卡尔曼滤波器

卡尔曼滤波器是贝叶斯滤波器在线性高斯系统下的一个特例。

假设线性高斯系统下运动模型和观测模型如下:

- $xk \in R \ N \ boldsymbol\{x\}_{k} \ in \ mathbb\{R\}^{N} \ xk \in RN \ 表示系统状态$ $vk \in R \ N \ boldsymbol\{v\}_{k} \ in \ mathbb\{R\}^{N} \ yk \in RN \ 表示输入$ $vk \in R \ N \ boldsymbol\{w\}_{k} \ in \ mathbb\{R\}^{N} \ yk \in RN \ xk \in RN \ xk \in RN \ yk \in RN \ xk \in RN \ yk \in RN \ xk \in RN \ yk \in RN \$
- y k ∈ R M \boklsymbol{y} _{k} \in \mathbb{R}^\M} yk ∈ RM 表示測量
 n k ∈ R M \boklsymbol{n}_{k} \in \mathbb{R}^\M nk ∈ RM 表示測量噪声, n k ~ N (0, R k) \boklsymbol{n}_{k} \sim N(\mathbb{R}^\R)_{k} \in \mathbb{R}^\R_{k} \nk ∈ RM 表示测量噪声, n k ~ N (0, R k) \boklsymbol{n}_{k} \sim N(\mathbb{R}^\R_{k} \sim N(\mathbb{R}^\R_{k}), \mathbb{R}^\R_{k} \sim N(\mathbb{R}^\R_{k}) \sim N(\mathbb{R}^\R_{k} \sim N(\mathbb{R}^\R_{k}), \mathbb{R}^\R_{k} \sim N(\mathbb{R}^\R_{k} \sim N(\mathbb{R
- kkk为时间下标,最大值为KKK

用(·) \ check {(\cdot)} (·) 表示先验,用(·) \ hat{(\cdot)} (·) 表示后验,卡尔曼滤波器则可以表示为:

预测

 $x^*k = Ak - 1 x^k - 1 + vk$ check \boldsymbol(x\) \ \{k\} = \boldsymbol(x\) \ \{k-1\} \boldsymbol(x\) \ \\{k-1\} + \boldsymbol(x\) \ \\{k\} \ x^*k = Ak-1x^k-1+vk

 $P^*k = A \\ k - 1 \\ P^*k = A \\$

卡尔曼增益

 $K\ k = P\ k\ C\ k\ T\ (C\ k\ P\ k\ C\ k\ T+R\ k)-1\ (\ boldsymbol\{K\}_{k}=boldsymbol\{P\}_{k}\ boldsymbol\{C\}_{k}^{mathrm}\{T\}\}\ (\ boldsymbol\{C\}_{k}\ boldsymbol\{R\}_{k}\ boldsymbol\{R\}_{k}$

更新

\check{\boldsymbol{x}}_{k}\right)}_{\text {更新量 }} x^k = x^k + Kk更新量

通过贝叶斯推断推导

预测

由于是线性高斯系统,因此直接将 k-1 k-1 k-1 时刻的后验分布通过线性运动模型传递,即可得到 k k k 时刻的先验:

 $x^*k = E[xk] = E[Ak - 1xk - 1 + vk + wk] = Ak - 1E[xk - 1] \\ \square xk - 1 + vk + E[wk] \\ \square 0 = Ak - 1x^k - 1 + vk \\ \text{begin{aligned}} \\ \text{check {boldsymbol}\{x\}} \\ \mathbb{Z}[k] \\ \text{for all } \\ \mathbb{Z}[k] \\ \mathbb$

E[xk-1]+vk+0

 $E[wk] = Ak-1x^k-1+vk$

 $P^*k = E[(xk - E[xk])(xk - E[xk])T] = E[(Ak - 1xk - 1 + vk + wk - Ak - 1x^k - 1 - vk) \times (Ak - 1xk - 1 + vk + wk - Ak - 1x^k - 1 - vk)T] = Ak - 1E[(xk - 1 - x^k - 1)(xk - 1 - x^k - 1)T] \\ = P^*k - 1Ak - 1T + E[wkwkT] \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Ak - 1P^k - 1Ak - 1T + Qk \\ = Qk = Qk - 1P^k - 1Ak - 1T + Qk \\ = Qk = Qk - 1P^k - 1Ak - 1T + Qk \\ = Qk - 1P^k - 1Ak - 1T + Q$ $Elefi[boklsymbol\{x\}_\{k\} right] vight] \\ Vefi(boklsymbol\{x\}_\{k\}-Elefi[boklsym$ Exeti (botksymbol(x) _ (k) - wight pight)ceti (botksymbol(x) _ (k) - botksymbol(x) _ (k-1) - botksymbol(x) _

