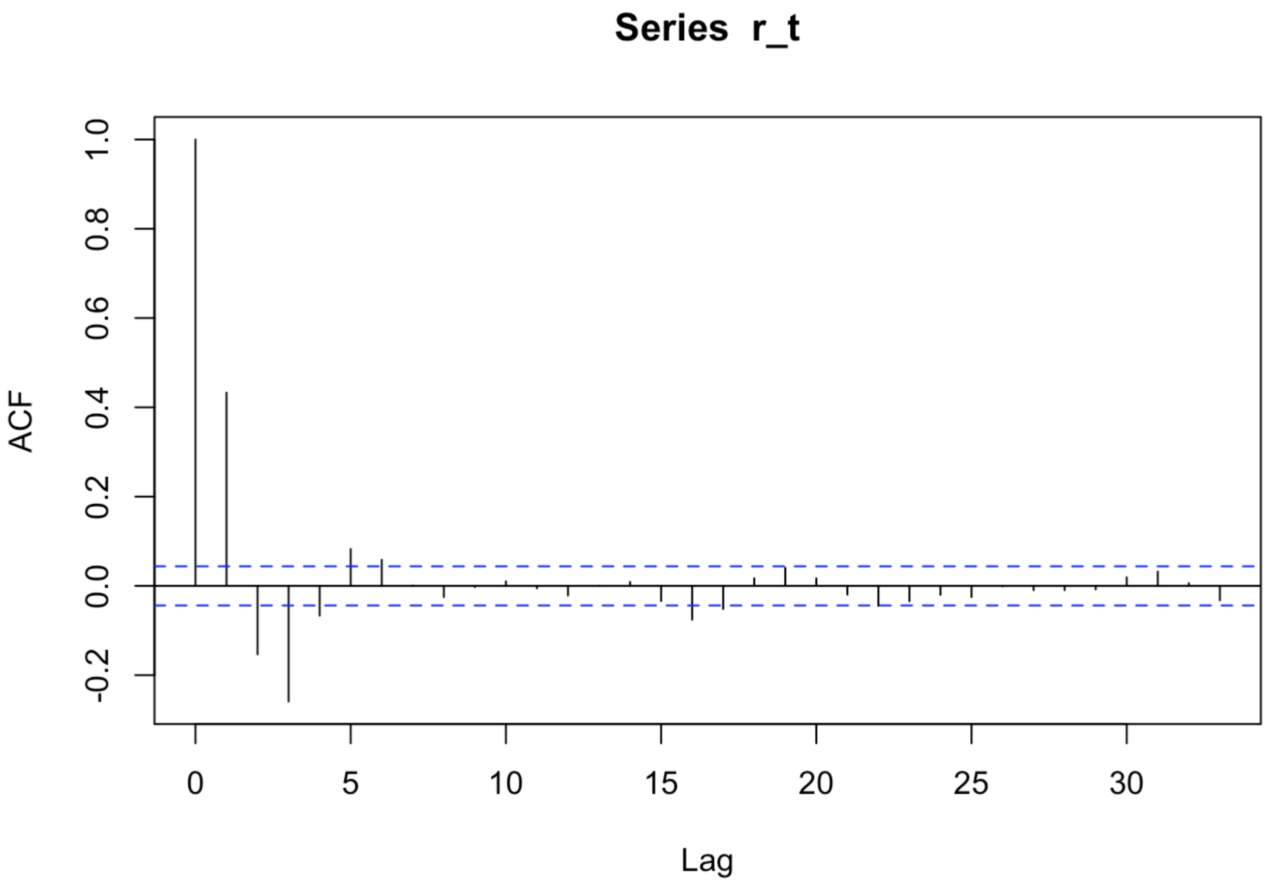
Let's now create a time series plot

```
plot.ts(r_t)
```



Sample autocorrelation plot

```
acf <- acf(r_t)
```




```
acf
```

```
##
## Autocorrelations of series 'r_t', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000  0.433 -0.153 -0.259 -0.067  0.083  0.058  0.001 -0.025 -0.003  0.010
##     11     12     13     14     15     16     17     18     19     20     21
## -0.005 -0.022  0.000  0.009 -0.034 -0.076 -0.052  0.017  0.040  0.017 -0.020
##     22     23     24     25     26     27     28     29     30     31     32
## -0.045 -0.034 -0.020 -0.025 -0.001 -0.010 -0.010 -0.008  0.019  0.032  0.006
##     33
## -0.032
```

