

# Non-Monetary Mechanism Design without Distributional Information: Using Scarce Audits Wisely

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# Challenge: Resource Allocation to Strategic Agents

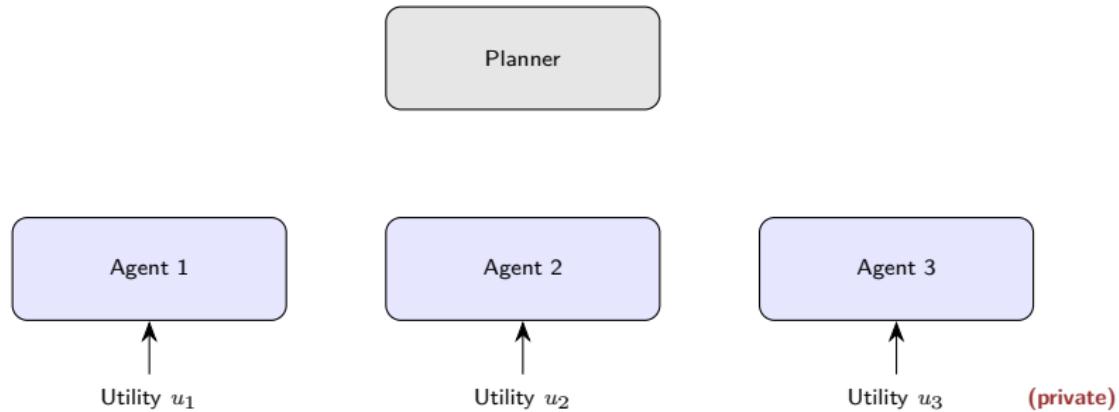
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- ②  $K$  strategic agents self-interested (*i.e.*, may lie)
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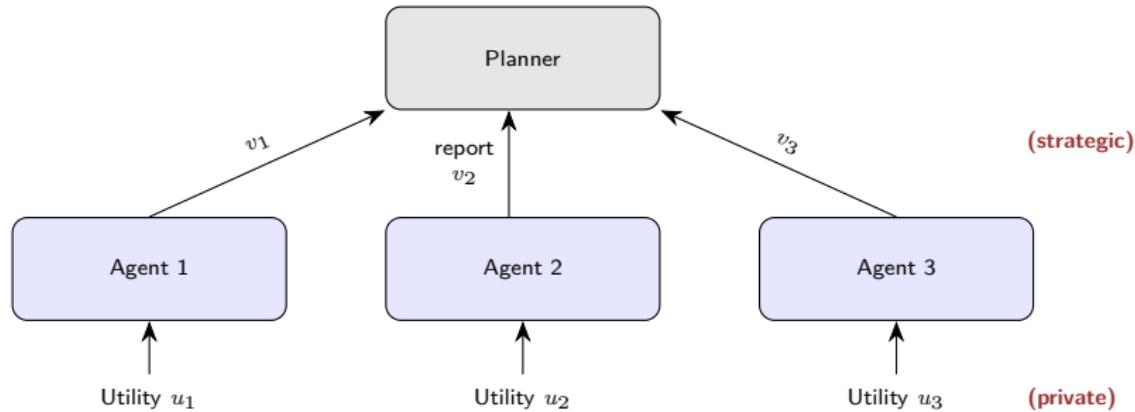
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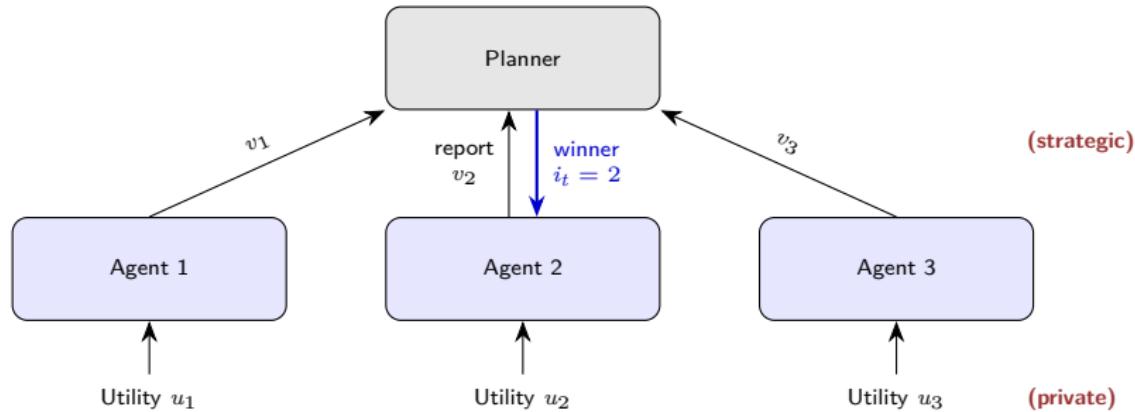
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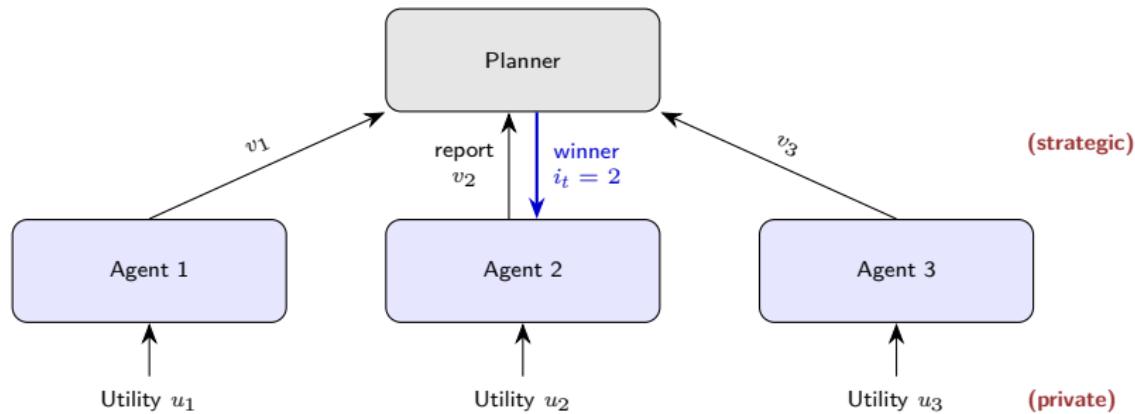
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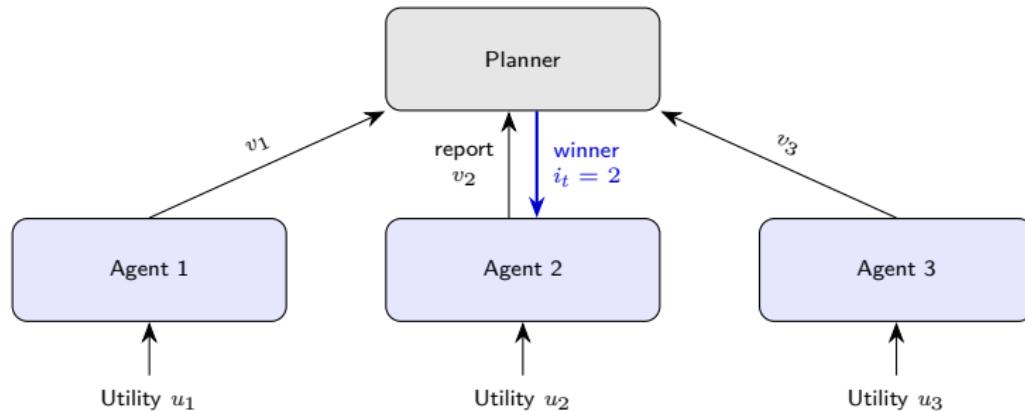
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## Two Objectives

