# 扩展卡尔曼滤波EKF

如果将置信度和噪声限制为高斯分布,并且对运动模型和观测进行线性化,计算贝叶斯滤波中的积分(以及归一化积),即可得到扩展卡尔曼滤波(EKF)。

 $p\left(x \mid x \uparrow 0, v \mid 1 : k, y \mid 0 : k\right) = N\left(x \land k, P \land k\right) \\ p\left(x \mid k \mid x \uparrow 0, v \mid 1 : k, y \mid 0 : k\right) \\ p\left(x \mid k \mid x \downarrow 0, v \mid 1 : k, y \mid 0 : k\right) \\ p\left(x \mid k \mid x \mid 0, v \mid 1 : k, y \mid 0 : k\right) \\ p\left(x \mid k \mid x \mid 0, v \mid 1 : k, y \mid 0 : k\right) \\ p\left(x \mid k \mid x \mid 0, v \mid 1 : k, y \mid 0 : k\right) \\ p\left(x \mid k \mid x \mid 0, v \mid 1 : k, y \mid 0 : k\right) \\ p\left(x \mid k \mid x \mid 0, v \mid 1 : k, y \mid 0 : k\right) \\ p\left(x \mid k \mid x \mid 0, v \mid 1 : k, y \mid 0 : k\right) \\ p\left(x \mid k \mid x \mid 0, v \mid 1 : k, y \mid 0 : k\right) \\ p\left(x \mid k \mid x \mid 0, v \mid 1 : k, y \mid 0 : k\right) \\ p\left(x \mid k \mid x \mid 0, v \mid 1 : k\right) \\ p\left(x \mid k \mid x \mid 0, v$ 

其中, x^k \hat{\boldsymbol{x}} {k} x^k 为均值, P^k \hat{\boldsymbol{P}} {k} P^k 为协方差。

并且假设噪声变量 w k \boldsymbol{w}\_{k} wk 和 n k \boldsymbol{n}\_{k} nk 也是高斯分布的:

 $w \ k - N \ (0 \ , Q \ k) \ n \ k - N \ (0 \ , R \ k) \ begin \{array\} \ \{l\} \ boldsymbol\{w\}_{k} \ bim \ mathcal\{N\} \ bif(mathbf\{0\}, boldsymbol\{Q\}_{k} \ bim \ mathcal\{N\} \ bif(mathbf\{0\}, boldsymbol\{Q\}_{k} \ bim \$  $\label{eq:continuous} $$ \boldsymbol{R}_{k} \rightarrow \boldsymbol{k} \cdot \boldsymbol{k} \cdot$ 

x = f(x + 1, v + w + k) y + g(x + k, v + k) when y + g(x + $\label{eq:local_symbol} $$ \left\{ n \right\}_{k} \right\} = d \left\{ a \text{ ligned} \right\} \ xk = f(xk-1,vk,wk)yk = g(xk,nk)$ 

由于  $f(\cdot)$  boldsymbol{f}(cdot) f(\cdot) 和  $g(\cdot)$  boldsymbol{g}(cdot)  $g(\cdot)$  是非线性函数,所以我们需要对其进行线性化。**在当前状态的均值处**展开,对运动和观测模型进行线性化:

 $g\left(x\,k\,,n\,k\right)\approx y^*k+G\,k\left(x\,k-x^*k\right)+n\,k\,'\lookdsymbol\{g\}\lefi(\lookdsymbol\{x\}\_\{k\},\lookdsymbol\{n\}\_\{k\}\lefi(\lookdsymbol\{y\}\}\_\{k\}+\lookdsymbol\{G\}\_\{k\}\lefi(\lookdsymbol\{x\}\_\{k\},\lookdsymbol\{n\}\_\{k\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lookdsymbol\{n\}-\lo$  $\label{eq:check-prime} $$ \left( boldsymbol\{x\} \right)_{k} \right) + boldsymbol\{n\}_{k}^{\infty} g(xk,nk) \approx y^k + Gk(xk-x^k) + nk$ 

