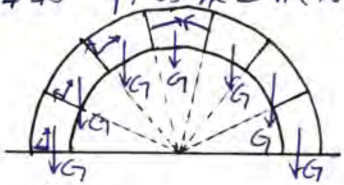
决赛专题

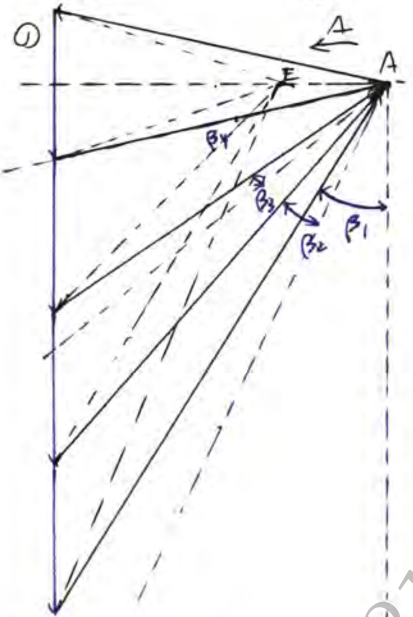
☆[例] $\alpha=\frac{\pi}{7}$, ① 最上块光滑, 求维持的 μ_{min} ② 最上块也有 μ, 讨论.

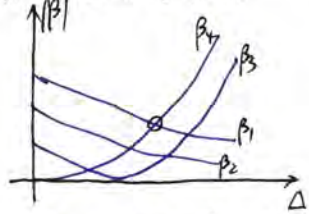


解: ① 作图: 最大 β: $\beta_1 \approx 32^\circ \Rightarrow \mu_{min}=0.63$

② 假设受力对称, 则A点水平向左平移.

分析各个角变化:



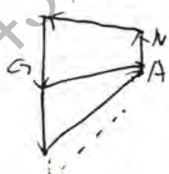


则 $\beta_4=-\beta_1$ 时对应有最小的 μ.

分析: (1) 实则应让E向外移动同时求 β_{max}

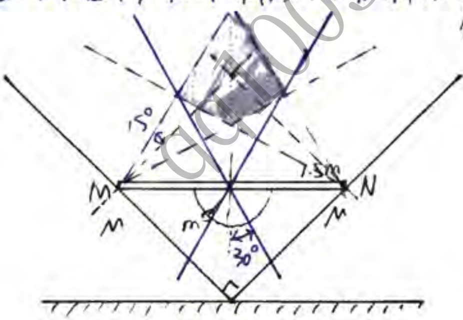
(2) 若问对顶块施最少多少力可使拱塌, 可如下处理:

作图如: (以下为 $\mu\,2\beta_0$)



$\Rightarrow$ 然后找 β_{max}.

☆[例] 平面内水平释放 m, 使杆不滑, 求 μ_{min}.



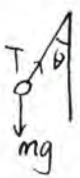
解:

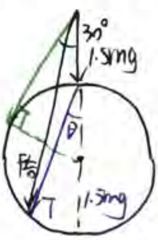
$$\begin{cases} \frac{1}{2}mv^2=mgr\cos\theta \\ T-mg\cos\theta=mv^2/r \end{cases}$$

$\therefore T=3mg\cos\theta$

对杆受力分析.

$\Rightarrow \mu_{min}=\tan15^\circ=2-\sqrt{3}$.





有：

$$\begin{vmatrix} \lambda-2 & 1 & 0 & 0 & 0 & 1 \\ 1 & \lambda-2 & 1 & 0 & 0 & 0 \\ 0 & 1 & \lambda-2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda-2 & 1 & 0 \\ 0 & 0 & 0 & 1 & \lambda-2 & 1 \\ 1 & 0 & 0 & 0 & 1 & \lambda-2 \end{vmatrix} = \lambda \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \lambda-2 & 1 & 0 & 0 & 0 \\ 0 & 1 & \lambda-2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda-2 & 1 & 0 \\ 0 & 0 & 0 & 1 & \lambda-2 & 1 \\ 1 & 0 & 0 & 0 & 1 & \lambda-2 \end{vmatrix} = \lambda \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \lambda-3 & 0 & -1 & -1 & -1 \\ 0 & 1 & \lambda-2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda-2 & 1 & 0 \\ 0 & 0 & 0 & 1 & \lambda-2 & 1 \\ 0 & -1 & -1 & -1 & 0 & \lambda-3 \end{vmatrix}$$

$$= \lambda \begin{vmatrix} \lambda-3 & 0 & -1 & -1 & -1 \\ 1 & \lambda-2 & 1 & 0 & 0 \\ 0 & 1 & \lambda-2 & 1 & 0 \\ 0 & 0 & 1 & \lambda-2 & 1 \\ -1 & -1 & -1 & 0 & \lambda-3 \end{vmatrix} = \lambda(\lambda-3) \begin{vmatrix} \lambda-3 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & \lambda-2 & 1 & 0 \\ 0 & 0 & 1 & \lambda-2 & 1 \\ -1 & -1 & -1 & 0 & \lambda-3 \end{vmatrix} = \lambda(\lambda-3) \begin{vmatrix} \lambda-3 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & \lambda-2 & 1 & -1 \\ 0 & 0 & 1 & \lambda-2 & 1 \\ -1 & -1 & -1 & 0 & \lambda-2 \end{vmatrix}$$

