Economics 640 San Diego State University
LIANG Fall 2025

Econ 640 - Problem Sets 3

Instructions: *For the theory section, submit your individual answers as a* ***physical copy****. For the Stata simulation part, each individual must submit their own GitHub repository containing the cleaned do-file and all figures, and invite me as a collaborator.* ***Due: by the beginning of class on November 11****.*

1. **Theory: OLS, Robust OLS, and GLS**

   Consider the linear model

   $$y = X\beta + u, \qquad \mathbb{E}[u \mid X] = 0, \quad \mathrm{Var}(u \mid X) = \Omega,$$

   (a) **Derive the variance of the OLS estimator.**
   Let $\widehat{\beta}_{\text{OLS}} = (X'X)^{-1}X'y$. Derive $\mathrm{Var}(\widehat{\beta}_{\text{OLS}} \mid X)$ when Ω is *not* necessarily proportional to I_N.

   (b) **Heteroskedasticity-robust and cluster-robust corrections.**
   (i) Write the White (heteroskedasticity-robust) "sandwich" estimator for $\mathrm{Var}(\widehat{\beta}_{\text{OLS}})$.
   (ii) Suppose observations are partitioned into G clusters $g = 1, \ldots, G$ (e.g., states). Write the *one-way cluster-robust* variance estimator. State clearly what correlation structure it allows and what it rules out.

   (c) **GLS: estimator and its variance.**
   (i) Derive the GLS estimator $\widehat{\beta}_{\text{GLS}}$ when Ω is known.
   (ii) Derive $\mathrm{Var}(\widehat{\beta}_{\text{GLS}} \mid X)$.

   (d) **Feasible GLS (FGLS).**
   Explain the steps to estimate GLS when Ω is unknown. Describe:

   1. How to obtain consistent estimates $\widehat{\sigma_{ij}}$ of the nuisance parameters (from OLS residuals).
   2. How to form $\widehat{\Omega} = \widehat{\Omega}(\widehat{\sigma_{ij}})$ and compute $\widehat{\beta}_{\text{FGLS}}$.

2. **Empirical Task: Monte Carlo on OLS vs. Clustered OLS vs. GLS/FGLS**

   You will simulate data with both heteroskedasticity and serial correlation *within clusters*. Use the following data-generating process (DGP) or you propose and document an alternative one:

$$
\begin{aligned}
&\text{Panel structure: } i = 1, \ldots, G \text{ clusters (states)}, \ t = 1, \ldots, T \text{ periods}, \ N = G \times T. \\
&\varepsilon_{it} \sim \mathcal{N}(0,1), \quad \text{all independent across } i, t. \\
&\sigma_i^2 \sim \text{Uniform}[0.5,\ 2] \quad \text{(heteroskedasticity across clusters)}. \\
&u_{it} = \rho\, u_{i,t-1} + \sigma_i\, \varepsilon_{it}, \qquad u_{i0} = 0, \qquad |\rho| < 1 \\
&y_{it} = \alpha + \beta\, x_{it} + u_{it}, \quad \alpha = 0,\ \beta = 1 \text{ (use these as the true values)}.
\end{aligned}
$$

   Recommended baseline sizes: $G = 50$, $T = 10$, $\rho = 0.5$. (You may also explore sensitivity to G, T, and ρ.)

   (a) **Point estimation (single sample).**
   Using a single Monte Carlo draw from the DGP:

   1. Estimate β by OLS.
   2. Estimate β by OLS with *group-level clustered* standard errors (cluster on i).
   3. Estimate β by GLS/FGLS under the maintained error structure:
      - In Stata: `xtset i t`; then `xtgls y x, panels(hetero) corr(ar1)`.

   Report $\widehat{\beta}$ and its standard error under each method and briefly interpret any differences.

   (b) **Monte Carlo comparison (sampling distributions).**
   Run $R = 1000$ independent replications of the DGP. For each replication, compute and store the estimate $\widehat{\beta}$ under each method:

   1. **OLS (conventional SE)**: estimate by OLS and store $\widehat{\beta}^{\text{OLS}}$.
   2. **OLS (cluster-robust SE)**: re-estimate by OLS and store $\widehat{\beta}^{\text{C-OLS}}$ (the *point estimate* is the same as OLS; the SE differs). Also store the cluster-robust SE.
   3. **GLS/FGLS**: estimate by GLS (or FGLS) and store $\widehat{\beta}^{\text{GLS}}$ (and its model-based SE).

   Deliverables:

   - Three histograms or density plots of $\widehat{\beta}$: OLS, clustered OLS (same $\widehat{\beta}$ as OLS, but include cluster SE summary), and GLS/FGLS.
   - Clean and well-documented Stata codes.
   - Create a repo on GitHub to host your codes and figures, and invite me as a collaborator to review.