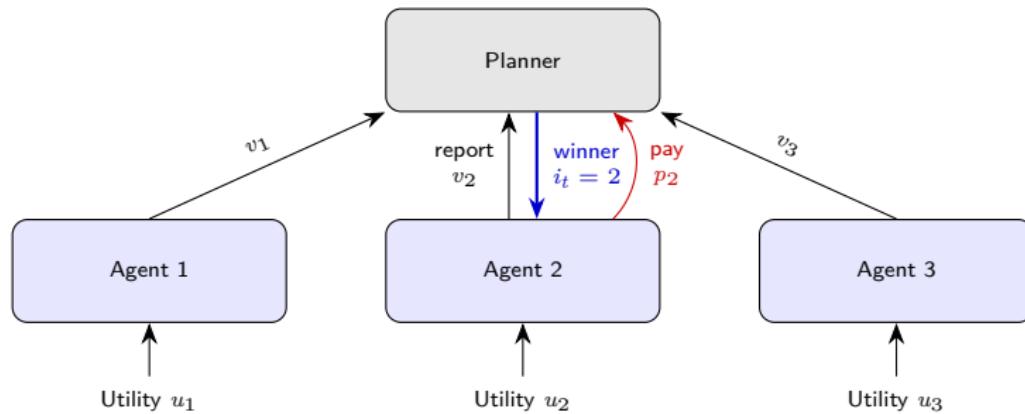
**Efficiency.** max established utility  $u_{i_t}$  (unknown!)

