1.

a) $y_{i+1} = y_i + hf(t_i, y_i)$, $f(t,y) = 1 + (y/t) + (y/t)^2$

i	t_i	y_i	$f(t,y)$	$y_{i+1} = y_i + hf(t,y)$
0	1	0	$1 + 0/1 + (0/1)^2 = 1$	$0 + 0.1 \cdot 1 = 0.1$
1	1.1	0.1	$1 + \frac{0.1}{1.1} + (\frac{0.1}{1.1})^2 = 1.0992$	$0.1 + 0.1 \cdot 1.0992 = 0.20992$
2	1.2	0.20992	$1 + \frac{0.20992}{1.2} + (\frac{0.20992}{1.2})^2 = 1.2055$	$0.20992 + 0.1 \cdot 1.2055 = 0.33047$
3	1.3	0.33047	1.3188	$0.33047 + 0.1 \cdot 1.3188 = 0.46235$
4	1.4	0.46235	1.4393	0.60628
5	1.5	0.60628	1.5695	0.76303
6	1.6	0.76303	1.7043	0.93346
7	1.7	0.93346	1.8506	1.11852
8	1.8	1.11852	2.0095	1.31929
9	1.9	1.31929	2.1965	1.53692
10	2.0	1.53692	2.3589	1.99284

Actual value: 1.66128 , Euler's method : 1.53692

b) $f(t,y) = 1 + y/t + (y/t)^2$, $f'(t,y) = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial y} \underbrace{y'}_{f(t,y)} = \frac{y'}{t} - \frac{y}{t^2} + \frac{2y}{t^3}(y't - y)$

i	t_i	y_i	$f(t,y)$	$f'(t,y)$	$y_{i+1} = y_i + hf(t,y) + \frac{h^2}{2} f'(t,y)$
0	1	0	1	1	0.105
1	1.1	0.105	1.1046	1.0925	0.2209
2	1.2	0.2209	1.218	1.1788	0.3486
3	1.3	0.3486	1.3401	1.2668	0.4887

i	x_i	y_i	$f(x_i,y_i)$	$f'(x_i,y_i)$	$y_{i+1}=y_i+hf(x_i,y_i)+\frac{h^2}{2}f'(x_i,y_i)$
4	1.4	0.4889	1.4712	1.3612	0.6428
5	1.5	0.6428	1.6122	1.4654	0.8113
6	1.6	0.8113	1.7642	1.5825	0.9956
7	1.7	0.9956	1.9286	1.7153	1.197
8	1.8	1.197	2.1092	1.8668	1.417
9	1.9	1.417	2.302	2.041	1.657
2	2.0	1.657			

by Taylor's method: $y=1.657$ > exact value: 1.66128

2, RK4: $k_1=h\cdot f(x_n,u_n)$

$k_2=h\cdot f(x_n+\frac{h}{2},u_n+\frac{k_1}{2})$

$k_3=h\cdot f(x_n+\frac{h}{2},u_n+\frac{k_2}{2})$

$k_4=h\cdot f(x_n+h,u_n+k_3)$

$u_{n+1}=u_n+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$

經程式計算

$h=0.05$ > when $t=0.1$ => RK4=[1.7269, -0.8326] > exact=[1.79306, -1.032]

$h=0.1$, when $t=0.1$ => RK4=[-3.052, 8.983] > exact=[1.79306, -1.032]

h=0.1的error非常大 > 須縮小h使解的error小一點