*Proceedings of ISSS-7*, 26-31 March, 2005

# One-dimensional Electromagnetic Particle Code

Yoshiharu Omura

*Research Institute for Sustainable Humanosphere, Kyoto University*
*omura@rish.kyoto-u.ac.jp*

The basic concept and techniques of the one-dimensional electromagnetic particle code:KEMPO1 [Omura and Matsumoto, 1993] are reviewed briefly. Special characteristics of the code are enhanced electrostatic thermal fluctuations that often interfere the physical processes to be reproduced in the particle code. From a simplified analysis on the fluctuations, a criteria for the grid spacing is given. A modification of the KEMPO1 for solution of the relativistic equation of motion is also described. Since the essential part of the code is very simple and short, the KEMPO1 code is implemented with the MATLAB software, using the powerful graphic library. Explanation of the input parameters for the relativistic KEMPO1/MATLAB code and several examples of the applications are given.

## 1. Basic Equations and Computation Methods

Electromagnetic processes in space plasmas are governed by Maxwell's equations:

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} \tag{1}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{2}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_o} \tag{3}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{4}$$

where $\boldsymbol{J}$, $\rho$, $c$, $\varepsilon_o$, and $\mu_0$ are the current density, charge density, light speed, electric permittivity, and magnetic permeability, respectively. In simulations the values of the permittivity $\varepsilon_0$ and permeability $\mu_0$ can be defined arbitrarily, as far as they satisfy the relation

$$\varepsilon_0 \mu_0 = \frac{1}{c^2} \tag{5}$$

In the KEMPO1, for simplicity, we adopt the following definition

$$\varepsilon_0 = 1, \qquad \mu_0 = \frac{1}{c^2}$$

We solve Maxwell's equations for the electric field $\boldsymbol{E} \equiv E_x, E_y, E_z)$ and magnetic field $\boldsymbol{B} \equiv (B_y, B_z)$ in a one-dimensional system. It is noted that $B_x$ is a constant in the one-dimensional system because of (4). In a vacuum without any charged particles, we have $\boldsymbol{J} = 0$. The set of Maxwell's equations (1) and (2) are solved by the standard FDTD (Finite Difference Time Domain) method, which has been widely used in the various field of radio science.

We introduce two sets of spatial grid systems along the x-axis. One is a full-integer grid system defined at $i\Delta x$ $(i = 1, 2, 3, ..., Nx)$ and the other is a half-integer grid system at $(i + 1/2)\Delta x$. We define $E_y$, $B_y$, $J_y$, and $\rho$ on the full-integer grids, and $E_x$, $E_z$, $B_z$, $J_x$ other on the half-integer grids. We replace spatial and time derivatives in Maxwell's equations with centered differences by $\Delta x$ and the time step $\Delta t$. Because of the centered difference scheme, the dispersion relation of electromagnetic waves in vacuum $\omega^2 = c^2 k^2$ is replaced by the modified dispersion relation

$$\Omega^2 = c^2 K^2 \tag{6}$$

where $\Omega$ and $K$ are given by

$$\Omega = \frac{\sin(\omega \Delta t/2)}{\Delta t/2}, \qquad K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

This gives the Courant condition for the time step and the grid spacing,

$$c\Delta t < \Delta x \tag{7}$$

If the Courant condition is violated, the electromagnetic field grows exponentially because of the imaginary part of the solution for $\omega$.

In the presence of charged particles, we need to compute the charge density and the current density to incorporate their effects on the electromagnetic field. The charge density $\rho_i$ on a grid point at $x = X_i$ is calculated by

$$\rho_i = \frac{1}{\Delta x}\sum_j^{N_p} q_j W(x_j - X_i) \tag{8}$$

where $W_x$ is a particle shape function given by

$$\begin{aligned} W(x) &= 1 - \tfrac{|x|}{\Delta x},\ |x| \le \Delta x \\ &= \quad 0, \qquad |x| > \Delta x \end{aligned} \tag{9}$$

The initial electric field $E_x$ is calculated from (3) in the difference form

$$\frac{E_{x,i+1/2} - E_{x,i-1/2}}{\Delta x} = \frac{\rho_i}{\varepsilon_0} \tag{10}$$

where $\varepsilon_0$ is the electric permittivity.

