GMM-IV Exmaples

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In-Class Exercise on IV and GMM (with Answers)

This exercise uses classic empirical examples to review the basics of instrumental variables, compliers, LATE, and the GMM formulation of IV and OLS.

A. Basic IV Setup

Consider the linear model:
$$Y_i = \beta D_i + \varepsilon_i, \qquad D_i = \pi Z_i + \nu_i.$$

Task: (1) Write the moment condition. (2) State assumptions. (3) If Z_i is binary, identify the causal parameter and who it applies to. (4) Give an example from a real empirical setting.

Answer:

Moment condition:
$$E[Z_i(Y_i - \beta D_i)] = 0.$$

Assumptions:

- **Relevance:** $E[Z_i D_i] \neq 0$.

- **Exclusion:** Z_i affects Y_i only through D_i.

- **Independence:** $Z_i \perp (\varepsilon_i, \nu_i)$.

- **Monotonicity:** No defiers.

Binary IV and LATE: IV identifies the **Local Average Treatment Effect (LATE)**:

$$\beta_{\text{IV}} = E[Y_i(1) - Y_i(0) \mid \text{compliers}].$$

Identified group: Individuals who take the treatment when $Z_i = 1$ and do not when $Z_i = 0$.

Example: Draft-lottery compliers are people who serve in the military *only* if they draw an eligible lottery number.

B. Angrist (1990) Draft Lottery IV

Tasks: Write model, moment condition, assumptions, identify compliers, interpret LATE.
    Answer:
    Model:

$$Y_i = \beta D_i + \varepsilon_i, \qquad D_i = \pi Z_i + v_i,$$

where Z_i = draft eligibility and D_i = veteran status.
    Moment condition:

$$E[Z_i(Y_i - \beta D_i)] = 0.$$

    Assumptions:

- Independence: lottery number is randomly assigned.

- Exclusion: eligibility affects earnings only via service.

- Monotonicity: no one serves *because* they are *not* eligible.

    Compliers: Men who serve in the military *only if* they are draft-eligible.
    LATE interpretation: The IV estimate reflects the effect of military service on earnings for draft-induced veterans.
    Non-identified groups: always-takers (volunteers), never-takers (refusers).

C. Tennessee STAR Class-Size Experiment

Tasks: Model, Wald estimator, assumptions, compliers, interpret LATE.
    Answer:
    Model:

$$Y_i = \beta D_i + \varepsilon_i,$$

where D_i = being in a small class; Z_i = randomly assigned offer.
    Wald estimator:

$$\beta_{\text{Wald}} = \frac{E[Y_i \mid Z_i = 1] - E[Y_i \mid Z_i = 0]}{E[D_i \mid Z_i = 1] - E[D_i \mid Z_i = 0]}.$$

    Assumptions: random assignment, exclusion, monotonicity.
    Compliers: Students whose class-size assignment follows the random offer (those who move to small class *only* when offered).
    LATE: The treatment effect of small classes for students who comply with random assignment.

D. Judge Leniency IV

Tasks: Model, moment condition, assumptions, compliers, causal interpretation.
    Answer:
    Model:

$$Y_i = \beta D_i + \varepsilon_i, \qquad D_i = \pi Z_i + v_i,$$

where Z_i = judge assignment and D_i = detention or sentence length.

Moment condition:

$$E[Z_i(Y_i - \beta D_i)] = 0.$$

Assumptions:

- Random judge assignment within court–time cell.

- Exclusion: judge affects Y_i only through sentencing/detention.

- Monotonicity: stricter judges never release someone a lenient judge would detain.

Compliers: Defendants whose detention status depends on the harshness of the randomly assigned judge.

LATE interpretation: Effect of detention on outcomes for defendants whose detention is determined by judge leniency.

E. Additional Classic IV Examples

Below are short answers for each model.

Card (1993): College Proximity as IV Model: $Y_i = \beta S_i + \varepsilon_i$. IV: distance to nearest college. Compliers: students who attend college only if they live near one. LATE: return to education for those induced by proximity.

Angrist & Krueger (1991): Quarter-of-Birth IV Model: schooling on wages. IV: quarter of birth affects compulsory schooling age laws. Compliers: individuals whose schooling is changed by compulsory schooling rules. LATE: returns to schooling for those who leave school at the minimum age.

Fulton Fish Market (Weather Shocks) Model: supply/demand estimation. IV: weather shocks affecting supply but not demand. Compliers: sellers whose supply changes with weather. LATE: effect of supply shifts on prices for weather-induced movers.

Minimum Wage Border Discontinuity (Not strictly IV but similar logic.) Instrument: geographic assignment to high vs. low minimum-wage regions. LATE: effect on firms located near the border whose wages are determined by location.

F. How IV Fits into GMM

Tasks: Derive GMM estimator, show 2SLS as GMM, efficient GMM under heteroskedasticity, show OLS as special case.

Answer:

Moment condition:

$$E[Z_i(Y_i - X_i'\beta)] = 0.$$

Sample moment:

$$g_n(\beta) = \frac{1}{n}Z'(Y - X\beta).$$

GMM estimator:

$$\hat{\beta}_{\text{GMM}} = (X'ZWZ'X)^{-1}X'ZWZ'Y.$$

2SLS as GMM: Choosing $W = (Z'Z)^{-1}$ yields

$$\hat{\beta}_{2SLS} = (X'P_Z X)^{-1} X' P_Z Y,$$

where $P_Z = Z(Z'Z)^{-1}Z'$.

Efficient GMM: With heteroskedasticity, optimal $W = \Omega^{-1}$, where

$$\Omega = E[Z_i Z_i' \varepsilon_i^2].$$

OLS as GMM: Setting $Z_i = X_i$ gives moment condition $E[X_i(Y_i - X_i'\beta)] = 0$, which yields

$$\hat{\beta}_{OLS} = (X'X)^{-1} X'Y.$$