**Incentive-Compatibility.** truthfully report  $v_i \approx u_i$

# Classical VCG Mechanisms

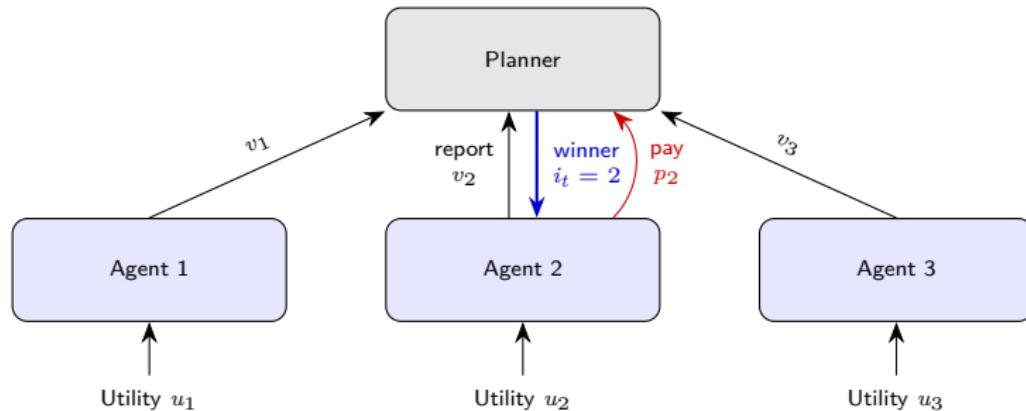


# Classical VCG Mechanisms



**Monetary** mechanisms: VCG family [Vic61; Cla71; Gro73]

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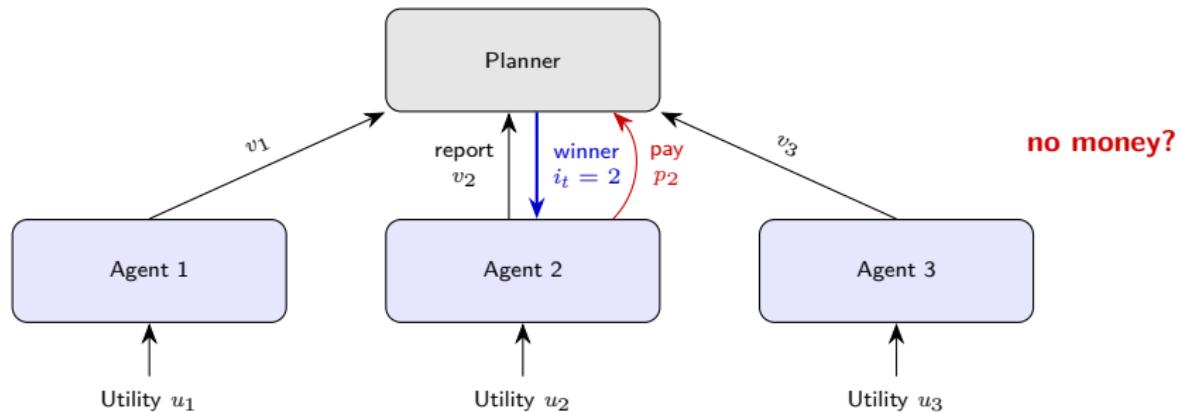


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Money isn't everything!

- ① Food bank allocations [Pre17; Pre22]
- ② Healthcare resources [PSÜY24; YBP23]
- ③ GPU in company [ABDVW22; PSMST22]

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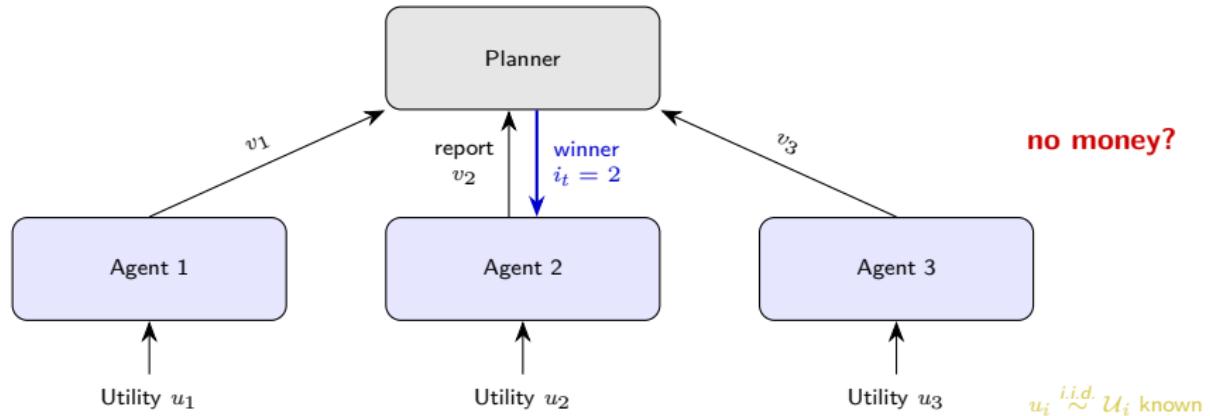


