Homework No.4 (MSDS 954:567) **Spring 2022**

Due date: 5/3/2022

Problem 1. Let $X_1, X_2, \cdots, X_6$ be a sequence of independent, identically distributed Bernoulli random variables with parameter θ, and suppose we observe $x_1 = x_2 = x_3 = x_4 = x_5 = 1$ and $x_6 = 0$. Derive the posterior probabilities for Model 0 (M_0): $\theta = \frac{1}{2}$ and Model 1 (M_1): $\theta > \frac{1}{2}$, assuming the following prior distributions:

(a) $P(M_0) = 0.5$, $P(M_1) = 0.5$, $\pi_1(\theta) = 2$; $\theta \in (\frac{1}{2}, 1)$.

(b) $P(M_0) = 0.8$, $P(M_1) = 0.2$, $\pi_1(\theta) = 8(1-\theta)$; $\theta \in (\frac{1}{2}, 1)$.

(c) $P(M_0) = 0.2$, $P(M_1) = 0.8$, $\pi(\theta) = 48(\theta - \frac{1}{2})(1-\theta)$; $\theta \in (\frac{1}{2}, 1)$.

In addition, compute the Bayes factor of Model 0 versus Model 1 in (a)-(c).

[*Remark: Use paper and pencil to solve this problem, write down the detailed calculation process.*]

Problem 2. Write your OWN code to simulate 100 samples from

(a) Exponential distribution: $Exp(\lambda)$ with $\lambda = 2.8$.

(b) Normal distribution using BOTH Box-Muller transformation AND central limit theorem: $N(\mu, \sigma^2)$ with (a) $(\mu, \sigma^2) = (0, 1)$ and (b) $(\mu, \sigma^2) = (3.5, 2)$.

(c) Log-normal distribution $LN(\mu, \sigma^2)$ with (a) $(\mu, \sigma^2) = (0, 1)$ and (b) $(\mu, \sigma^2) = (-4, 2)$.

(d) Binomial Distribution: $Binomial(n, p)$ with $n = 10$, $p = 0.24$.

In each case, plot the density of sample sets to illustrate (validate) your simulated samples.

For part (b), read the attached slides to learn the two methods.

[*Remark: Use a computer to simulate those data; explain your code and results carefully*]

Problem 3. Write a computing code and use the rejection sampling method to simulate a set of 150 samples from the following distribution with density

$$f(x) = \begin{cases} \frac{1}{c}\phi(x)\big(1 - \sin(20x)/4\big), \text{ if } |x| < 3 \\ 0, \text{ if } |x| > 3. \end{cases}$$

where $\phi(x)$ is the density function of standard normal distribution $N(0, 1)$ and $c = \int_{-3}^{3} \phi(x)\big(1 - sin(20x)/4\big)dx$. Draw the density of those 150 samples.

[*Remark: Use a computer to simulate those data; explain your code and results carefully*]