Progress of last week

1 – Completed debugging of last week's code

inappropriate **weight factor** in eval function ($\mathrm{eval}=(1-\alpha)\,(1-E)+\alpha\big||a_1|^2-|a_{-1}|^2\big|$)

2 – Used new evaluation function to express the optimization problems (see next slide)

3 – Run optimization for several 2D scalar gratings

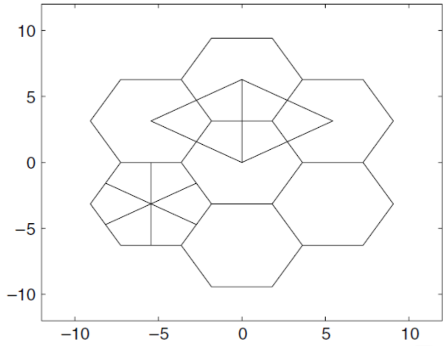
2D structure: not necessarily a rectangular lattice
Lattice's symmetry $\Rightarrow$ diffraction orders' symmetry

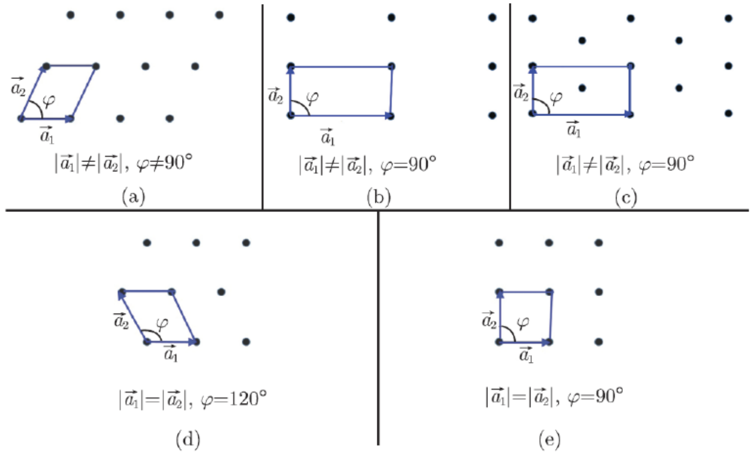
- **oblique:** Diamond-shaped diffraction orders
- **rectangular/centered rectangular:** 4/5/8/9 symmetric beam splitting
- **hexagonal:** 6/7/12/13 symmetric beam splitting
- **square:** 4/5/9 symmetric beam splitting

Describe periodicity: See solid state physics:

$$\mathbf{q_k} = m_1\mathbf{q}_1 + m_2\mathbf{q}_2 \qquad \mathbf{k} = (m_1, m_2).$$

$$f(\mathbf{x}) = \sum_{\mathrm{m}} a_{\mathrm{m}} e^{i\mathbf{q_m}\cdot\mathbf{x}}. \qquad a_{\mathrm{m}} = \frac{1}{A}\int_{\Omega} f(\mathbf{x}) e^{-i\mathbf{q_m}\cdot\mathbf{x}} d\mathbf{x},$$





There are **only 5** 2D lattices

Use new evaluation function

Take 1D scalar grating splitting K Beams for example

$\mathrm{eval} = (1-\alpha)\,(1-e) + \alpha\,\mathrm{Variance}(a_k)$

Constraint Optimization Problem:

Given **a set of indices K**, find the **2π real periodic function ϕ(x)** that **maximizes the normalized energy e** in the modes $k \in K$

$$e(\phi) = \frac{\sum_{k\in K} |a_k|^2}{\sum_{k=-\infty}^{\infty} |a_k|^2}$$

where a_k are the **Fourier coefficients** of $e^{i\phi(x)}$

subject to the **constraint** that the amplitude of the coefficients a_k for $k \in K$ are all the same.

The Least Squares Optimization Problem:

Given a set of indices K find the **2π real periodic function ϕ(x)**, **the phases α_k**, $k \in K$, and **a real positive number λ** such that we maximize the quantity (**the "distance"** between the actual function and ideal function)

$$e(\phi, \lambda, a_k) = ||e^{i\phi(x)} - \lambda \sum_{k\in K} e^{ikx} e^{i\alpha_k}||$$

Where $||f(x)|| = \frac{1}{2\pi}\int_{-\pi}^{\pi} |f(x)|^2 dx$, represents the "distance"

Notes: (1) for equal-energy-splitting case, theoretically $\lambda = \frac{1}{\sqrt{N}}$ by Parseval's theorem

(2) No need to manually adjust the weight factor α

Plan for next week

1 – Finish Optimization calculation for 1D/2D scalar/vector

2 – Run simulation for 2D ideal vector gratings

3 – Run simulation for 2D sampled vector gratings (metalens)

4 – Prepare a overall report on this