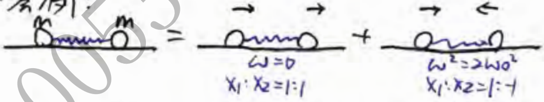
$$= \lambda(\lambda-3) \begin{vmatrix} \lambda-3 & -1 & -1 & -1 \\ 0 & \lambda-2 & 1 & -1 \\ 0 & 1 & \lambda-2 & 1 \\ -1 & -1 & 0 & \lambda-2 \end{vmatrix} = \lambda(\lambda-3)^2 \begin{vmatrix} \lambda-3 & -1 & -1 & -1 \\ 0 & \lambda-2 & 1 & -1 \\ 1 & 0 & 1 & 0 \\ -1 & -1 & 0 & \lambda-2 \end{vmatrix} = \lambda(\lambda-3)^2 \begin{vmatrix} \lambda-2 & -1 & -1 & -1 \\ -1 & \lambda-2 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & \lambda-2 \end{vmatrix}$$

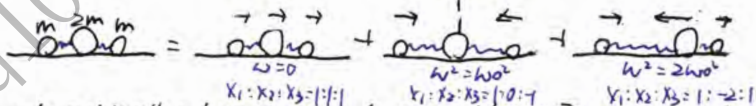
$$= \lambda(\lambda-3)^2 \begin{vmatrix} \lambda-2 & -1 & -1 \\ -1 & \lambda-2 & -1 \\ -1 & -1 & \lambda-2 \end{vmatrix} = \lambda(\lambda-3)^2(\lambda-4) \begin{vmatrix} 1 & 1 & 1 \\ -1 & \lambda-2 & -1 \\ -1 & -1 & \lambda-2 \end{vmatrix} = \lambda(\lambda-3)^2(\lambda-4) \begin{vmatrix} 1 & 1 & 1 \\ 0 & \lambda-1 & 0 \\ 0 & 0 & \lambda-1 \end{vmatrix}$$

$$= \lambda(\lambda-3)^2(\lambda-4) \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-1 \end{vmatrix} = \lambda(\lambda-3)^2(\lambda-4)(\lambda-1)^2$$

分析：①这是循环行列式,有特别解法。亦可用特征值处理。
②更重要的是"猜"出简正模！

→举例：

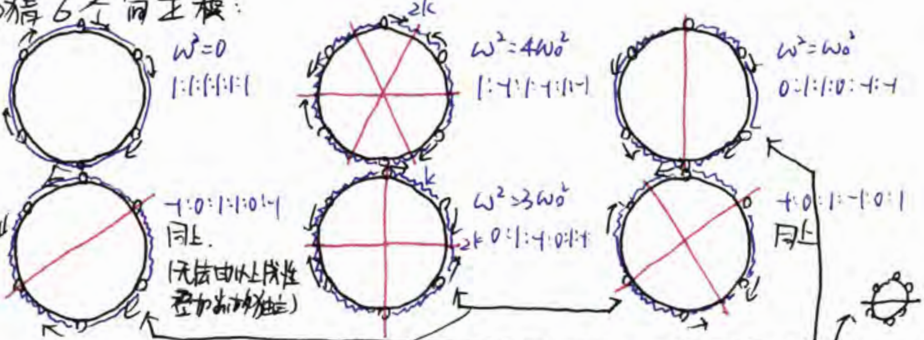




理解：①线性代数正交性 ②2m拆成2个m分别和左右同动，需$(-1)\times(-1)=-2$。
←同样可得$\omega^2=2\omega_0^2$！

法二：

⇒猜6个简正模：



可组合出以线性叠加到的图…

△振子波：

核心：由轨换对称性进行分析.

☆[例]弹簧绳波，求运动.

l, m, k 解：猜：$x_n = Ae^{-i\omega t + in\varphi} \Rightarrow x(\xi, t) = Ae^{-i\omega t + i\xi\frac{\varphi}{l}}$ 波数

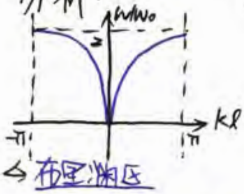
$x_{n-1}\ x_n\ x_{n+1}$，ξ

牛二：$m\ddot{x}_n = -k(2x_n - x_{n+1} - x_{n-1})$，令 $\frac{k}{m} = \omega_0^2$

代回：$-\frac{\omega^2}{\omega_0^2} = -(2 - e^{i\varphi} - e^{-i\varphi}) = -4\sin^2\frac{\varphi}{2}$

$\Rightarrow \omega = \pm 2\omega_0 \sin\frac{\varphi}{2} = \pm 2\omega_0 \sin\frac{kl}{2}$（左/右传播）$\Rightarrow x_n = x_n(t)$

分析：



↳ 布里渊区

有：一个波矢对应一个频率，周期：$T = \frac{2\pi}{l}$

① φ 较小时（k 也小），$\omega = kl\omega_0$，相速度 $v_p = \frac{\omega}{k} = \omega_0 l$ 不变

—— 低频无色散. —— 对应实际：m, k 极密.

$\lambda \gg l$ 的绳波，无色散存在.

② 正常的 ω（包括复数）：

由：$\frac{\omega^2}{\omega_0^2} = 2 - 2\cos\varphi$

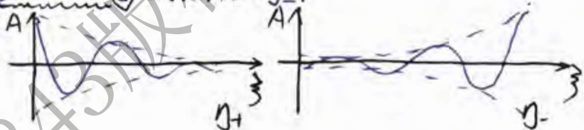
1° $\omega \in \mathbb{R}$，$\omega \in [0, 2\omega_0] \Rightarrow \varphi = \pm\arcsin\frac{\omega}{2\omega_0}$，是有色散的行波

2° $\omega \in \mathbb{R}$，$\omega \in [2\omega_0, +\infty)$，代 $\cos\varphi = (e^{i\varphi} + e^{-i\varphi})/2$

$\Rightarrow \varphi = -i\ln\left[\frac{2 - \omega^2/\omega_0^2 \pm \sqrt{(\omega^2/\omega_0^2)^2 - 4}}{2}\right]$，结果记为：$\eta_\pm$.