Calculate lag1

```
acf[1]
```

```
##
## Autocorrelations of series 'r_t', by lag
##
##     1
## 0.433
```

Calculate lag2

```
acf[2]
```

```
##
## Autocorrelations of series 'r_t', by lag
##
##      2
## -0.153
```

Calculate lag3

```
acf[3]
```

```
##
## Autocorrelations of series 'r_t', by lag
##
##      3
## -0.259
```

1. Suppose that the daily log return of a security follows the model

$$r_t = 0.01 + 0.6 r_{t-1} - 0.4 r_{t-2} + a_t$$

where $\{a_t\}$ is a white noise series with mean zero and variance 0.02.

(a) What is the mean of the return series r_t?

$$\mu = E[r_t] = \frac{\phi_0}{1-\phi_1-\phi_2-\dots-\phi_p} = \frac{0.01}{1-0.6+0.4} = 0.0125$$

(b) Compute the lag-1, lag-2, and lag-3 autocorrelations of r_t.

Recall:

$$\gamma_h = \mathrm{Cov}(r_t, r_{t+h})$$

$$= \mathrm{Cov}(r_t,\ 0.01 + 0.6 r_{t-1+h} - 0.4 r_{t-2+h} + a_{t+h})$$

$$= \underbrace{\mathrm{Cov}(r_t, 0.01)}_{0} + \mathrm{Cov}(r_t, 0.6 r_{t-1+h}) - \mathrm{Cov}(r_t, 0.4 r_{t-2+h}) + \underbrace{\mathrm{Cov}(r_t, a_{t+h})}_{0}$$

$$= 0.6\gamma_{h-1} - 0.4\gamma_{h-2}$$

$$\rho_h = \frac{\gamma_h}{\gamma_0}$$

Thus, we can write ρ_h like γ_h

$$\rho_h = 0.6\rho_{h-1} - 0.4\rho_{h-2}$$

Solve for ρ_1:

$$\rho_1 = 0.6\rho_0 - 0.4\rho_1$$

We know $\rho_0 = 1$:

$$\rho_1 = 0.6 - 0.4\rho_1 \Rightarrow 1.4\rho_1 = 0.6 \Rightarrow \boxed{\rho_1 = 0.4285}$$

$$\rho_2 = 0.6\rho_1 - 0.4\rho_0 \Rightarrow \rho_2 = 0.6\rho_1 - 0.4 \Rightarrow \boxed{\rho_2 = -0.1429}$$

$$\rho_3 = 0.6\rho_2 - 0.4\rho_1 \Rightarrow \boxed{\rho_3 = -0.25714}$$

d). Bonus: Compute the variance, lag-1 and lag-2 autocovariances of r_t. What are the corresponding sample autocovariances?

$$\rho_h = \frac{\gamma_h}{\gamma_0}$$

$$\gamma_h = \rho_h * \gamma_0$$

$$\gamma_0 = 1 + \sum_{i=1}^{\infty} \psi_i^2 = 1 + (0.6)^2 + (-0.4)^2 = 1.52$$

$$\gamma_1 = 1.52 * 0.4285 = 0.65132$$
$$\gamma_2 = 1.52 * (-0.1429) = -0.2172$$

$$\hat{\gamma}_1 = 1.52 * 0.433 = 0.685816$$
$$\hat{\gamma}_2 = 1.52 * (-0.153) = -0.23256$$

2. Suppose we have the following estimates from the data:

$\hat{\mu} = 0.5768$, $\hat{\gamma}_0 = 1.7379$, $\hat{\gamma}_1 = 1.4458$, $\hat{\gamma}_2 = 1.0600$

Find the Yule-Walker estimates for the AR(2) model.

$$\begin{bmatrix} \gamma_0 & \gamma_1 \\ \gamma_1 & \gamma_0 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}$$

$$\begin{bmatrix} 1.7379 & 1.4458 \\ 1.4458 & 1.7379 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} 1.4458 \\ 1.0600 \end{bmatrix}$$

$AB = y$

$B = A^{-1} y$

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} 1.8688 & -1.5547 \\ -1.5547 & 1.8688 \end{bmatrix} \begin{bmatrix} 1.4458 \\ 1.0600 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0539 \\ -0.2686 \end{bmatrix}$$