Yandex

SCHOOL OF DATA ANALYSIS

Digital Signal Processing for Speech

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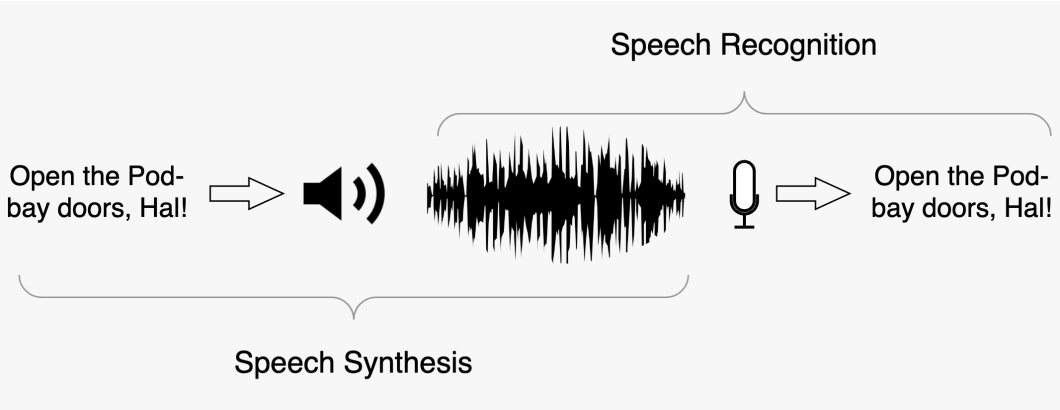
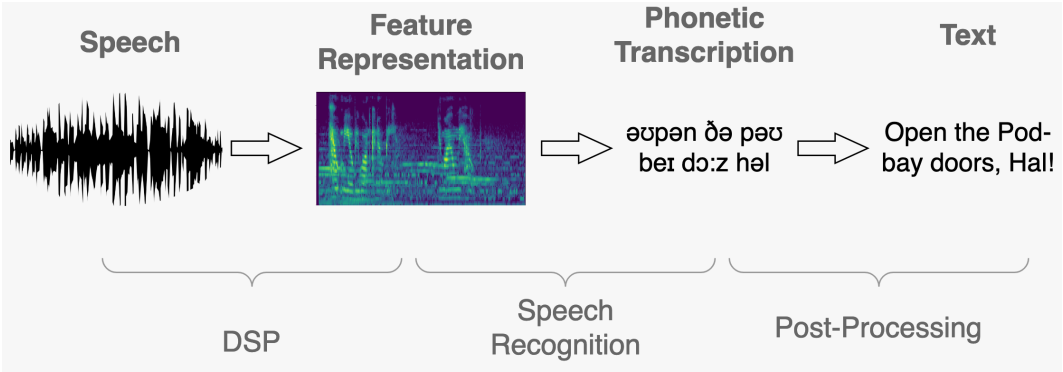
14th February 2022

In this previous episode...

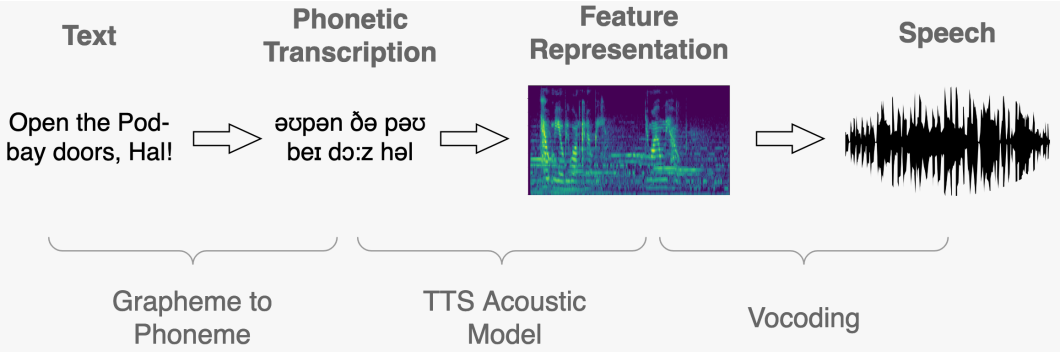
Story so far


Speech Recognition
Open the Pod-
bay doors, Hal!
Open the Pod-
bay doors, Hal!
Speech Synthesis

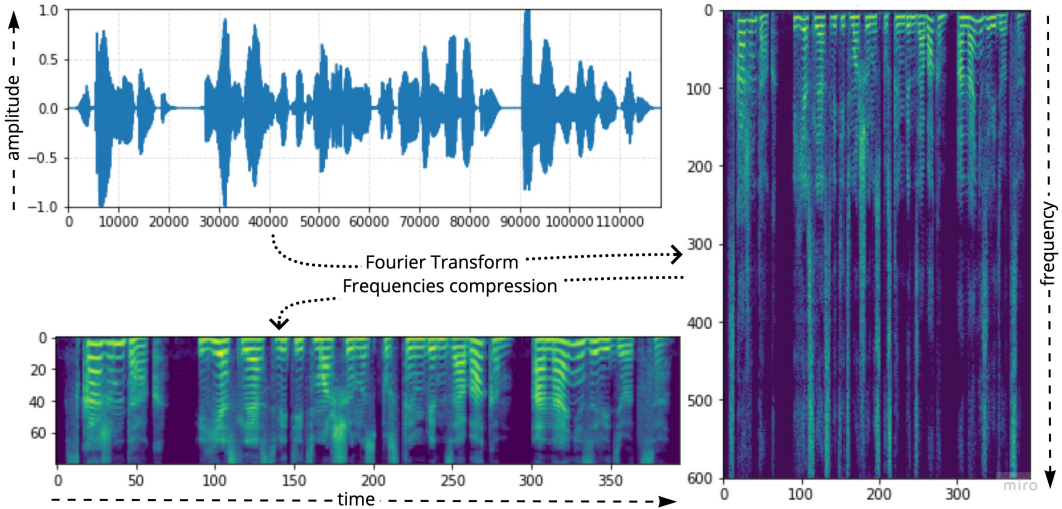
Recap - High-level Speech Recognition Pipeline



Recap - High-Level Speech Synthesis Pipeline



DSP basics - Time and Frequency Compression



How can we extract useful representations of speech?

How can we reconstruct speech from these representations?

Lecture Structure

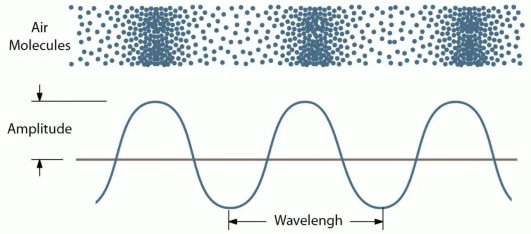
- Representing sound as discrete digital signal
- Mathematical Foundations of DSP
- Spectral representations of speech
- Reconstructing speech from spectral representations

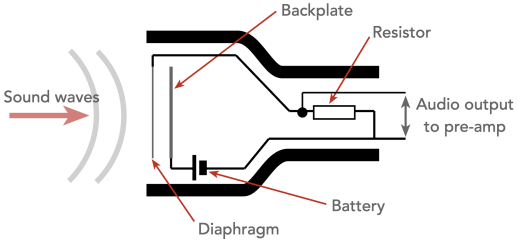
Lecture Structure

- **Representing sound as discrete digital signal**
  - Discretization of time and amplitude
- Mathematical Foundations of DSP
- Spectral representations of speech
- Reconstructing speech from spectral representations

What is sound?

