

1.To compute the 95% confidence interval for the population mean μ using classical methods, we use the formula:

$$\text{Confidence Interval} = \bar{y} \pm Z_{\alpha/2} \cdot \sigma / \sqrt{n}$$

Variables:

- Sample mean (\bar{y}): 176
- Population standard deviation (σ): 3
- Sample size (n): 10
- Confidence level: 95% so $Z_{\alpha/2} = 1.96$

Steps:

1. Compute the standard error of the mean:

$$SE = \sigma / \sqrt{n} = 3 / \sqrt{10} \approx 0.949$$

2. Compute the margin of error:

$$\text{Margin of Error} = Z_{\alpha/2} \cdot SE = 1.96 \cdot 0.949 \approx 1.86$$

3. Compute the confidence interval:

$$\text{Confidence Interval} = \bar{y} \pm \text{Margin of Error} = 176 \pm 1.86$$

$$\text{Lower bound: } 176 - 1.86 = 174.14$$

$$\text{Upper bound: } 176 + 1.86 = 177.86$$

Result:

The 95% confidence interval for μ using classical methods is:

$$[174.14, 177.86]$$

2. Obtain 95% posterior credible intervals for μ for each of the cases:

(a) $\theta = 176$, $\tau = 8$; (b) $\theta = 176$, $\tau = 1000$ (c) $\theta = 0$, $\tau = 1000$. The result is below:

(a) $\theta = 176$, $\tau = 8$, *Prior1*:

```
> posteriorSummaryExact
$posterior_mean
[1] 176
```

```
$posterior_standard_deviation  
[1] 0.9420824
```

```
$posterior_quantiles  
  0.50 0.025 0.975  
[1] 176.0000 174.1536 177.8464
```

(b) $\theta = 176, \tau = 1000$

```
$posterior_mean  
[1] 176
```

```
$posterior_standard_deviation  
[1] 0.9486829
```

```
$posterior_quantiles  
[1] 176.0000 174.1406 177.8594
```

(c) $\theta = 0, \tau = 1000$

```
> posteriorSummaryExact  
$posterior_mean  
[1] 175.9998
```

```
$posterior_standard_deviation  
[1] 0.9486829
```

```
$posterior_quantiles  
[1] 175.9998 174.1405 177.8592
```

3.Comparison

Classical Model Confidence Interval:

[174.14,177.86]

Bayesian Credible Intervals Comparison:

1. (a):

[174.1536,177.8464]

Very close to the classical interval but slightly shifted.

2. (b):

[174.1406, 177.8594]

Almost identical to the classical interval.

3. (c):

[174.1405, 177.8592]

Also very close to the classical model, but slightly influenced by the prior mean ($\theta=0$).

4. Conclusion:

- Case (b): $\theta=176, \tau=1000$ is the closest to the classical result.

Case (b): $\theta = 176, \tau = 1000$ is the closest to the classical method, almost exactly the same. This is because the prior mean (θ) matches the sample mean (\bar{y}) perfectly, and the prior variance (τ^2) is very large, minimizing the influence of the prior on the posterior.