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NATIONAL RESEARCH UNIVERSITY

SCHOOL OF DATA ANALYSIS

Skoltech

Skolkovo Institute of Science and Technology

Convolutional Neural Networks

Convolutions, pooling and CNNs. Neural architectures for computer vision.

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Lecture overview

- Digital images and processing by neural networks
- Image processing operations: convolutions and pooling
- Convolutional neural networks from scratch
- Modern computer vision architectures: AlexNet, VGG, Inception and ResNets

Images as inputs to neural networks

Digital representation of an image

- Grayscale image is a matrix of pixels (picture elements)
- Dimensions of this matrix are called image resolution (e.g. 300 x 300)
- Each pixel stores its brightness (or intensity) ranging from 0 to 255, 0 intensity corresponds to black color:

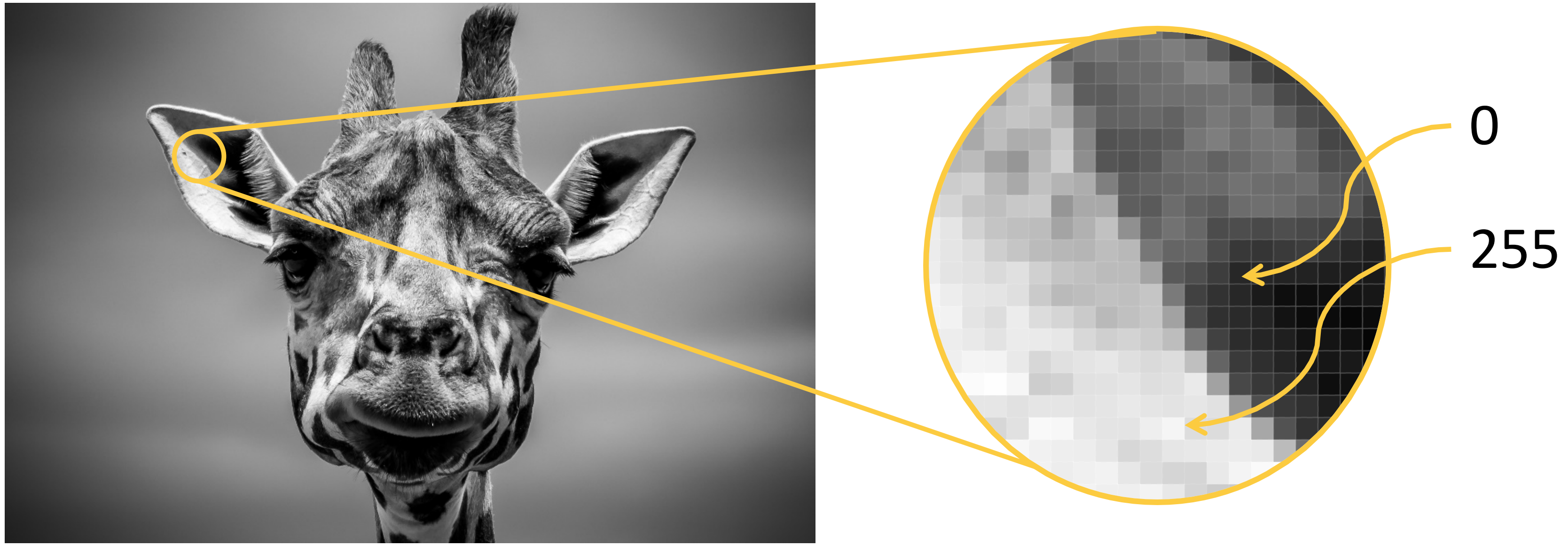

Image as a neural network input

- Normalize input pixels: $x_{norm} = \frac{x}{255} - 0.5$

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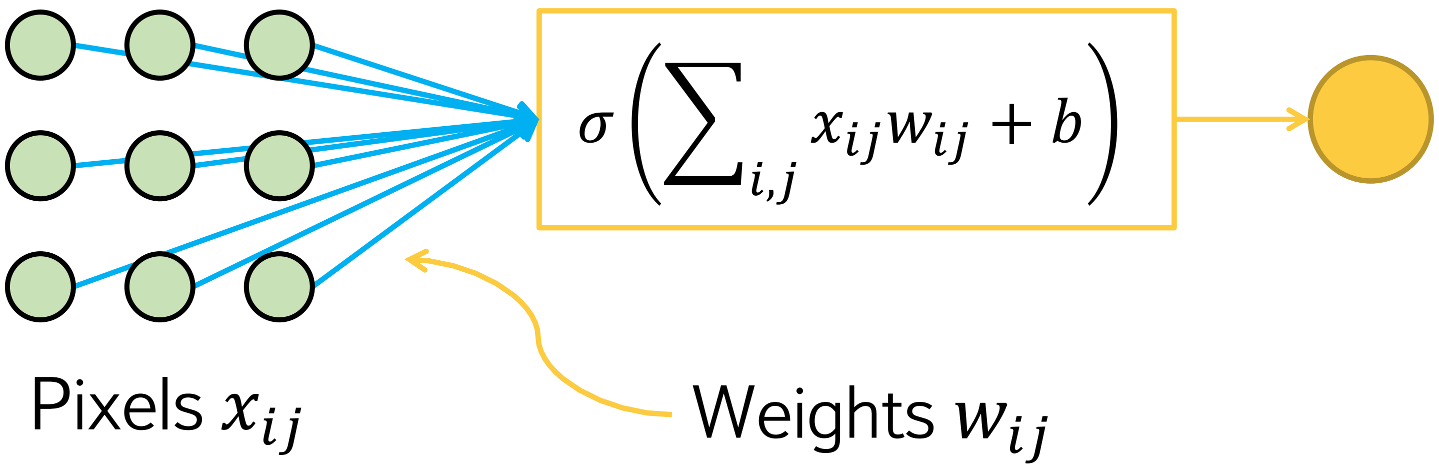
- Maybe MLP will work?



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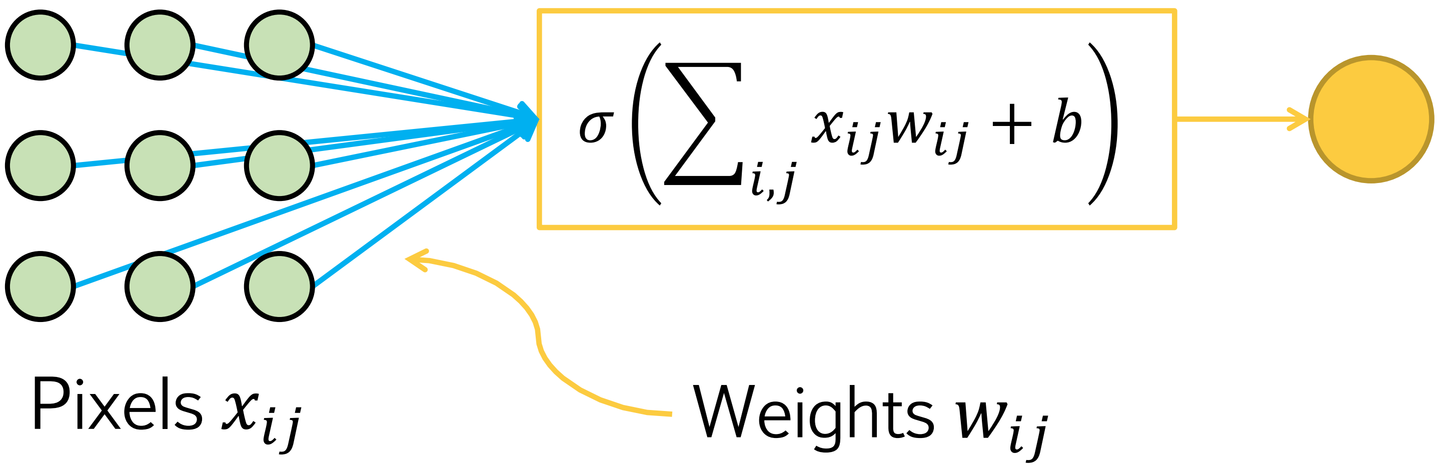
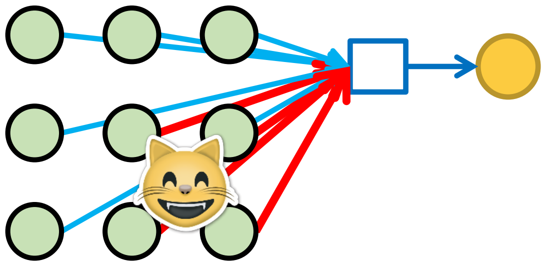


- Actually, no!

Why not MLP?

- Let's say we want to train a "cat detector"



On this training image red weights w_{ij} will change a little bit to better detect a cat

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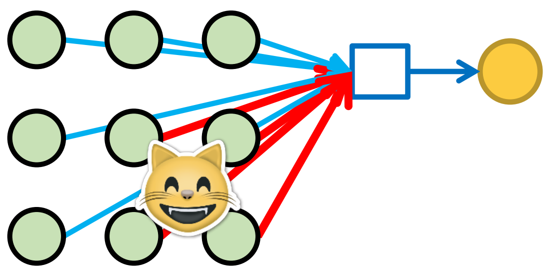

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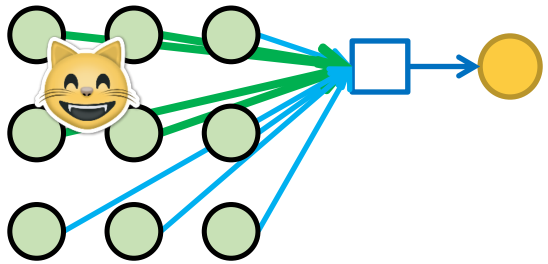

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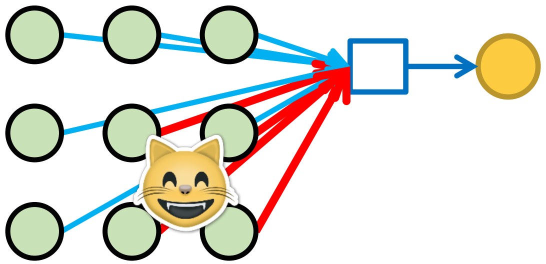

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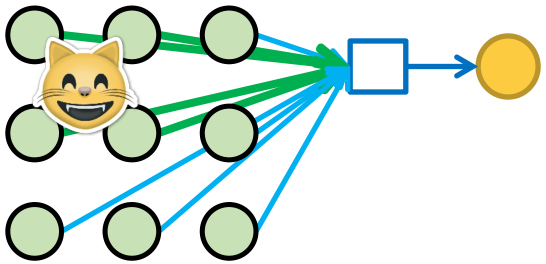
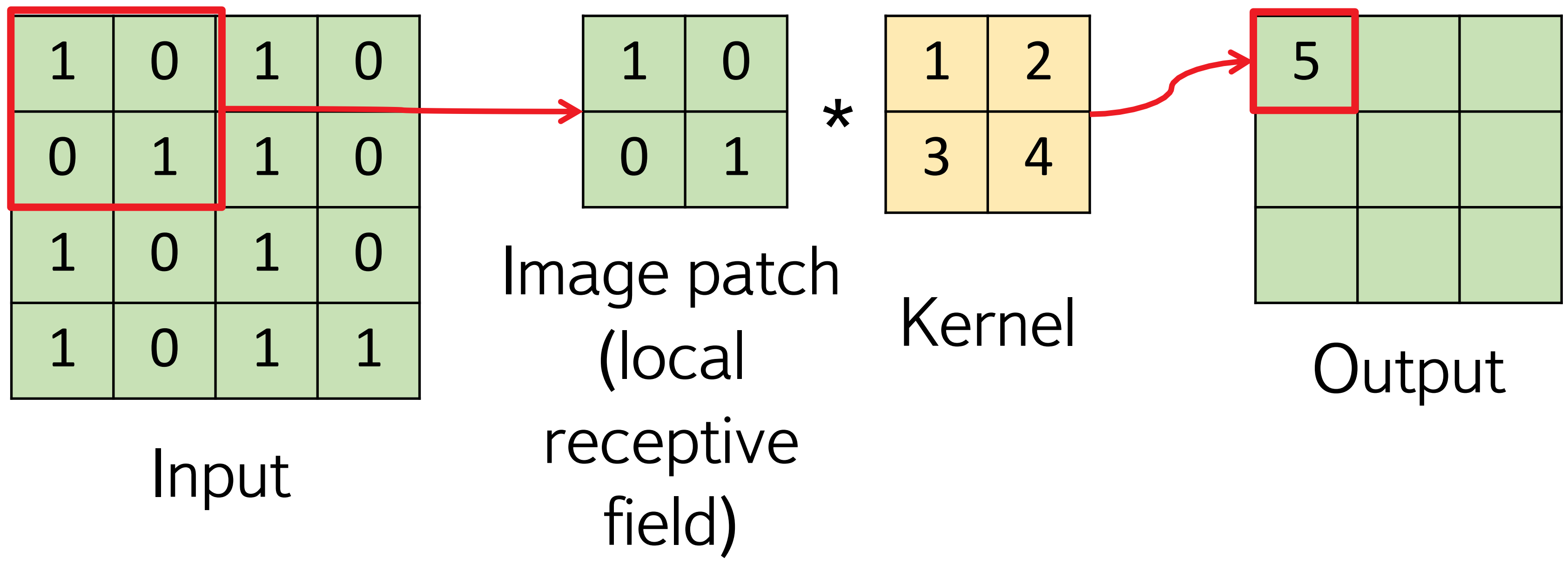


On this training image green *weights* w_{ij} will change…

- We learn the same "cat features" in different areas and don't fully utilize the training set!
- What if cats in the test set appear in different places?