→ 对 η_+，策动力在原点：

→ 对 η_-，策动力在 ∞ 处：



☆[例]反射波，求运动.

k, m, l

$\cdots\ n=2\ n=1\ n=0$

解：方程线性，故可叠加.

入射：$x_{n_1} = \tilde{A} \cdot e^{i(-\omega t + n\varphi)}$

反射：$x_{n_2} = \tilde{r} \cdot \tilde{A} \cdot e^{i(-\omega t - n\varphi)}$

$\Rightarrow x_n = \tilde{A}(e^{i(-\omega t + n\varphi)} + \tilde{r} \cdot e^{i(-\omega t - n\varphi)})$；牛二：$m\ddot{x}_n = -k(2x_n - x_{n+1} - x_{n-1})$，$n \le -1$

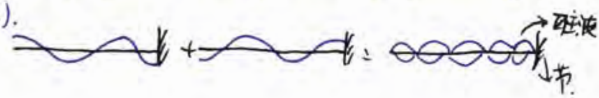
扩展定义域：对 $n=1$ 也成立，边界条件为 $x_0 = 0$. 有：$m\ddot{x}_1 = -k(2x_1 - x_2 - \underbrace{x_0}_{\to 0})$.

故有：$x_0 = \tilde{A}(e^{i(-\omega t)} + \tilde{r} \cdot e^{i(-\omega t)}) = 0 \Rightarrow \tilde{r} = -1 = e^{i\pi}$ —— 半波损失.

再代入一般式：$m \cdot (-\omega^2) \cdot \tilde{A}(e^{in\varphi} + \tilde{r} \cdot e^{-in\varphi}) = -k[2\tilde{A}(e^{in\varphi} + \tilde{r} \cdot e^{-in\varphi})$
$- \tilde{A}(e^{i(n+1)\varphi} + \tilde{r} \cdot e^{-i(n+1)\varphi}) - \tilde{A}(e^{i(n-1)\varphi} + \tilde{r} \cdot e^{-i(n-1)\varphi})]$

$\Rightarrow \omega = f(\varphi) \Rightarrow x_n = x_n(t)$.

分析：半波损失原因为：



☆[例]空气边界，求运动.

m, k　M

$\cdots\ n=2\ n=1\ n=0$

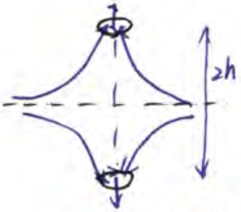
解：入+反：$x_n = \tilde{A}e^{i(n\varphi - \omega t)} + \tilde{A}\tilde{r} \cdot e^{i(-n\varphi - \omega t)}$，今定义域延拓

牛二：$m\ddot{x}_n = -k(2x_n - x_{n+1} - x_{n-1})$ 令 $\frac{M}{m} = \eta$，$\omega_0^2 = \frac{k}{m}$

对 $n=0$：$-\eta\frac{\omega^2}{\omega_0^2}(1 + \tilde{r}) = -(1 + \tilde{r} - e^{-i\varphi} - \tilde{r}e^{i\varphi})$

$\Rightarrow \tilde{r} = -\frac{\eta\frac{\omega^2}{\omega_0^2} - 1 + e^{-i\varphi}}{\eta\frac{\omega^2}{\omega_0^2} - 1 + e^{i\varphi}} = -\frac{\eta\frac{\omega^2}{\omega_0^2} - 1 + \cos\varphi - i\sin\varphi}{\eta\frac{\omega^2}{\omega_0^2} - 1 + \cos\varphi + i\sin\varphi} = e^{i(\pi + \alpha)}$

↳ $\alpha = 2\arctan\frac{\sin\varphi}{\eta\frac{\omega^2}{\omega_0^2} - 1 + \cos\varphi}$



(2) $\vec{B}=\frac{\mu_0}{4\pi}\cdot\frac{2m}{z^3}\hat{z}$

有：$|F|=\left|u\cdot\frac{dB}{dz}\Big|_{z=2h}\right|=\left|I\cdot\pi r^2\frac{\mu_0}{4\pi}\cdot\frac{2m}{(2h)^4}(-3)\right|=mg$

$\Rightarrow h=\sqrt[4]{\frac{3I^2\pi r^4\mu_0}{32mg}}$

故先对B求导后再代入.

分析：再求振动：$\ddot{x}\cdot m=-\frac{4\cdot 6\cdot\mu_0\cdot u^2}{4\pi\cdot 16\cdot x_0^5}\cdot x\Rightarrow \omega$

量子力学初步(8.3)

△黑体辐射：

·全频段叠加：

辐射本领 $P(\nu)=\frac{dE}{dSdt}\cdot\frac{1}{d\nu}\xrightarrow[\text{分布}]{\int_0^{+\infty}}$ 黑体：$J=\frac{dE}{dSdt}=\sigma T^4$

↳ 反射下：$J'=(1-r)\sigma T^4$ （r：反射率）

☆[例]已知 r_S. r_{ES}. T_S. (1) 求 T_E (2) 地表包一层黑体壳，求 T_E'.