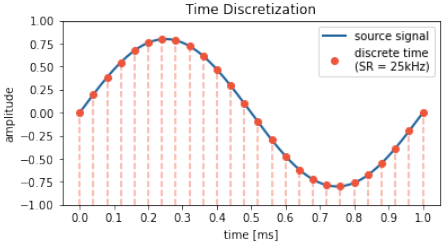
- Sound waves – longitudinal waves travelling through the air
  - Defined by **Amplitude** (pressure) and **Frequency** (inverse of Wavelength)
- A microphone captures these variations converting them into analog signal
  - Analog signal amplified, filtered and passed to ADC (analog-digital converter)

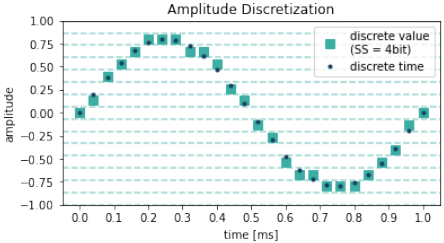




Waveform as Pulse-Code Modulation

- Digital signals are discrete in time and amplitude
- Time discretization $\rightarrow$ sample analog signal at constant sample rate
  - Sample rate – number of audio samples per second (8kHz, 22.05kHz, 44.1kHz)
- Amplitude discretization - round continuous amplitude to nearest discrete value
  - Precision depends on bit rate – number of bits per sample (eg. 8, 16, 24, 32 bits)
- Number of channels – number of signals recorded in parallel (ex: mono vs. stereo)



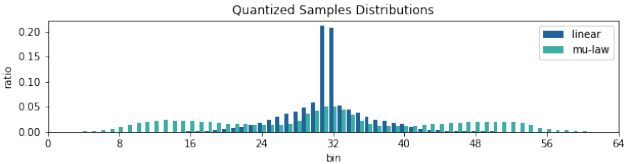


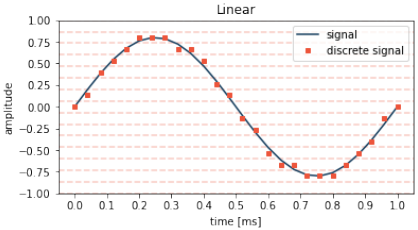
Non-linear Quantizations

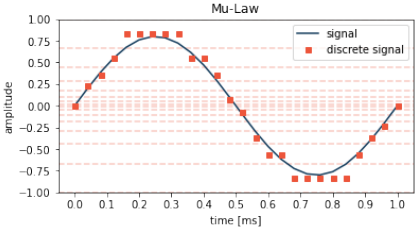
- Linear quantization encodes values uniformly across dynamic range, but most of information is contained near 0.
- μ-law encoding tries to solve the problem by re-scaling quantization bins:

$$f_{encode}(x) := sign(x)\frac{\log(1+\mu|x|)}{\log(1+\mu)}$$

$$\mu = 2^{\text{bits}} - 1, x \in (-1, 1)$$







Audio Formats

- **Non-compressed** formats: `WAV`, `raw` (header-less PCM), etc.
- **Lossless** compression formats: `FLAC`, `ALAC`, etc. (a compression ratio is about 2:1)
- **Lossy** compression formats: `MP3`, `Opus`, etc. (10:1)
  - Lossy compression formats are exploiting **psychoacoustics**
- **Bit rate** and **Sample rate** measure degrees of compression

Properties of Signals - Power

- Consider a discrete-time signal $\{f_n\}_{1:T}$ of length T.
  - The energy and power of the signal are defined as:

$$E = \sum_{n=1}^{T} f_n^2, \quad P = \frac{1}{T} \sum_{n=1}^{T} f_n^2$$

- The signal energy and power is typically computed for a fixed window τ:

$$E_t = \sum_{n=t}^{t+\tau} f_n^2, \quad P_t = \frac{1}{\tau} \sum_{n=t}^{t+\tau} f_n^2$$

- The *signal mean* is negligible, so the variance of amplitude is equal to power

$$P_t = \frac{1}{\tau} \sum_{n=t}^{t+\tau} f_n^2 = \mathrm{Var}\left[\{f_n\}_{n=t}^{t+\tau}\right] = \sigma_t^2$$

Properties of Signals - Decibels and Signal-to-Noise Ratio (SBR)

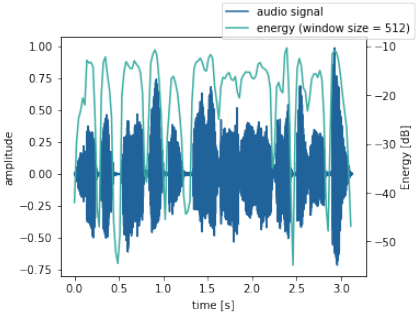
- Signal energies have a large dynamic range
  - A logarithmic scale is more informative
- **Decibels** are a logarithmic unit of energy
  - Closer to human perception of acoustic energy.
- Decibels defined in terms of energy (power):

$$P_t(dB) = 10 \log_{10} \sigma_t^2 = 20 \log_{10} \sigma_t$$

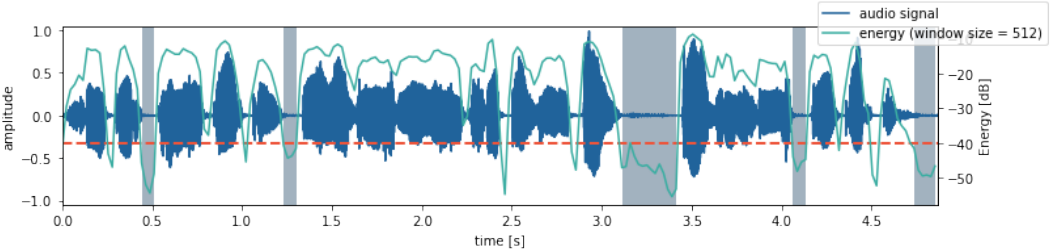
- Decibels often used as a relative power:

$$SNR_{dB} = 20 \log_{10} \frac{\sigma_t(f_{signal})}{\sigma_t(f_{noise})}$$

- Here, noise power is used as a reference



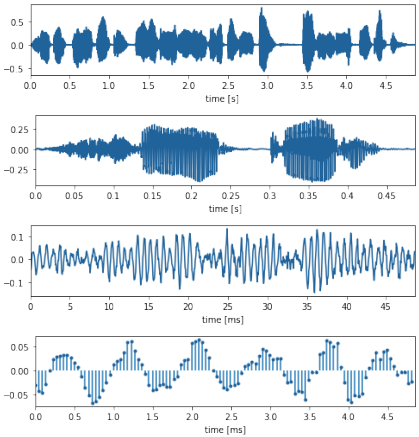
Energy-based Voice Activity Detection (Naïve approach)



- Find intervals (ignore the short ones) with energy lower than the threshold
  - Simple (+)
  - Not robust when SNR is low and fails at low-energy (unvoiced) sounds (-)
- ▸ Audio – audio with beeped silence regions

Are waveforms enough for sound representation?

- Advantages of waveforms:
  - East to obtain
  - Easy to synthesize via DAC
- Disadvantages of waveforms:
  - Extreme dimensionality (44K samples / sec)!
  - Periodic structure are different time scales
  - Inaudible variations (time shift, amplitude scaling) produce different audio signals.
- Need a more compact and robust representation!