Convolutions will help!

- Convolution is a dot product of a kernel (or filter) and a patch of an image (local receptive field) of the same size



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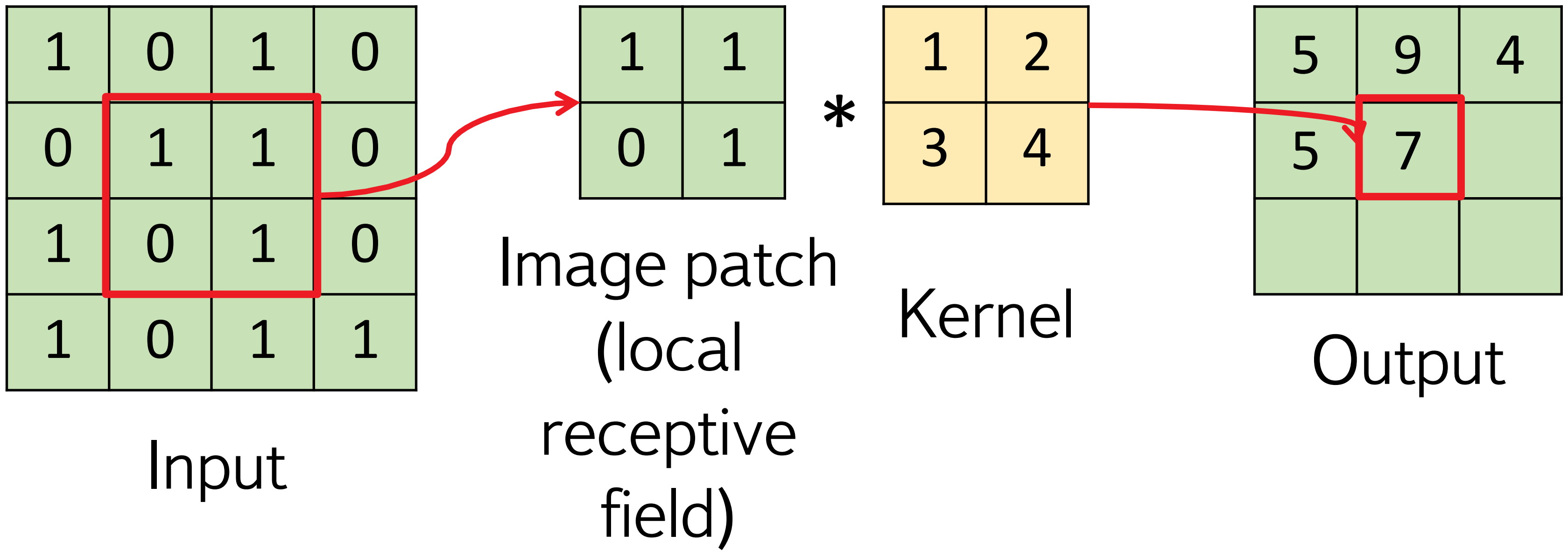


Convolutions have been used for a while

Kernel

Original image

*

-1	-1	-1
-1	8	-1
-1	-1	-1

=

Edge detection

Sums up to 0 (black color) when the patch is a solid fill

Convolutions have been used for a while

Original image

Kernel

$*$

-1	-1	-1
-1	8	-1
-1	-1	-1

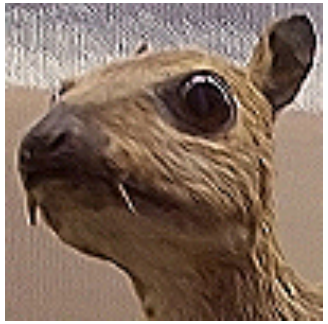
$=$

Edge detection

$*$

0	-1	0
-1	5	-1
0	-1	0

$=$



Sharpening

Doesn’t change an image for solid fills

Adds a little intensity on the edges

Convolutions have been used for a while

Kernel

Original image

$*$

-1	-1	-1
-1	8	-1
-1	-1	-1

$=$

Edge detection

$*$

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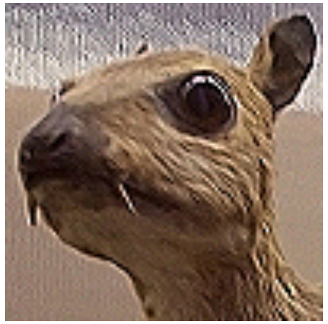

Sharpening

$* \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$=$

Blurring

Convolution is similar to correlation

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Input}} * \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{Kernel}} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{\text{Output}}$$

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$$\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} * \begin{bmatrix}1&0\\0&1\end{bmatrix} = \begin{bmatrix}0&0&0\\0&1&0\\0&0&2\end{bmatrix}$$

Input Kernel Output

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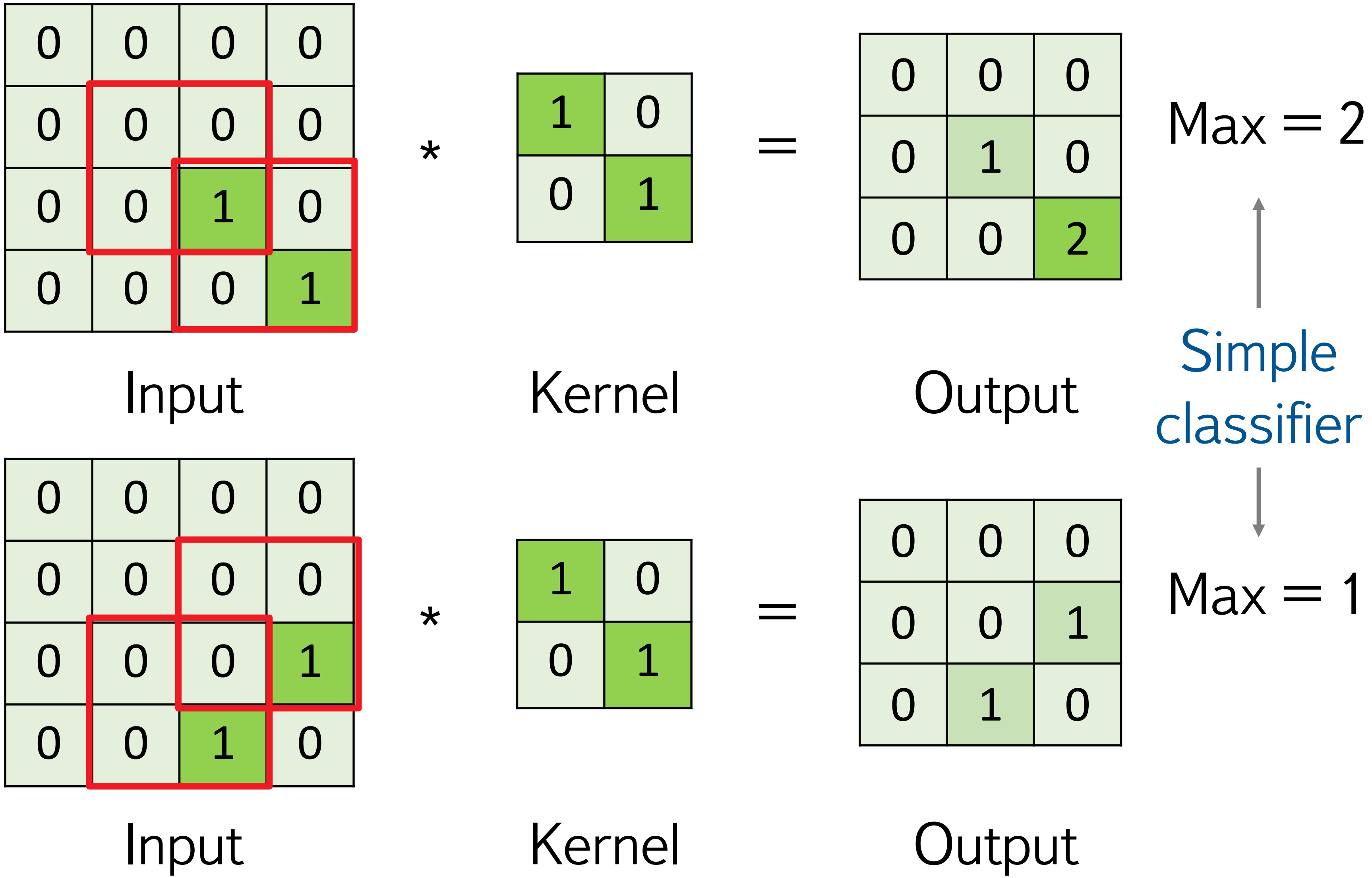

Convolution is translation equivariant

$$\underset{\text{Input}}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} * \underset{\text{Kernel}}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \underset{\text{Output}}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}$$

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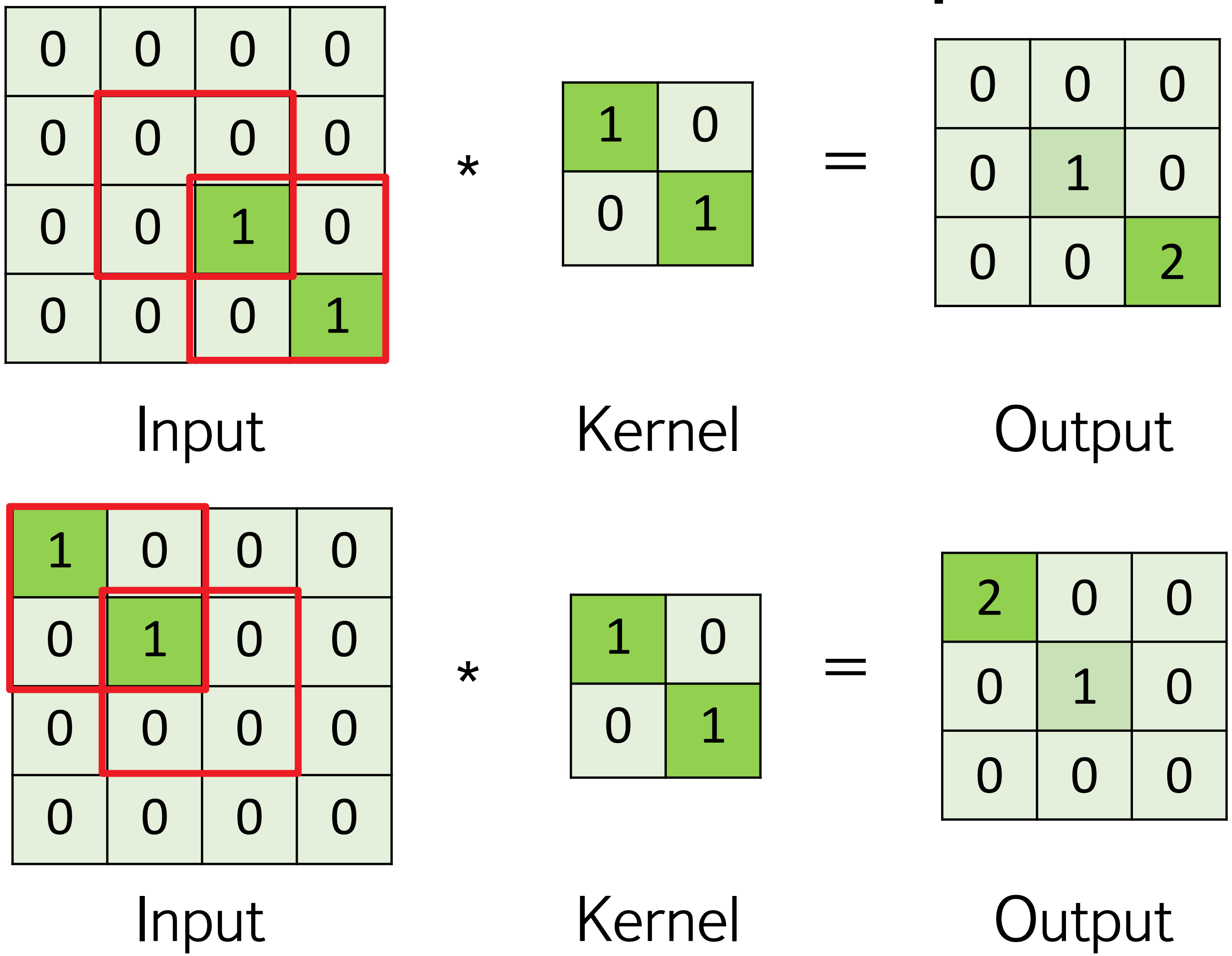