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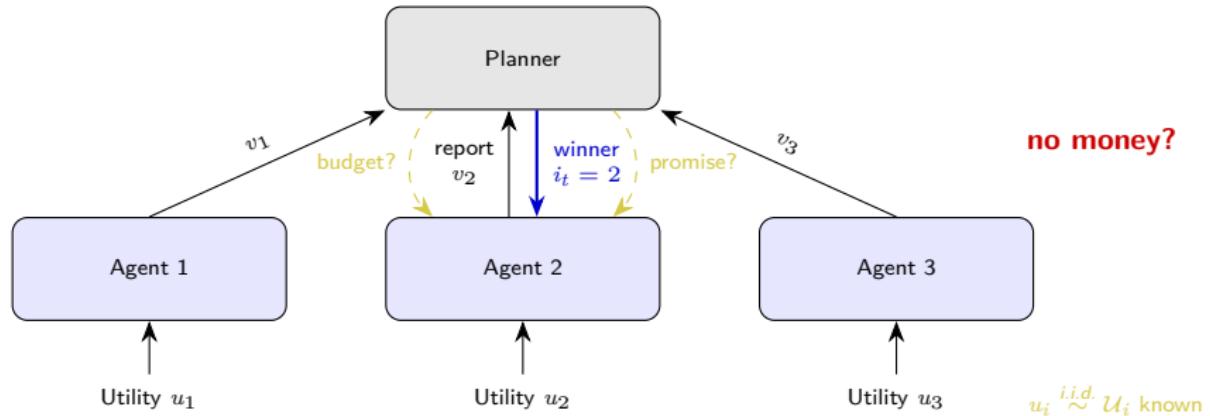
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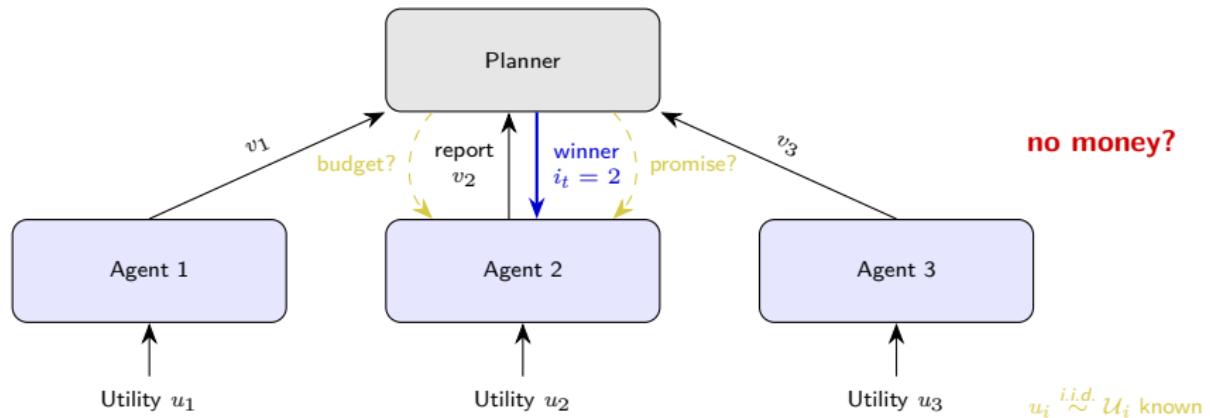


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**Distribution-Aware** approaches [BGS19; GBI21; BJ24]

# Non-Monetary Mechanisms?

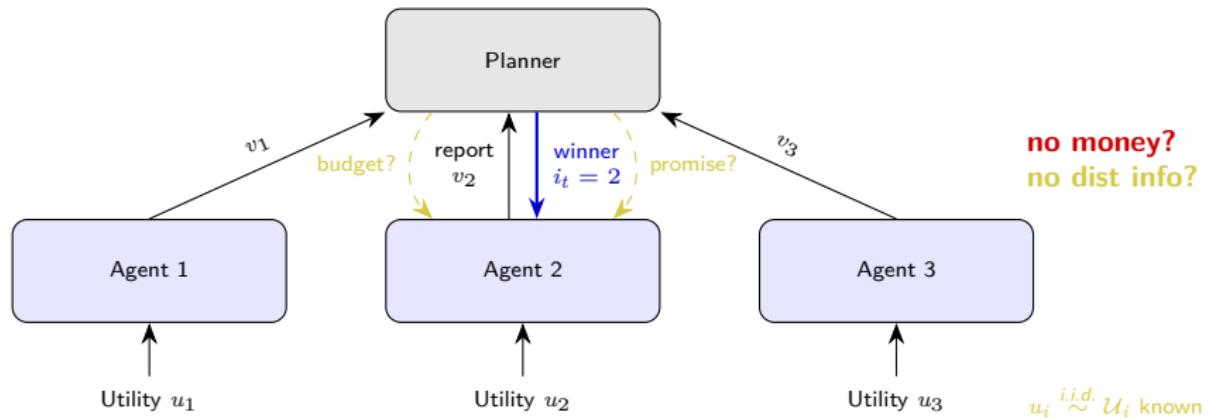


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Distributional info *a-priori* is hard!