The current density $J_x$ is calculated based on the charge conservation method [Villasenor and Buneman, 1992; Umeda et al., 2003] satisfying the continuity equations of the charge,

$$J^{t+\Delta t/2}_{x,i+1/2} - J^{t+\Delta t/2}_{x,i-1/2} = -\frac{\Delta x}{\Delta t}(\rho_i^{t+\Delta t} - \rho_i^t) \tag{11}$$

The current densities $J_y$ and $J_z$ are calculated by

$$\boldsymbol{J}_{i+1/2}^{t+\Delta t/2} = \frac{1}{\Delta x}\sum_{j}^{N_p} q_j \boldsymbol{v} W(x_j - X_{i+1/2}) \tag{12}$$

The $J_y$ calculated at the half-integer grids are relocated to the full-integer grids by the following procedure.

$$J_{y,i} = \frac{J_{y,i-1/2} + J_{y,i+1/2}}{2} \tag{13}$$

With these components of the current density $\boldsymbol{J}$, we can trace time evolution of electromagnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ by solving Maxwell's equations with the FDTD method. Since the current density exactly satisfies the continuity equation of the charge density, $E_x$ updated by the current density $J_x$ automatically satisfy (10), if the difference equation is satisfied initially.

When we have an electron beam or an ion beam flowing with a drift velocity, we have spatially uniform components of $J_x$, $J_y$, or $J_z$. The uniform components give rise to uniformly oscillating solutions of $E_x$, $E_y$, or $E_z$ in the one-dimensional periodic system. Such uniform oscillations are not of our interest, and they strongly disturb physical processes we want to reproduce in the simulation system. Therefore, we calculate the uniform components by taking spatial averages of the current densities $J_x$, $J_y$, and $J_z$, and subtract them from current densities on each grid point. Since the uniform component of $J_x$ does not change the charge density as is obvious from the continuity equation, the cancellation of the uniform component does not affect the electrostatic solution.

The equations of motion for a particle with a charge $q$ and a mass $m$ are the following.

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{q}{m}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{14}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v_x \tag{15}$$

The equation (14) is solved by the Buneman-Boris method [Hockney and Eastwood, 1981; Birdsall and Langdon, 1985]. The advantage of the method is strict conservation of the kinetic energy in calculation of cyclotron motion. Even with a relatively large time step $\omega_c \Delta t$, the kinetic energy is strictly conserved, while the angular cyclotron frequency $\Omega_c$ in the simulation is slightly decreased as the time step becomes large.:

$$\Omega_c = \frac{\tan^{-1}\omega_c \Delta t/2}{\Delta t/2} \tag{16}$$

In most of the applications the decrease can be neglected because $\Omega_c/\omega_c = 0.9967$ with $\omega_c \Delta t = 0.2$. In advancing the velocity $\boldsymbol{v}$ from $t - \Delta t/2$ to $t + \Delta t/2$, we need the electric and magnetic field $\boldsymbol{E}$ and $\boldsymbol{B}$ at the time $t$ at the particle position $x(t)$. We interpolate the field linearly from those values at the adjacent grid points. It is interesting that the linear interpolation can be expressed by the following two equations with the same shape function $W(x)$ used for calculation of the charge and current densities. We use

$$F(x) = \sum_{i=1}^{N_x} F_i W(x - X_i) \tag{17}$$

for the fields defined at the full-integer grids, while we use

$$H(x) = \sum_{i=1}^{N_x} H_{i+1/2} W(x - X_{i+1/2}) \tag{18}$$

for the fields defined at the half-integer grids.

The electrostatic component $E_x$ defined at the half-integer grids, however, has to be relocated to the full-integer grids before the interpolation by the same procedure as in (18). This is to cancel the electrostatic self-force [Matsumoto and Omura, 1985]. Since the source of the electrostatic field is the charge density $\rho$ defined at the full-integer grids, the force interpolation must also be made from the full-integer grids by (17). A particle should not be influenced by the field due to its own charge. Similarly the magnetic field $B_y$ has to be relocated before the interpolation from the full-integer grids to the half-integer grids by

$$B_{y,i+1/2} = \frac{B_{y,i} + B_{y,i+1}}{2} \tag{19}$$

for cancellation of the magnetostatic force due to the current density $J_z$, i.e., Ampere's law:

$$\frac{B_{y,i+1} - B_{y,i}}{\Delta x} = \mu_o J_{z,i+1/2} \tag{20}$$

It is noted that both $J_y$ and $J_z$ are calculated at the half-integer grids. In summary, $E_x$ and $E_y$ are interpolated from the full-integer grids, while $E_z$, $B_y$, and $B_z$ are interpolated from the half-integer grids.