Lecture Structure

- Representing sound as discrete digital signal
- **Mathematical Foundations of DSP**
  - Continuous and Discrete Fourier Transforms
  - Sampling and Aliasing
  - Windowing and Short-Time Fourier Transform
- Spectral representations of speech
- Reconstructing speech from spectral representations

Lecture Structure

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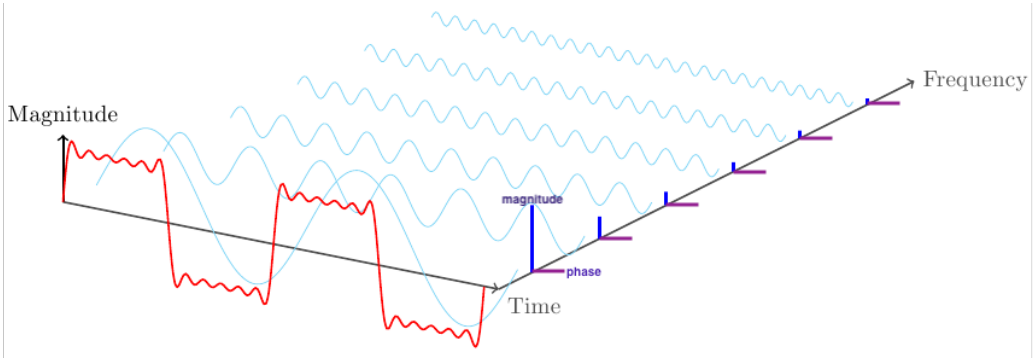
Periodic functions basis



- We can try to represent functions (from time argument) in basis of periodic functions (from frequency argument) parameterized by **phase** and **magnitude**.

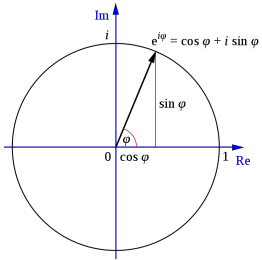
▸ Cool Demo!

Euler's Formula and the Complex function basis

- Complex numbers of the form $x + i \cdot y$ can be represented in magnitude-phase space via Euler's formula

$$\rho e^{i\phi} = \rho \cos\phi + i \cdot \rho \sin\phi$$

- Here magnitude $\rho = \sqrt{x^2 + y^2}$ and phase ϕ is $\arctan(\frac{y}{x})$



Fourier Transform

- Fourier Transform maps a **time domain** signal to the **frequency domain**.

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i t \omega} \, dt \qquad f(t) = \int_{-\infty}^{\infty} F(\omega) e^{2\pi i t \omega} \, d\omega$$

time $\rightarrow$ **frequency** **frequency** $\rightarrow$ **time**

- Signals encountered in practice are usually real $f(t) \in \mathbb{R}$ (e.g. audio)
  - *Fourier Transform* maps a real signal into complex space $F(\omega) \in \mathbb{C}$.
- **Convolution theorem**: convolution in one domain (e.g., time domain) equals point-wise multiplication in the other domain (e.g., frequency domain).

Discrete Fourier Transform (DFT)

- Discrete FT maps a discrete-time signal to the discrete-frequency domain
- Consider a function of period N:
  - Given a sequence $\{x_n\}_{n=0}^{N-1}$ the **Discrete Fourier Transform** $\mathcal{F}$ is defined by:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi}{N}kn}$$

- The inverse transform is defined by:

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{i\frac{2\pi}{N}kn}$$

- **In practice** - given a vector `x[0:N-1]` we can take DFT via `np.fft.rfft(x)`.

Discrete Fourier Transform (DFT) - Power Spectrum

- The DFT of a sequence $\{x_n\}$ produces the spectrum $\{X_k\}_{k=1}^{K}$ of the signal
  - Each X_k is a complex number is defined by magnitude $|X_k|$ and phase ϕ_k

$$X_k = |X_k| e^{i\phi_k}$$

- In speech processing we only look at the spectral magnitudes and ignore phase.
  - It is more information to consider the *power spectrum* $|X_k|^2$
- **Power Spectrum** - power of each frequency component $|X_k|^2$ of our signal.

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Theorem

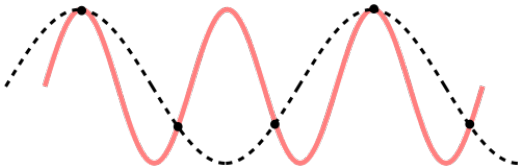
If a function $x(t)$ contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced $\frac{1}{2B}$ seconds apart.

Example: If signal contains frequency 100Hz, the sampling rate for this signal needs to be 200Hz at least.

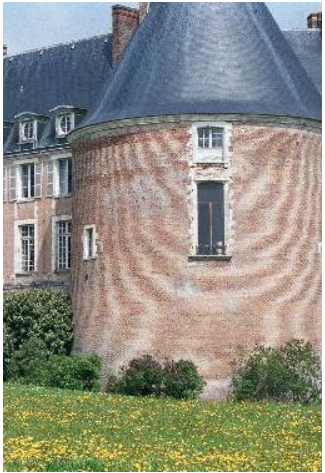
But what happens if we take sampling rate equal to 150Hz?

Aliasing

Aliasing – effect that causes different signals to become indistinguishable (or aliases of one another) when sampled.



Solution: low-pass filtering or sampling rate increase.



Sampling Rate

Recall - when digitising speech, we sample signal at a *sample rate*

- Choice of sample rate depends on which frequencies you want to retain!

Human speech contains 20Hz - 20kHz frequencies

- Must sample *at least* at 40kHz in order to retain all informationn
- In practice, we need a transition band where low-pass filter works
- Common sample rate - 44.1kHz (CD, retro hi-fi) or 48kHz (modern hi-fi)

Other common sampling rates:

- 8kHz - telephone narrowband - adequate for human speech but without sibilance
- 16kHz - Wideband extension over standard telephone narrowband.
- 22.05kHz - MPEG audio, AM radio, digitization of old records (pre-vinyl).
- 96kHz - DVD-Audio, Blu-ray Disc audio tracks, professional audio editing

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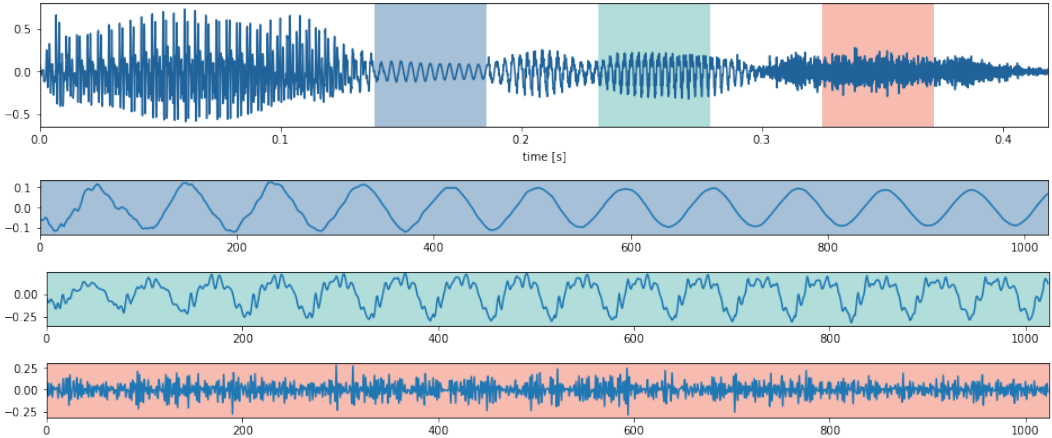
Audio signals

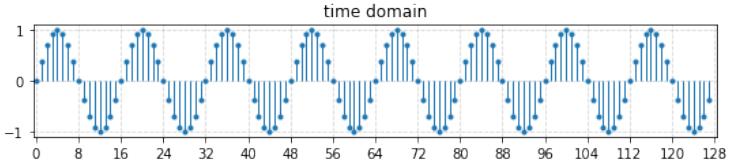
- We could take our digitized audio signal and convert into the frequency domain
  - Produces spectral decomposition of out entire speech signal.
- However, speech varies and is non-stationary
  - Whole-signal DFT does not helps us to retrieve information about local structures.
- Need to examine local spectrum to see how it changes!