- $\bullet \ y \ k = g(x \ k \ , 0) \ (\text{hock} \ (\text{hoklsymbol}\{y\} \ \{k\} = \text{hock} \ (\text{hoklsymbol}\{x\} = \text{hock} \ (\text{hoklsymbol}\{x\} = \text{hock} \ (\text{hoklsymbol}\{x\} \ \{k\} = \text{hock} \ (\text{hoklsymbol}\{x\} = \text{hock} \ (\text{hock} \ (\text{hoklsymbol}\{x\} = \text{hock} \ (\text{hock} \ (\text{hock}$  $\mathsf{Mathbf}\{0\}\$   $\mathsf{Gk} = \partial \mathsf{xk} \partial \mathsf{g}(\mathsf{xk},\mathsf{nk}) \mid \ \ | \ \ \mathsf{x}^*\mathsf{k},0$
- $nk = \partial g\left(xk, nk\right) \partial nk + x^*k, 0 nk^* boldsymbol\{n\}_{k}^{\text{prime}}=\left(\frac{x}{nk}\right) boldsymbol\{g\} \left(\frac{x}{nk}, boldsymbol\{n\}_{k}^{\text{prime}}\right) \left(\frac{x}{nk}, bo$

给定过去的状态和最新输入,则当前状态  $x \times boldsymbol\{x\}_{k} x \times box计学特性为$ :

 $x \\ k \\ = x \\ k \\ + F \\ k \\ - 1 \\ (x \\ k \\ - 1 \\ - x \\ k \\ - 1 \\ ) \\ + w \\ k \\ \text{boldsymbol}\{x\} \\ \\ \{k \\ - 1\} \\ \text{w.} \\ \text{boldsymbol}\{x\} \\ \\ \{k \\ - 1\} \\ \text{w.} \\ \text{boldsymbol}\{x\} \\ \\ \{k \\ - 1\} \\ \text{w.} \\ \text{boldsymbol}\{x\} \\ \\ \{k \\ - 1\} \\ \text{w.} \\ \text{boldsymbol}\{x\} \\ \\ \text{bo$  $1\} \land w = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk - 1 - x \cdot k - 1) + wk' = x \cdot k + F \cdot k - 1 \cdot (xk -$ 

 $E\left[x\,k\right] \approx x^*k + F\,k - 1\left(x\,k - 1 - x^k - 1\right) + E\left[w\,k'\right] = 0 \text{ Elleft[boldsymbol}\{x\}_{k}] \left[x\right] \left[x\,k\right] \approx x^*k + F\,k - 1\left(x\,k - 1 - x^k - 1\right) + E\left[w\,k'\right] = 0 \text{ Elleft[boldsymbol}\{x\}_{k}] \left[x\,k\right] \approx x^*k + F\,k - 1\left(x\,k - 1 - x^k - 1\right) + E\left[w\,k'\right] \left[x\,k\right] \approx x^*k + F\,k - 1\left(x\,k - 1 - x^k - 1\right) + E\left[w\,k'\right] \left[x\,k\right] = 0 \text{ Elleft[boldsymbol}\{x\}_{k}] \left[x\,k - 1\right] \left[x\,k\right] = 0 \text{ Elleft[boldsymbol}\{x\}_{k}] \left[x\,k - 1\right] \left[x\,k\right] = 0 \text{ Elleft[boldsymbol}\{x\}_{k}] \left[x\,k - 1\right] \left[x\,k\right] \left[x\,k\right] = 0 \text{ Elleft[boldsymbol}\{x\}_{k}] \left[x\,k - 1\right] \left[x\,k\right] \left[x\,k\right]$ 

 $E\left[\left(x\,k-E\left[x\,k\right]\right)(x\,k-E\left[x\,k\right])T\right] \approx E\left[w\,k'\,w\,k'\,T\right] \ \square \ Q\,k'\,E.left[left] \ heft[left] \ heft[left] \ heft[left] \ heft] \ heft[left] \ heft] \ heft[left] \ heft] \ heft] \ heft[left] \ heft] \ heft] \ heft[left] \ heft] \$ 

## E[wk'wk'T]

 $p\left(x \ | \ x \ k-1 \ , v \ k\right) \approx N\left(x^* \ k+F \ k-1 \ (x \ k-1-x^k-1) \ , Q \ k'\right) \\ p\left(x \ | \ x \ k-1 \ , v \ k\right) \approx N\left(x^* \ k+F \ k-1 \ (x \ k-1-x^k-1) \ , Q \ k'\right) \\ p\left(x \ | \ x \ k-1 \ , v \ k\right) \approx N\left(x^* \ k+F \ k-1 \ , v \ k\right) \\ \left(x \ | \ x \ k-1 \ , v \ k\right) \approx N\left(x^* \ k+F \ k-1 \ , v \ k\right) \\ \left(x \ | \ x \ k-1 \ , v \ k\right) \approx N\left(x^* \ k+F \ k-1 \ , v \ k\right) \\ \left(x \ | \ x \ k-1 \ , v \ k\right) \approx N\left(x^* \ k+F \ k-1 \ , v \ k\right) \\ \left(x \ | \ x \ k-1 \ , v \ k\right) \approx N\left(x^* \ k+F \ k-1 \ , v \ k\right) \\ \left(x \ | \ x \ k-1 \ , v \ k\right) \approx N\left(x^* \ k+F \ k-1 \ , v \ k\right) \\ \left(x \ | \ x \ k+F \$  $\label{thm:prop:linear:linea$  $p(xk \mid xk-1, vk) \approx N(x^*k+Fk-1(xk-1-x^*k-1), Qk')$ 

