Important notes. Please upload your homework to Canvas or email your homework to our TA: ys688 at stat.rutgers.edu. For the simulation and data analysis problems, please put the code you developed at the end of the homework report (no separated files).

- (I) Without using any computer software, make 1 step to 3 step ahead forecast for the following models. Assume $\hat{\phi}_1 = 0.7$, $\hat{\phi}_2 = 0.4$, $\hat{\theta}_1 = 0.6$, $\hat{\theta}_2 = 0.4$, $\hat{\mu} = 40$, $x_{n-2} = 30$, $x_{n-1} = 20$, $x_n = 25$, $\hat{\varepsilon}_{n-2} = 2$, $\hat{\varepsilon}_{n-1} = 1$, $\hat{\varepsilon}_n = 3$.

  1. $(x_t - \mu) = \phi_1(x_{t-1} - \mu) + \varepsilon_t$
  2. $x_t - \mu = \varepsilon_t + \theta_1\varepsilon_{t-1}$
  3. $(1 - \phi_1 B)\Delta x_t = (1 + \theta_1 B)\varepsilon_t$

- (III) For the following models, simulate 400 observations, estimate the model, obtain 1-step ahead to 12-step ahead predictions and their standard errors, and plot the last 24 observed observations with the predictions and their 95% confidence intervals.

  1. $(x_t - 20) = -0.7(x_{t-1} - 20) + \varepsilon_t$, $\varepsilon_t \sim N(0, 5^2)$.
  2. $(1 - 1.4B + 0.48B^2)(x_t - 20) = (1 + 1.2B + 0.35B^2)\varepsilon_t$, $\varepsilon_t \sim N(0, 5^2)$.
  3. $(1 - 0.8B)\Delta x_t = (1 + 0.6B)\varepsilon_t$, $\varepsilon_t \sim N(0, 5^2)$.
  4. $(1 - B^{12})x_t = (1 + 0.8B)(1 + 0.8B^{12})\varepsilon_t$, $\varepsilon_t \sim N(0, 5^2)$.