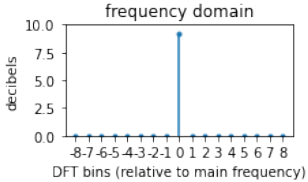
Audio signals

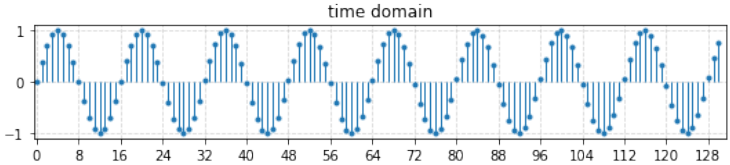


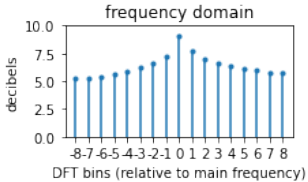
Partition and than DFT? Is it really that simple? **Not really.**

DFT from $\sin(x)$









- DFT from part of the signal may cause **spectral leakage**.
- It comes out in form of spreading out values from one bin (representing frequency of signal) to neighboring.
- Periodic extension of the function becomes the other function with another

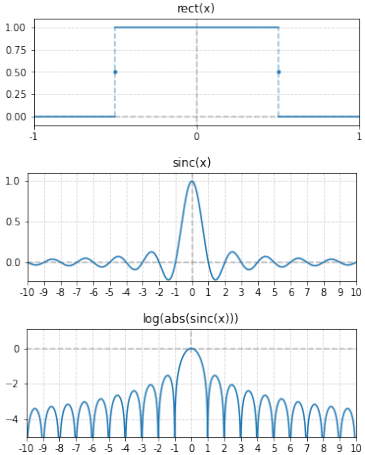
Let's take a look on a simple **rectangular function**:

$$\Pi(x) = \begin{cases} 0, & \text{if} |t| > \frac{1}{2} \\ \frac{1}{2}, & \text{if } |t| = \frac{1}{2} \\ 1, & \text{if} |t| < \frac{1}{2} \end{cases}$$

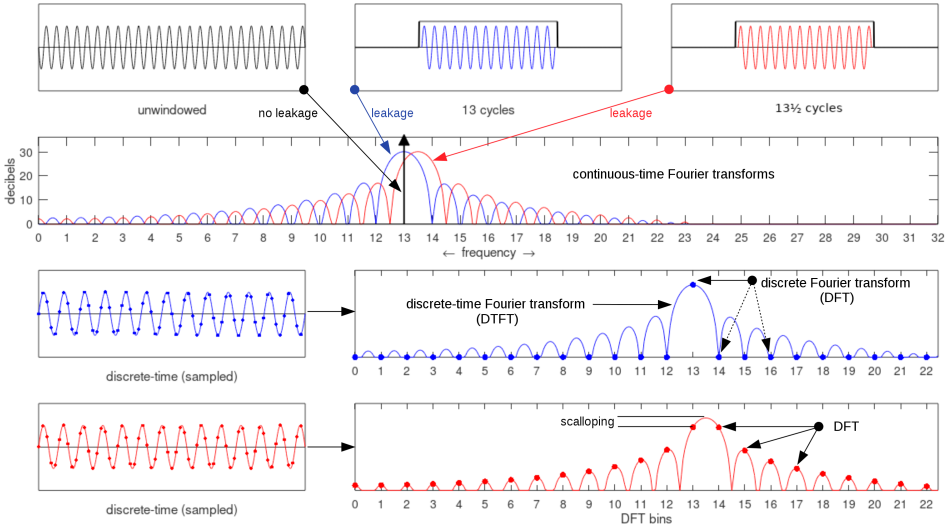
FT of this function is equal **sine cardinal** function:

$$\int_{-\infty}^{\infty} \Pi(x) e^{-j2\pi fx}\, dx = \frac{\sin(\pi f)}{\pi f} = sinc(\pi f)$$

The sinc function is widely used in DSP, and here is why:



Spectral leakage



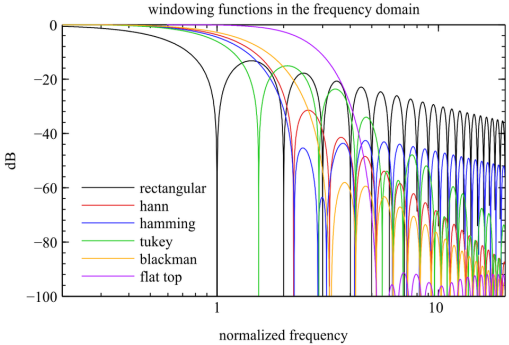
Windowing Functions (Hann Window)

Mostly used window function is **Hann** window function:

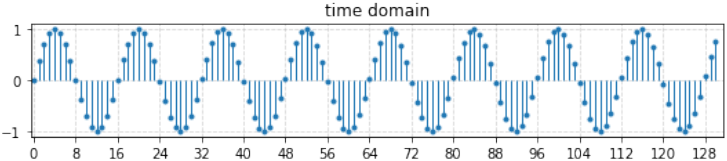
$$w_{\text{hann}}(x) = \begin{cases} \frac{1}{2}(1+\cos(\frac{2\pi n}{M})), & \text{if} |x| < \frac{M}{2} \\ 0, & \text{if} |x| > \frac{M}{2} \end{cases}$$

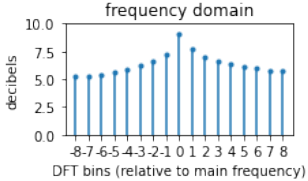
$$\frac{1}{2}(1+\cos(\frac{2\pi n}{M})),\ n \in [-\frac{M}{2}, \frac{M}{2}]$$

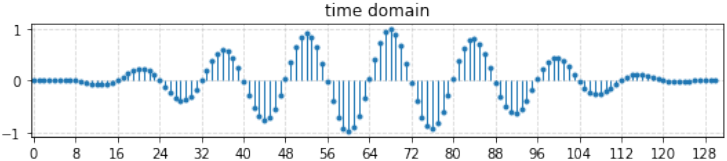
General criteria of selection: minimization of spectral leakage.