## 2. Relativistic KEMPO1

For high energy particles with velocities close to the speed of light, We have to solve the relativistic equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\boldsymbol{v}) = q\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{21}$$

where $m = \gamma m_0$, $m_0$ is the mass at rest, and $\gamma$ is the Lorentz factor given by

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{22}$$

We define $\boldsymbol{u} = \gamma \boldsymbol{v}$

$$\boldsymbol{u} = \frac{c}{\sqrt{c^2 - |\boldsymbol{v}|^2}}\boldsymbol{v} \tag{23}$$

Solving for $\boldsymbol{v}$, we have

$$\boldsymbol{v} = \frac{c}{\sqrt{c^2 + |\boldsymbol{u}|^2}}\boldsymbol{u} \tag{24}$$

Equation (21) is rewritten as

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \frac{q}{m_0}\left(\boldsymbol{E} + \frac{c}{\sqrt{c^2+|\boldsymbol{u}|^2}}\boldsymbol{u}\times\boldsymbol{B}\right) \tag{25}$$

Defining a modified magnetic field as

$$\boldsymbol{B}_u = \frac{c}{\sqrt{c^2+|\boldsymbol{u}|^2}}\boldsymbol{B} \tag{26}$$

we obtain

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \frac{q}{m_0}\left(\boldsymbol{E} + \boldsymbol{u}\times\boldsymbol{B}_{\boldsymbol{u}}\right) \tag{27}$$

which is the same form with (14). The difference form of the equation is

$$\frac{\boldsymbol{u}^{t+\Delta t/2} - \boldsymbol{u}^{t-\Delta t/2}}{\Delta t} = \frac{q}{m_0}\left(\boldsymbol{E}^t + \frac{\boldsymbol{u}^{t+\Delta t/2} + \boldsymbol{u}^{t-\Delta t/2}}{2}\times\boldsymbol{B}_u^t\right) \tag{28}$$

We can apply the Buneman-Boris method as listed below.

Step 1:

$$\boldsymbol{u}^{t-\Delta t/2} = \frac{c}{\sqrt{c^2-\left|\boldsymbol{v}^{t-\Delta t/2}\right|^2}}\boldsymbol{v}^{t-\Delta t/2} \tag{29}$$

Step 2:

$$\boldsymbol{u}_1^t = \boldsymbol{u}^{t-\Delta t/2} + \frac{q}{m_0}\frac{\Delta t}{2}\boldsymbol{E}^t \tag{30}$$

Step 3:

$$\boldsymbol{B}_u^t = \frac{c}{\sqrt{c^2+|\boldsymbol{u}_1^t|^2}}\boldsymbol{B}^t \tag{31}$$

Step 4:

$$\boldsymbol{u}^{t'} = \boldsymbol{u}_1^t + \frac{q}{m_0}\frac{\Delta t}{2}\boldsymbol{u}_1^t\times\boldsymbol{B}_u^t \tag{32}$$

Step 5:

$$\boldsymbol{u}_2^t = \boldsymbol{u}_1^t + \frac{2}{1+\left(\boldsymbol{B}_u^t\frac{q}{m_0}\frac{\Delta t}{2}\right)^2}\boldsymbol{u}^{t'}\times\boldsymbol{B}_u^t\frac{q}{m_0}\frac{\Delta t}{2} \tag{33}$$

Step 6:

$$\boldsymbol{u}^{t+\Delta t/2} = \boldsymbol{u}_2^t + \frac{q}{m_0}\frac{\Delta t}{2}\boldsymbol{E}^t \tag{34}$$

Step 7:

$$\boldsymbol{v}^{t+\Delta t/2} = \frac{c}{\sqrt{c^2+\left|\boldsymbol{u}^{t+\Delta t/2}\right|^2}}\boldsymbol{u}^{t+\Delta t/2} \tag{35}$$

In (31), $\boldsymbol{B}_u^t$ is computed from $\boldsymbol{u}_1^t$ and $\boldsymbol{B}^t$, because

$$\left|\boldsymbol{u}^{t+\Delta t/2} + \boldsymbol{u}^{t-\frac{\Delta t}{2}}\right| = \left|\frac{\boldsymbol{u}_1^t + \boldsymbol{u}_2^t}{2}\right| \tag{36}$$

and $|\boldsymbol{u}_1^t| = |\boldsymbol{u}_2^t|$.

In the relativistic code, we initialize particle velocities so that the following distribution function is realized in the momentum space $(u_x, u_y, u_z)$

$$f(u_\parallel, u_\perp) \propto \exp(-\frac{(u_\parallel - V_{d\parallel})^2}{2V_{t\parallel}^2} - \frac{(u_\perp - V_{d\perp})^2}{2V_{t\perp}^2}) \tag{37}$$

With $V_{d\perp} = 0$, the distribution function is reduced to the shifted bi-Maxwellian distribution, while a finite $V_{d\perp}$ ($>> V_{t\perp}$) realizes a ring distribution. Using a random number generator for the standard normal distribution, we assign each particle a momentum $\boldsymbol{u} = (u_x, u_y, u_z)$, which is converted to a velocity $\boldsymbol{v} = (v_x, v_y, v_z)$ by (24)

$$\boldsymbol{v} = \boldsymbol{u}/\gamma = \frac{c}{\sqrt{c^2+u_x^2+u_y^2+u_z^2}}\boldsymbol{u} \tag{38}$$

As one of the diagnostics, the kinetic energy of each particle is calculated by

$$\begin{aligned} T_E &= mc^2 - m_0c^2 \\ &= (\gamma-1)m_0c^2 \end{aligned} \tag{39}$$

The sum of the kinetic energy is taken for all particles in the simulation system. We divide the sum by the length of the system to obtain the averaged kinetic energy density. The sums of the electric and magnetic field energies are taken for all grid points forming the simulation system. Dividing them by the number of grid points, we obtain the averaged electric and magnetic field energy densities in the system. We exclude the energy of the static magnetic field $B_o$ from the diagnostics.

