Statistical Learning HW1 Applied

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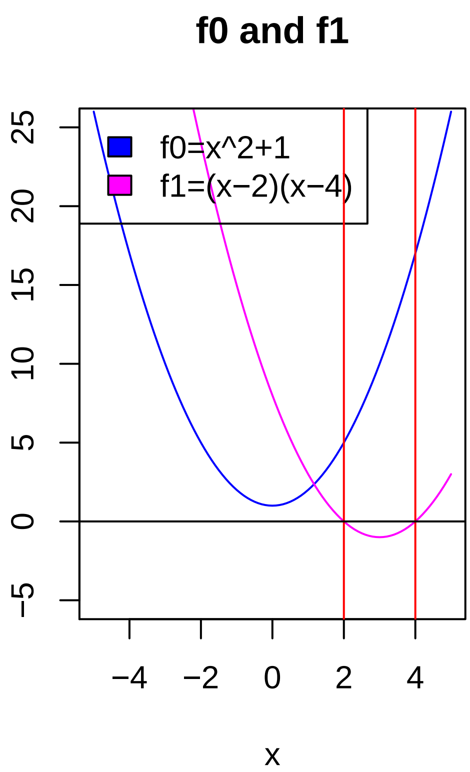
(a). *Analysis of Primal Problem* Give the feasible set, the optimal value, and the optimal solution

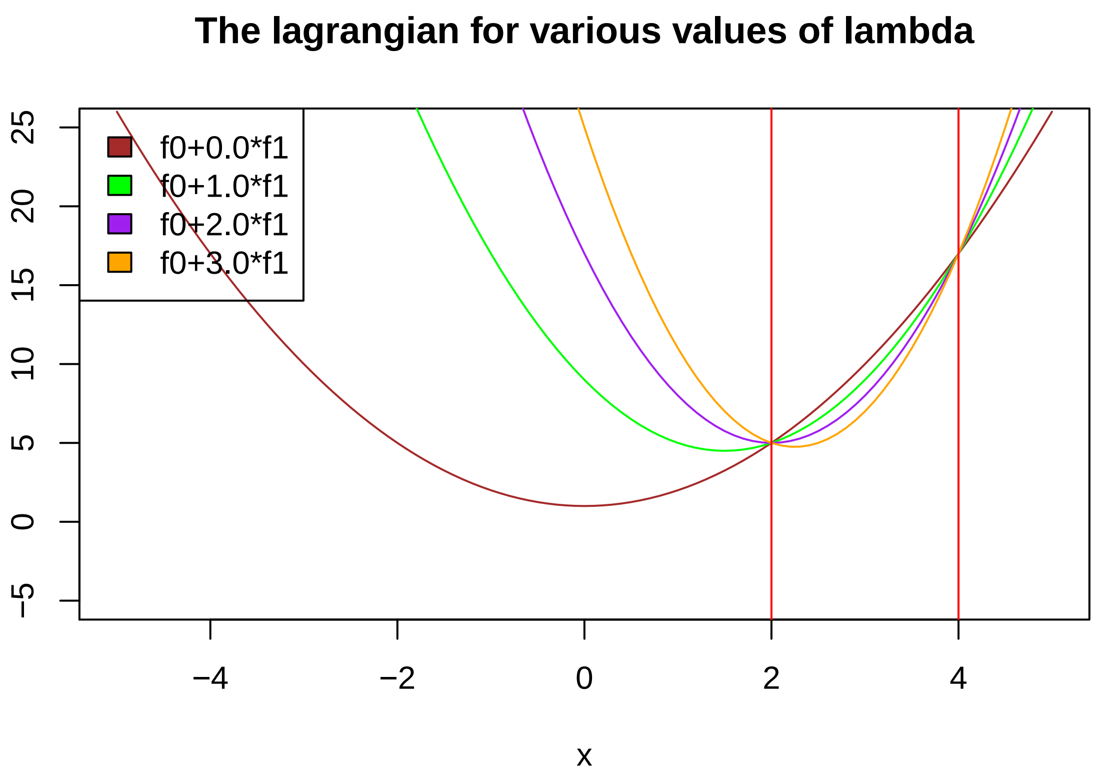
Feasible Set: The interval [2,4] $(x-2)(x-4) \leq 0$ $x-2 \leq 0$ and $x-4 \leq 0$ $x \leq 2$ and $x \leq 4$ Thus, the optimal point is x*=2 The optimal value is $2^2 + 1 = 5$