 $E[(xk-1-x^k-1)(xk-1-x^k-1)T]Ak-1T+Qk$

 $E[wkwkT]Ak-1P^k-1Ak-1T+Qk$

更新

对于更新部分,我们将状态与 k k k 时刻的测量写成联合高斯分布的形式:

 $p\left(xk,yk \mid x^*0,v1:k,y0:k-1\right) = N\left(\left[\mu x\mu y\right],\left[\Sigma xx\Sigma xy\Sigma yx\Sigma yy\right]\right) = N\left(\left[x^*kCkx^*k\right],\left[P^*kP^*kCkTCkP^*kCkT^*kCkT^*k\right]\right) \\ \text{begin (aligned) p\left(boldsymbol(x)_{-}k),}$

由高斯推断可以直接得到 k k k 时刻的条件分布 (即后验概率):

 $p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1\Sigma yx \mid P^k) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1\Sigma yx \mid P^k) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1\Sigma yx \mid P^k) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1\Sigma yx \mid P^k) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1\Sigma yx \mid P^k) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1\Sigma yx \mid P^k) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1\Sigma yx \mid P^k) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1\Sigma yx \mid P^k) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1\Sigma yx \mid P^k) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1\Sigma yx \mid P^k) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \Sigma xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1(yk - \mu y) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \mu xy \Sigma yy - 1(yk - \mu y) \mid x^k, \Sigma xx - \Sigma xy \Sigma yy - 1(yk - \mu y) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \mu xy \Sigma yy - 1(yk - \mu y) \mid x^k \Sigma yy - 1(yk - \mu y) \\ p(xk \mid x^*0, v1:k, y0:k) = N(\mu x + \mu xy \Sigma yy - 1(yk - \mu y) \mid x^k \Sigma yy - 1(yk - \mu y) \\ p(xk \mid x^*0, v1:k) = N(\mu x + \mu xy \Sigma yy - 1(yk - \mu y) \mid x^k \Sigma yy - 1(yk - \mu y) \\ p(xk \mid x^*0, v1:k) = N(\mu x + \mu xy \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \\ p(xk \mid x^*0, v1:k) = N(\mu x + \mu xy \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy - 1(yk - \mu y) \quad x^k \Sigma yy$ $\label{thm:conditional} $$ \operatorname{boldsymbol}(Sigma_{x x}-\operatorname{boldsymbol}(Sigma_{x y}-\operatorname{boldsymbol}(Sigma_{y y})^{-1} \operatorname{boldsymbol}(Sigma_{y x})_{\operatorname{thm}(\operatorname{boldsymbol}(P}_{x y}-\operatorname{boldsymbol}(Sigma_{y y})^{-1}) \operatorname{boldsymbol}(Sigma_{y x})_{\operatorname{thm}(\operatorname{boldsymbol}(P}_{x y}-\operatorname{boldsymbol}(Sigma_{y y})^{-1}) \operatorname{boldsymbol}(Sigma_{y x})_{\operatorname{thm}(\operatorname{boldsymbol}(P)_{x y}-\operatorname{boldsymbol}(Sigma_{y y}))} \\$

 $\mu x + \Sigma xy \Sigma yy - 1 (yk - \mu y), P^k$

 $\Sigma xx - \Sigma xy \Sigma yy - 1 \Sigma yx$

\boklsymbol{\C}_{\k}^{\mathrm{T}}\\eft\(\chi\)\chig\(\c 益),则有:

 $x^k = x^k + K k (yk - Ckx^k) \cdot \{k\} = x^k + K k (yk - Ckx^k) \cdot \{k\} \cdot \{k$ $x^k = x^k + Kk(yk - Ckx^k)$

 $P^k = (1 - K \ k \ C \ k) \ P^k \ \text{hat} \ \text{holdsymbol} \ \{P\}_{k} = \text{heff}(\text{mathbf} 1\} - \text{holdsymbol} \ \{K\}_{k} \ \text{holdsymbol} \ \{K\}_{k} \ P^k = (1 - K \ k \ C \ k) \ P^k = (1 - K \ k \ C \ k$

即证。