## 3. Thermal Fluctuation

Each superparticle in the particle code has a much larger kinetic energy than a real charged particle in real space plasmas. The number of particles in the Debye length is much smaller than that of real space plasmas. This results in enhanced thermal fluctuations of the electrostatic field. The electrostatic field energy density of the thermal fluctuation $F_E$ is given by

$$\begin{aligned} F_E &= \frac{T}{2}\int_{-\infty}^{\infty}\frac{1}{1+k^2\lambda_D^2}\frac{dk}{2\pi} \\ &= \frac{T}{4\lambda_D} \end{aligned} \tag{40}$$

where $T(\equiv mV_t^2)$ is the temperature in the dimension of energy. The thermal energy density, on the other hand, is given by,

$$T_E = \frac{1}{2}nT \tag{41}$$

We obtain the ratio of the electrostatic field energy density to the thermal energy density as expressed in terms of the number of particles per cell and the ratio of the Debye length to the grid spacing.

$$\begin{aligned} \frac{F_E}{T_E} &= \frac{1}{2}\frac{1}{n\lambda_D} \\ &= \frac{1}{2}\frac{N_x}{N_p}\frac{\Delta x}{\lambda_D} \end{aligned} \tag{42}$$

In the presence of a fluctuation $\delta\phi$ of the electrostatic potential, we have the density fluctuation approximated by

$$n \sim n_0 \exp(-\frac{q\delta\phi}{T}) \tag{43}$$

If we insert the relation into Poisson's equation, we have

$$\frac{\partial^2 \delta\phi}{\partial x^2} \sim \frac{q^2 n}{\varepsilon_o T}\delta\phi = \frac{\omega_p^2}{V_t^2}\delta\phi = \frac{1}{\lambda_D^2}\delta\phi \tag{44}$$

Applying Fourier transformation, we obtain

$$(k^2 - \frac{1}{\lambda_D^2})\delta\phi_k \sim 0 \tag{45}$$

In order to have a finite $\delta\phi_k$, we have

$$|k| \sim \frac{1}{\lambda_D} \tag{46}$$

Because of the difference scheme in calculating the spatial derivatives, the wavenumber $k$ is replaced by $K$. Multiplying $\Delta x/2$, we obtain

$$|\sin(k\Delta x/2)| \sim \frac{\Delta x}{2\lambda_D} \tag{47}$$

In order to satisfy the above relation, the following condition must be satisfied.

$$\Delta x < 2\lambda_D \tag{48}$$

A detailed study of thermal fluctuations has been conducted with the KEMPO1 code [Ueda et al., 1994]. If the condition is violated, there arises a nonphysical numerical heating with a large growth rate. The heating continues, and the effective Debye length increases until the condition (48) is satisfied.

## 4. Input Parameters

The original FORTRAN code of KEMPO1 [Omura and Matsumoto, 1993] has been modified to incorporate the relativistic effect, and it has been implemented with the MATLAB software library. The following is the list of parameters, which can be specified through the graphical user interface program. The key parameters are the same as the original KEMPO1 code.

- $DX$: Grid spacing.
- $DT$: Time step.
- $CV$: Speed of light.
- $WC$: Cyclotron frequency of species 1, $\omega_{c1}$. From this cyclotron frequency, we compute the magnitude of the static magnetic field $B_o$ from the relation of $B_o = \omega_{c1}/(q/m)_1$.
- $ANGLE$: Angle between the static magnetic field $\boldsymbol{B}_o$ and the wave Vector $\boldsymbol{k}$. The static magnetic field $\boldsymbol{B}_o$ is taken in the x-y plane.
- $NX$: Number of grid points $N_x$.
- $NTIME$: Number of time steps in a simulation run.
- $NS$: Number of particle species.
- $QM(i)$: Charge-to-mass ratio $q_i/m_i$ of species $i$
- $WP(i)$: Plasma frequency of species $i$ defined by

$$\omega_{pi} = \sqrt{\frac{n_i q_i^2}{m_i}} \tag{49}$$

  where $n_i$, $q_i$, and $m_i$ are number density, charge and mass of species $i$, respectively. In a system with ions as mobile particles, the following charge neutrality condition must be satisfied.

$$\sum_i \rho_i = \sum_i \frac{\omega_{pi}^2}{q_i/m_i} = 0 \tag{50}$$

  In a system with mobile electrons only, the charge density of the background immobile ions is automatically computed to establish the charge neutrality.

