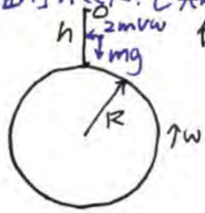
复赛专题下

★[例] $h \ll R$. 已知 h, w, g. 初速 0. 求落地点距楼 x.



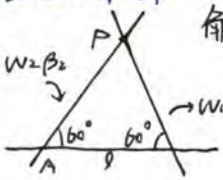
解: $v_y = gt$

∴ $a_x = 2wgt \Rightarrow v_x = wgt^2 \Rightarrow x = \frac{1}{3}wgt^3$

由 $t_0 = \sqrt{2h/g}$

$\Rightarrow x = \frac{1}{3}wg\left(\frac{2h}{g}\right)^{\frac{3}{2}}$

★[例] 求 v_P, a_P.



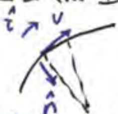
解: (1) $v_P = \frac{\sqrt{(w_1 l)^2 + (w_2 l)^2 - 2(w_1 l)(w_2 l)\cos\theta}}{\sin\theta}$

(2) I杆: $a_{1\parallel} = \ddot{P}_{1x} - w_1^2 l_v$　　$a_{1\perp} = \beta_1 l_v + 2w_1 \dot{P}_{1v}$

II杆: $a_{2\parallel} = \ddot{P}_{2x} - w_2^2 l_v$　　$a_{2\perp} = \beta_2 l_v + 2w_2 \dot{P}_{2v}$

$\Rightarrow a_P = \frac{\sqrt{a_{1\perp}^2 + a_{2\perp}^2 - 2a_{1\perp}a_{2\perp}\cos\theta}}{\sin\theta}$

△自然坐标:

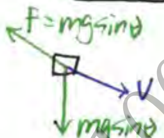


$\vec{a} = \frac{dv}{dt}\hat{\tau} + v\frac{d\hat{\tau}}{dt} = \dot{v}\cdot\hat{\tau} + vw\cdot\hat{n}$

有: $\begin{cases} a_\tau = \dot{v} \\ a_n = v\cdot w = w^2\rho = \frac{v^2}{\rho} \end{cases}$ —— 分别对应2个方向牛二.

★[例] $\mu = \tan\theta$. 求 $v_末$.

俯:



解: 直: $\begin{cases} x: ma_x = mg\sin\theta\sin\alpha & \cdots ① \\ y: ma_y = mg\sin\theta(1-\cos\alpha) & \cdots ② \end{cases}$

自: $\begin{cases} \tau: mg\sin\theta(1-\cos\alpha) = -ma_\tau & \cdots ③ \\ n: mg\sin\theta\sin\alpha = mwv. & \cdots ④ \end{cases}$

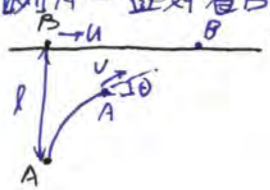
①②/③④ 联立可解一般.

但: ②, ③ 联立 ⇒ $v_末$:

$\Rightarrow a_y = -a_\tau \Rightarrow v' = v_0 - v'$

∴ $v' = \frac{1}{2}v_0$.

★[例] A 一直对着 B, 求 $t=?$ 追上.



解: 直: $\begin{cases} x: \int v\cos\theta\,dt - \int u\,dt = 0 & \cdots ① \\ y: \int v\sin\theta\,dt = l \end{cases}$

自: $\begin{cases} \tau: \int v\,dt - \int u\cos\theta\,dt = l & \cdots ② \\ n: 略. \end{cases}$

①,② $\Rightarrow$ $t = \frac{vl}{v^2 - u^2}$

轨道运动(7.2)

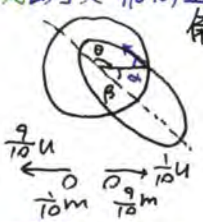
△椭圆轨道:

$$\begin{cases}\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\\ \rho=\frac{p}{1+e\cos\theta}\end{cases}\quad 由:\begin{cases}E=-\frac{GMm}{2a}\\ L=mb\sqrt{\frac{GM}{a}}\end{cases}\Rightarrow\begin{cases}a=-\frac{GMm}{2E}\\ b=L\sqrt{\frac{-1}{2mE}}\\ c=\sqrt{\frac{G^2M^2m^2}{4E^2}+\frac{L^2}{2mE}}\\ e=\sqrt{1+\frac{2EL^2}{G^2M^2m^3}}\\ p=\frac{L^2}{GMm^2}\end{cases}$$

在 θ 处: $\tan\alpha=\frac{\rho d\theta}{d\rho}=\frac{1+e\cos\theta}{e\sin\theta}$

面积速度: $S_v=\frac{1}{2}\frac{L}{m}$.