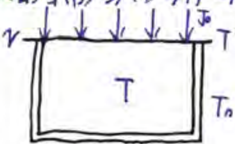
解：(1) $\sigma T_S^4\cdot 4\pi r_S^2\cdot\frac{\pi r_E^2}{4\pi r_{ES}^2}=\sigma T_E^4\cdot 4\pi r_E^2 \Rightarrow T_E=\left(\frac{r_S^2}{4r_{ES}^2}\right)^{\frac{1}{4}}T_S$

(2) 地球平衡：地→壳＝壳→地

壳+地球平衡：太阳→壳＝壳→外 $\Rightarrow J'=J_0$. $T_E'=T_E$

薄壳：壳→地＝壳→外

☆[例]高频反射率 r_H，低频反射率 r_L，求 T，T'（散热 $\frac{dQ}{dt}=K(T-T_0)$）



（房间黑体）

解：房顶：$J_0(1-r_H)S+\sigma T^4S(1-r_L)=\sigma T'^4(1-r_L)\cdot 2S$

房间：$\sigma T'^4(1-r_L)S=\sigma T^4(1-r_L)S+K(T-T_0)$

$\Rightarrow T.\ T'$

·频率分布细化：

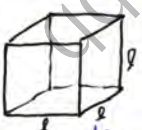
单位频率能量密度：$u(\nu,T)=\frac{dE}{dV\cdot d\nu}=G(\nu)\cdot\varepsilon(\nu,T)$ ← 一个模式下的平均能

↳ 模式密度：$G(\nu)=\frac{dM(\nu)}{dV\cdot d\nu}$ ← 模式个数，与T无关

单位频率能流密度：$\sigma=\frac{dE}{dSdtd\nu}\Rightarrow P=\frac{dE}{dSdt}$

☆[例]使用驻波条件，使用离散的 k_x. k_y. k_z. 由最大频率 ν_0. c. l

求驻波模式个数 M. 模式密度 G.



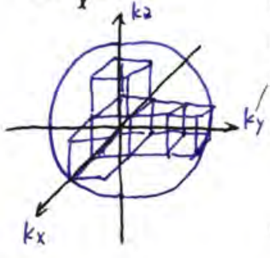
解：一个方向上有2个驻波！

$\begin{cases}E_y(x,t)=E_0\cos(k_x\cdot x-\omega t)\\ E_z(x,t)=E_0\cos(k_x\cdot x-\omega t)\end{cases}$ c.c. 这里已设 $k_i>0$.

驻波条件：$k_xl=n_x\pi$；$k_yl=n_y\pi$；$k_zl=n_z\pi$

作波矢空间：$k^2=k_x^2+k_y^2+k_z^2$ $\quad k_{max}=\frac{\omega_{max}}{c}=\frac{2\pi\nu_0}{c}$

模式个数：$\frac{1}{8}$球中有N个顶点⟺含N个正方体.



有：$M=\underbrace{2}_{\text{一方向2个驻波}}\times\frac{\frac{1}{8}\cdot\frac{4}{3}\pi\left(\frac{2\pi\nu_0}{c}\right)^3}{\left(\frac{\pi}{l}\right)^3}=\frac{8}{3}\pi\nu_0^3\frac{l^3}{c^3}$

$\Rightarrow G=\frac{dM}{d\nu}\cdot\frac{1}{l^3}=\frac{8\pi\nu_0^2}{c^3}$

在 Δt 一个周期内计算它：

$$\Delta\vec{B}=\int_0^T\left(\frac{d\vec{B}}{dt}\right)dt=\int_0^{2\pi}\hat{\theta}\cdot\frac{GMm}{r_0^3}\alpha r^3 d\theta=\int_0^{2\pi}(-\sin\theta,\cos\theta)\cdot\frac{GMm\alpha}{r_0^3}\frac{r_0^3}{(1+e\cos\theta)^3}d\theta$$

$$=\int_0^{2\pi}(-\sin\theta,\cos\theta)\cdot GMm\alpha\cdot(1-3e\cos\theta)d\theta=-\frac{3}{2}\cdot eGMm\cdot 2\pi\,\hat{y}$$

$$\Rightarrow\overline{\left(\frac{d\vec{B}}{dt}\right)}=-\frac{3}{2}eMmG\cdot\alpha\omega_0\hat{y}=\left(-\frac{3}{2}\alpha\cdot\omega_0\hat{z}\right)\times\vec{B}$$

$\vec{B}$ 进动(与行星进动反向).

☆[例] 在 $e\ll 1$ 时，求轨迹变化.（有 $\omega_{电子}=\omega_{E}$）

$\vec{E}=E_0\cos\omega t$

解：$\frac{d\vec{B}}{dt}=\frac{d\vec{v}}{dt}\times\vec{L}-GMm\dot{\hat{r}}=\frac{1}{m}qE_0\cos\omega t\times mr^2\dot{\theta}\hat{y}=qE_0\cos\omega t\,r^2\dot{\theta}\hat{x}$

$$\therefore\Delta\vec{B}=\int_0^T\left(\frac{d\vec{B}}{dt}\right)dt=\int_0^T qE_0\cos\omega t\cdot\frac{e^2p^2}{(1+e\cos\omega t)^2}d\omega t\,\hat{x}$$

$$=\int_0^{2\pi}qE_0\cdot e^2p^2\times\cos\theta(1-2e\cos\theta)d\theta\,\hat{x}=-qE_0\cdot e^2p^2\times 2e\pi=-2qE_0e^3p^2\pi\hat{x}$$

$$\Rightarrow\overline{\left(\frac{d\vec{B}}{dt}\right)}=2qE_0e^3p^2\omega\hat{x}=2qE_0\omega\times\frac{L^2}{kq^2m}\times\frac{1}{kq^2}\vec{B}=\frac{2E_0\omega L^2}{k^2q^3m}\vec{B}$$

$\therefore$ 有大致关系：$\vec{B}=\vec{B}_0\exp\left(\frac{2E_0\omega L^2}{k^2q^3m}t\right)$ 含义：量子下能级跃迁.

☆[例] 用方向向量处理轨迹.