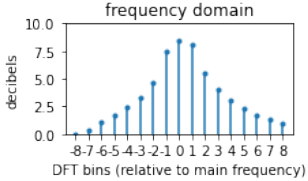


Spectral leakage with Hann



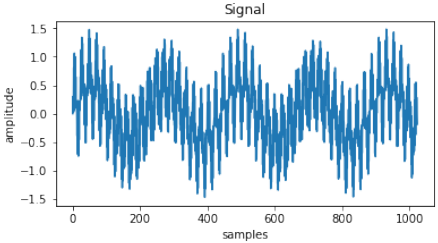


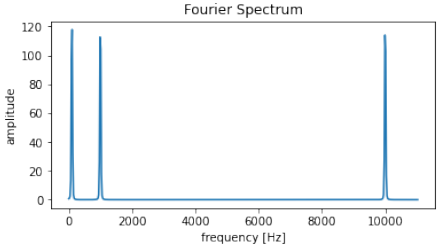


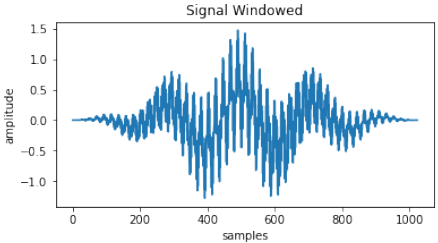


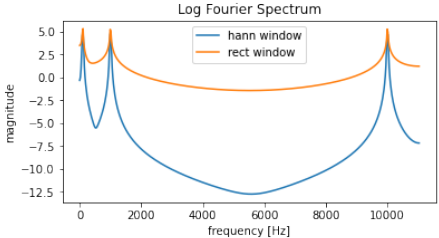
Spectrum Example

Three Sinusoidal Signals (100Hz, 1kHz, 10kHz)



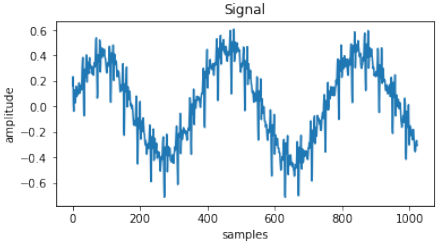


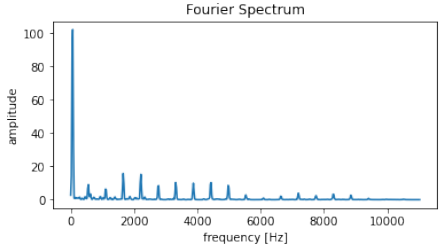


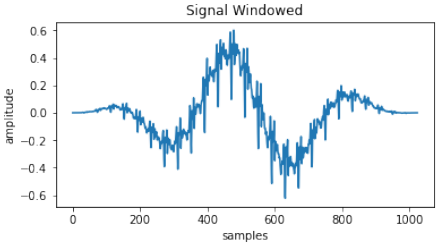


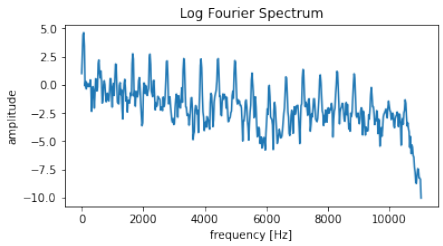
Spectrum Example

Saxophone



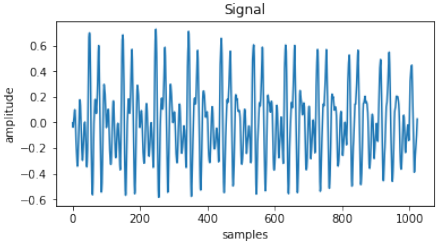


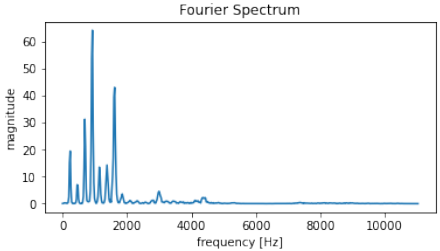


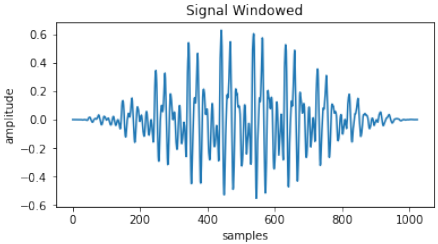


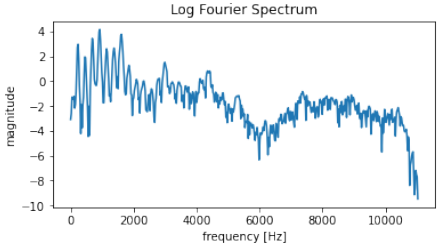
Spectrum Example

Human Voice









Lecture Structure

- Representing sound as discrete digital signal
- Mathematical Foundations of DSP:
- **Spectral representations of speech**
  - Spectrograms
  - The Mel-Scale and Mel-Spectrograms
  - Cepstrum and MFCCs
- Reconstructing speech from spectral representations

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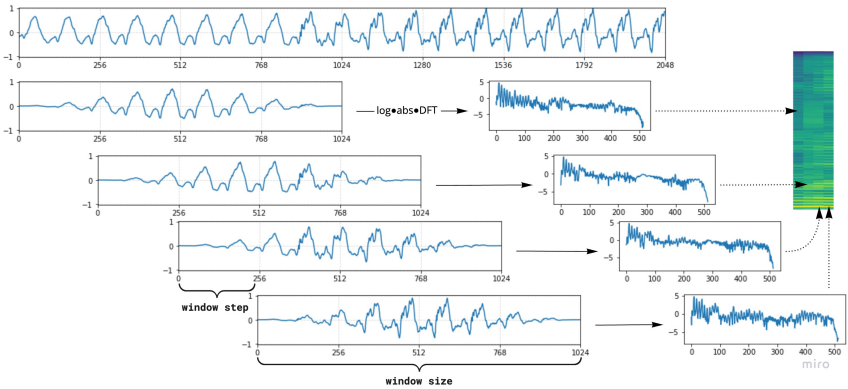
Limitations of waveforms

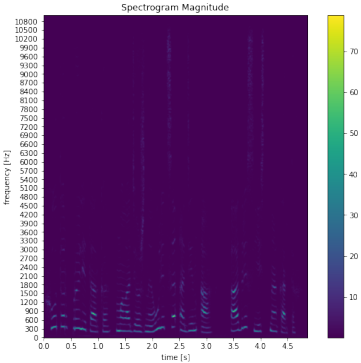
- Recall limitations of waveforms
  - Extreme dimensionality
  - Periodic structure are different time scales
  - Inaudible variations (time shift, amplitude scaling) produce different audio signals.
- We want a representations which is:
  - Compact in time
  - Disentangles structure at different time scales (frequencies)
  - Robust to inaudible variables
- Use the power spectrogram!

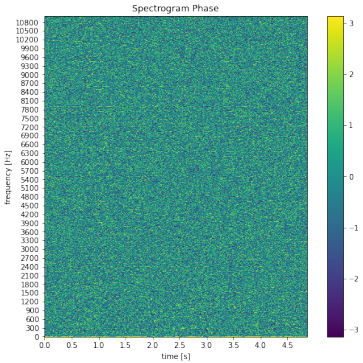
Spectrograms



- Consider stacking STFTs of a window sliding over the signal
  - Each window is called an acoustic frame

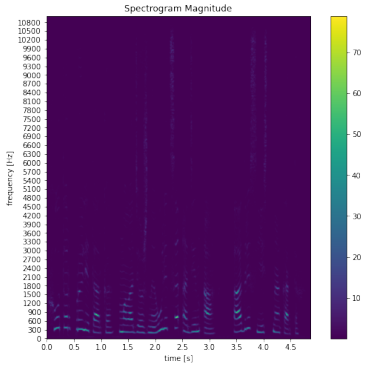
Spectrogram magnitude and phase

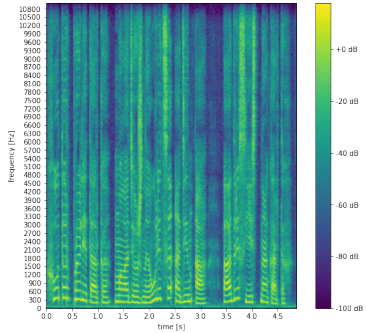




- Retain magnitude, discard phase information.