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Didn't change

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{Input}} * \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{Kernel}} = \underbrace{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{Output}} \qquad \text{Max} = 2$$

Convolutional layer in neural network

Shared bias:

b

Shared kernel:

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

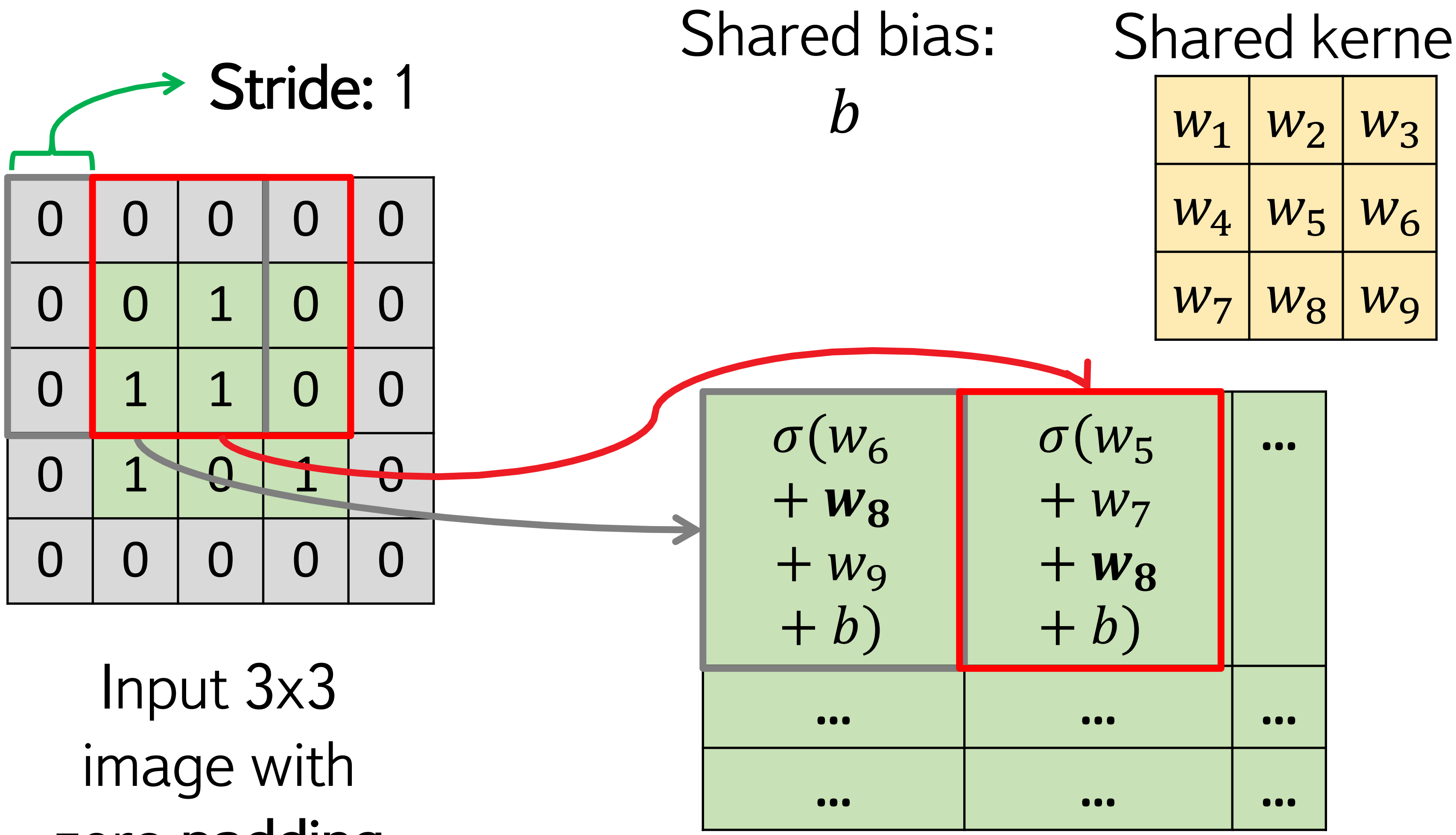
0	0	0	0	0
0	0	1	0	0
0	1	1	0	0
0	1	0	1	0
0	0	0	0	0

Input 3x3 image with zero **padding** (grey area)

$\sigma(w_6 + \boldsymbol{w_8} + w_9 + b)$	...	...
...	...	...
...	...	...

9 output neurons (**feature map**) with only 10 parameters

Convolutional layer in neural network



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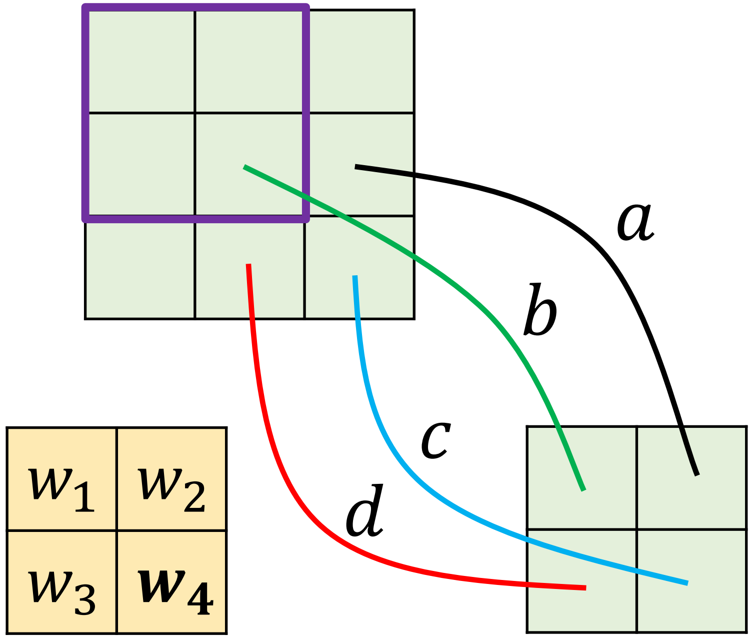
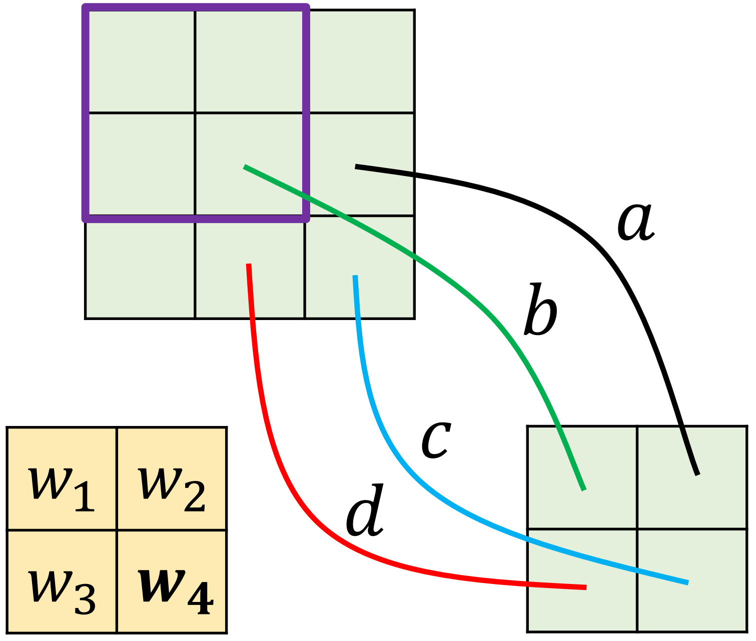
Backpropagation for CNN



Gradients are first calculated as if the kernel weights were not shared:

Backpropagation for CNN



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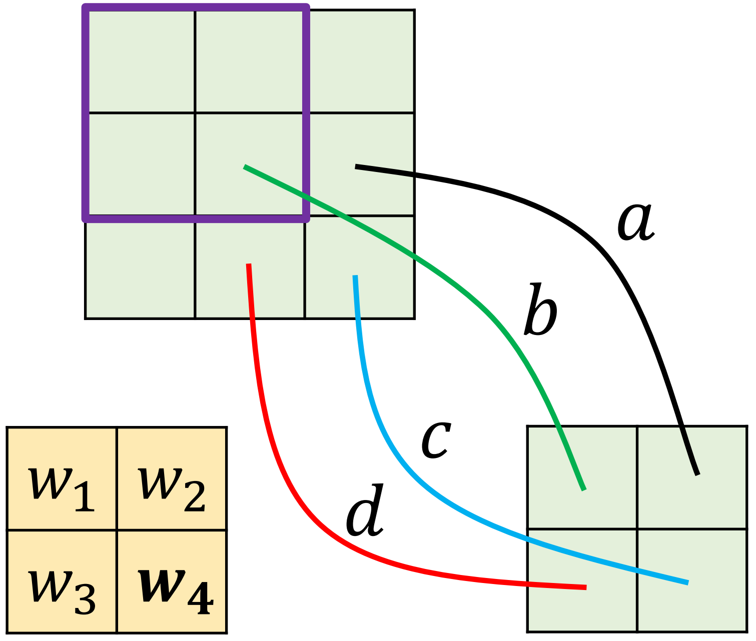
$$a = a - \gamma \frac{\partial L}{\partial a} \qquad b = b - \gamma \frac{\partial L}{\partial b}$$

$$c = c - \gamma \frac{\partial L}{\partial c} \qquad d = d - \gamma \frac{\partial L}{\partial d}$$

Backpropagation for CNN



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$$w_4 = w_4 - \gamma \left(\frac{\partial L}{\partial a} + \frac{\partial L}{\partial b} + \frac{\partial L}{\partial c} + \frac{\partial L}{\partial d} \right)$$

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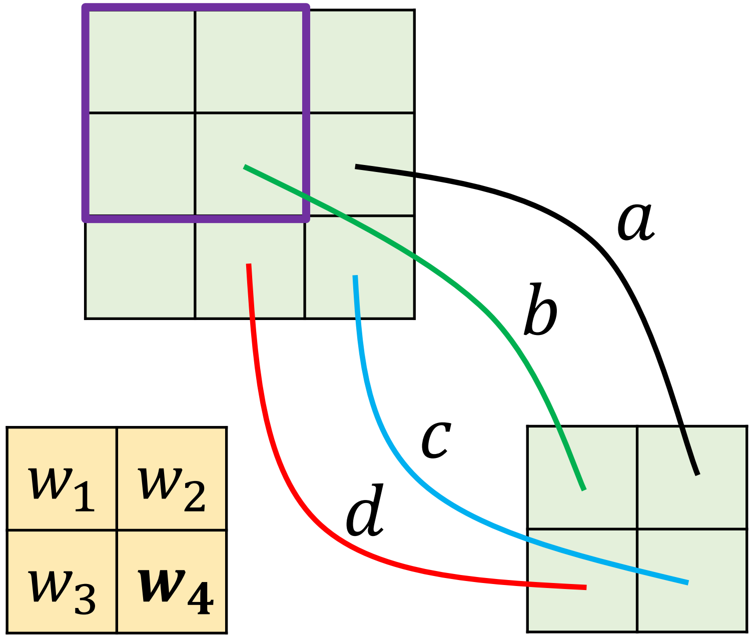

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$$w_4 = w_4 - \gamma \left(\frac{\partial L}{\partial a} + \frac{\partial L}{\partial b} + \frac{\partial L}{\partial c} + \frac{\partial L}{\partial d} \right)$$

Gradients of the same shared weight are summed up!

Convolutional vs fully connected layer

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Convolutional vs fully connected layer

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- 300x300 input, 300x300 output, 5x5 kernel — 26 parameters in convolutional layer and 8.1×10^9 parameters in fully connected layer (each output is a perceptron);
- Convolutional layer can be viewed as a special case of a fully connected layer when all the weights outside the local receptive field of each neuron equal 0 and kernel parameters are shared between neurons.

Intermediate summary

- We’ve introduced a convolutional layer which works better than fully connected layer for images: it has fewer parameters and acts the same for every patch of input.
- This layer will be used as a building block for larger neural networks!