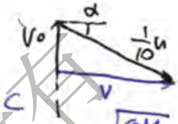
☆[例]变轨问题. 喷射 $\frac{1}{10}m$(切向 α 角, u), 求轨道.



解: $V^2=V_0^2+(\frac{1}{10}u)^2-2V_0\cdot\frac{1}{10}u\cdot\sin\alpha$.

$$\begin{cases}\frac{1}{2}m'V^2-\frac{GMm'}{r_0}=-\frac{GMm'}{2a}\Rightarrow a\\ m'(\frac{1}{10}u\sin\alpha-V_0)r_0=m'b\sqrt{GM/a}\Rightarrow b\Rightarrow c\end{cases}$$

$\Rightarrow r_0=\frac{\frac{b^2}{a}}{1+\frac{c}{a}\cos\theta}$ (回代可出 θ_0) → 得 β ⇒ 主轴转角.



$V_0=\sqrt{\frac{GM}{r_0}}$

☆[例]原: T. 近地点 r_0. M, 卫星 m. 点火: 反向喷出. 耗能 ΔE

在 θ 处点火. 求轨道.

解: $\frac{a^3}{T^2}=\frac{GM}{4\pi^2}\Rightarrow a$; $r_0=a-c\Rightarrow c\Rightarrow b$

$\frac{1}{2}\frac{0.9m\times0.1m}{0.9m+0.1m}u^2=\Delta E\Rightarrow u$

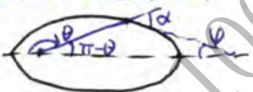
代入原来方程: $\rho=\frac{p}{1+e\cos\theta}$; $\frac{1}{2}mv^2-\frac{GMm}{\rho}=-\frac{GMm}{2a}$

新轨: $\frac{1}{2}m'(V+\frac{1}{10}u)^2-\frac{GMm'}{\rho}=-\frac{GMm'}{2a'}\Rightarrow a'\Rightarrow\rho V$.

$b'\sqrt{GM/a'}/b\sqrt{GM/a}=(V+\frac{1}{10}u)/V\Rightarrow b'\Rightarrow c'$

回代 ⇒ θ' ⇒ 主轴转角 $\alpha=\theta'-\theta$.

☆[例] θ 时速度方向为 φ. 周期 T. 中心天体 M. 求 $\theta_1\to\theta_2$ 历时.

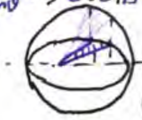


解: $\frac{a^3}{T^2}=\frac{GM}{4\pi^2}\Rightarrow a$; $\tan(\pi-\theta+\varphi)=\frac{1+e\cos\theta}{e\sin\theta}\Rightarrow e, c, b$

时间: $S=\frac{1}{2}b\sqrt{\frac{GM}{a}}t$.

利用仿射变换: $\frac{S_{椭}}{S}=\frac{b}{a}$

$\Rightarrow\frac{S_{椭}}{S_{圆}}=\frac{b}{a}\Rightarrow t$



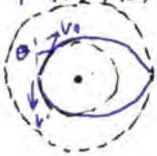
△双曲线轨道:

$$\begin{cases}E=\frac{GMm}{2a}\\ L=mb\sqrt{\frac{GM}{a}}\\ \tan\alpha=\frac{1+e\cos\theta}{e\sin\theta}\end{cases}\quad\begin{cases}\rho=\frac{p}{1+e\cos\theta}\\ \frac{x^2}{a^2}-\frac{y^2}{b^2}=1\end{cases}$$

瞄准距 b 即是双曲线的 b!