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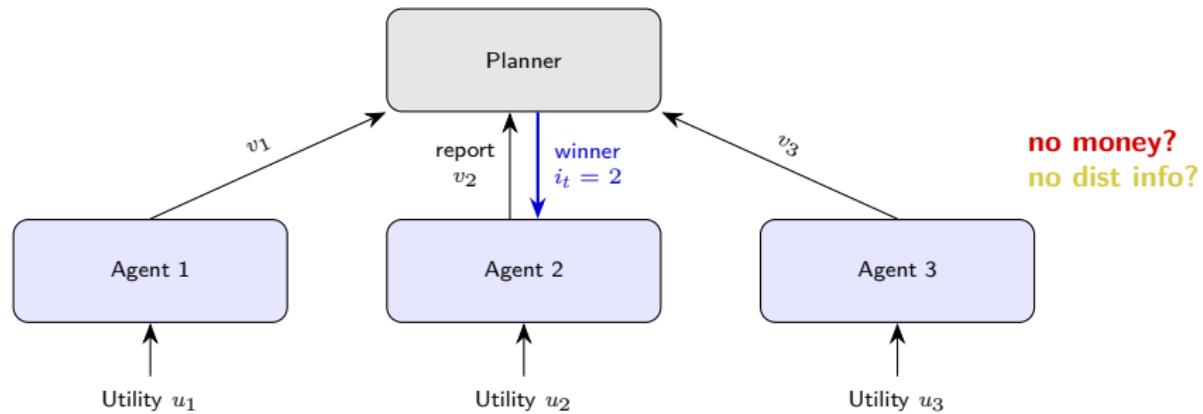


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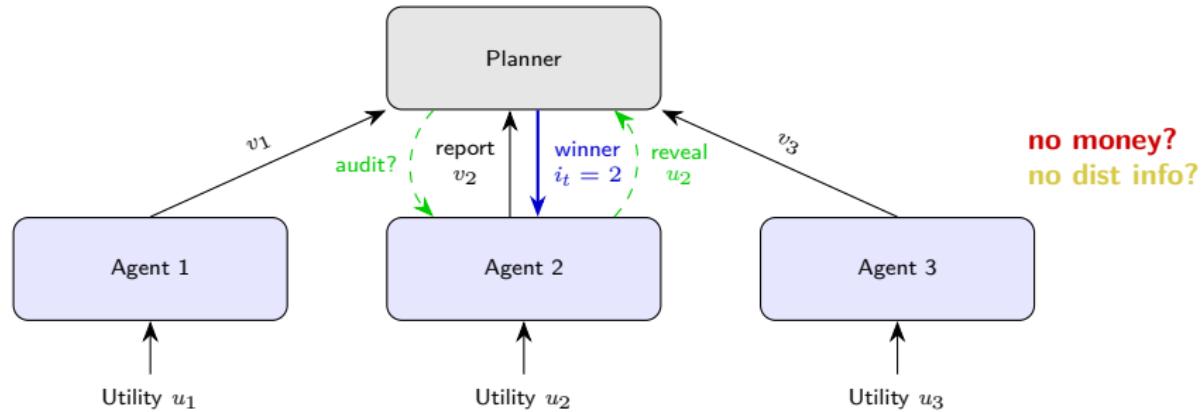
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# Our Prior-Free Mechanism

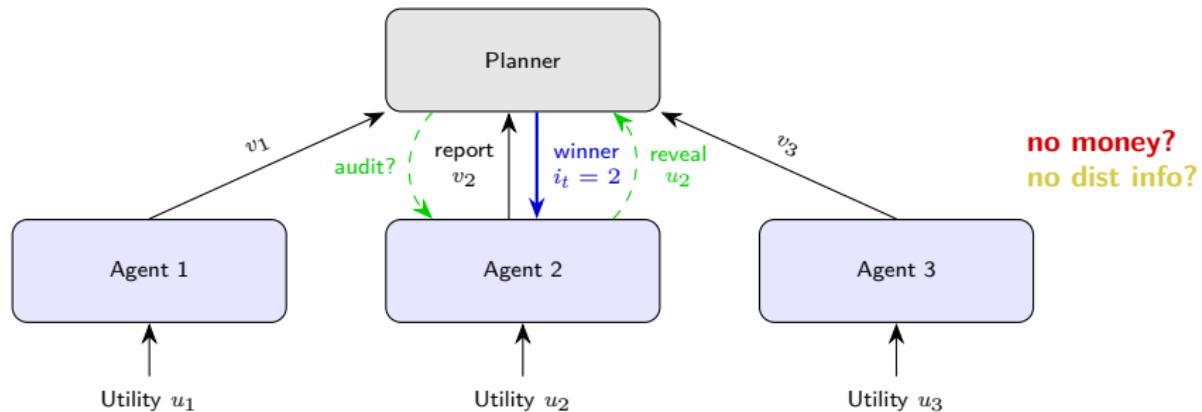


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Prior-Free mechanism via scarce & powerful “audits”