- $VPE(i)$; Perpendicular thermal velocity of species $i$.
- $VPA(i)$; Parallel thermal velocity of species $i$.
- $VD(i)$: Drift velocity of species $i$. In combination with the pitch angle $\phi = PCH(i)$, the drift velocities of parallel and perpendicular components are determined.

$$V_{d\perp} = V_d \sin\phi \tag{51}$$

$$V_{d\parallel} = V_d \cos\phi \tag{52}$$

- $PCH(i)$: Pitch angle $\phi$ (degrees) of species $i$ defining parallel and perpendicular drift velocities $V_{d\parallel}$ and $V_{d\perp}$.
- $NP(i)$: Number of superparticles for species $i$ in the simulation system.
- $AJAMP$ : The amplitude of an external current $J_{z,ext}$ placed at the center of the simulation system. When a finite value other than 0 is specified, The cancellation of uniform components of the current densities is suppressed.
- $WJ$ : The frequency of the external current $J_{z,ext}$.
- $IEX$: Control parameter for electrostatic option. If $IEX = 0$, the electrostatic component $E_x$ is not solved. If $IEX = 1$, both electromagnetic and electrostatic components are solved. If $IEX = 2$, the electromagnetic components are not solved. Instead of Maxwell's equations, Poisson's equation is solved along with calculation of the charge density.
- $NPLOT$: Number of diagnostics to be made throughout the simulation run. In the KEMPO1/MATLAB interface windows, we can specify four different diagnostics to be made at a time interval of $\Delta t \times NTIME/NPLOT$. The following diagnostics are available.
  - 2D phase space plot of particles in $(V_x, x)$, $(V_y, x)$, or $(V_z, x)$

- 3D velocity space plot of particles in $(V_x, V_y, V_z)$
- Plot of $E_x(x)$, $E_y(x)$, $E_z(x)$, $B_y(x)$, or $B_z(x)$
- 3D plots of transverse fields $(E_y, E_z)$, $(B_y, B_z)$, and perpendicular particle velocities $(V_y, V_z)$ along the $x$-axis. The transverse scales for velocities, electric fields, and magnetic fields, are normalized by $V_{max}$, $E_{max}$, and $B_{max}$, respectively.
- Wavenumber spectrum of $E_x(k)$, $E_y(k)$, $E_z(k)$, $B_y(k)$, or $B_z(k)$
- $\omega - k$ diagram of $E_x$, $E_y$, $E_z$, $B_y$, and $B_z$.
- Contour plot on $(t, x)$ of $E_x$, $E_y$, $E_z$, $B_y$, and $B_z$.
- Contour plot on $(t, k)$ of $E_x$, $E_y$, $E_z$, $B_y$, and $B_z$.
- Energy history plot of kinetic, electric, magnetic, and total energy densities
- Distribution function of $f(V_x)$, $f(V_y)$, or $f(V_z)$ for electrons and ions.

In the panel of the diagnostics parameters, options of "P.Color" and "Param" can be specified. "P.Color" makes particles plotted in color, and "Param" makes the parameters such as the speed of light, the initial drift velocity, the cyclotron frequency, the plasma frequencies, and the initial velocity distributions plotted in the diagnostics.

## 5. Exercises

We study several examples of particle simulations using the KEMPO1/MATLAB code. Thanks to the advanced capability of personal computers and efficiency of the MATLAB software, we can perform various test runs that facilitate our understanding about some of the basic processes in space plasmas such as wave instabilities and their nonlinear evolutions. In the following, we will go through basic tests of the simulation code with emphasis on its numerical property. In running the code, some of the graphic outputs can be modified interactively at the time of execution. We can stop execution of a simulation run by typing any input key such as "SPACE" key. We then modify the configurations of the figures by clicking "Zoom In","Zoom Out" and "Rotate 3D" icons in the tool bar of the figure window. Especially these tools are very useful in observing particle motions in the "$(V_x, V_y, V_z)$ phase space. We can also read the values of any data points by "Data Cursor". We can resume the simulation run by typing any input key after turning off the tools. To terminate the simulation run, type the "Esc" key.