☆[例]已知 r_1. r_2. 太阳 M. 飞船初 V_0. 从木星偷能后变轨椭圆

求 (1) θ, (2) b.



解: (1) 飞船在太阳系中 θ. V_0.

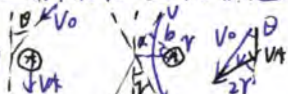
$V^2=V_木^2+V_0^2-2V_木V_0\cos\theta$; $V'=V_木+V$

有: $\frac{1}{2}mV'^2-\frac{GMm}{r_1}=-\frac{GMm}{r_1+r_2}\Rightarrow V'$

又: $V_木=\sqrt{GM/r_1}\Rightarrow V.\theta$.

(2) 双: $\frac{1}{2}mV^2+0=\frac{GMm}{2a}\Rightarrow a$

$\tan\gamma=a/b\qquad\Rightarrow b$



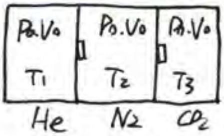
充气、漏气、混合问题 (9.4)

混合气体：共用V；T相同；$P=\sum P_i$

各自：$P_iV=n_iRT \Rightarrow PV=\sum n_i RT$

有：等效热容：$C_V=\frac{n_1C_{V1}+n_2C_{V2}+\cdots}{n_1+n_2+\cdots}$

☆例 求(1)混后P,T (2)绝热压$3V_0\to V_0$，求P' (3)ΔS(混合的)



解：(1) $\begin{cases}\Delta n=0. & \frac{P_0V_0}{T_1}+\frac{P_0V_0}{T_2}+\frac{P_0V_0}{T_3}=\frac{P\cdot 3V_0}{T}\\ \Delta E=0. & \frac32P_0V_0+\frac32P_0V_0+\frac62P_0V_0=\frac32P\cdot3V_0+\frac32P\cdot3V_0+\frac62P3\end{cases}$

$\Rightarrow P, V.$

(2) $C_V'=\frac{n_1\cdot\frac32R+n_2\cdot\frac32R+n_3\cdot\frac62R}{n_1+n_2+n_3}$ $\quad\gamma'=\frac{C_V'+R}{C_V'}$

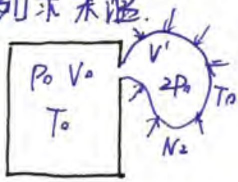
有：$P(3V_0)^{\gamma'}=P'\cdot V_0^{\gamma'}$

(3) $\Delta S=nR\ln\frac{V}{V_0}+nC_V\ln\frac{T}{T_0}$

则：$\begin{cases}\Delta S_{He}=n_1\frac32R\ln\frac{T}{T_1}+n_1R\ln3\\ \Delta S_{N_2}=n_2\frac32R\ln\frac{T}{T_2}+n_2R\ln3\\ \Delta S_{CO_2}=n_3\frac62R\ln\frac{T}{T_3}+n_3R\ln3\end{cases}$ $\Rightarrow \Delta S=\Delta S_{He}+\Delta S_{N_2}+\Delta S_{CO_2}$

· 整体法：假设多了个气球

☆例 求末温.

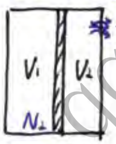


解：$\begin{cases}\Delta n=0. & \frac{P_0V_0}{T_0}+\frac{2P_0V'}{T_0}=\frac{2P_0V_0}{T}=nR\\ \Delta E=0. & nC_V(T-T_0)=2P_0V'\end{cases}$

$\Rightarrow T, V'$

· 隔离法：假设多了个挡板，其余气体绝热.

☆例 初P_0, V_0, T_0，漏到$P=\frac12P_0$，求$\Delta V/V_0$



解：$V_0=\frac{P_0V_0}{RT_0}$.

$P_0\cdot V_1^{\gamma}=(\frac12P_0)V_0^{\gamma}\Rightarrow V_1=(\frac12)^{\frac1\gamma}\cdot V_0$

$\therefore \frac{\Delta V}{V_0}=1-(\frac12)^{\frac1\gamma}$

$\Delta Q=0.$

$W=P\Delta V$

$=$不过压.

· 多孔塞：

细小孔，细长管，缓慢漏气.