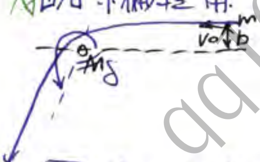
解：$mr^2\dot{\theta}=L\Rightarrow r^2d\theta=\frac{L}{m}dt$　$\begin{cases}d\hat{r}=d\theta\hat{\theta}\\ d\hat{\theta}=-d\theta\hat{r}\end{cases}$

由 $d\vec{v}=-\frac{\vec{F}}{m}dt=-\frac{GMm}{L}d\theta\cdot\hat{r}$

$\Rightarrow d\vec{v}=+\frac{GMm}{L}d\hat{\theta}$ 积分 $\Rightarrow\vec{v}-v_0\hat{y}=+\frac{GMm}{L}(\hat{\theta}-\hat{y})$，点乘 $\hat{\theta}$

$\Rightarrow\frac{L}{mr}=\frac{GMm}{L}+\left(v_0-\frac{GMm}{L}\right)\cos\theta\Rightarrow r=\frac{ep}{1+e\cos\theta}$

☆[例] 求偏转角.



解：$d\vec{v}=\frac{GMm}{L}d\hat{\theta}$ 有 $v_0\cdot 2\cdot\cos\frac{\delta}{2}=\frac{GMm}{L}\cdot 2\sin\frac{\delta}{2}$

$\Rightarrow\tan\frac{\delta}{2}=\frac{v_0^2b}{GM}$

或 $\vec{B}=\vec{v}\times\vec{L}+\alpha\hat{r}=v_0bmv_0\hat{y}+(-\alpha)\hat{x}\propto[\sin\frac{\delta}{2},\cos\frac{\delta}{2}]$

$\Rightarrow\tan\frac{\delta}{2}=-\frac{mbv_0^2}{\alpha}=\frac{v_0^2b}{GM}$　　☆后者更推荐.

☆[例] 用LRL矢量证明动量空间为一个圆.

证：$GMm\hat{r}=\vec{v}\times\vec{L}-\vec{B}\Rightarrow G^2M^2m^2=v^2L^2-(\vec{v}\cdot\vec{L})^2+B^2-2\vec{B}\cdot\vec{v}\times\vec{L}$

$\therefore\left(\frac{GMm}{L}\right)^2=P_x^2+\left(P_y+\frac{Mm}{L}\right)^2$

有：$\operatorname{sh}k=\frac{at}{c}\Rightarrow \operatorname{sh}\frac{a\tau}{c}=\frac{at}{c}\Rightarrow$ 世界线：$\begin{cases} ct=\frac{c^2}{a}\cdot\operatorname{sh}\frac{a\tau}{c} \\ x=\frac{c^2}{a}(\operatorname{ch}\frac{a\tau}{c}-1)\end{cases}$

分析：观察同方向LT：$\begin{bmatrix}ct'\\x'\end{bmatrix}=\begin{bmatrix}\gamma_1 & -\gamma_1\beta_1\\ -\gamma_1\beta_1 & \gamma_1\end{bmatrix}\begin{bmatrix}ct\\x\end{bmatrix}\overset{记}{=}\begin{bmatrix}\operatorname{ch}k_1 & -\operatorname{sh}k_1\\ -\operatorname{sh}k_1 & \operatorname{ch}k_1\end{bmatrix}\begin{bmatrix}ct\\x\end{bmatrix}$

$(\operatorname{ch}k=\gamma.\ \operatorname{sh}k=\gamma\beta)$ $\quad\begin{bmatrix}ct''\\x''\end{bmatrix}=\begin{bmatrix}\gamma_2 & -\gamma_2\beta_2\\ -\gamma_2\beta_2 & \gamma_2\end{bmatrix}\cdot\begin{bmatrix}ct'\\x'\end{bmatrix}\overset{记}{=}\begin{bmatrix}\operatorname{ch}k_2 & -\operatorname{sh}k_2\\ -\operatorname{sh}k_2 & \operatorname{ch}k_2\end{bmatrix}\begin{bmatrix}ct'\\x'\end{bmatrix}$

$\Rightarrow\begin{bmatrix}ct''\\x''\end{bmatrix}=\begin{bmatrix}\operatorname{ch}(k_1+k_2) & -\operatorname{sh}(k_1+k_2)\\ -\operatorname{sh}(k_1+k_2) & \operatorname{ch}(k_1+k_2)\end{bmatrix}\begin{bmatrix}ct\\x\end{bmatrix}$ —— $K=k_1+k_2$：快度.

今用 ict 表达：

$\begin{bmatrix}ict'\\x'\end{bmatrix}=\begin{bmatrix}\operatorname{ch}k_1 & -i\operatorname{sh}k_1\\ +i\operatorname{sh}k_1 & \operatorname{ch}k_1\end{bmatrix}\begin{bmatrix}ict\\x\end{bmatrix}$ $\begin{cases}\operatorname{ch}ik_1=\cos k_1\\ -i\operatorname{sh}ik_1=\sin k_1\end{cases}$ ⟶ 复数意义下转动叠加.

今本例：$\overset{S}{\underset{}{}}\ dk=\frac{a}{c}d\tau \quad \overset{S_k}{}\quad \therefore K=\int dk=\frac{a}{c}\tau.$

$S_{\tau_1}\to S_{\tau_2}\to\cdots\to S_{\tau_n}$

$\frac{ad\tau}{c}=\operatorname{th}dk=dk$ —— 解决了火箭等问题

·增加问题：(5) 求 $S_\tau\to S_0$ 换系.

(6) S 看T距离.

