贝叶斯滤波器

定义以下运动和观测模型:

运动方程: xk=f(xk-1,vk,wk),k=1,···,K观测方程: yk=g(xk,nk),k=0,···,K\begin{aligned}&\text{运动方程: }\boldsymbol{x}_{k}-{k}=fleft(\boldsymbol{x}_{k}-{k-1}, \boklsymbol{v}_{k}\boklsymbol{w}_{k}\boklsymbol{w}_{k}\boklsymbol{k}, \quad k=1, \cdots, K \&\text{观测方程: }\boklsymbol{y}_{k}=g\left(\boklsymbol{x}_{k}), \boklsymbol{x}_{k}\boklsymbol{n}_{k}\b 程: xk = f(xk-1, vk, wk), $k = 1, \dots, K$ 观测方程: yk = g(xk, nk), $k = 0, \dots, K$

- kkk 为时间下标,最大值为 KKK
- 函数 $f(\cdot)$ $f(\cdot)$ 为非线性的运动模型
- 函数 g() g(cdot) g() 为非线性的观测模型

贝叶斯滤波仅使用过去以及当前的测量,通过构造一个完整的概率密度函数PDF,来计算当前状态 x k \boklsymbol{x} {k} xk 的置信度:

 $P\left(x \mid x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (1) P \left(x \mid x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (2) P \left(x \mid x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (3) P \left(x \mid x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (4) P \left(x \mid x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (4) P \left(x \mid x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (5) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (6) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (6) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (7) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (8) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (8) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (8) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (8) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (8) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (8) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (9) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (9) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (9) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (9) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (9) P \left(x \mid x \mid 0, v \mid 1 \mid k, y \mid 0 \mid k\right) \\ (9) P \left(x \mid x \mid 0, v \mid 1 \mid k\right) \\ (9) P \left(x \mid$

其中, $x \, \hat{} \, 0 \, \text{check} \, \{ \text{boldsymbol} \, \{x\} \}_{0} \, x \, \hat{} \, 0 \,$ 表示 $0 \, \text{时刻系统状态的先验}.$

贝叶斯滤波器可以用下面公式表示:

p(xk | x 0, v1:k, y0:k)□后验置信度 = ηp(yk | xk)□利用 g(·) 进行更新 ∫p(xk | xk-1, vk)□利用 f(·) 进行预测 p(xk-1 | x 0, v1:k-1, v0:k-1)□先验置信度 dxk-1(2) | Negin [aligned] & \underbrace [p\left[\boldsymbol[x]_{k}] \underbrace [p\left[\bold 1}\end{aligned}\tag{2} =后验置信度

p(xk | x ˙0, vl :k, y0:k)η利用 g(·) 进行更新

p(xk | xk-1,vk)先验置信度

 $p(xk-1 \mid x^0, v1:k-1, v0:k-1)dxk-1(2)$

推导过程

对(1)式进行贝叶斯展开,有:

 $p(xk \mid x^*0,v1:k,y0:k) = p(xk,x^*0,v1:k,y0:k) p(x^*0,v1:k,y0:k) = p(yk \mid xk,x^*0,v1:k,y0:k-1) p(xk \mid x^*0,v1:k,y0:k-1) p(x^*0,v1:k,y0:k-1) p(yk \mid x^*0,v1:k,y0:k-1) p(yk$ $\{p \setminus \{b \mid (k_1, k_2, k_3), (k_3, k_4)\} \} = \{p \mid (k_1, k_2, k_3), (k_3, k_4)\} \} = \{p \mid (k_2, k_3), (k_3, k_4)\} \} = \{p \mid (k_3, k_4), (k_3, k_4)\} \} = \{p \mid (k_3, k_4), (k_4, k_4)\} \} = \{p \mid (k_3, k_4), (k_4, k_4)\} \} = \{p \mid (k_4, k_4), (k_4, k_4), (k_4, k_4)\} \} = \{p \mid (k_4, k_4), (k_4, k_4), (k_4, k_4), (k_4, k_4)\} \} = \{p \mid (k_4, k_4), (k_4, k_4), (k_4, k_4), (k_4, k_4), (k_4, k_4)\} \} = \{p \mid (k_4, k_4), (k_4, k$