特点：① 左右有Δp，但仍然准静态.

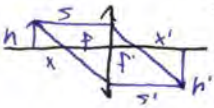
② 物质交换，但无热交换：$dQ=0$.

③ 余下气体温度均匀，绝热膨胀.

注：物质流：$\nabla\cdot\vec{J}=0$；$\nabla^2T=0$ 符从唯一性定理——猜$T(\vec{r},t)=T(t)$

→满足规律&边界$\Rightarrow$T均匀——内部部分间不吸放热；剩下气体绝

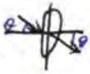
透镜(10.3)



· $\frac{f}{s}+\frac{f'}{s'}=1$；$\frac{h'}{h}=\frac{s'}{s}\frac{f}{f'}$

· $xx'=ff'$；$\frac{h'}{h}=\frac{f}{x}=\frac{x'}{f'}$

· 光心处：



$\theta=\frac{h}{s}$ $\theta'=\frac{h'}{s'}$

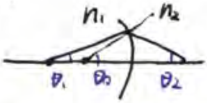
$\sin\theta\cdot n=\sin\theta_0 n_0=\sin\theta'\cdot n'$

$\Rightarrow \frac{h}{s}n=\frac{h'}{s'}n'\Rightarrow\frac{f}{f'}=\frac{n}{n'}$

· 横向放大率：$\beta=\frac{y'}{y}$ · 纵向放大率：$\gamma=\frac{dx'}{dx}$

· 常见元件：

□球面：



由：$(\theta_0-\theta_1)n_1=(\theta_0+\theta_2)n_2$ $\theta_0=\frac{h}{R}$, $\theta_1=\frac{h}{s}$, $\theta_2=\frac{h}{s'}$

$\Rightarrow \frac{n}{s}+\frac{n'}{s'}=\frac{n'-n}{R}$ $\beta=\frac{h'}{h}=-\frac{s'}{s}\frac{n}{n'}$

□平面：$R\to\infty$：$\frac{s'}{s}=-\frac{n'}{n}$(成像在同侧) $\beta=\frac{H}{h}=1$(横向不变)

□球面反射：重新定义 s' 正方向，$n'\to -n$

① 凹：$\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$

② 凸 $\frac{1}{s}+\frac{1}{s'}=-\frac{2}{R}\Rightarrow\frac{1}{u}+\frac{1}{v}=-\frac{2}{R}$

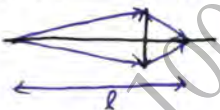
□薄透镜：$s_2=-s_1'\Rightarrow\frac{n_1}{s_1}+\frac{n_2}{s_2'}=\frac{n-n_1}{r}+\frac{n-n_2}{r}$

→当 $n_1=n_2$ 时：$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}$，$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}$

$\Rightarrow\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f_1}+\frac{1}{f_2}$ (光焦度相加：$\Phi=\Phi_1+\Phi_2$)

· 特殊情况：

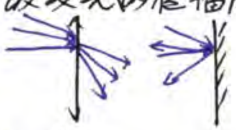
□共轭：$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$



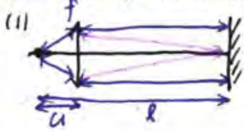
$\begin{cases}\frac{1}{u}+\frac{1}{v}=\frac{1}{f}\\ u+v=l\end{cases}\Rightarrow u^2-lu+fl=0$

$\Delta=l^2-4fl$ 则：$\begin{cases}l>4f & \text{大小像}\\ l=4f & \text{一个像}\\ l<4f & \text{无像}\end{cases}$

□成像在镜子处：不改变像的大小，只改变光的传播方向。



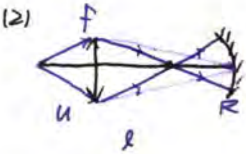
☆[例]物体发光像成在自身处。



解：(1) 情况一：在焦点

情况二：反射像

$\begin{cases}u+v=l\\ \frac{1}{u}+\frac{1}{v}=\frac{1}{f}\end{cases}\Rightarrow\begin{cases}l>4f & \text{二解}\\ l=4f & \text{一解}\\ l<4f & \text{0解}\end{cases}$