Building convolutional neural networks for vision

A color image input

- Let's say we have a color image as an input, which is $W\times H\times C_{in}$ **tensor** (multidimensional array), where
  - W – is an image width,
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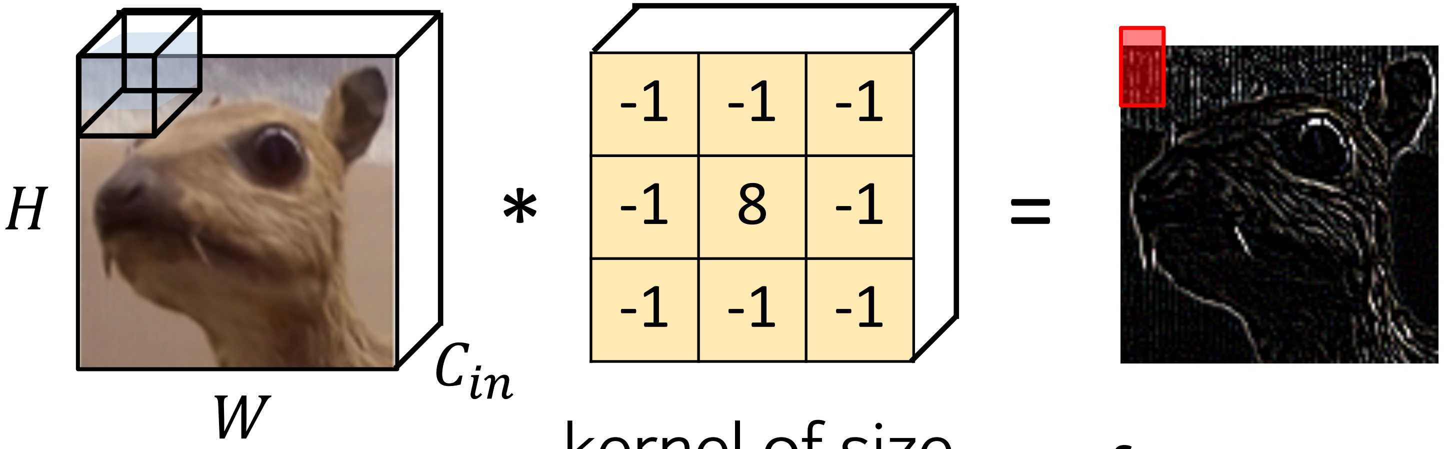

kernel of size $W_k \times H_k \times C_{in}$

feature map

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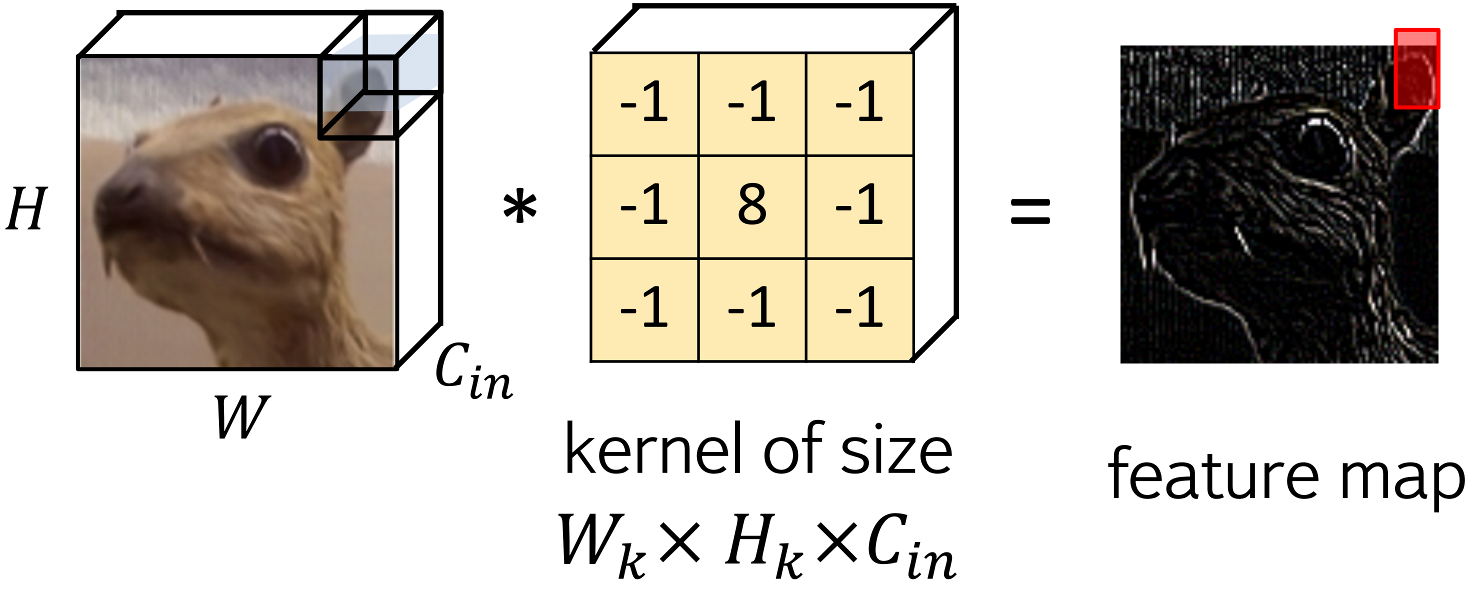
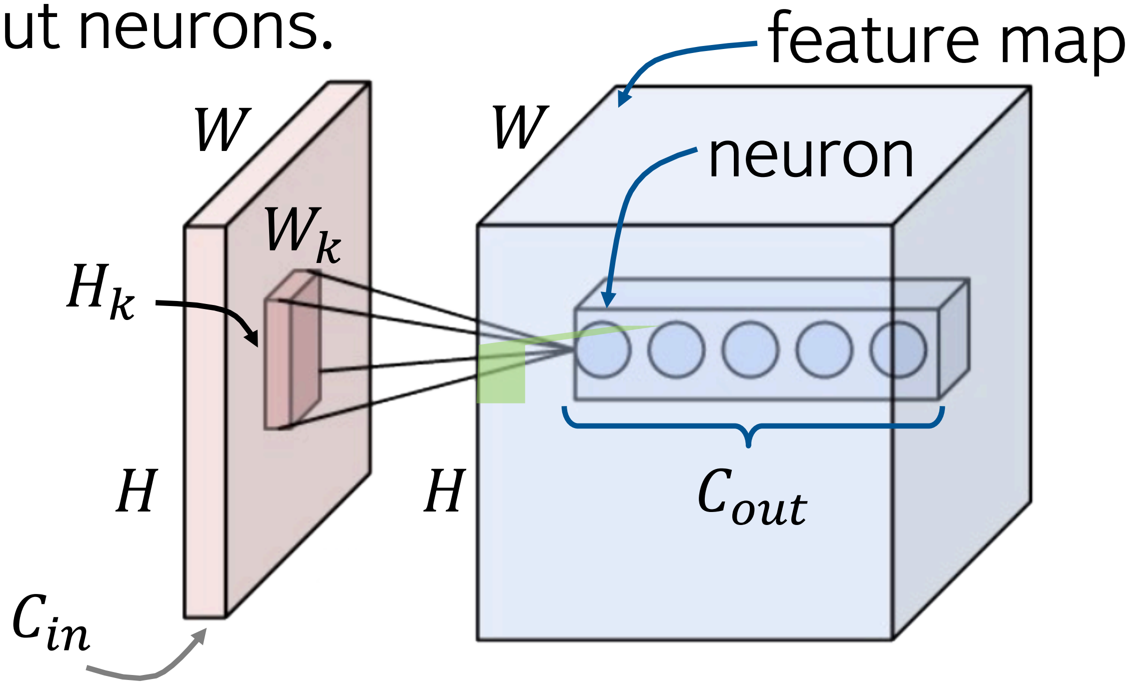


One kernel is not enough!

- We want to train C_{out} kernels of size $W_k \times H_k \times C_{in}$.
- Having a stride of 1 and enough zero padding we can have W$\times$ $H \times C_{out}$ output neurons.



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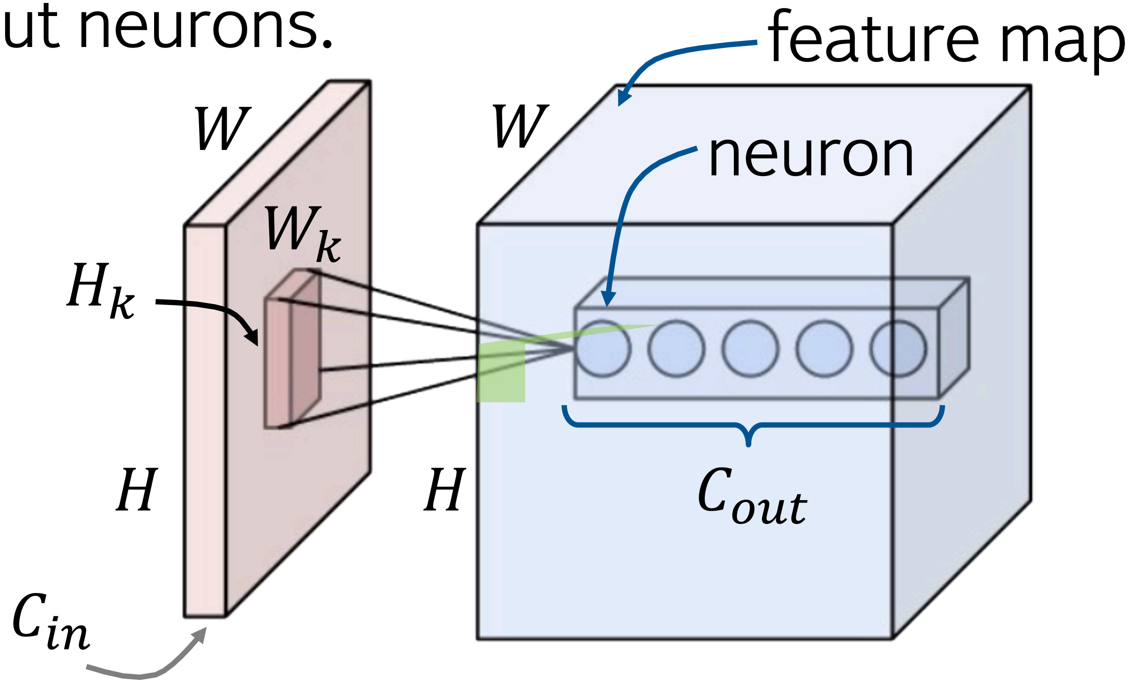

- Using $(W_k * H_k * C_{in} + 1) * C_{out}$ parameters.

One convolutional layer is not enough!

- Let's say neurons of the 1st convolutional layer look at the patches of the image of size 3x3.
- What if an object of interest is bigger than that?
- We need a 2^{nd} convolutional layer on top of the 1^{st}!