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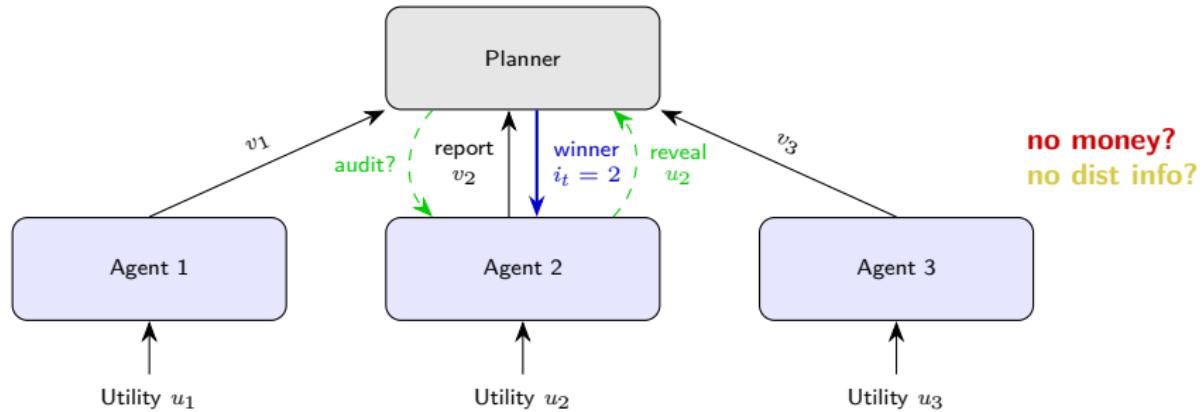
**Prior-Free mechanism** via scarce & powerful “audits”

Tradeoff between Regret & #Audits

Repeated allocation for  $T$  rounds:

① **Social Welfare Regret.**  $\mathcal{R}_T := \mathbb{E}[\sum_{t=1}^T (\max_i u_{t,i} - u_{t,i_t})]$

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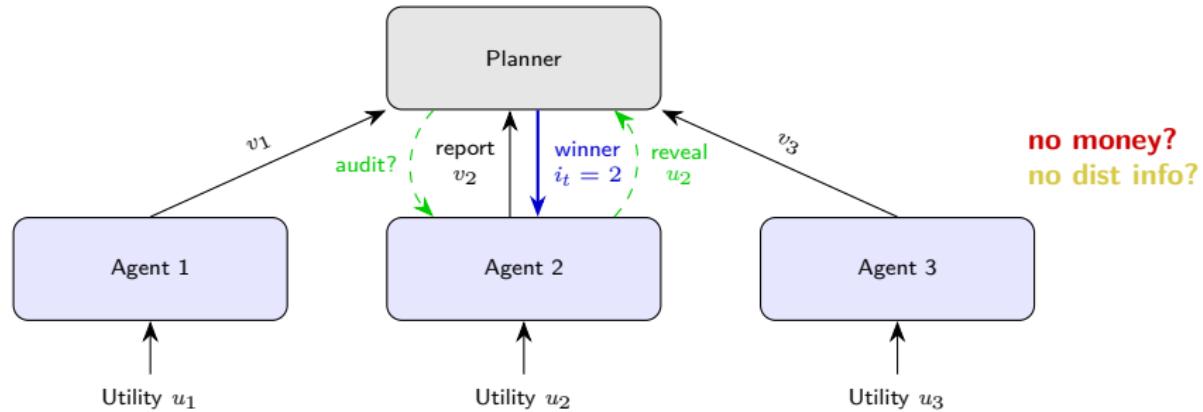
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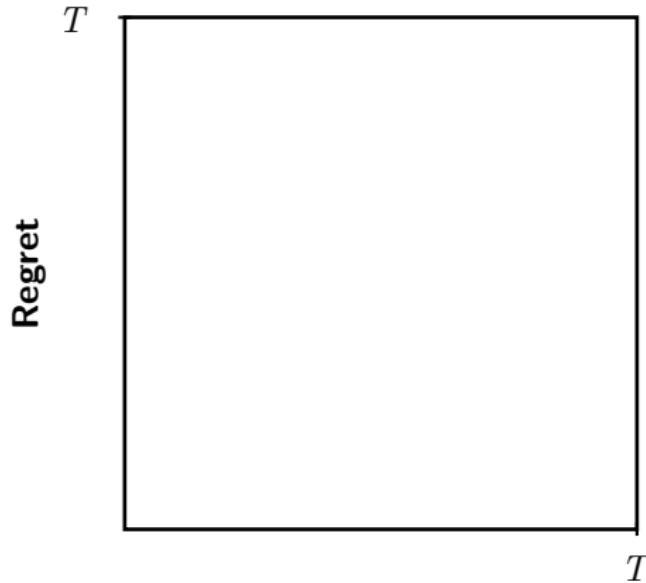
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**(Different from Online Learning regret!)**
- ② **Expected Number of Audits.**  $\mathcal{B}_T := \mathbb{E}[\sum_{t=1}^T \mathbb{1}[\text{audit}]]$

# Tradeoff between Regret & #Audits

$\exists$  Perfect Bayesian Equilibrium (PBE)  $\pi^*$ , s.t.



