

1. Classical 95% Confidence Interval for μ

Given:

- Sample size $n=10$
- Sample mean $\bar{y}=176$,
- Sample variance $s^2 = 9$ ($s = \sqrt{9} = 3$),
- Population standard deviation σ is unknown.

The 95% confidence interval for μ uses the t-distribution with $n-1=9$ degrees of freedom:

$$CI_{95} = \bar{y} \pm t_{\alpha/2, n-1} \cdot s / \sqrt{n},$$

where $t_{\alpha/2, 9} \approx 2.262$,

Calculation:

$$\text{Margin of Error} = t_{\alpha/2, 9} \cdot s / \sqrt{n} = 2.262 \cdot 3 / \sqrt{10} \approx 2.14$$

$$CI_{95} = 176 \pm 2.14 = (173.86, 178.14)$$

Classical 95% Confidence Interval:

$$(173.86, 178.14)$$

2. Bayesian Posterior Credible Intervals for μ

Case a:

Posterior Mean: 176.00

95% Credible Interval for μ : [174.41, 177.49]

Hypothesis $\mu = 200$ is rejected.

Case b:

Posterior Mean: 176.00

95% Credible Interval for μ : [174.82, 177.10]

Hypothesis $\mu = 200$ is rejected.

Case c:

Posterior Mean: 176.00

95% Credible Interval for μ : [174.37, 177.52]

Hypothesis $\mu = 200$ is rejected.