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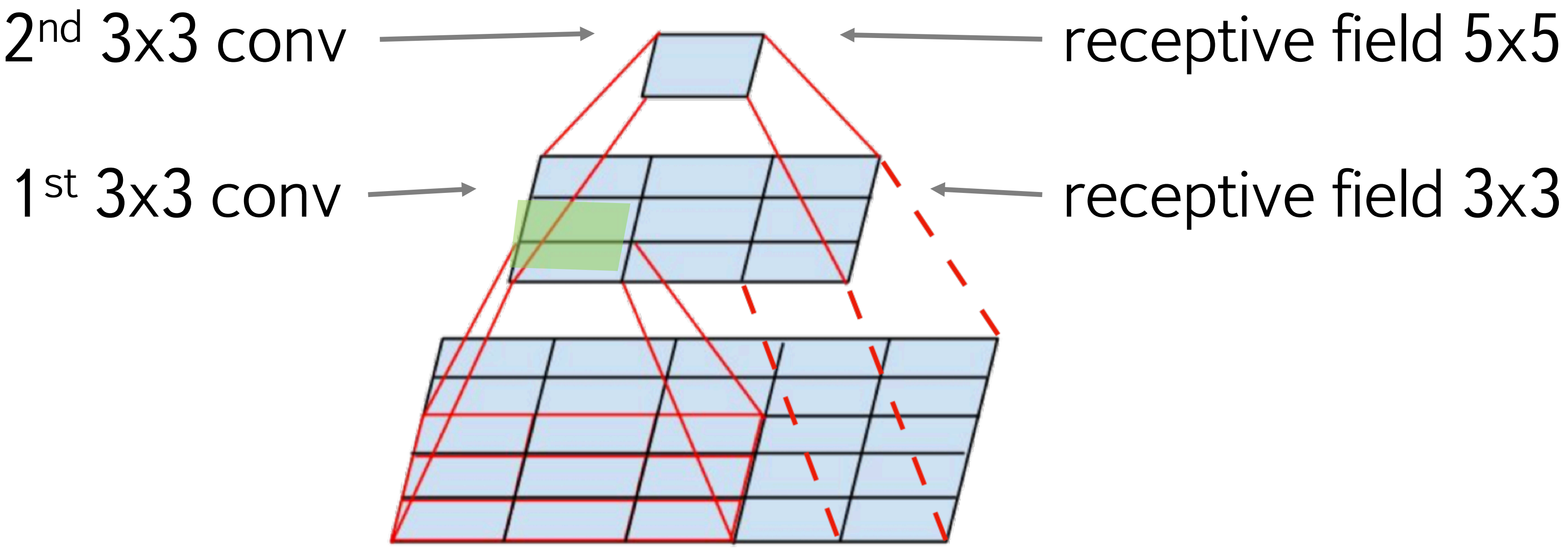
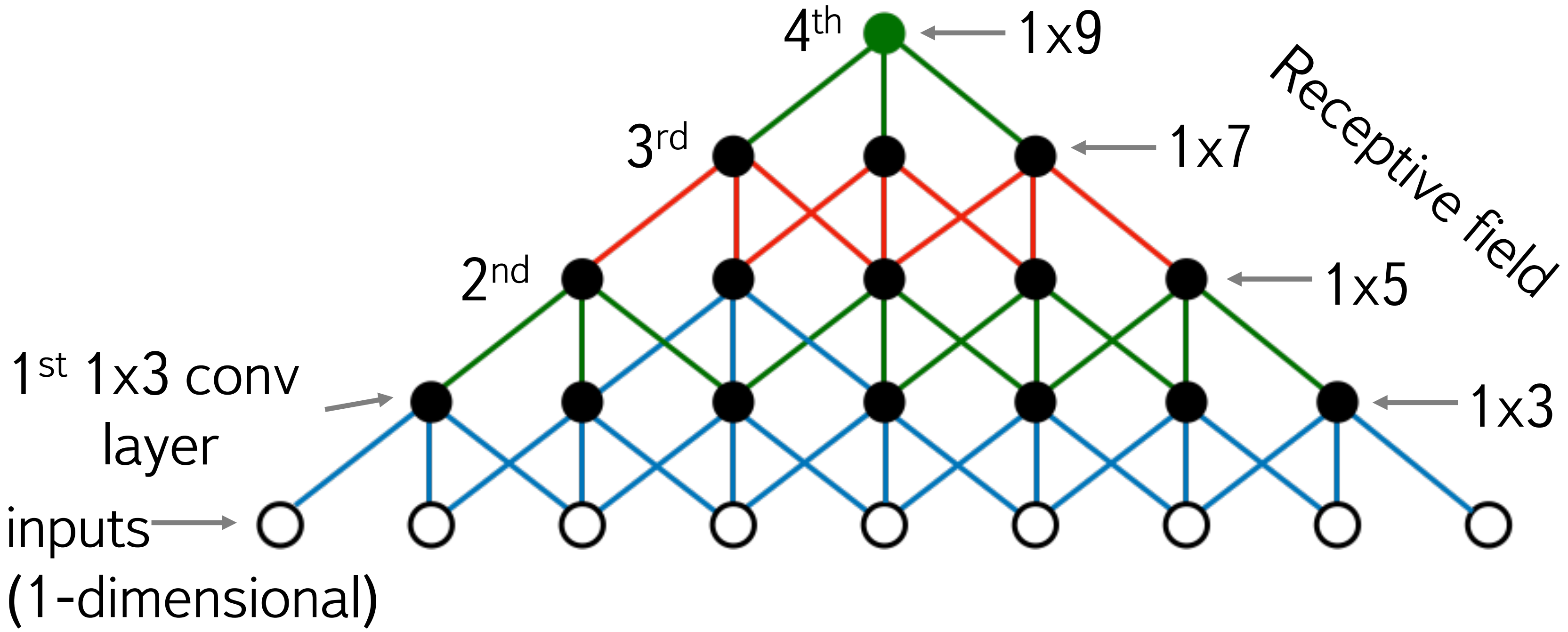


Receptive field after N convolutional layers



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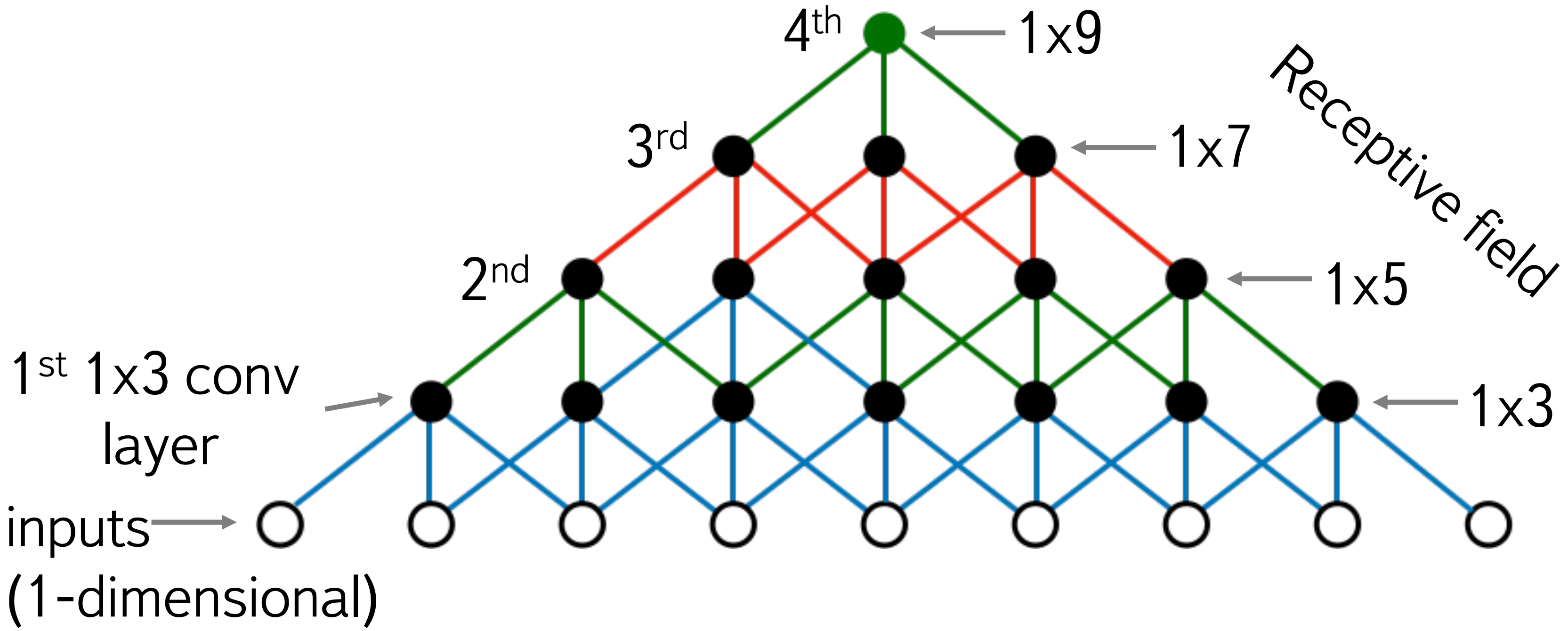
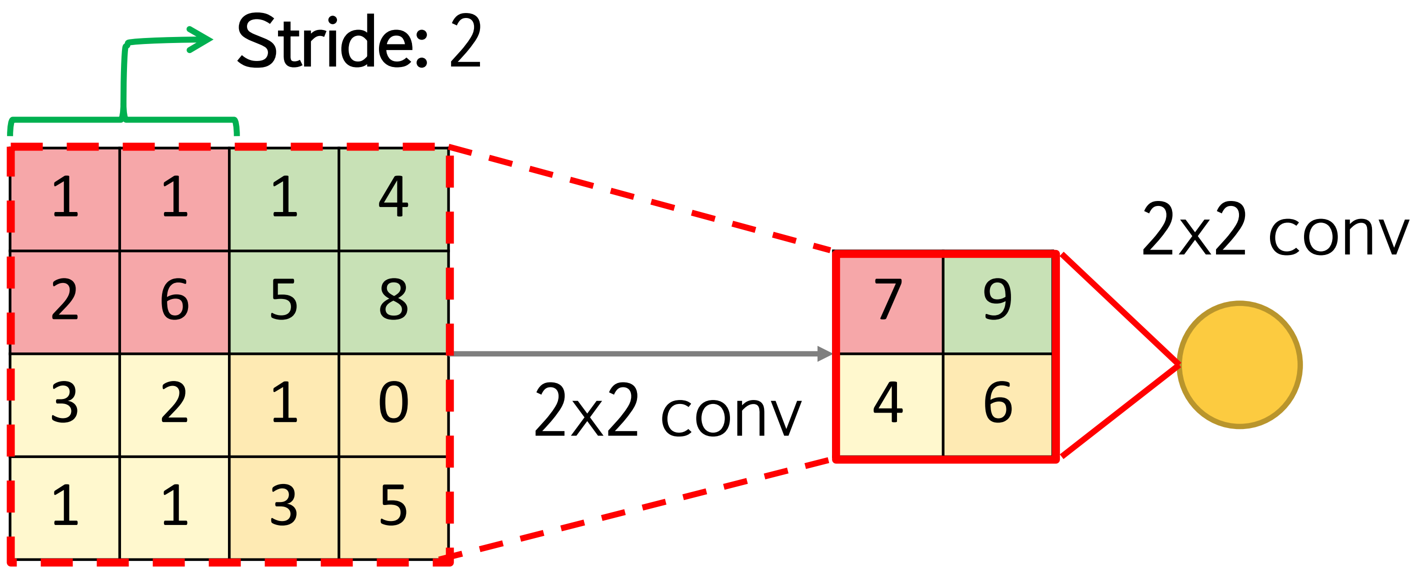


- If we stack N convolutional layers with the same kernel size 3x3 the receptive field on N-th layer will be $2N + 1 \times 2N + 1$.
- It looks like we need to stack a lot of convolutional layers!
  To be able to identify objects as big as the input image 300x300 we will need 150 convolutional layers!

We need to grow receptive field faster!

- We can increase a stride in our convolutional layer to reduce the output dimensions!



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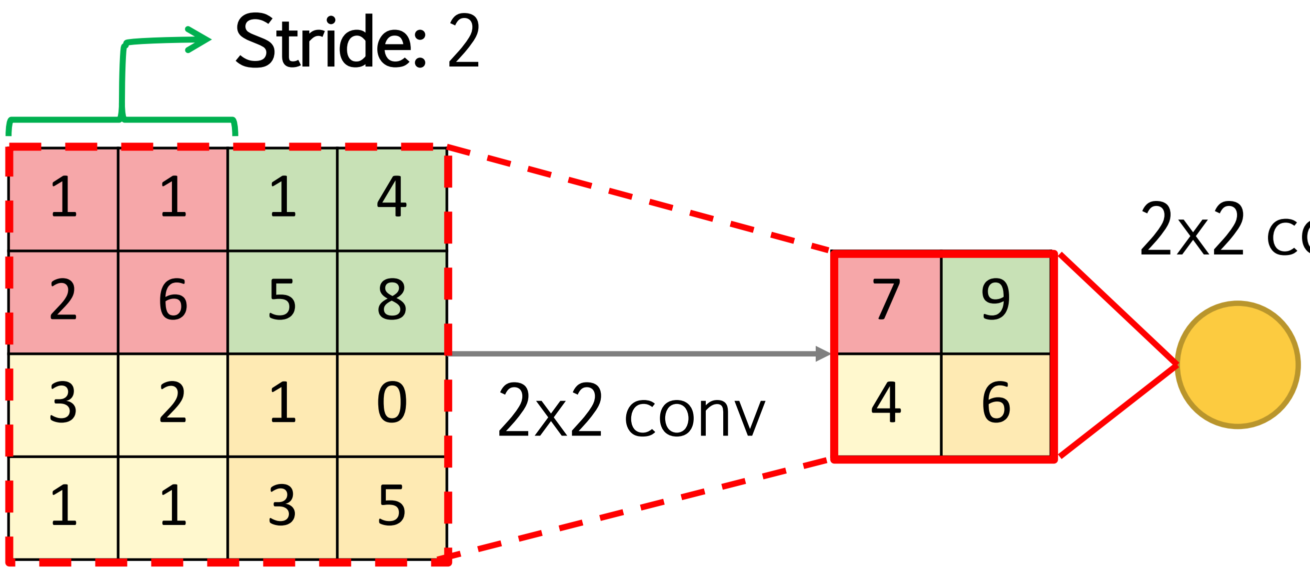


- Further convolutions will effectively double their receptive field!