解：(5) 对于点 $(ct',x)=(c\tau,0)$ 时满足：$\begin{cases}ct=\frac{c^2}{a}\operatorname{sh}\frac{a\tau}{c}\\ x=\frac{c^2}{a}(\operatorname{ch}\frac{a\tau}{c}-1)\end{cases}$

代入含 x_0,t_0 的变换：$\begin{cases}c\tau=\operatorname{ch}k\cdot\frac{c^2}{a}\operatorname{sh}\frac{a\tau}{c}-\operatorname{sh}k\cdot\frac{c^2}{a}(\operatorname{ch}\frac{a\tau}{c}-1)+ct_0\\ 0=-\operatorname{sh}k\cdot\frac{c^2}{a}\operatorname{sh}\frac{a\tau}{c}+\operatorname{ch}\cdot\frac{c^2}{a}(\operatorname{ch}\frac{a\tau}{c}-1)+x_0\end{cases}$

$\therefore\begin{cases}ct_0=c\tau-\frac{c^2}{a}\operatorname{sh}\frac{a\tau}{c}\\ x_0=-\frac{c^2}{a}+\frac{c^2}{a}\operatorname{ch}\frac{a\tau}{c}\end{cases}$ ⟶ 用 $\tau\to0$ 检验. $\begin{cases}t_0\to0\\ x_0\to\frac{1}{2}a\tau^2.\end{cases}$

$\Rightarrow$ L.T.：$\begin{bmatrix}ct'\\x'\end{bmatrix}=\begin{bmatrix}\operatorname{ch}k & -\operatorname{sh}k\\ -\operatorname{sh}k & \operatorname{ch}k\end{bmatrix}\begin{bmatrix}ct\\x\end{bmatrix}+\begin{bmatrix}ct_0\\x_0\end{bmatrix}$

(6). 在 S_τ 系中. 时间为 τ 时找T的 τ.

处理：写T的世界线：$\begin{cases}ct=\frac{c^2}{a}\cdot\operatorname{sh}\frac{a\tau}{c}\\ x=\frac{c^2}{a}\cdot(\operatorname{ch}\frac{a\tau}{c}-1)+l\end{cases}$

$\Rightarrow$ 换系：$\begin{cases}c\tau_s=\operatorname{ch}\frac{a\tau_s}{c}\cdot\frac{c^2}{a}\operatorname{sh}\frac{a\tau}{c}-\operatorname{sh}\frac{a\tau_s}{c}[\frac{c^2}{a}(\operatorname{ch}\frac{a\tau}{c}-1)+l]+c\tau_s-\frac{c^2}{a}\operatorname{sh}\frac{a\tau_s}{c}\\ l_T=-\operatorname{sh}\frac{a\tau_s}{c}\cdot\frac{c^2}{a}\operatorname{sh}\frac{a\tau}{c}+\operatorname{ch}\frac{a\tau_s}{c}[\frac{c^2}{a}(\operatorname{ch}\frac{a\tau}{c}-1)+l]+\frac{c^2}{a}[\operatorname{ch}\frac{a\tau_s}{c}-1]\end{cases}$

表明同时

看到的距离

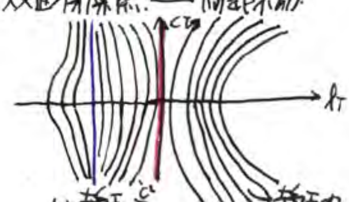
$\Rightarrow\begin{cases}\operatorname{sh}\frac{a(\tau_T-\tau_s)}{c}=\frac{al}{c^2}\operatorname{sh}\frac{a\tau_s}{c}\\ l_T=\frac{c^2}{a}\operatorname{ch}\frac{a(\tau_T-\tau_s)}{c}-\frac{c^2}{a}+l\cdot\underbrace{\operatorname{ch}\frac{a\tau_s}{c}}_{\gamma}\end{cases}$ ⟶ 当 $\frac{a\tau_s}{c}\ll1$, $\frac{al}{c^2}\ll1$

$\Rightarrow l_T=\frac{c^2}{a}\sqrt{1+(\frac{al}{c^2}\operatorname{sh}\frac{a\tau_s}{c})^2}-\frac{c^2}{a}+l\operatorname{ch}\frac{a\tau_s}{c}$

分析：当 $l=-\frac{c^2}{a}\Rightarrow l_T=-l$ —— S中双曲线原点.—— 间距不动.

当 $l=0\Rightarrow l_T=0$ —— 自己不动

当 $l>0\Rightarrow l_T\nearrow$

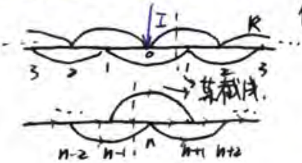


(六)电路

基尔霍夫方程(6.1)

处理：节点多，回路少：设电流，列回路。
节点少，回路多：设电压，列节点。

☆[例] 求电流分布.



解：支路多节点少，设电压。设电势如图。
→ 列基尔霍夫第一定律：对点 n：

$$\frac{U_{n-1}-U_n}{R}+\frac{U_{n-2}-U_n}{R}-\frac{U_n-U_{n+1}}{R}-\frac{U_n-U_{n+2}}{R}=0$$

$\therefore U_{n-2}+U_{n-1}-4U_n+U_{n+1}+U_{n+2}=0$ 　　看不出重根

猜：$U_n=A\cdot\lambda^n \Rightarrow 1+\lambda-4\lambda^2+\lambda^3+\lambda^4=0$ ⟶ 不用CASIO. — $\boxed{\text{处理一}}$

→ 也可以利用对称性：$\frac{1}{\lambda^2}+\frac{1}{\lambda}-4+\lambda+\lambda^2=0$.（$a$ 为解，$\frac{1}{a}$ 也为解）

$\boxed{\text{处理二}}$：$I=0$ 也为解 $\Rightarrow \lambda=1$ 为解，$\lambda=\frac{1}{1}$ 也为解.（是重根）

$\therefore$ 降幂 $\Rightarrow (\lambda-1)^2(\lambda^2+3\lambda-1)=0 \Rightarrow \lambda$

$\boxed{\text{处理三}}$：令 $\lambda+\frac{1}{\lambda}=a$ $\therefore a+a^2=6 \Rightarrow a_1=2\quad a_2=-3 \Rightarrow \lambda$.