(2) 情况一：$\begin{cases}u+v=l-R\\ \frac{1}{u}+\frac{1}{v}=\frac{1}{f}\end{cases}\Rightarrow\begin{cases}l-R>4f & \text{2解}\\ l-R=4f & \text{一解}\\ l-R<4f & \text{0解}\end{cases}$

情况二：$\begin{cases}u+v=l\\ \frac{1}{u}+\frac{1}{v}=\frac{1}{f}\end{cases}\Rightarrow\begin{cases}l>4f & \text{2解}\\ l=4f & \text{一解}\\ l<4f & \text{0解}\end{cases}$

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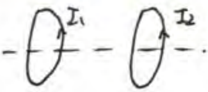
电感(7.4)

$\varepsilon=-L\frac{dI}{dt}$；$\Phi=LI$ ； $w=\frac{1}{2}\mu_0\mu_r H^2=\frac{B^2}{2\mu_0\mu_r}$

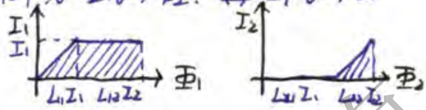
$B=\mu_0 nI$；$\Phi=NBS\Rightarrow L=\mu_0 n^2 V$

有：$dW=u\,dq=\frac{d\Phi}{dt}dq=I\,d\Phi$.

☆[例]已知 $L_{11}.L_{12}.L_{21}.L_{22}$. (1) 先 $I_1:0\to I_1$，再 $I_2:0\to I_2$.
求电源做功. (2) 先 $I_2:0\to I_2$，再 $I_1:0\to I_1$

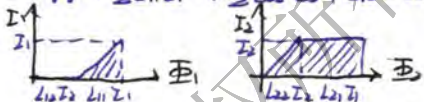


解：(1)



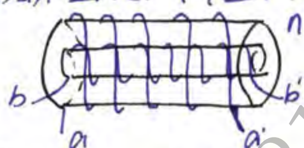
$W=\frac{1}{2}L_{11}I_1^2+\frac{1}{2}L_{22}I_2^2+L_{12}I_1I_2$ ⇒ $L_{21}=L_{12}=M$ (互感)

(2)



$W=\frac{1}{2}L_{11}I_1^2+\frac{1}{2}L_{22}I_2^2+L_{21}I_1I_2$

☆[例]外圈R. 内圈r. l. 求：(1)内圈L_1. 外圈L_2. 互感M.
(2) a'接b; a.b'外接. 求L.
(3) a'接b'; a.b外接. 求L.



解：(1) $L_1=\mu_0 n^2\cdot\pi r^2 l$ $L_2=\mu_0 n^2\cdot\pi R^2 l$. $M=\mu_0 n^2\cdot\pi r^2 l$.

(2) 内：$B=2\mu_0 nI$

内外之间：$B=\mu_0 nI$

$\therefore\Phi=\underbrace{2\mu_0 nI(\pi r^2)nl}_{内圈}+\underbrace{\mu_0 nI(\pi R^2-\pi r^2)nl+2\mu_0 nI\pi r^2 nl}_{外圈}$.

$\Rightarrow L=\mu_0 n^2(\pi R^2+3\pi r^2)=L_1+L_2+2M$

(3) 内：$B=0$

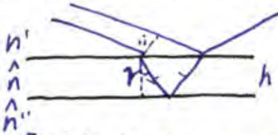
内外之间：$B=\mu_0 nI$

$\therefore\Phi=\mu_0 nI(\pi R^2-\pi r^2)$

$\Rightarrow L=\mu_0 n^2(\pi R^2-\pi r^2)=L_1+L_2-2M$

[或]：用场能：以(3)为例：$E=\frac{1}{2}LI^2=\frac{1}{2\mu_0}\cdot B^2(\pi R^2-\pi r^2)l\Rightarrow L$

△等厚干涉(表面干涉): → 上下表面不一定平行,这里近似.

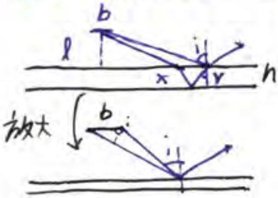


$\Delta \doteq 2nh\frac{1}{\cos r} - 2h\tan r\sin i\cdot n'$

$= 2nh\frac{1}{\cos r} - 2h\frac{\sin^2 r}{\cos r}\cdot n$

$= 2nh\cos r$ r=内部的角. 注意半波损!