Power (dB) Spectrogram of human voice

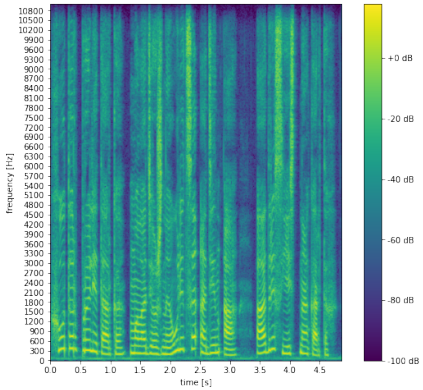




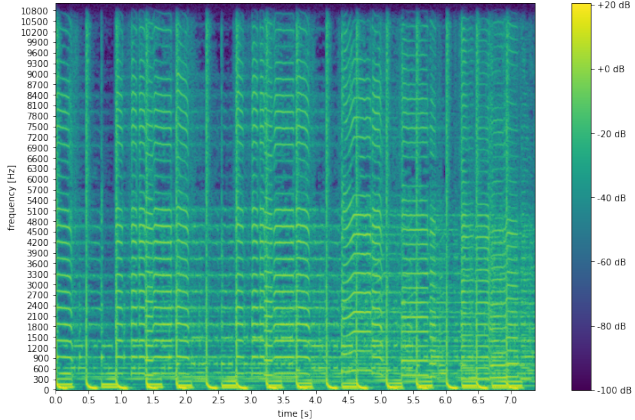
- Spectrograms usually represented as energies of corresponding frequencies
  Logarithmic scale (decibels) emphasize high frequency structure

Power (dB) Spectrogram of human voice

- $0 - 4kHz$ frequencies covers most of the speech signal
  - $8kHz$ (telephone codec) sampling rate is enough for intelligible speech
  - ... but sounds bad.
- Voiced sounds have non-constant fundamental frequency (intonation) (▸ Audio)

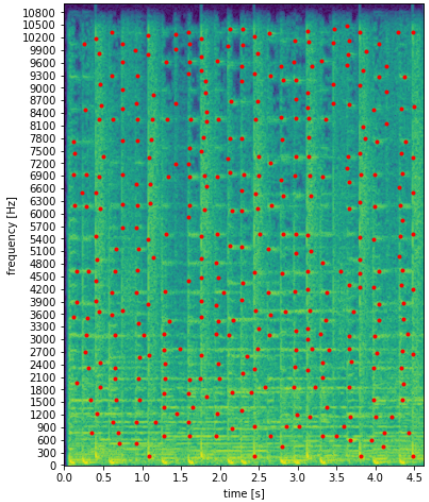


STFT Example (Saxophone)



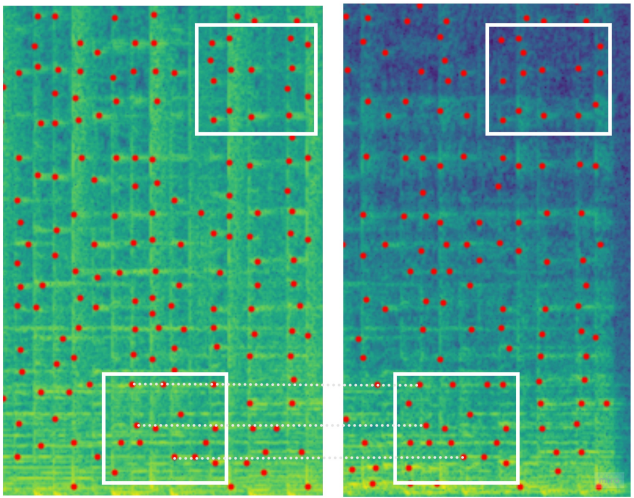
- High harmonics are more visible (comparing to speech signal)
  - Harmonics are mostly constant in time (notes) (Audio)

Songs Fingerprinting



- In "play $\rightarrow$ record" transform frequency corresponding amplitudes are changed (smoothly) and some background noise is added.
- $\Rightarrow$ most of local maximums will be preserved and their relative location is constant.
- ▸ Song played from speaker
- ▸ Recording from smart-phone's microphone

Song Fingerprinting



Fingerprinting algorithm:

- obtain local maximums,
- for each point: we iterate over all pairs with this point in given quadrant,
- for each pair: frequency and time differences are hashed,
- $\Rightarrow$ each song is represented as a set of hashes;

How to trick this algorithm?

Spectrograms vs Waveforms

- Spectrograms overcome some limitations of waveforms
  - Compresses time axis by the size of window-shift (x256 or x512 time-steps)
  - Expresses structure at different frequences as an additional axis
  - More robust to minor time shift and amplitude scaling
- However, we still are very large in frequency dimension
  - Human hearing 20Hz - 20kHz + transition band.
- Make use of Psychoacoustics to compress frequency axis!

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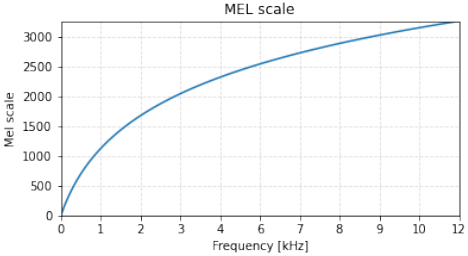
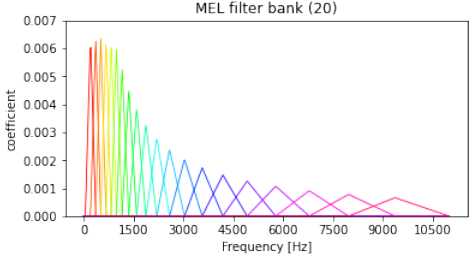
Mel Scale and Filter Bank

- Human auditory perception which is logarithmic
  - We are better at discriminating between low frequencies
- The Mel scale is an approximate scale of human frequency perception
  - Pays more attention to lower part of spectrum.
- Frequency-to-mel transform is defined as $m = 2595 \log_{10}(1 + \frac{f}{700})$

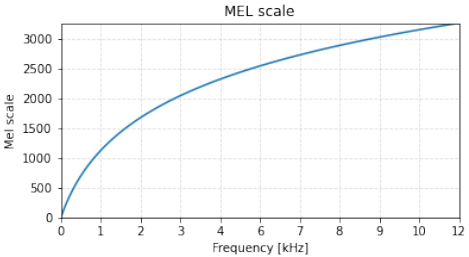


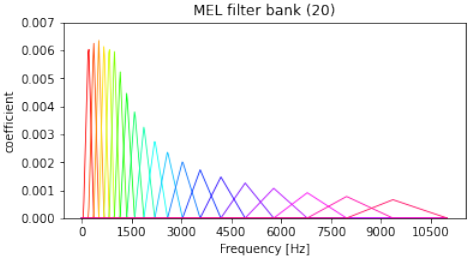


Mel Scale and Filter Bank

- Mel spectrum is produced by linear projection with Mel filter bank.
  - For a filter bank $\boldsymbol{F}$ and spectrum $\boldsymbol{s}$, Mel spectrum $\boldsymbol{m}$ is produced as $\boldsymbol{m} = \boldsymbol{F}\boldsymbol{s}$
  - Given a spectrogram $\boldsymbol{S}$, the Mel-Spectrogram $\boldsymbol{M}$ is produced as $\boldsymbol{M} = \boldsymbol{F}\boldsymbol{S}$
- The filter bank is defined by frequency range and number of filters (usually 80).
- The spectrogram can be approximately reconstructed via Pseudo-Inverse:

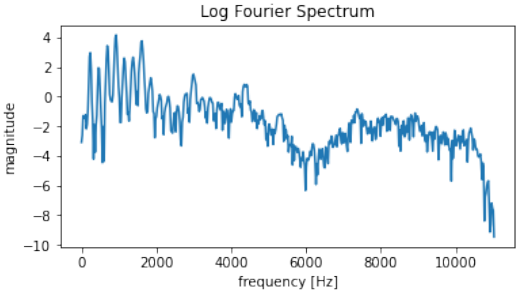
$$\hat{\boldsymbol{S}} = (\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F})^{-1}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{M}$$

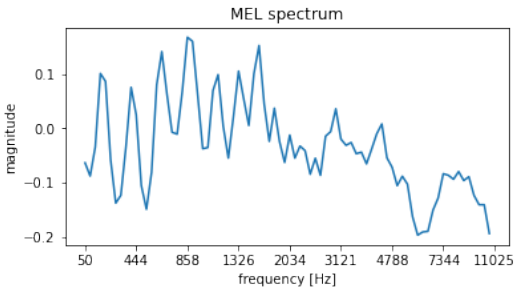




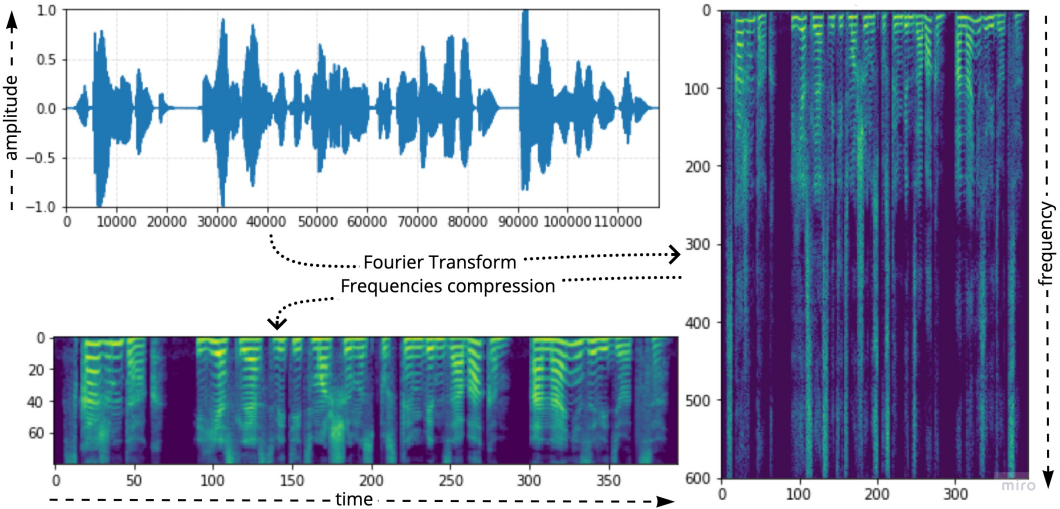
Mel Spectrum Example

- We have now greatly compressed the Spectrogram along the frequency axis
  - Overall structure is retained, but high-frequency fidelity is lower

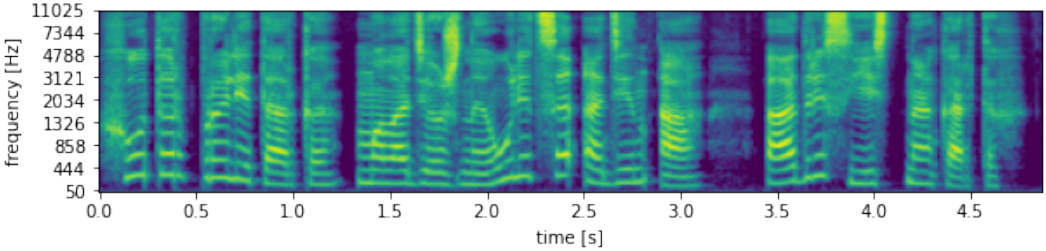




Time and Frequency Compression



Mel Spectrogram



- Mel-Spectrogram → compact, efficient, but lossy representation of speech
  - Compression in time via STFT, compression in Frequency via Mel-Scale
  - Lossy in time (phase) and frequency (linear projection)
- Can be reconstructed via Griffin-Lim Algorithm - used in speech synthesis! ▸ Audio
- We can easily detect vowels sounds on Mel-Spectrogram
  - Phoneme classification needs deep expertise in phonetics (formants recognition).

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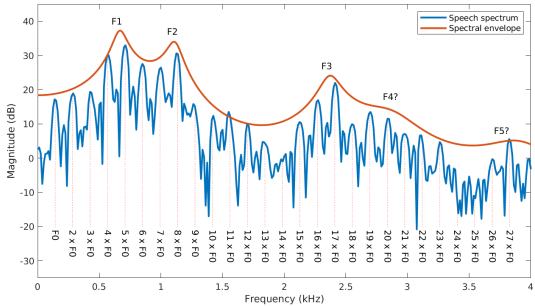
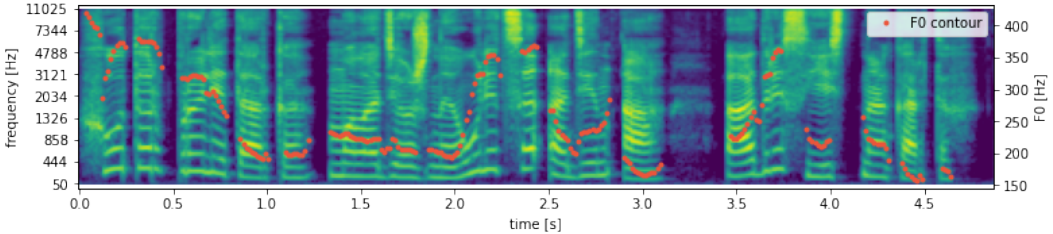
Fundamental frequency (F_0) and formants



- F_0 – *fundamental frequency*, $k \times F_0$ – *harmonics*
- Peaks on envelope curve (**red**) – formants
- *Pitch* is perceptual value, F_0 is physical (correlated)
- Changing (in time) pitch forms voice intonation
- How can we find F_0?