$\Rightarrow$ 得 $\lambda_{1,2}=1.\quad \lambda_{3,4}=\frac{-3\pm\sqrt{13}}{2}$

$\therefore U_n=(A_1+nA_2)+A_3\cdot\lambda_3^n+A_4\cdot\lambda_4^n$

→ 边界定系数：① 当 $n\to\infty$，U_n 应以最大 $n'\to\infty$. $\therefore A_4=0$.（λ_4 为 $+$）/ 实质：左右 g' 对称.

② 差值有意义．令 $A_1=0$

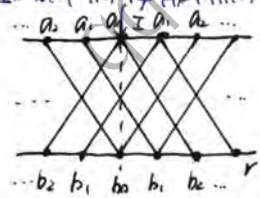
$\Rightarrow U_n=n\cdot A_2+A_3\cdot\left(\frac{-3+\sqrt{13}}{2}\right)^n$

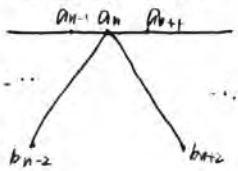
③ 找一截线：$\frac{U_{n-1}-U_{n+1}}{R}+\frac{U_{n-1}-U_n}{R}+\frac{U_{n-1}-U_n}{R}=I/2$

（或直接看1处）$(U_1-U_0)+(U_1-U_2)+(U_1-U_{-1})+(U_1-U_3)=0$

④ 端点：$2(U_0-U_1)+2(U_0-U_2)=I$ 　故边界从端点和对称考虑.

☆[例]（中间接触点不连通）求电流分布.





解：基一：$(a_n-a_{n-1})+(a_n-a_{n+1})+(a_n-b_{n-2})+(a_n-b_{n+2})=0$

$$\Rightarrow\begin{cases}a_{n-1}-4a_n+a_{n+1}+b_{n-2}+b_{n+2}=0\\ a_{n-2}+a_{n+2}+b_{n-1}-4b_n+b_{n+1}=0\end{cases}$$

猜：$\begin{pmatrix}a_n\\b_n\end{pmatrix}=\begin{pmatrix}a\\b\end{pmatrix}\lambda^n \Rightarrow \begin{cases}(\frac{1}{\lambda}-4+\lambda)A+(\lambda^2+\frac{1}{\lambda^2})B=0\\(\lambda^2+\frac{1}{\lambda^2})A+(\frac{1}{\lambda}-4+\lambda)B=0\end{cases}$

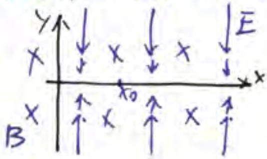
要求 $\det=0 \Rightarrow$ 用 $X=\lambda+\frac{1}{\lambda}$ 换元 $\Rightarrow \lambda_{1,2\cdots 8}$.

$\Rightarrow$ 由端点值和对称性待定出系数

注：一般极限分析 $\Rightarrow$ 都只剩 $nA+B\cdot\lambda^n$ 之2项.

△连续对称性:
找守恒量，消自由度得有效势.

☆[例] $\vec{B} = B_0(-\hat{z})$, $\vec{E} = -\alpha y\hat{y}$. 初 $\vec{v}_0 = v_0\hat{y}$. 求解运动.



解: x方向平移不变: $\frac{\partial V}{\partial x} = -E_x = 0$; $\frac{\partial \vec{B}}{\partial x} = 0$

有: $P_x = p_x + qBy = mv_x + qBy = 0$.

$\Rightarrow \frac{1}{2}mv_y^2 + \frac{1}{2}mv_x^2 + \frac{1}{2}\alpha q y^2 = \frac{1}{2}mv_0^2$

代入 $\Rightarrow \frac{1}{2}mv_y^2 + \underbrace{\frac{1}{2}\frac{q^2B^2}{m}y^2 + \frac{1}{2}\alpha q y^2}_{V_{eff}} = \frac{1}{2}mv_0^2$

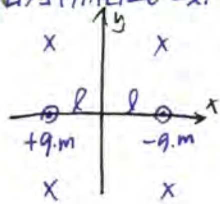
$\Rightarrow$ 有效势得y方向振动: $\omega = \sqrt{\left(\frac{qB}{m}\right)^2 + \frac{\alpha q}{m}}$

$\therefore v_y = v_0\cos\omega t$, $y = \frac{v_0}{\omega}\sin(\omega t)$

$\therefore v_x = -\frac{qB}{m}\frac{v_0}{\omega}\sin\omega t$, $x = x_0 + \frac{qBv_0}{m\omega^2}(\cos(\omega t) - 1)$

—— 正椭圆.

☆[例] 初相距2l, 静止释放, 弹性碰撞, $v \ll c$. 求解运动.



解: 只看一边: $\frac{d\vec{p}}{dt} = -q\vec{v}\times\vec{B}$

$\Rightarrow P_y = qBl - qBx$

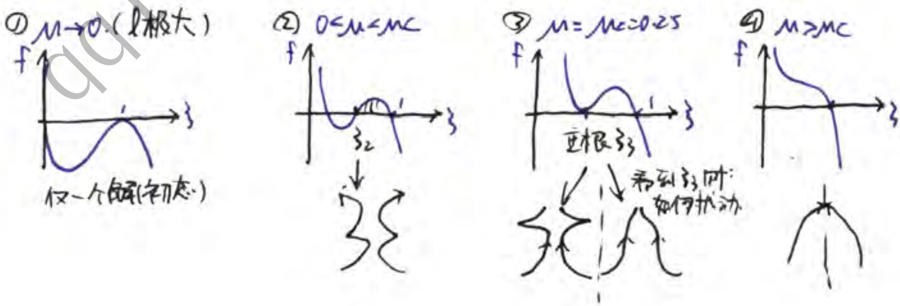
能: $2\cdot\frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2x}kq^2 = -\frac{1}{2l}kq^2$

$\Rightarrow \frac{1}{2}m\dot{x}^2 + \underbrace{\frac{1}{2}\frac{q^2B^2}{m}(l-x)^2 - \frac{kq^2}{4x}}_{V_{eff}} = -\frac{kq^2}{4l}$

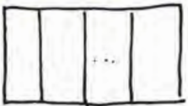
→无量纲化 令 $\frac{x}{l} = \xi$, $\frac{kq^2}{4l}\cdot\frac{2m}{q^2B^2l^2} = \mu$.