### 5.1 Electrostatic thermal fluctuations

As we have seen in equation (40), thermal motion of electrons gives a fluctuation of electrostatic field. We check the condition on Debye length and the grid spacing by varying the ratio $\lambda_D/\Delta x = VPA/(WPDX)$. We can find a distinct difference between 0.5 and 0.2 after running the code with $WP = 1$, $DT = 0.1$, $DX = 1$, $NX = 128$, $NS = 1$, $NP = 128$, and $NTIME = 4096$. In both cases, the thermal energy increases due to the electrostatic fluctuations. The rates of the energy increase are, however, clearly different. The energy increase indicates that the fluctuation with the small $\lambda_D/\Delta x$ is nonphysical, resulting in stochastic heating of particles. Since the code can be run with the electrostatic option $IEX = 2$, while it may also be run with the full electromagnetic option $IEX = 1$. The Courant condition (7) must strictly be satisfied, i.e., $CV < DX/DT = 10$. As a test, one may try violation of the condition. It is also useful to confirm that the ratio of the electrostatic energy to the thermal energy is inversely proportional to $NP/NX$. To study coherent wave phenomena in space plasmas by the particle code, we generally need a large number of superparticles.

### 5.2 $\omega - k$ dispersion relations of plasma waves

We have prepared a spectrum analysis program that generates $\omega - k$ diagrams by applying Fourier transformations in space and time. The forward traveling wave and backward traveling waves are separated by positive and negative wavenumbers, respectively. The technique is described in Matsumoto and Omura [1985]. We can reproduce the dispersion relations of various plasma waves by giving enough resolutions in space and time. We need to give an enough number of superparticles to form the Maxwellian velocity distribution functions within one wavelength of a wave of interest. Try the following parameters with the options for diagnostics "Ex(w,k)", "Ey(w,k)", "By(w,k)", and "Energy" for panels 1, 2, 3, and 4, respectively. The code must be executed as the full electromagnetic code with option of $IEX = 1$. A set of interesting $\omega - k$ diagrams can be obtained with $DX = 1$ $NX = 256$, $DT = 0.1$, $NTIME = 1024$, $CV = 8$, $WC = -1$, $NS = 1$, $WP = 2$, $NP = 1024$ and $NPLOT = 1024$. We can reproduce a variety of plasma dispersion relations by varying the propagation angle $ANGLE$ from 0 to 90 degrees. By varying the thermal velocities $VPA = VPE$ from 1 to 4, we can study plasma waves in hot and relativistic plasmas. Execution of the code with these parameters takes a while with Pentium or Athlon processors. We can monitor the execution by the energy plot, but the $\omega - k$ diagrams appears only after completion of the run. It is also noted that dispersion relation of the light mode with its phase velocity approaching to the speed of light is distorted at the high wavenumber range close to the wavenumber $k_{max} = \pi/\Delta X$. This is due to the centered difference scheme as expressed in (6).

### 5.3 Propagation of electromagnetic waves

We can radiate electromagnetic waves from an antenna in a plasma. We have prepared an external current source at the center of the simulation system. The current source is directed in the z-axis direction. Give a finite amplitude to $AJAMP$ to turn on the current source. The current source is mathematically expressed by the Dirac delta-function $\delta$ as

$$J_{z,ext} = J_0 \Delta x \, \delta(x - X_{Nx/2}) \sin(\omega_J t) \qquad (53)$$

where $I_0$ and $\omega_J$ are specified by $AJAMP$ and $WJ$, respectively. The solution of Maxwell's equations with the current (53) in vacuum can be obtained analytically, and we can es-

timate the amplitude by

$$|E_{z,max}| = \frac{J_0 \Delta x}{2c} \tag{54}$$

In a plasma, however, the electrons and ions responds to the field excited by the external current, the various normal modes we have seen in the $\omega - k$ diagrams can be excited depending on the frequency $\omega_J$. In order to suppress the effect of the enhanced electrostatic fluctuation, we set $E_x = 0$ throughout the runs by specifying $IEX = 0$. We use the following parameters: $DX = 1$, $NX = 1024$, $DT = 0.04$, $CV = 20$, $NS = 1$, $WP = 2$, $VPE = 0.01$, $VPA = 0.01$, $EMAX = 1$, $BMAX = 0.05$, $VMAX = 1$, and $NTIME = 1024$. By comparing response of an unmagnetized plasma ($WC = 0$) and the that of a magnetized plasma ($WC = -1.0$ and $ANGLE = 0$), we can find different polarization of the electromagnetic waves. In the magnetized plasma, a whistler mode wave can be excited with $WJ = 0.5$. In the three dimensional plot of the diagnostics specified by "VyzEByz-X", we can find the spiral structure of the whistler mode wave. In these runs with particles very low temperature, the particles works as a fluid without any kinetic effects.

