ECON 640 Causal Models and Instrumental Variable

Dr. Yang Liang

1 Introduction to Causal Models

Formal Setup: Potential Outcome Causal Model

Binary Treatment for Simplicity:

- Let $Y_i(1)$ denote the individual i's outcome if they *receive* treatment $(D_i = 1)$.
- Let $Y_i(0)$ denote the individual i's outcome if they *do not receive* treatment $(D_i = 0)$.

Treatment Effect:

$$\delta_i = Y_i(1) - Y_i(0)$$

Switching Equation:

$$Y_i = D_i \cdot Y_i(1) + (1 - D_i) \cdot Y_i(0)$$

Key Problem:

- We only observe either $Y_i(1)$ or $Y_i(0)$, not both. The unobserved counterpart is referred to as the *counterfactual*.

Terminology: Average Treatment Effects

Definition:

- **Average Treatment Effect (ATE):**

$$\text{ATE} = \mathbb{E}[\delta_i] = \mathbb{E}[Y_i(1) - Y_i(0)] = \mathbb{E}[Y_i(1)] - \mathbb{E}[Y_i(0)]$$

- **Average Treatment Effect on the Treated (ATT):**

$$\text{ATT} = \mathbb{E}[\delta_i \mid D_i = 1] = \mathbb{E}[Y_i(1) - Y_i(0) \mid D_i = 1]$$
$$= \mathbb{E}[Y_i(1) \mid D_i = 1] - \mathbb{E}[Y_i(0) \mid D_i = 1]$$

- Key Question: *Who sorts into $D_i = 1$?*

Selection on Observables vs Unobservables

Causal Inference:

$$Y = \beta X + \gamma Z + u$$

- Y: Outcome
- X: Treatment
- Z: Observed covariates
- u: Error term (unobserved confounders)

Conditions:

- If $\mathbb{E}[u \mid X, Z] = 0$, then Z provides sufficient control to address selection on observables.
- Otherwise, $\mathbb{E}[u \mid X] \neq 0$, leading to *endogeneity*:
  - Omitted Variable Bias (OVB)
  - Model misspecification
  - Inconsistency of OLS, regardless of the sample size.

Average Treatment Effect on the Controlled (ATC)

$$\text{ATC} = \mathbb{E}[\delta_i \mid D_i = 0] = \mathbb{E}[Y_i(1) - Y_i(0) \mid D_i = 0]$$

However, we only observe:

$$\mathbb{E}[Y_i \mid D_i = 1] - \mathbb{E}[Y_i \mid D_i = 0] = \mathbb{E}[Y_i(1) \mid D_i = 1] - \mathbb{E}[Y_i(0) \mid D_i = 1] + \text{Selection Bias}$$

Key Insight: Understand the source of variation and what makes the treatment group different.

2 Randomized Controlled Trials (RCTs) and Instrumental Variables (IVs)

- Randomized experiments provide exogenous variation to identify causal effects.
- Instrumental Variables (IV) can address endogeneity when randomization is infeasible:

$$\text{Cov}(Z, X) > 0 \quad \& \quad \text{Cov}(Z, u) = 0 \quad \Rightarrow \quad Z \text{ affects } X \text{ exogenously.}$$

IV in Action: Two-Stage Least Squares (2SLS)

Structural Equations:

$$Y_i = \alpha + \beta \hat{X}_i + u_i \quad \text{(Second Stage)}$$

$$X_i = \pi + \delta Z_i + \epsilon_i \quad \text{(First Stage)}$$

Reduced Form:

$$Y_i = \mu + \omega Z_i + D_i$$

$$\hat{\beta} = \frac{\omega}{\delta}$$

Consistency of IV Estimators

$$\hat{\beta}_{\text{IV}} = (Z'X)^{-1}Z'Y = (Z'X)^{-1}Z'(X\beta) + (Z'X)^{-1}Z'u$$

- Unbiased if $\mathbb{E}[u \mid Z] = 0$.
- Consistent if:

$$\hat{\beta}_{\text{IV}} = \beta + (Z'X)^{-1}Z'u \quad \Rightarrow \quad \frac{1}{N}Z'u \to 0$$

LATE: Local Average Treatment Effect

$$\beta_{\text{IV}} = \frac{\mathbb{E}[Y \mid Z = 1] - \mathbb{E}[Y \mid Z = 0]}{\mathbb{E}[D \mid Z = 1] - \mathbb{E}[D \mid Z = 0]}$$

Example:

- Z: Lottery to charter school
- D: Admission

Relevance is key: The power of your instrument determines the strength of the causal estimate.

The Lottery Example: Groups and Identification

Setup:

- Z: Lottery assignment (instrument, binary: $Z = 1$ if assigned, $Z = 0$ if not).
- D: Admission (treatment, binary: $D = 1$ if admitted, $D = 0$ otherwise).
- Y: Outcome (e.g., test scores, future earnings, etc.).

Population Groups:

- **Never Takers:** Individuals who never take the treatment ($D_i = 0$) regardless of lottery assignment (Z).
$$D_i(Z = 1) = 0 \quad \text{and} \quad D_i(Z = 0) = 0$$
- **Always Takers:** Individuals who always take the treatment ($D_i = 1$) regardless of lottery assignment (Z).
$$D_i(Z = 1) = 1 \quad \text{and} \quad D_i(Z = 0) = 1$$
- **Compliers:** Individuals who take the treatment ($D_i = 1$) only when assigned through the lottery ($Z = 1$), but not otherwise.
$$D_i(Z = 1) = 1 \quad \text{and} \quad D_i(Z = 0) = 0$$
- **Defiers:** Individuals who take the treatment ($D_i = 1$) only when *not* assigned through the lottery ($Z = 0$), but not otherwise.
$$D_i(Z = 1) = 0 \quad \text{and} \quad D_i(Z = 0) = 1$$

Treatment Effect Identification:

- **Key Insight:** LATE is identified only for **Compliers**.
- Why? The instrument (Z) creates exogenous variation in D only for compliers, as their treatment status changes with Z.
$$\beta_{\text{IV}} = \frac{\mathbb{E}[Y \mid Z = 1] - \mathbb{E}[Y \mid Z = 0]}{\mathbb{E}[D \mid Z = 1] - \mathbb{E}[D \mid Z = 0]}$$
- Never Takers and Always Takers do not contribute to identifying variation because their treatment status does not change with Z.
- Defiers are typically excluded by assuming monotonicity: $D_i(Z = 1) \geq D_i(Z = 0)$.

Practical Considerations:

- Relevance of the instrument is critical: $\text{Cov}(Z, D) \neq 0$ ensures that Z affects D.
- Strength of the instrument matters for precision and bias in IV estimation.
- LATE applies to compliers only, so generalization to the entire population requires careful interpretation.