How do we maintain translation invariance?

$$\underset{\text{Input}}{\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}} * \underset{\text{Kernel}}{\begin{bmatrix}1&0\\0&1\end{bmatrix}} = \underset{\text{Output}}{\begin{bmatrix}0&0&0\\0&1&0\\0&0&2\end{bmatrix}}$$

Max = 2

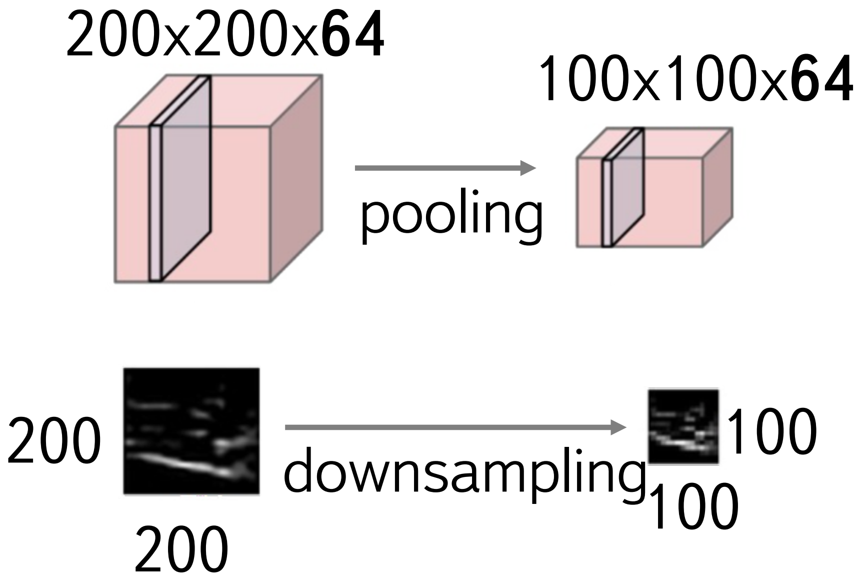
Didn't change

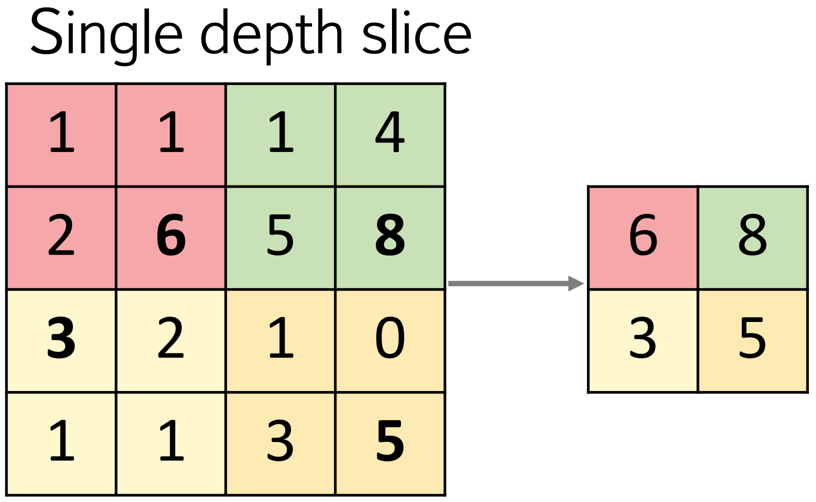
Max = 2

$$\underset{\text{Input}}{\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}} * \underset{\text{Kernel}}{\begin{bmatrix}1&0\\0&1\end{bmatrix}} = \underset{\text{Output}}{\begin{bmatrix}2&0&0\\0&1&0\\0&0&0\end{bmatrix}}$$

Pooling layer will help!

- This layer works like a convolutional layer but doesn't have kernel, instead it calculates maximum or average of input patch values.





2x2 **max pooling** with stride 2

Backpropagation for max pooling layer

- Strictly speaking: maximum is not a differentiable function!

Backpropagation for max pooling layer

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6	8
3	5

→ Maximum = 8

7	8
3	5

→ Maximum = 8

- There is no gradient with respect to non maximum patch neurons, since changing them slightly does not affect the output.

Backpropagation for max pooling layer

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6	8
3	5

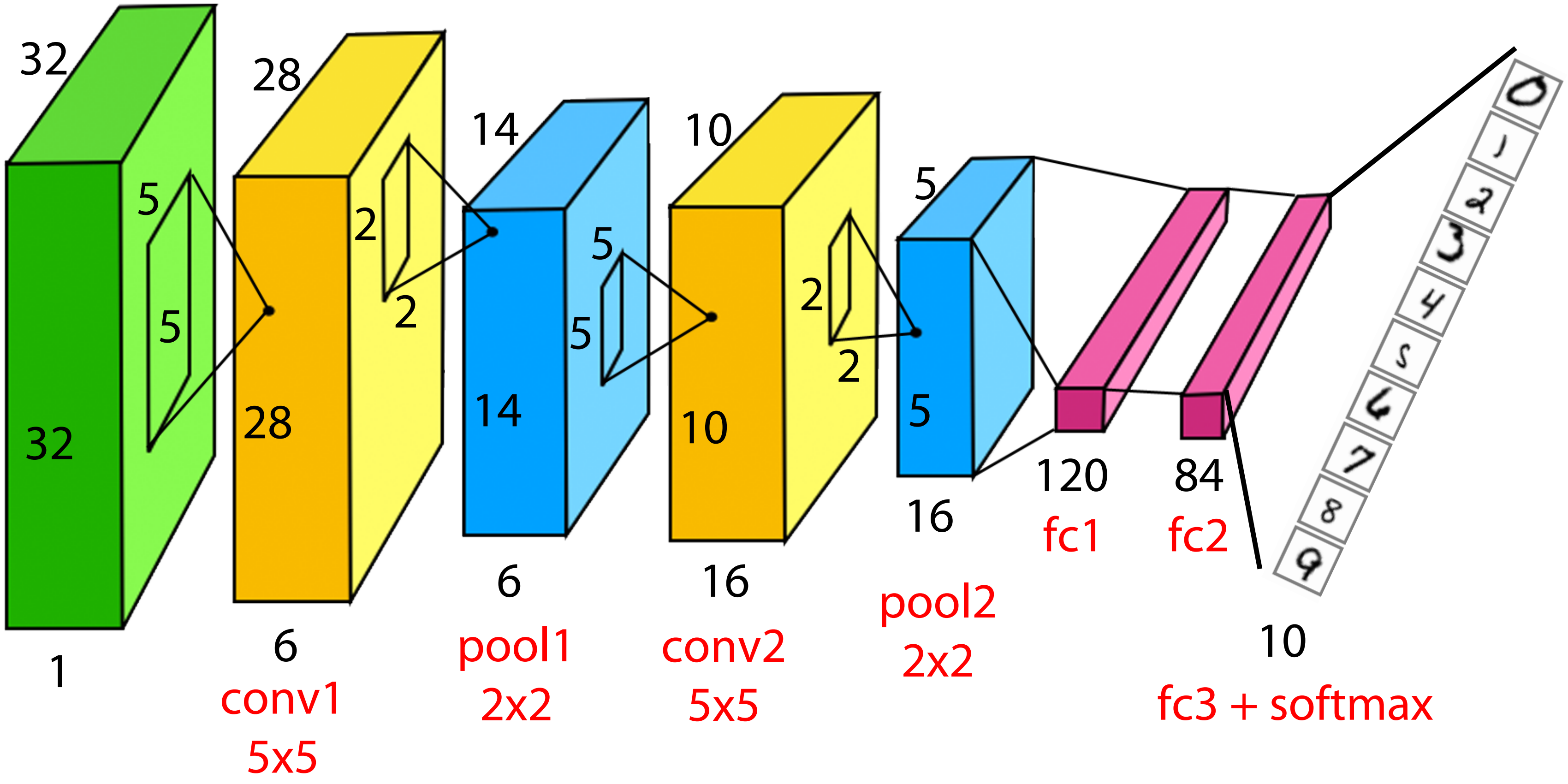
→ Maximum = 8

7	9
3	5

→ Maximum = 9

Putting it all together into a simple CNN

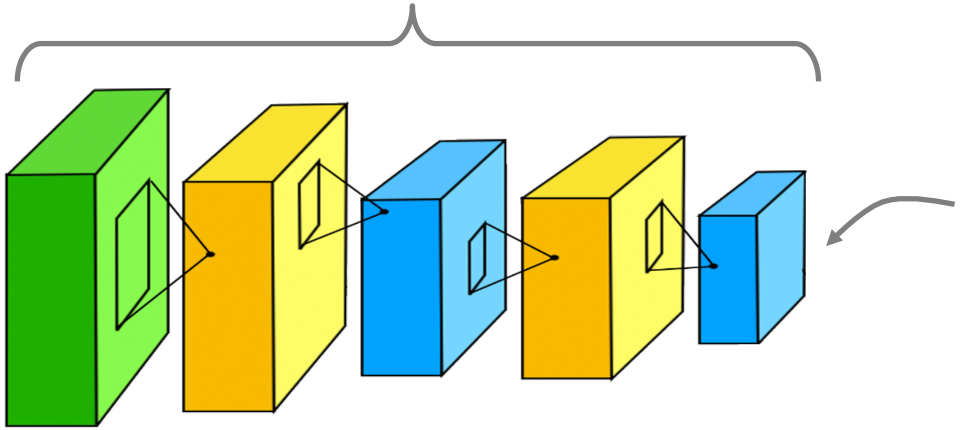
- LeNet-5 architecture (1998) for handwritten digits recognition on MNIST dataset:



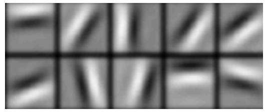
http://yann.lecun.com/exdb/publis/pdf/lecun-98.pdf

Learning deep representations

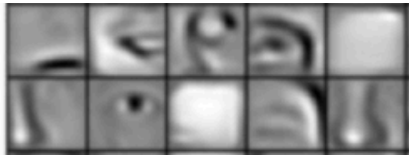
- Neurons of deep convolutional layers learn complex representations that can be used as features for classification with MLP.



Inputs that provide highest activations:



conv1



conv2

conv3

http://web.eecs.umich.edu/~honglak/icml09-ConvolutionalDeepBeliefNetworks.pdf

Summary

- Using convolutional, pooling and fully connected layers we've built our first network for handwritten digits recognition!

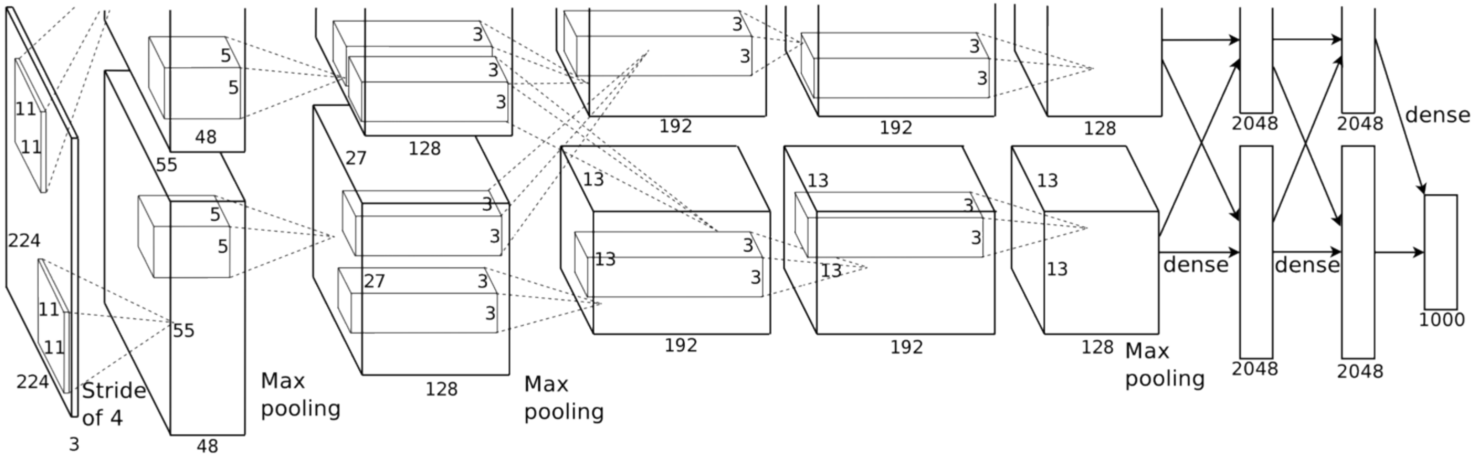
Neural architectures for computer vision