### 5.4 Two-stream instability and electron holes

We now introduce two different groups of electrons, while the ions are assumed to be the neutralizing background as in the preceding test runs. The two groups of electrons have different drift velocities in the direction parallel to the static magnetic field. If the thermal velocities of the electrons are much smaller than the relative drift velocity between the two groups of electrons, there arises a strong electrostatic instability. Since the instability is purely electrostatic, the code can be run with the electrostatic option $IEX = 2$ as well as the electromagnetic option $IE = 1$. The growth rate of the instability is large enough so that it can be demonstrated in spite of large thermal fluctuations. Try the following parameters: $DX = 1$, $NX = 64$, $DT = 0.04$, $CV = 20$, $NS = 2$, $WP1 = WP2 = 2$, $VPE1 = VPA1 = 1$, $VPE2 = VPE2 = 1$, $VD1 = 0$, $VD2 = 10$, $PCH1 = PCH2 = 0$, $VMAX = 20$, and $EMAX = 10$. First we study growth and saturation of the two-stream instability with $NP1 = NP2 = 256$ and $NTIME = 256$. We find that a coherent electrostatic potential grows to trap most of the electrons. The electrons undergo nonlinear oscillation in the potentials forming vortices. The instability is saturated because of the mixing of the two streams of electrons. Second, we try a longer run with $NTIME = 2048$, keeping the same number of particles $NP1 = NP2 = 256$. We find that the phase mixing continues through the dissipation of the trapping potentials. Third, we increase the number of particles in order to lower the thermal noise level by setting $NP1 = NP2 = 4096$, and run the code with $NTIME = 2048$. We find formation of very stable potential structures through coalescence of smaller potentials. These potential structures are called electron holes, or electrostatic solitary waves (ESW). Many observations, simulations, and theories on ESW have been reported [Matsumoto et al., 1994; Omura et al., 1994, 1996; Krasovsky et al., 2003].

### 5.5 Whistler mode instability driven by temperature anisotropy

In the presence of highly anisotropic electrons as in the radiation belt, the whistler mode waves propagating parallel to the static magnetic field become unstable, diffusing the pitch angles of the energetic electrons through cyclotron resonance.

$$\omega - k v_{\parallel} = \omega_{ce}/\gamma \tag{55}$$

where $\omega_{ce}$ is the electron cyclotron frequency. If the parallel velocity $v_{\parallel}$ of a high energy electron satisfies (55), the electron sees the stationary transverse electric field, exchanging energy with the transverse field. The parallel velocity $v_{\parallel}$ is, however, affected by the enhanced thermal fluctuations of the parallel electrostatic fields that disturb an effective wave-particle interaction through the cyclotron resonance. In stead of suppressing the thermal fluctuations by assigning a large number of superparticles per grid, we can enforce $E_x = 0$ by setting the option $IEX = 0$. Compare two runs with $IEX = 0$ and $IEX = 1$. We assume two different species of electrons. One is cold dense electrons, and the other is hot tenuous electrons. Try the following parameters. $DX = 1$, $NX = 256$, $DT = 0.04$, $NTIME = 4096$, $CV = 20$, $WC = -1$, $ANGLE = 0$, $NS = 2$, $QM = -1$, $WP1 = 2$, $WP2 = 0.5$, $VPE1 = VPA1 = 1$, $VPE2 = 20$, $VPA2 = 5$, $NP1 = NP2 = 4096$. We can find growth of whistler mode waves propagating both forward and backward along the magnetic field, i.e., the $x$ axis, which we can confirm from the $\omega - k$ diagram of the transverse components $B_y$ and $B_z$. A comparison with the linear growth rate of the whistler mode instability is found in Omura and Summers [2004].

### 5.6 Competing process of electrostatic instability and whistler mode instability

When an electron beam with the ring distribution in the perpendicular velocity is drifting along the static magnetic field, both longitudinal electrostatic field and transverse electromagnetic field are excited owing to an electrostatic beam instability and a whistler mode beam instability, respectively. By plotting the $\omega - k$ diagram for $E_x$ and $B_z$, we can find the directions of propagation are different for the electrostatic wave ad the whistler mode wave. The following parameters are suggested: $DX = 1$, $NX = 256$, $DT = 0.04$, $NTIME = 2048$, $CV = 20$, $WC = -1$, $ANGLE = 0$, $NS = 2$, $QM = -1$, $WP1 = 2$, $WP2 = 0.5$, $VPE1 = VPA1 = VPE2 = VPA2 = 1$, $VD1 = 0$, $VD = 20$, $PCH2 = 60$, and $NP1 = NP2 = 4096$. Study the diffusion process of the electron beam in the $(V_x, V_y, V_z)$ phase space. Also check the structure of the wave magnetic field, specifying the three-dimensional plot of $v_{y,z}$, $B_{y,z}$, and $E_{y,z}$ along the $x$ axis.

Try another run with $ANGLE = 90$. We find excitation of the perpendicular electrostatic field $E_x$, and the cold plasma is thermalized in the perpendicular direction. Although the growth rate is smaller than the instabilities in the parallel propagation, we need to compare the competing processes among different propagation directions. This underlines the limitation of the one-dimensional model.

