Econ 640 - Problem Set 5

Instructions: *Submit your individual answers in printed form. This problem set is due **at the beginning of class on **December 11*****.

1. Consider the structural model
$$Y_i = \beta X_i + u_i,$$
where X_i is endogenous. Suppose an instrument Z_i is available.

   (a) **IV Assumptions**
   State and briefly explain the two key assumptions required for a valid instrument:
      1. Relevance
      2. Exogeneity (Exclusion Restriction)

   (b) **Population and Sample IV Estimator**
   Write the population moment condition that defines the IV estimator. Then write the sample IV estimator for the case with a single instrument.

   (c) Suppose Z_i is binary, what causal effect does the IV estimator identify? State the name of this causal parameter using the appropriate terminology. Then, propose a concrete real-world example (e.g., education, health, labor, public policy) in which:
      1. the instrument is binary,
      2. the treatment changes only for a subset of individuals,
      3. and you clearly identify who the "compliers" are in your example.

   Briefly explain why the IV estimator in your example identifies the causal effect for this group.

2. Suppose a model implies moment conditions
$$\mathbb{E}[m(W_i, \theta_0)] = 0,$$
with sample analog
$$g_n(\theta) = \frac{1}{n}\sum_{i=1}^{n} m(W_i, \theta).$$
The GMM estimator solves
$$\hat{\theta}_{GMM} = \arg\min_{\theta} \; g_n(\theta)' W_n g_n(\theta),$$
where W_n is a positive definite weighting matrix.

(a) **GMM Intuition**
Explain the intuition behind GMM. What is the estimator trying to match, and why?

(b) **OLS as a Special Case of GMM**
Show that OLS is a special case of GMM by specifying:

1. the moment condition $m_i(\beta)$,
2. the sample moment $g_n(\beta)$,
3. the choice of weighting matrix W_n that yields the OLS estimator.

(c) **IV / 2SLS as a Special Case of GMM**
Show that IV (2SLS) is a special case of GMM by specifying:

1. the moment condition using instruments Z_i,
2. the sample moment,
3. the choice of weighting matrix W_n that yields the 2SLS estimator.

(d) **Unified Framework Interpretation**
In one or two sentences, explain how different choices of moment conditions and weighting matrices allow GMM to unify OLS, GLS, IV, and efficient GMM.