由于 y k \boldsymbol{y} k yk 的状态只与 x k \boldsymbol{x} k xk 相关, 所以上式可化简为:

 $p(xk \mid x^*0, v1:k, y0:k) = p(yk \mid xk)p(xk \mid x^*0, v1:k, y0:k-1)p(yk)(4) \land p(k)(4) \land$ k}\right)\\\&=\frac{p\left(\boldsymbol{y}_{k}\right)\boldsymbol{x}_{k}\ $\{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k) = p(yk)p(yk \mid xk)p(xk \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k, y0:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \} \\ \{p \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \mid (boldsymbol\{y\}_k \mid x'0, v1:k-1)(4) \}$

 $\diamondsuit p-1(yk)=\eta p^{-1}\left(\frac{yk}{p-1}\right)=\eta p^{-1}\left(\frac{yk}{p-1}\right)=\eta$, 则有:

 $p\left(xk \mid x^*0, v1:k, y0:k\right) = \eta\left(yk \mid xk\right)p\left(xk \mid x^*0, v1:k, y0:k-1\right) (5) \left(xk \mid x^*0, v1:k, y0:k-1\right) (6) \left(xk \mid x^*0, v1:k, y0:k-1\right) (6) \left(xk \mid x^*0, v1:k, y0:k-1\right) (7) \left(xk \mid x^*0, v1:k-1\right) (7) \left(xk \mid x^*0, v$ eta p\left(\boldsymbol\{y}_{\ k}\ \mid \boldsymbol\{x}_{\ k}\ \right)\ p\left(\boldsymbol\{x}_{\ k}\ \mid \check \boldsymbol\{x}_{\ k}\ \mid \check \boldsymbol\{y}_{\ k}\ \mid \check \boldsym $p(xk \mid x^0, v1:k, y0:k) = \eta p(yk \mid xk) p(xk \mid x^0, v1:k, y0:k-1)(5)$

由全概率公式和贝叶斯公式有:

 $p(xk \mid x^*0, v1:k, y0:k-1) = \int p(xk, xk-1 \mid x^*0, v1:k, y0:k-1) \, dxk-1 = \int p(xk \mid xk-1, x^*0, v1:k, y0:k-1) \, p(xk-1 \mid x^*0, v1:k, y0:k-1) \, dxk-1 \, (6) \cdot (6)$ $p(xk \mid x \mid 0, \forall 1.k, y \mid 0.k = 1) = p(xk \mid x \mid 1 \mid x \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0, \forall 1.k, y \mid 0.k = 1) p(xk \mid 1 \mid 0$

然后,由一**阶马尔可夫性假设**(k时刻的状态只和k-1时刻状态有关)有:

 $p\left(x \mid x \mid x \mid 1, x \mid 0, v \mid 1 : k, y \mid 0 : k \mid 1\right) = p\left(x \mid x \mid x \mid 1, v \mid k\right) (7) \\ p\left(x \mid x \mid x \mid 1, v \mid 1, v \mid 1\right) \\ p\left(x \mid x \mid x \mid 1, v \mid 1, v$

 $p\left(x \\ k-1 \\ + x^*0, \\ v1: \\ k, y0: \\ k-1\right) = p\left(x \\ k-1 \\ + x^*0, \\ v1: \\ k-1, \\ y0: \\ k-1\right)\left(8\right) \\ \text{with (boldsymbol(x)_{k-1} \\ with (boldsymbol(x)_{k-1} \\ with (boldsymbol(x)_{k-1}) \\ with (boldsymbol(x)_{k-1}) \\ \text{with (boldsymbol(x)_{k-1}) } \\ \text{with (bold$ $plefi(boldsymbol\{x\}_{k-1}\} \cdot (x_0, x_1 + x_0, x_1 + x$

结合(5)、(6)、(7)、(8) 式,即可得到:

 $p(xk \mid x^*0, v1:k, y0:k) = \eta p(yk \mid xk) \\ \int p(xk \mid xk-1, vk) p(xk-1 \mid x^*0, v1:k-1, y0:k-1) \\ dxk-1(9) \\ \text{begin a ligned } \\ \& p \\ \text{left(boldsymbol}\{x\}_{k} \\ \text{wrid \check \boldsymbol}\{x\}_{k} \\ \text{p(short)} \\$

即证