则: 令: $f(x) = -\frac{kq^2}{4l} + \frac{kq^2}{4x} - \frac{q^2B^2}{2m}(l-x)^2$

$= \frac{q^2B^2l^2}{2m}\left(-\mu + \frac{\mu}{\xi} - (\xi-1)^2\right) = \frac{1}{2}m\dot{x}^2 > 0$



☆[例] 不同理气混合. $C_V=\frac{3}{2}R$. $V_i=\frac{V}{N}$. $P_i=\alpha_i P_0$ $T_i=t_iT_0$. 求ΔS.



解: $\Delta E=0$: $\sum(\nu_i C_{V_i} T_i)=(\sum\nu_i)C_V T_f$

$$\Rightarrow T_f=\frac{\sum\nu_i T_i}{\sum\nu_i}=T_0\frac{\sum\alpha_i}{\sum\frac{\alpha_i}{t_i}}$$

看某一组分: $\Delta S(V,T)=nC_V\ln\frac{PV^\gamma}{P_0V_0^\gamma}=nR\ln\left((\frac{T}{T_0})^{\frac{3}{2}}\frac{V}{V_0}\right)$

∴ $\Delta S_i=\nu_i R\ln\left((\frac{T_f}{t_iT_0})^{\frac{3}{2}}\cdot N\right)$

$\Rightarrow \Delta S=\sum\nu_i R\ln\left((\frac{T_f}{t_iT_0})^{\frac{3}{2}}N\right)$

分析: 取 $n=2$: 令 $\alpha_1=\frac{3}{2}$. $\alpha_2=\frac{1}{2}$. $t_1=\frac{3}{2}$. $t_2=\frac{1}{2}$.

则: $T_f=T_0$. $\Delta S_1=\frac{1}{2}\nu_0R\ln\left((\frac{1}{3/2})^{\frac{3}{2}}\cdot2\right)$ $\Delta S_2=\frac{1}{2}\nu_0R\ln\left((\frac{1}{1/2})^{\frac{3}{2}}\cdot2\right)$

$$\Rightarrow \Delta S=\underbrace{\nu_0R\ln2}_{混合}+\underbrace{\frac{1}{2}\nu_0C_V\ln4/3}_{传热}$$

$\boxed{或}$: 先传热, 再抽板:

$\Delta S=\int\frac{dQ}{T}=\nu C_V\ln\frac{T}{T_0}$ $\Delta S_左=\frac{\nu_0}{2}C_V\ln\frac{2}{3}$ $\Delta S_右=\frac{\nu_0}{2}C_V\ln2$

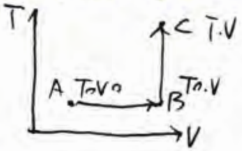
⇒ 混前: $\Delta S_1=\frac{1}{2}\nu_0C_V\ln4/3$

再抽板: ① A.B不同种: $\Delta S_2=\frac{\nu_0}{2}R\ln2+\frac{\nu_0}{2}R\ln2=\nu_0R\ln2$

② A.B同种气体: $\Delta S_2=0$.

↳ "全同粒子"不能区分同种气体的2个分子.

☆[例] 范氏气体: $(P+\frac{a}{V^2})(V-b)=RT$ $U=C_VT-\frac{a}{V}$. 求绝热方程与熵表达式.



解: $\boxed{法一}$: dQ: $PdV+dU$ 积分.

$\boxed{法二}$: $S_B-S_A=\int\frac{dQ}{T}=\int\frac{PdV+(dU)_T}{T_0}$

$=\frac{1}{T_0}\int_{V_0}^{V}(\frac{RT_0}{V-b}-\frac{a}{V^2})dV-d(-\frac{a}{V})$

$=R\ln\frac{V-b}{V_0-b}$.

$S_C-S_B=\int\frac{dQ}{T}=\int\frac{(dU)_V}{T}=\int_{T_0}^{T}\frac{C_VdT}{T}=C_V\ln\frac{T}{T_0}$.

$$\Rightarrow S_C-S_A=R\ln\frac{(V-b)T^{C_V/R}}{(V_0-b)T_0^{C_V/R}}$$

∴ 绝热线: $(V-b)T^{C_V/R}=Const$.

☆[例] A: m. 100℃的水; B: m. 0℃的水. ① 若只能热交换(取水靠上)能否将0℃水升到60℃? ② 不限制传热, 可用热机, 可否?

解: ① 尽量让A的水传更多热: 大温差.

∴ 将A分N分. m/N. 则n次后: $T_B=T_n$

则: $T_{n+1}(m+\frac{m}{N})=\frac{m}{N}\cdot100+m\cdot T_n$

$\Rightarrow 100-T_{n+1}=\frac{N}{N+1}(100-T_n)$

$\Rightarrow 100-T_N=(\frac{N}{N+1})^N(100-T_0)$ ∴ 极限下: $T_N=100-\frac{1}{e}\cdot100\approx63.2℃$