The drift velocity given to the ring distribution results in

a uniform components of the current density in the parallel direction, which is automatically cancelled in the calculation of the current density. With a negligibly small value $AJAMP = 0.0001$ of the external current, the cancellation of uniform component is suppressed. Study the effect of the uniform components for two cases with $ANGLE = 0$ and 90.

### 5.7 Buneman instability

When there arises a large scale parallel electric field with a very low frequency, electrons are accelerated along the magnetic field forming a field-aligned current. In the presence of a large relative drift velocity $V_{d\|}$ between the thermal electrons and the thermal ions, a strong electrostatic instability called "Buneman instability" arises. Try the following parameters. $DX = 1$, $NX = 128$, $DT = 0.04$, $NTIME = 4096$, $CV = 20$, $WC = 0$, $ANGLE = 0$, $NS = 2$, $QM1 = -1$, $QM2 = 0.01$, $WP1 = 1$, $WP2 = 0.1$, $VD1 = 10$, $VD2 = 0$, $VPE1 = VPA1 = 1$, $VPE2 = VPA2 = 0.1$, and $NP1 = NP2 = 2048$. As we find in the two-stream instability, the electrons are trapped by the growing electrostatic potential forming large electron holes or ESW, which subsequently decay into ion acoustic waves. Try runs with different propagation angles such as $ANGLE = 20$. The instability becomes electromagnetic, and a strong magnetic field grows along with the electrostatic field.

## 6. Concluding Remarks

There are many other interesting physical problems that can be studied by the KEMPO1/MATLAB code. I hope the code will be utilized for obtaining better understanding of nonlinear processes in space plasmas.

**Acknowledgments.** I thank Koichi Shin for his assistance in programming the KEMPO1/MATLAB code. I also thank Hideyuki Usui, Marcio Augusto, Yuto Katoh, Ken Tsubouchi, Toshinari Kimura, and Hiroshi Matsumoto for their comments and discussion in developing the code and the present manuscript. The KEMPO1 code has been developed on KDK computer system at Research Institute for Sustainable Humanosphere, Kyoto University, and the development of the KEMPO1/MATLAB code was partially supported by a Grant-in-Aid for the 21st Century COE Program (Kyoto University, G3).

## References

C. K. Birdsall and B. K. Langdon, in *Plasma Physics via Computer Simulation*, McGraw-Hill, New York, 1985.

R. W. Hockney and J. W. Eastwood, in *Computer Simulation Using Particle*, McGraw-Hill, New York, 1981.

V. L. Krasovsky, H. Matsumoto, and Y. Omura, Electrostatic solitary waves as collective charges in a magnetospheric plasma: Physical structure and properties of Vernstein-Greene-Kruskal (BGK) solitons, *J. Geophys. Res.*, vol. 108, no. 1117, doi:10.1029/2001JA000277, 2003.

H. Matsumoto and Y. Omura, Particle simulations of electromagnetic waves and their applications to space plasmas, *Computer Simulations of Space Plasmas*, Edited by: H. Matsumoto and T. Sato, Terra Pub. and Reidel Co., 1985.

H. Matsumoto, H. Kojima, S. Miyatake, Y. Omura, Mokada, I. Nagano and M. Tsutsui, Electrostatic Solitary Waves (ESW) in the Magnetotail: BEN Wave Forms Observed by GEOTAIL, *Geophys. Res. Lett.*, **21**, 2,915-2,918, 1994.

Y. Omura and H. Matsumoto, KEMPO1: Technical guide to one-dimensional electromagnetic particle code, in *Computer Space Plasma Physic: Simulation Techniques and Softwares*, Edited by: H. Matsumoto and Y. Omura, Terra Scientific Pub., Tokyo, 1993.

Y. Omura, H. Kojima and H. Matsumoto, Computer Simulation of Electrostatic Solitary Waves: A Nonlinear Model of Broadband Electrostatic Noise, *Geophysical Research Letters*, **21**, pages 2,923-2,926, 1994.

Y. Omura, D. Summers, Computer simulations of relativistic whistler-mode wave-particle interactions, *Physics of Plasmas*, **11**, 3530-3534, 2004.

H. Ueda, Y. Omura, H. Matsumoto and T. Okuzawa, A Study on the Numerical Heating in Electrostatic Particle Simulations, *Comput. Phys. Comm.*, **79**, 249-259, 1994.

Umeda, T., Y. Omura, T. Tominaga, and H. Matsumoto, A new charge conservation method in electromagnetic particle-in-cell simulations, *Comput. Phys. Comm.*, **156**, pp.73-85, 2003.

J. Villasenor, J., and O. Buneman, Rigorous charge conservation for local electromagnetic field solvers, *Comput. Phys. Comm.*, **69**, 306-316, 1992.