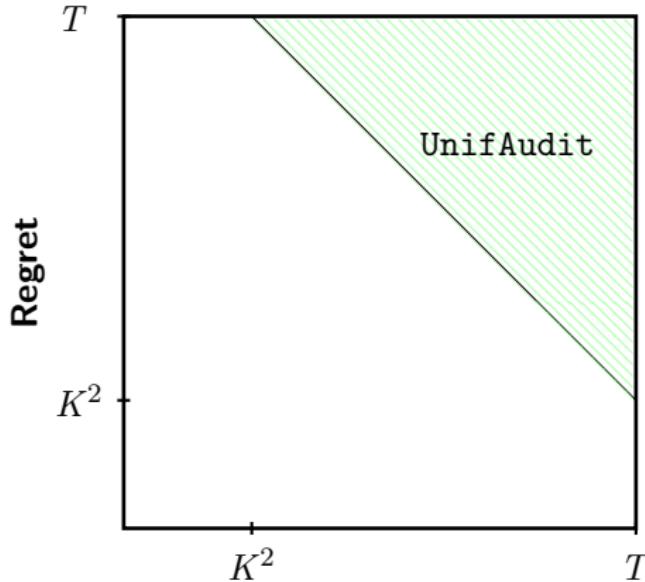
$$\mathbb{E}[\#\text{audits}]$$

Figure: Regret vs  $\mathbb{E}[\#\text{audits}]$

green possible; red impossible

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**Simple Mechanism:**  
**UnifAudit.**  
 $\mathcal{R}_T \leq \frac{K^2}{p}$  &  
 $\mathcal{B}_T \leq pT$ .

$\mathbb{E}[\#\text{audits}]$

**Figure:** Regret vs  $\mathbb{E}[\#\text{audits}]$

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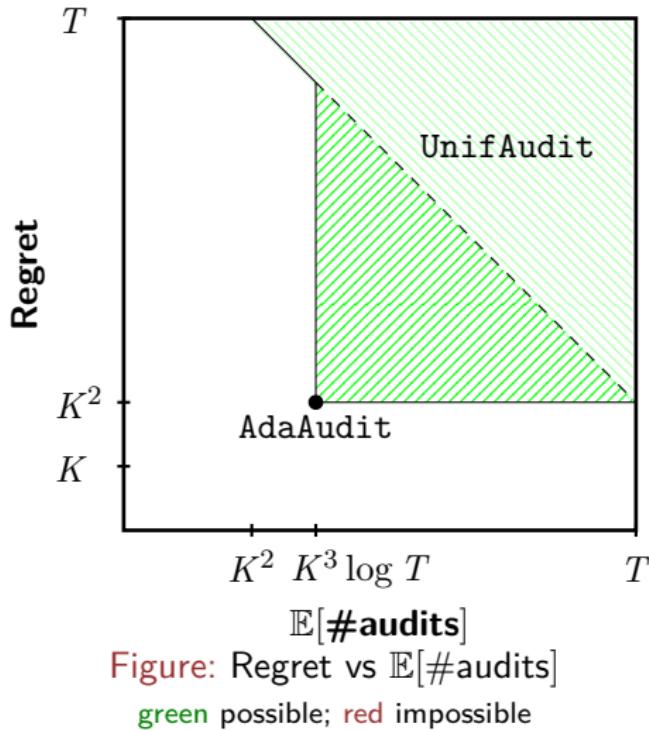


Figure: Regret vs  $E[\# \text{audits}]$

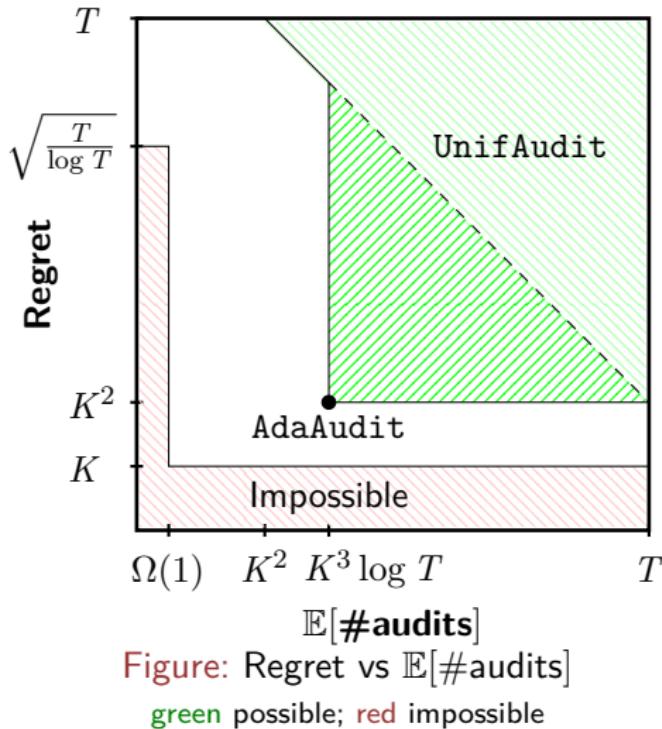
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Main  
Mechanism:  
AdaAudit.