给定当前状态,则当前观测 y k \check{\boldsymbol{v}} {k} v k 的统计学特性为:

 $y \ k \approx y^* \ k + G \ k \ (x \ k - x^* \ k) + n \ k' \ boldsymbol\{y\}_{\{k\}} \ approx \ check \ boldsymbol\{y\}_{\{k\}} + boldsymbol\{G\}_{\{k\}} \ boldsymbol\{x\}_{\{k\}} + boldsymbol\{x\}_$  $yk \approx y k + Gk(xk - x k) + nk'$ 

 $E\left[\,y\,k\,\right] \approx y^*\,k + G\,k\,(\,x\,k\,-\,x^*\,k\,) \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{y\}_{k}\right] \ approx\ \check\ \boldsymbol\{y\}_{k}+\boldsymbol\{G\}_{k}\eff(\boldsymbol\{x\}_{k}-\check\boldsymbol\{x\}_{k}+\check\,\{x\,k\,-\,x^*\,k\,) \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,) \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,) \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,) \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,) \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,) \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,) \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsymbol\{x\}_{k}-\check\,\{x\,k\,-\,x^*\,k\,\} \\ + E\left[\,n\,k^\prime\,\right] \ \square \ 0 \ Eleff(\boldsym$ 

 $E\left[\left(yk-E\left[yk\right]\right)\left(yk-E\left[yk\right]\right)T\right]\approx E\left[nk'nk'T\right] \\ = Rk' \\ \text{Eleft[boldsymbol\{y\}\_\{k\}-Eleft[boldsymbo$ 

# E[nk'nk'T]

## 由贝叶斯滤波器有:

 $p(xk \mid x^*0, v1:k, y0:k) □ 后验置信度 = \eta p(yk \mid xk) □ 利用 g(\cdot) 进行更新 \int p(xk \mid xk-1, vk) □ 利用 f(\cdot) 进行预测 p(xk-1 \mid x^*0, v1:k-1, y0:k-1) □ 先验置信度 dxk-1 \end{boldsymbol} \{x\}_{0}, \end{boldsymbol} \{y\}_{0:k} \end{boldsymbol} \{y\}_{0:k} \end{boldsymbol} \{x\}_{k} \end{boldsymbol} \{x\}_$ \text {进行预测}} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}) \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace} \underbrace} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace} \underbrace} \underbrace} \underbrace{p\left(boldsymbol{x}\_{k-1} \underbrace}} \underbrace} \und 1}\end{aligned} = 后验置信度

p(xk | x ˙0, vl:k, y0:k)η利用 g(·) 进行更新

p(yk | xk) 利用 f(·) 进行预测

p(xk | xk-1,vk)先验置信度

 $p(xk-1 \mid x^0, v1:k-1, y0:k-1)dxk-1$ 

将线性化之后的运动和观测模型代入到贝叶斯滤波器中,有:

 $p\left(xk \mid x^*0, v1:k, y0:k\right) \\ \square N\left(x^*k, P^*k\right) \\ = \eta p\left(yk \mid xk\right) \\ \square N\left(y^*k + Gk\left(xk - x^*k\right), Rk'\right) \\ \times \int p\left(xk \mid xk - 1, vk\right) \\ \square N\left(x^*k + Fk - 1\left(xk - 1 - x^*k - 1\right), Qk'\right) \\ p\left(xk - 1 \mid x^*0, v1:k - 1, yk\right) \\ \square N\left(x^*k - 1, P^*k - 1\right) \\ \square N\left(x^$ 

 $\label{eq:linear_part} $$1$ \rightarrow \frac{k}^{\text{prime}}\right] \widerbrace_{p}\left(\frac{k-1}{k-1}\right) \widerbrace_$ 