F0 over time

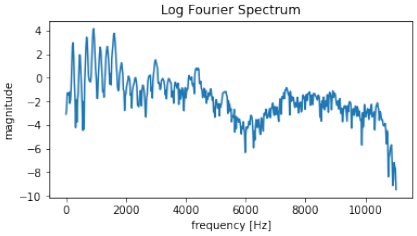
Demonstration of PRAAT algorithm for F0 extraction (one more beautiful image):



- cepstrum analysis algorithms
- signal auto-correlation algorithms
- hybrid algorithms
- neural networks

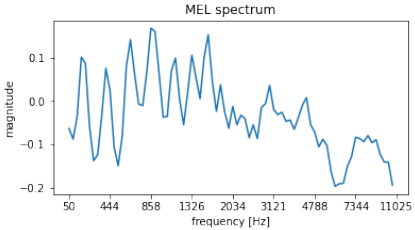
Cepstrum

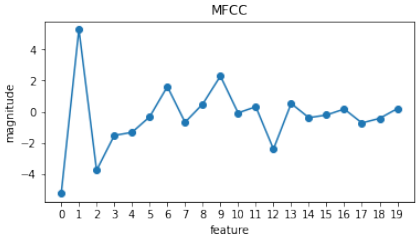
- Fourier Spectrum of voice has periodic structure (period is equal to F_0)
- $\Rightarrow$ let's get spectrum from spectrum (Discrete Cosine Transform) – brings us to the **Cepstrum**
- Two time-frequency transforms leads us back to time-domain (not signal)
- Low quefrencies encode information about formants (non-trivial to extract)
- Peak in the cepstrum should be located at $\frac{1}{F_0}$



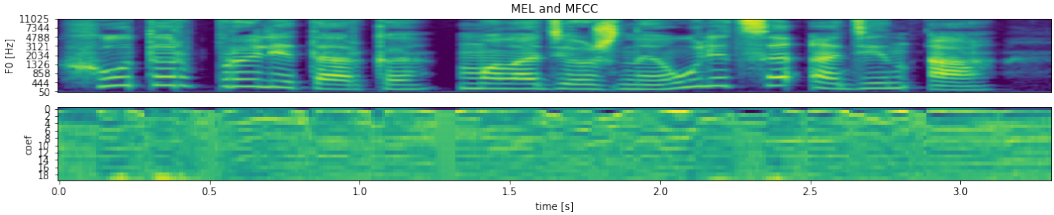
Mel-Frequency Cepstral Coefficients (MFCCs)

- Taking the **Discrete Cosine Transform (DCT)** of mel spectrum, we obtain the representation known as **mel-frequency cepstral coefficients (MFCCs)**.
- Mel-scale transform decorrelates F_0 harmonics.
- Phone definition is based on macro-shapes in the spectrum (preserves that information)
- MFCC separates the impact of **source** and **filter** in a speech signal.





MFCC over time



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  - Griffin-Lim Algorithm for phase reconstruction

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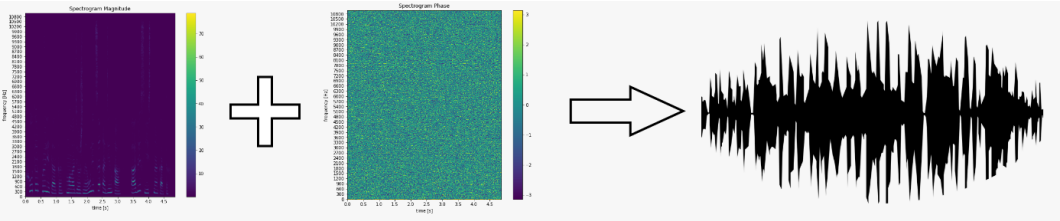
inverse Short-time Fourier Transform

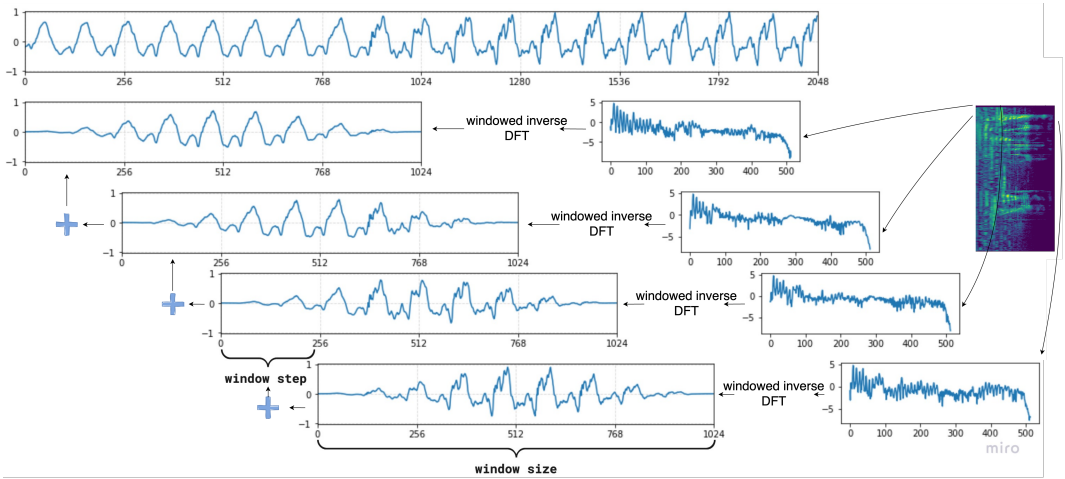


- Let's examine reconstructing audio from **magnitude** and **phase** of the spectrum
  - Remember to convert from decibels to linear power and from power to magnitude!

Overlap and Add inverse STFT

- Recall - each frame is an STFT of a hanh-windowed signal shifted by a hop-length
  - We can individually take the inverse DFT of each frame using `np.fft.irfft()`
- How do we combine all the restored overlapping signal windows together?
  - Overlap, window and add!

Overlap and Add inverse STFT



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  - **Griffin-Lim Algorithm for phase reconstruction**

Phase reconstruction

- Ok, but how do we get the phase information? We threw it away
  - Use the **Griffin-Lim Phase reconstruction algorithm**!
- STFT produced by **overlapping** windows – **redundant representation** of signal.
  - Iteratively take STFT and iSTFT, enforcing magnitude consistency
  - Eventually will recover a consistent phase.

Input: $\rho = |STFT(f)|$ – absolute of STFT f
Result: $\tilde{f} \approx f$ – reconstructed signal f
begin
  $\phi \sim \mathbb{U}(0, 2\pi)$ – random initialization
  $\tilde{f} = ISTFT(\rho * e^{i\phi})$ – first approximation
  for *number of iterations* **do**
    $\phi = \arg STFT(\tilde{f})$ – calculate angle
    $\tilde{f} = ISTFT(\rho * e^{i\phi})$ – new approximation
  end
end

Algorithm 1: Griffin-Lim

Griffin-Lim Algorithm

- GLA is a simple and very useful procedure – used as a simple vocoder in TTS
  - Simple, converges to decent audio in a few interation.
- However, GLA has limitations:
  - GLA is not robust even to float-arithmetic (metallic sound effect): Audio
  - GLA is not robust to noise – Audio reconstruction of slightly blurred spectrogram
- If our **synthesized** spectrogram is not perfect, GLA will generate weird artifacts.
  - GLA good starting point, but need better vocoders (TTS lectures).

Lecture Summary

- Covered how waveforms are recorded:
  - Sample at discrete time intervals and record quantized amplitude value
- Had a brief recap of DSP
  - Fourier Transform $\rightarrow$ Discrete Fourier Transform $\rightarrow$ Short-Time Fourier Transform
  - Aliasing and Sample Rate
- Used the STFT to obtain a compact spectral representation of sounds
  - STFT $\rightarrow$ Spectrogram dB $\rightarrow$ Mel Spectrogram $\rightarrow$ MFCC
- Learned how to reconstruct audio from spectrograms
  - Griffin-Lim Phase reconstruction algorithm