ImageNet classification dataset

Olga Russakovsky, https://arxiv.org/pdf/1409.0575.pdf

AlexNet (2012)

- First deep convolutional neural net for ImageNet
- Significantly reduced top 5 error from 26% to 15%

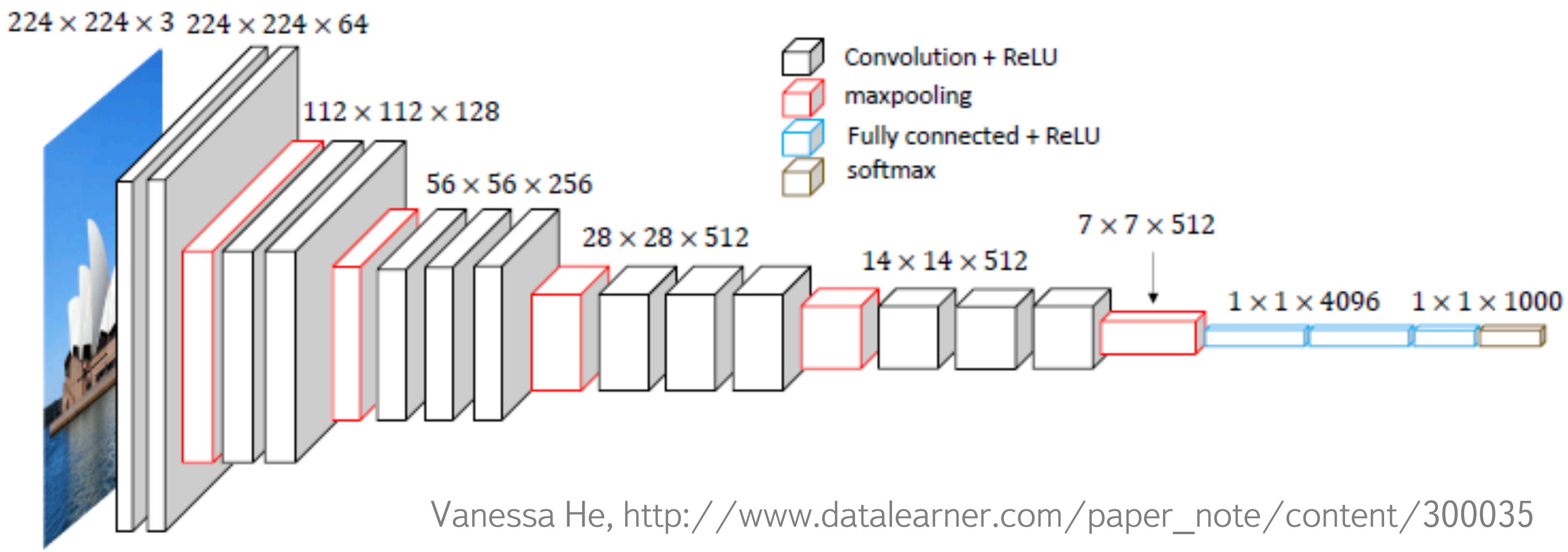


Alex Krizhevsky, https://papers.nips.cc/paper/4824-imagenet-classification-with-deep-convolutional-neural-networks.pdf

- 11x11, 5x5, 3x3 convolutions, max pooling, dropout, data augmentation, ReLU activations, SGD with momentum
- 60 million parameters
- Trains on 2 GPUs for 6 days

VGG (2015)

- Similar to AlexNet, only 3x3 convolutions, but lots of filters!
- ImageNet top 5 error: 8.0% (single model)

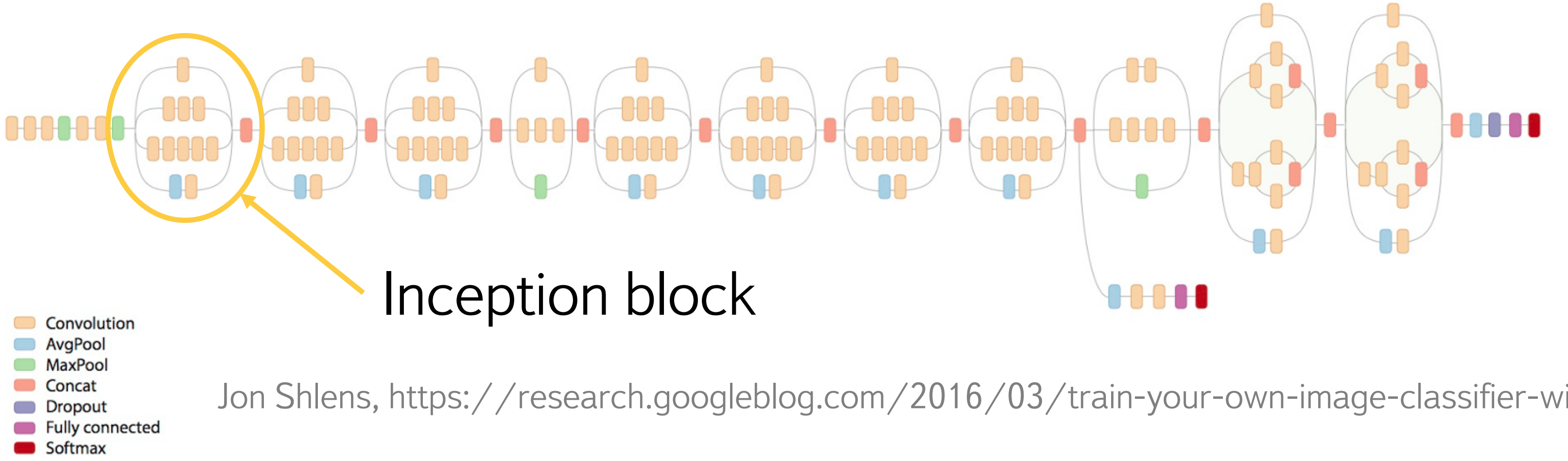


Vanessa He, http://www.datalearner.com/paper_note/content/300035

- Training similar to AlexNet with additional multi-scale cropping.
- 138 million parameters
- Trains on 4 GPUs for 2-3 weeks

Inception V3 (2015)

- Similar to AlexNet? Not quite, uses Inception block introduced in GoogLeNet (a.k.a. Inception V1)
- ImageNet top 5 error: 5.6% (single model), 3.6% (ensemble)

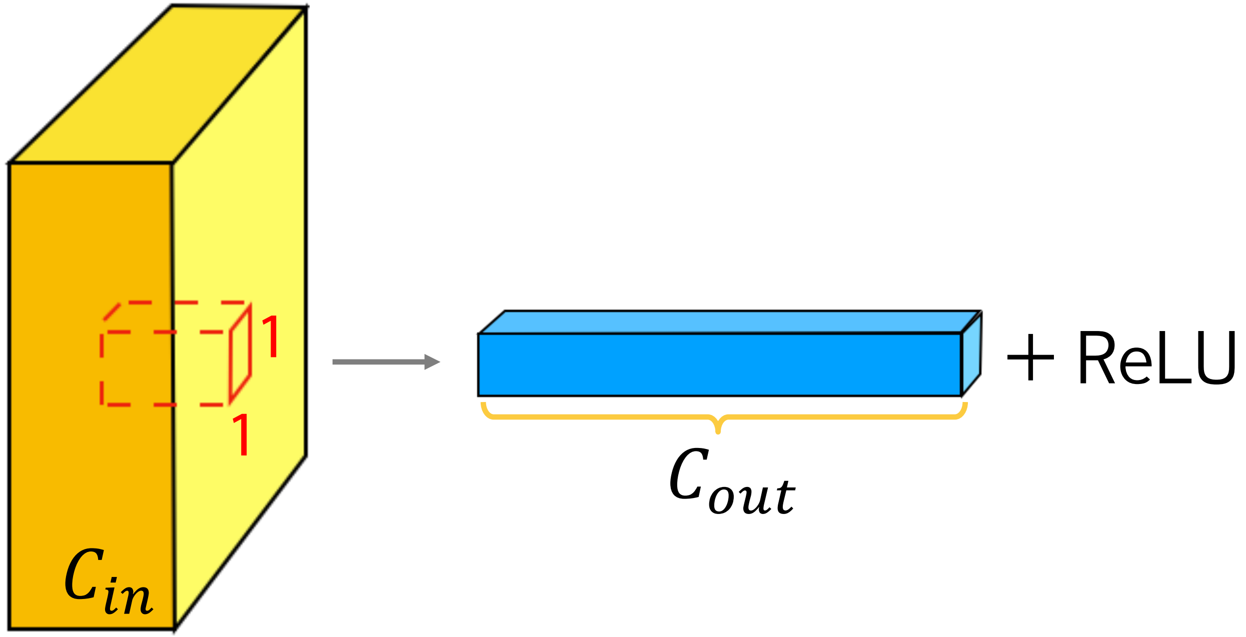


Jon Shlens, https://research.googleblog.com/2016/03/train-your-own-image-classifier-with.html

- Batch normalization, image distortions, RMSProp
- 25 million parameters!
- Trains on 8 GPUs for 2 weeks

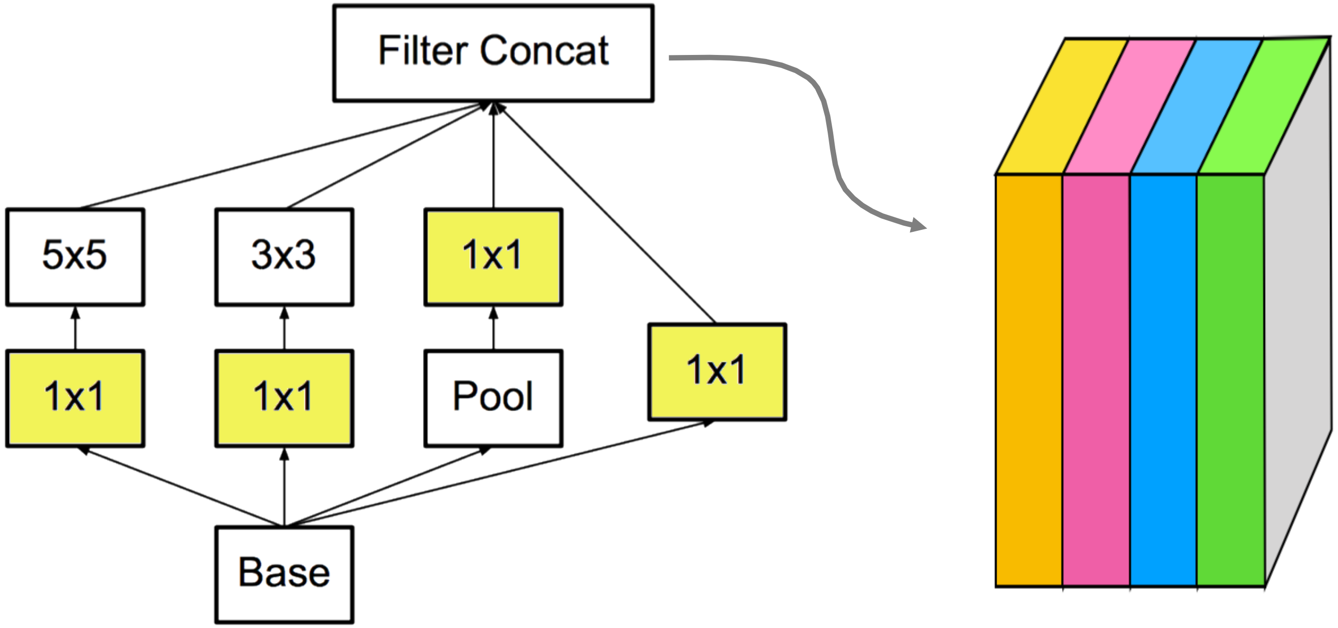
1x1 convolutions

- Such convolutions capture interactions of input channels in one "pixel" of feature map
- They can reduce the number of channels not hurting the quality of the model, because different channels can correlate
- Dimensionality reduction with added ReLU activation