 $1 \right) - \{ \max\{N\} \left( N\} \left( N\} \left( N\} \left( N\} \left( N\} \right) \right) - \{ N\} \left( N\} \left( N\} \left( N\} \left( N\} \right) \right) \right) - \{ N\} \left( N\} \left( N\} \left( N\} \left( N\} \right) \right) - \{ N\} \left( N\} \left( N\} \left( N\} \left( N\} \right) \right) - \{ N\} \left( N\} \left( N\} \left( N\} \left( N\} \left( N\} \right) \right) \right) - \{ N\} \left( N\} \left( N\} \left( N\} \left( N\} \left( N\} \right) \right) - \{ N\} \left( N\} \left( N\} \left( N\} \left( N\} \left( N\} \left( N\} \right) \right) \right) - \{ N\} \left( N\right) \left( N\right) \left( N\right) \right) \right) \right) \right) - \{ N\} \left( N\right) \left( N\right) \left( N\right) \left( N\right) \left( N\right) \right) \right) - \{ N\} \left( N\} \left( N\} \left( N\} \left( N\right) \left( N\right)$ 

 $\underbrace{p(xk \mid xk-1,vk)N(x^{\wedge}k-1,P^{\wedge}k-1)}$ 

p(xk-1 | x\*0,v1:k-1,y0:k-1)dxk-1

将服从高斯分布的变量传入到非线性函数中,积分之后仍然服从高斯分布:

 $p(xk \mid x^*0, v1:k, y0:k) \mid N(x^k, P^k) = \eta p(yk \mid xk) \mid N(y^*k + Gk(xk - x^*k), Rk') \times \int p(xk \mid xk - 1, vk) p(xk - 1 \mid x^*0, v1:k - 1, y0:k - 1) dxk - 1 \mid N(x^*k, Fk - 1P^k - 1Fk - 1T + Qk') begin{array} {|| underbrace {p\left(boldsymbol{x}_{k} \mid vid) boldsymbol{x}_{k} \mid vid} boldsymbol{y}_{1:k}, bo$ 

 $p(xk \mid x^*0, v1:k, y0:k) = \eta N(y^*k + Gk(xk - x^*k), Rk')$ 

 $p(yk \mid xk) \times N(x^*k, Fk-1P^k-1Fk-1T+Qk')$ 

再利用高斯概率密度函数的归一化积的性质,有:

 $p(xk \mid x^*0,v1:k,y0:k) \ \ \, | \ \ (x^k,P^k) = \eta p(yk \mid xk) \ \ \, | \ \ (x^k,k-1,vk) \ \, | \ \ (x^k,k-1,vk)$ 

 $p(xk + x^{-}0, v1:k, y0:k) = N(x^{-}k + Kk(yk - y^{-}k), (1 - KkGk)(Fk - 1P^{-}k - 1Fk - 1T + Qk'))$ 

 $\eta p(yk \mid xk) \int p(xk \mid xk-1, vk) p(xk-1 \mid x'0, v1:k-1, y0:k-1) dxk-1$ 

其中,  $K \ k \ boldsymbol\{K\}_{\{k\}} \ Kk \ 为卡尔曼增益。比较上面式子的左右两侧,有:$ 

### 预测

 $x^*k = f(x^k-1,vk,0) \cdot (k+1), vk,0) \cdot (k+1), vk+1 = f(x^k-1,vk,0) \cdot (k+1), vk+1 = f(x^k-1,vk,0$ 

## 卡尔曼增益

 $K \ k = P \ k \ G \ k \ T \ (G \ k \ P \ k \ G \ k \ T + R \ k') - 1 \ boldsymbol\{K\} \ \{k\} = \ boldsymbol\{P\} \ \{k\} \ boldsymbol\{G\} \ \{k\} \ boldsymbol\{G\} \ \{k\} \ boldsymbol\{G\} \ \{k\} \ \{k\} \ boldsymbol\{G\} \ \{k\} \ \{k\}$ 

### 更新

 $x^k = x^k + K \\ k (y k - g(x^k, 0)) \Box$  更新量  $\hat{x}_{k} = \hat{x}_{k} = \hat{x}_{k} + \hat{y}_{k} = \hat{x}_{k} + \hat{y}_{k} = \hat{x}_{k} + \hat{y}_{k} + \hat{y}_{k} + \hat{y}_{k} = \hat{x}_{k} + \hat{y}_{k} + \hat{y}_{k}$ 

 $(yk - g(x^*k, 0))$ 

 $P \wedge k = (1 - K \ k \ G \ k \ ) \ P \wedge k + \text{lat}\{boldsymbol\{P\}\}_{\{k\}} = \text{left}(\text{mathbf}\{1\}-boldsymbol\{K\}_{\{k\}} \ boldsymbol\{G\}_{\{k\}} \ \text{left}(\text{mathbf}\{1\}-boldsymbol\{G\}_{\{k\}} \ \text{left}(\text{math$