☆[例] 求亮纹的x值; x_0时亮纹消失, 求b.



解: (1) $\Delta = 2nh\cos r + \frac{\lambda}{2} = N\lambda \Rightarrow r = r(k)$

$\sin i = \sin r\cdot n \Rightarrow i = i(k)$

$\tan i\, l = x \Rightarrow x = x(k)$

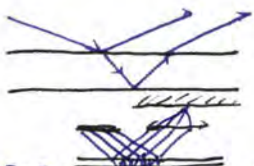
(2) $\delta\Delta = \lambda = 2nh\sin r\cdot\delta r$

$b\cos i = \delta i\sqrt{x_0^2+l^2}$

$\sin i\cdot 1 = \sin r\cdot n \Rightarrow \cos i\,\delta i = \cos r\,\delta r\cdot n$

$\Rightarrow b = \frac{\cos r\, n l}{\cos^3 i}\cdot\frac{\lambda}{\sin r\cdot 2nh}$

△等倾干涉(无穷远干涉):

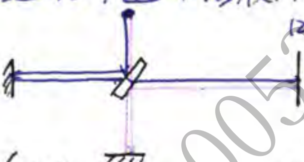


$\Delta = 2nh\cos r$ → ∞处干涉 ⇒ 平移平行 ⇒ 平移解.

→ 扩展光源不会使消失,反会使I↑: (无波谷)

$\delta\Delta = \delta(2nh\cos r)$ 而h.r均相同.

☆[例] 迈克尔逊干涉仪. (1) 等效点光源, z向错开 l, y向错开 h. (2) l变大,如何变 (3) 2种频率, 平均波长λ. 图样吞吐了Δk个环从清晰变模糊, 问Δλ?

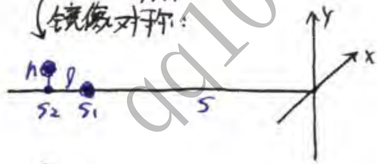


解: (1) $\Delta = \sqrt{x^2+(y+h)^2+(s+l)^2} - \sqrt{x^2+y^2+s^2}$

$= \frac{s}{2}\left[\left(\frac{x}{s+l}\right)^2+\left(\frac{x}{s}\right)^2\right] + \left[\left(\frac{y+h}{s+l}\right)^2-\left(\frac{y}{s}\right)^2\right]$

$+ l\left[1+\frac{1}{2}\left(\frac{x}{s+l}\right)^2+\frac{1}{2}\left(\frac{y+h}{s+l}\right)^2\right]$ (偏心圆)

镜像对称:



(2) $\Delta = l\cos\theta$ $l\uparrow \Rightarrow \cos\theta\downarrow \Rightarrow \theta\uparrow$ (吐环)

(3) 清: $\Delta = k_1\lambda_1 = k_2\lambda_2$

模: $\delta\Delta = (k_1+\Delta k)\lambda_1 - (k_2+\Delta k)\lambda_2 = \frac{1}{2}\lambda$.

$\Rightarrow \Delta k\cdot\Delta\lambda = \frac{1}{2}\lambda$

或

$\therefore \Delta\lambda = \frac{\lambda}{2\Delta k}$ 即差的级数=吞吐环数

☆[例] 等倾干涉中几乎垂直2个入射光(h, $n=1$) λ_1的2.5环在$r_{1,2}$, $r_{1,5}$处, λ_2的则在$r_{2,2}$, $r_{2,5}$处. 求Δλ. (要考虑不完全反射Δ')

解: λ_1环从级数 k_1+f_1 (N+非N) 有: $2h+\Delta' = (k_1+f_1)\lambda_1$ N级: $k_1-(N-2)$

有: $2h\cos\theta+\Delta' = \lambda_1(k_1-N+2) \Rightarrow 2h(1-\cos\theta) = h\theta^2 = (f_1+N-2)\lambda_1 \therefore \frac{r_{1,2}^2}{r_{1,5}^2} = \frac{2-2+f_1}{5-2+f_1} \Rightarrow f_1$