Basic Inception block

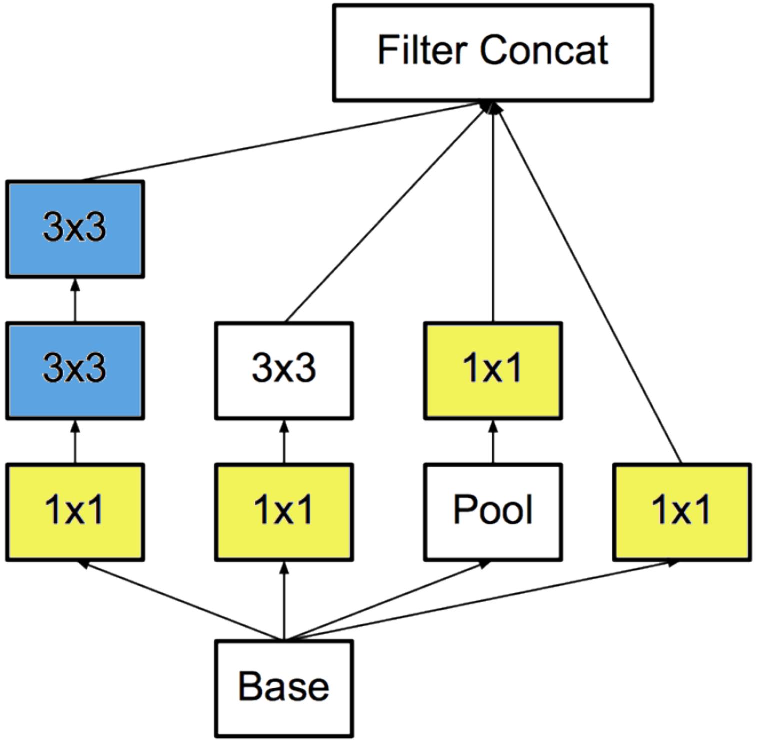
- All operations inside a block use stride 1 and enough padding to output the same spatial dimensions $(W \times H)$ of feature map.
- 4 different feature maps are concatenated on depth at the end

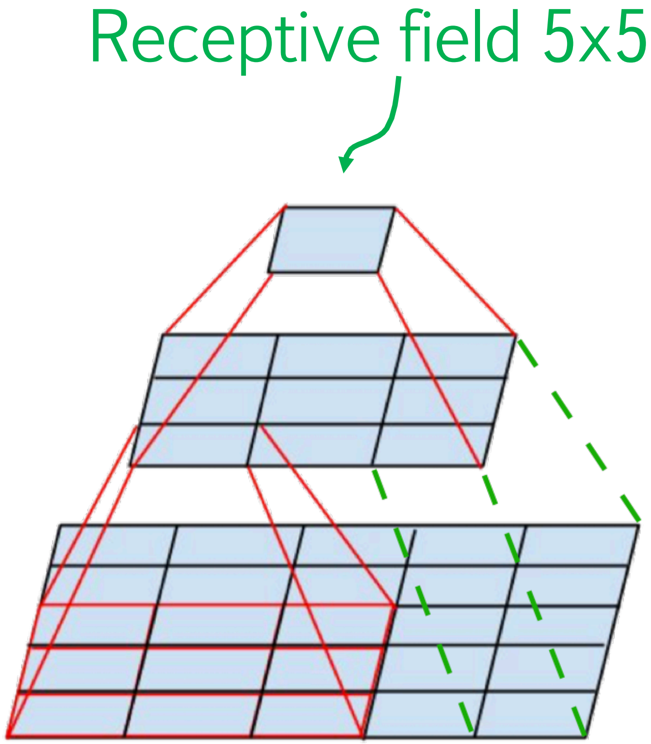


Christian Szegedy, https://arxiv.org/pdf/1512.00567.pdf

Replace 5x5 convolutions

- 5x5 convolutions are expensive! Let’s replace them with two layers of 3x3 convolutions which have an effective receptive field of 5x5.

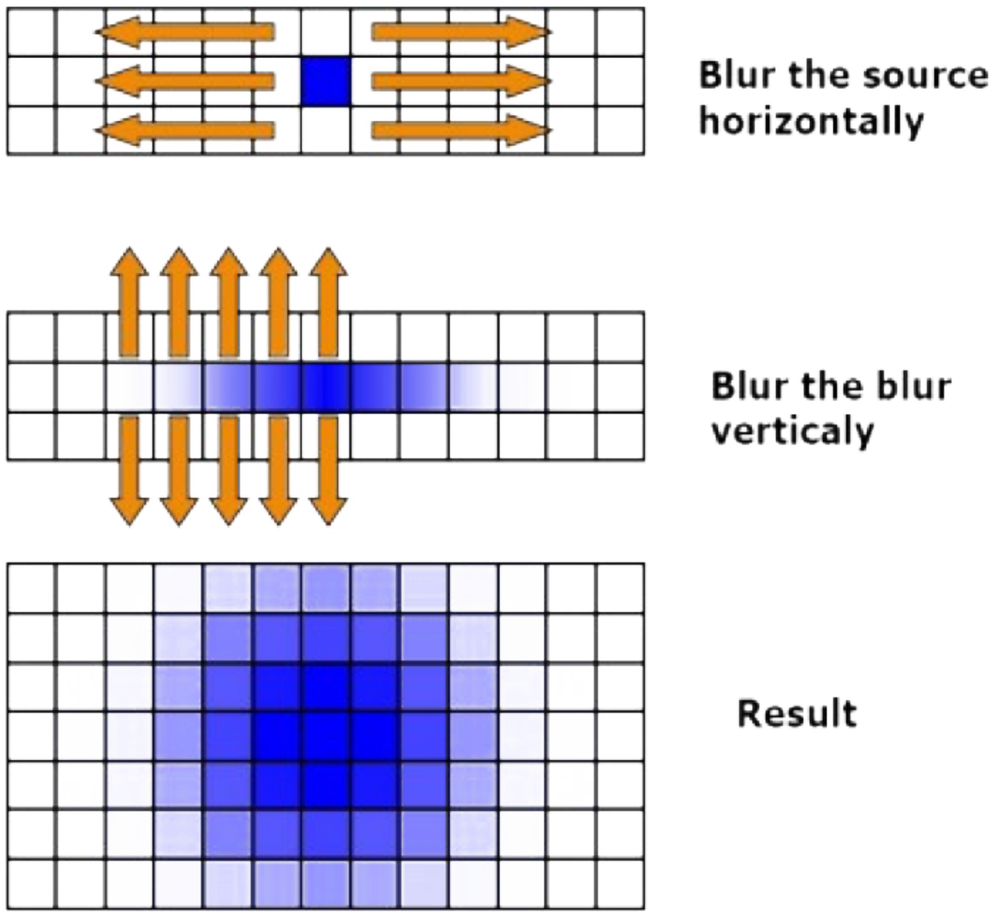




Christian Szegedy, https://arxiv.org/pdf/1512.00567.pdf

Filter decomposition

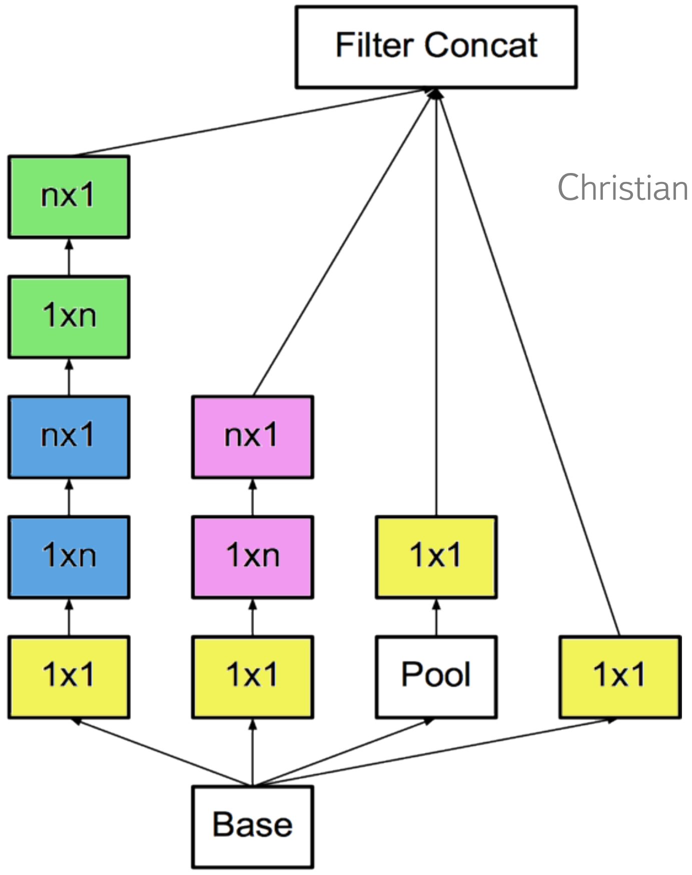
- It’s known that a Gaussian blur filter can be decomposed in two 1 dimensional filters:



Ati’s presentation, http://www.florian-oeser.de/rtr/ue5/

Filter decomposition in Inception block

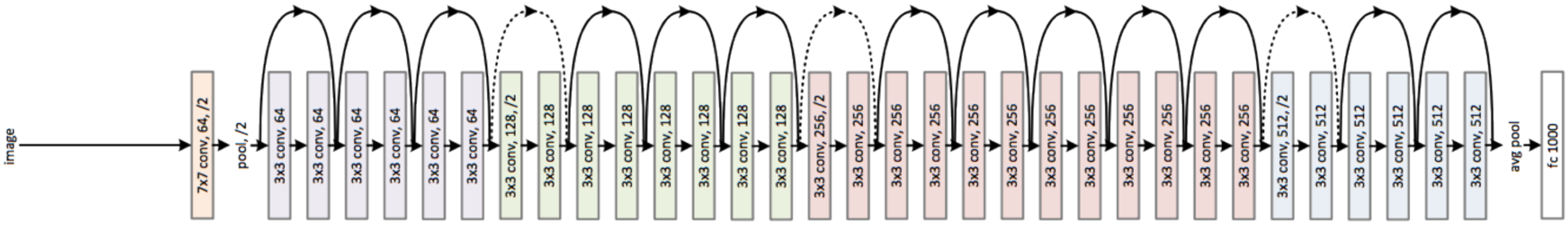
- 3x3 convolutions are currently the most expensive parts!
- Let's replace each 3x3 layer with 1x3 layer followed by 3x1 layer.



Christian Szegedy, https://arxiv.org/pdf/1512.00567.pdf

ResNet (2015)

- Introduces residual connections
- ImageNet top 5 error: 4.5% (single model), 3.5% (ensemble)

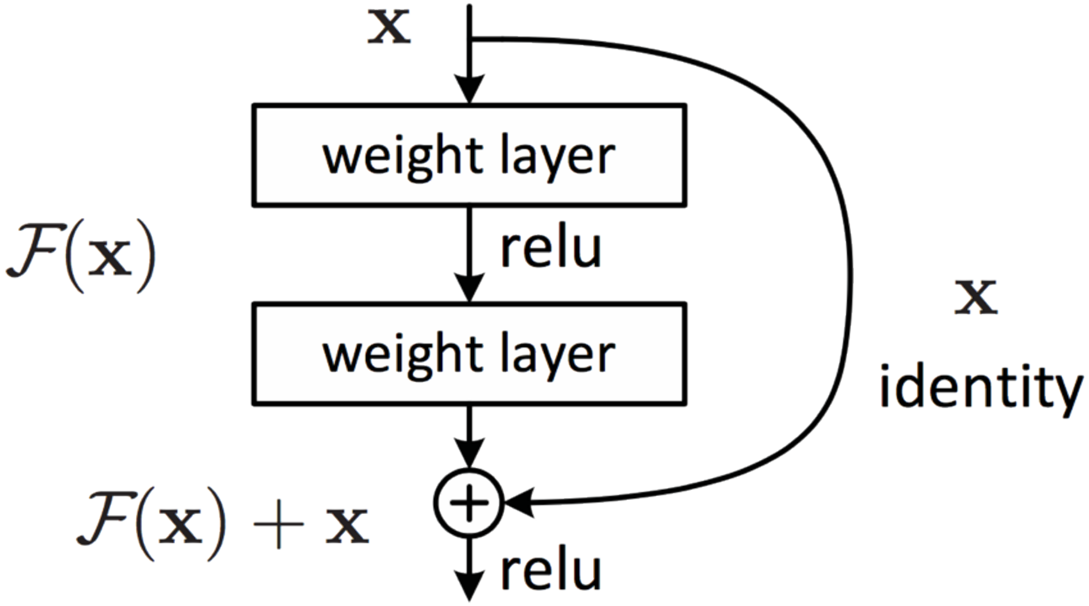


Kaiming He, https://arxiv.org/pdf/1512.03385.pdf

- 152 layers, few 7x7 convolutional layers, the rest are 3x3, batch normalization, max and average pooling.
- 60 million parameters
- Trains on 8 GPUs for 2-3 weeks.

Residual connections

- We create output channels adding a small delta $F(x)$ to original input channels x:



Kaiming He, https://arxiv.org/pdf/1512.03385.pdf

- This way we can stack thousands of layers and gradients do not vanish thanks to residual connections

Summary

- By stacking more convolution and pooling layers you can reduce the error! Like in AlexNet or VGG.
- But you cannot do that forever, you need to utilize new kind of layers like Inception block or residual connections.
- You’ve probably noticed that one needs a lot of time to train her neural network!