(b). *Lagrangian and dual Function* Plot the objective $x^2 + 1$ versus x. On the same plot, show the feasible set, optimal optimal point and value, and plot the Lagrangian $L(x, \lambda)$ versus x for a few positive values of λ. Verify the lower bound property $p* \geq inf L(x, \lambda)$ for $\lambda \geq 0$. Derive and sketch the Lagrange dual function g.

```r
x <- seq(-5,5, 0.1)
f0 <- x^2+1
f1 <- (x-2)*(x-4)
par(mfrow=c(1,2))
plot(
  x=x,
  y=f0,
  main="f0 and f1",
  ylab="",
  xlab="x",
  type="l",
  ylim = c(-5,25),
  col="blue"
)
lines(x=x, y=f1, col="magenta")
legend(
  "topleft",
  c("f0=x^2+1","f1=(x-2)(x-4)"),
  fill=c("blue","magenta")
)

abline(v=2, col='red')
abline(v=4, col='red')
abline(h=0, col='black')
```

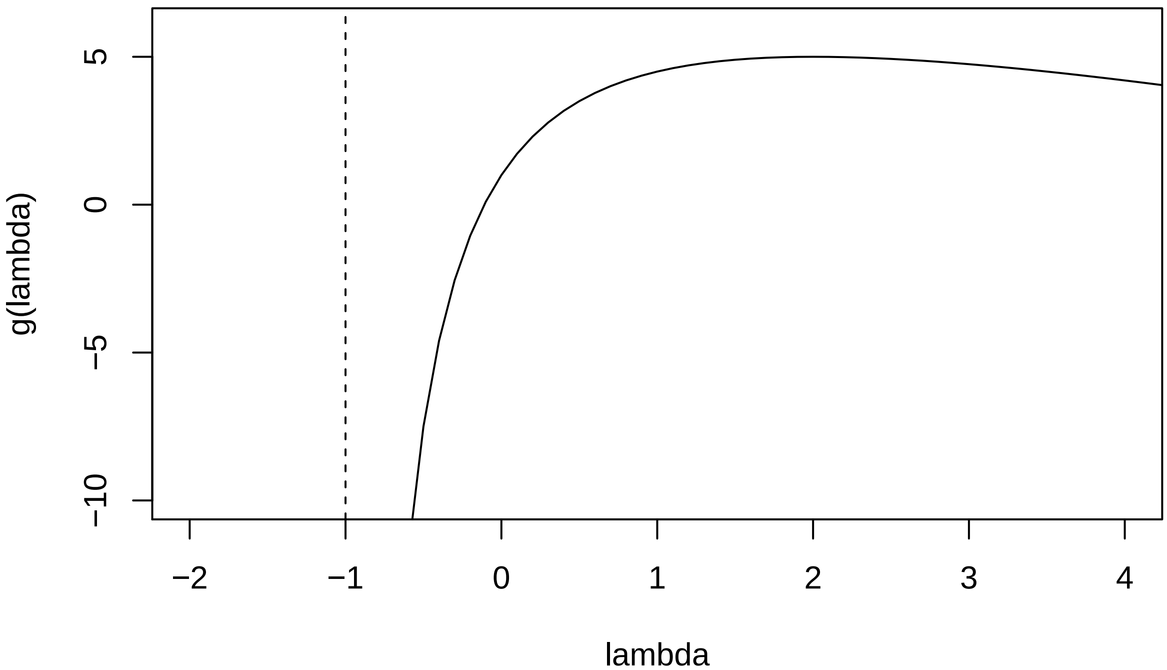



```
plot(x=x,
     y=f0,
     ylab="",
     xlab="x",
     type="l",
     ylim = c(-5,25),
     col="brown",
     main="The lagrangian for various values of lambda"
)
lines(x,f0+1.0*f1, col="green")
lines(x,f0+2.0*f1, col="purple")
lines(x,f0+3.0*f1, col="orange")

legend(
  "topleft",
  c("f0+0.0*f1","f0+1.0*f1", "f0+2.0*f1", "f0+3.0*f1"),
  fill=c("brown","green", "purple", "orange")
)

abline(v=2, col='red')
abline(v=4, col='red')
```



The overlayed plot above demonstrates the Lagrangian with input x and λ as the sum of f_0 and f_1 times a constant λ The minimum value of the Lagrangian is always less than p*. The maximum is reached at a λ value of 2 and decreases after that.

```
lambda <- seq(-0.9,16/3,0.1);
g <- (-9*lambda^2)/(1+lambda) + 1 + 8*lambda
plot(x=lambda,
     y=g,
     ylab="g(lambda)",
     xlab="lambda",
     type="l",
     xlim=c(-2,4),
     ylim=c(-10,6)
)
abline(v=-1, lty='dashed')
```


g(lambda)
5
0
−5
−10
−2
−1
0
1
2
3
4
lambda