$$\mathcal{R}_T \leq K^2 \text{ &} \\ \mathcal{B}_T = \mathcal{O}(K^3 \log T).$$

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**Impossible.**

$$\begin{aligned}\mathcal{R}_T &= \Omega(K); \\ \mathcal{B}_T &= \mathcal{O}(1) \implies \\ \mathcal{R}_T &= \Omega\left(\sqrt{\frac{T}{\log T}}\right).\end{aligned}$$

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# One-Slide Technical Overview

1. **Future Punishment.** Audit reveals  $v_{t,i} \neq u_{t,i} \Rightarrow$  never alloc again
2. **Adaptive Audits.**
3. **Learn via Flagging.**
4. **Auxiliary Games.**

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When  $i$  win in round  $t$ , audit w.p.  $p_{t,i} := 1/V_i^{\text{alive}}$   
 $(V_i^{\text{alive}} := \mathbb{E}_{\text{all agents truthful}}[\sum_{\text{future round}} \text{gain of agent } i])$   
 $\Rightarrow$  (almost) always truthful &  $\mathcal{B}_T = \tilde{\mathcal{O}}(1)$   
(truthful: get  $\geq 0 + V_i^{\text{alive}}$ ; lie: get  $\leq 1 + (1 - p_{t,i}) V_i^{\text{alive}}$ )
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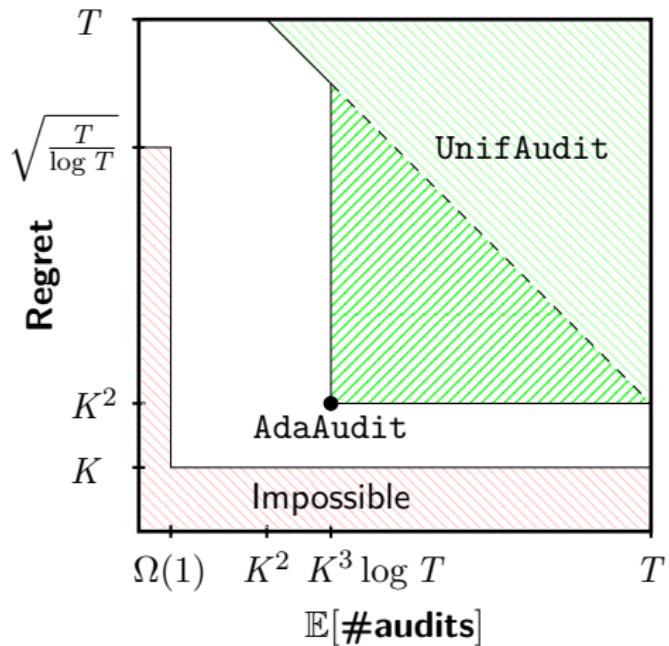
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2. **Adaptive Audits.** Need to estimate  $\mathbb{E}_{\text{all agents truthful}}[\text{gain of agent } i]$  empirically but can't "condition on" concentration  
(since agents can strategize early; happy to explain offline)  
Idea: Let agents "flag" others for biased estimates  
("victims" benefit from truthfully flagging  $\Rightarrow$  incentives aligned)
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Idea: Let agents "flag" others for biased estimates  
("victims" benefit from truthfully flagging  $\Rightarrow$  incentives aligned)
3. **Learn via Flagging.** Can't use revelation principle due to unknown / non-unique distributions (happy to explain offline)  
How to characterize PBE? Define a "well-behaved" aux game, show aux PBE  $\xrightarrow{\text{induce}}$  actual PBE
4. **Auxiliary Games.**

# Main Results & Takeaway

For resource allocation without money & without dist info...



## Technical Ingredients

- **Future Punishment**
- **Adaptive Audits** ( $\mathcal{O}(1)$  regret via  $\tilde{\mathcal{O}}(1)$  audits)
- **Learn via Flagging**  
("condition on" argument is problematic when strategic)
- **Auxiliary Games**  
(revelation principle is inapplicable w/o dist info)

Thank you!

Paper link: <https://arxiv.org/abs/2502.08412>

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