环从λ_1: $k_1+f_1 = \frac{2h}{\lambda_1}+\frac{\Delta'}{\lambda_1} = \frac{2h}{\lambda_1}+\frac{\varphi}{2\pi}$ λ_2: $k_2+f_2 = \frac{2h}{\lambda_2}+\frac{\varphi}{2\pi}$

$\Rightarrow f_1-f_2 = 2h\frac{\Delta\lambda}{\lambda^2}$ $\therefore \Delta\lambda = (f_1-f_2)\frac{\lambda^2}{2h}$

对称性破缺(3.7)

△势函数：

$U=U(x_0)+U'\Delta x+U''/2!\cdot(\Delta x)^2+\cdots \Rightarrow \omega_0=\sqrt{\dfrac{\frac{d^2U}{dx^2}\big|_{x=x_0}}{m_{eff}}}$

· 对称性破缺：$U(x)=A(\eta)x^2+B(\eta)x^4$ $A(\eta)=\begin{cases}>0, & \eta>\eta_0\\ <0, & \eta<\eta_0\end{cases}$ $B(\eta)>0$.

令 $\dfrac{dU(x)}{dx}=0 \Rightarrow x=\begin{cases}x_0=0\\ x_{1,2}=\pm\sqrt{-A/2B}\end{cases}$ ▫ $\eta>\eta_0$, $x=x_0$ 为稳定平衡解 ▫ $\eta<\eta_0$, $x_{1,2}$ 为实数，是平衡解 → 扭变.

☆[例]倒摆，螺旋弹簧使它支撑到 $\theta=0$ 的平衡位置上.

解：$U(\theta)=\frac{1}{2}k\theta^2+mgl(\cos\theta-1)=\frac{1}{2}mgl[(\eta-1)\theta^2+\frac{1}{12}\theta^4]$ $\eta=k/mgl$

$\Rightarrow \theta_0=0.$ $\theta_{1,2}=\pm\sqrt{6(1-\eta)}$.

☆[例]旋转摆.

解：$U(\theta)=-\frac{1}{4}mR^2\omega(1-\cos2\theta)+mgl(1-\cos\theta)=\frac{1}{2}mR^2[(\omega_0^2-\omega^2)\theta^2+(\omega^2-\frac{\omega_0^2}{4})\theta^4/\ldots]$

$\Rightarrow \theta_{1,2}=\pm\sqrt{2(\omega^2-\omega_0^2)/(4\omega^2-\omega_0^2)}$. $\omega_0=\sqrt{g/R}$

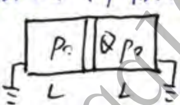
$\Rightarrow$ 特点：势能以偶次方为级(2,4…) $U=0x^2+0x^4$.

☆[例]讨论稳定性.

解：近似 $\Rightarrow \ddot{\theta}+\dfrac{mg(R-h/2)}{[m(h/2)^2+I_c]}\theta=0.$ $R>h/2$ 时平衡.

分析：$R=h/2$ 时 $\theta=0$ 处4阶导 $<0 \Rightarrow$ 相变.

☆[例]中间板带 Q，左右初始 P_0，求稳定点.



解：$C=C_1+C_2=\dfrac{\varepsilon_0S}{L+x}+\dfrac{\varepsilon_0S}{L-x}=\dfrac{2\varepsilon_0SL}{L^2-x^2}$ $\therefore W_e=\dfrac{Q^2}{2C}=\dfrac{Q^2(L^2-x^2)}{4\varepsilon_0SL}$

有：$F_e=-\dfrac{\Delta W_e}{\Delta x}=\dfrac{Q^2x}{2\varepsilon_0SL}$

又：$P_1=\dfrac{P_0L}{L+x}$，$P_2=\dfrac{P_0L}{L-x} \Rightarrow F_e=(P_2-P_1)S$ $x=\pm L\sqrt{1-\dfrac{4P_0\varepsilon_0S^2}{Q^2}}$

要求：$Q\geq 2S\sqrt{\varepsilon_0P_0}$.

(2) $\alpha>0$, 必 $E>0$

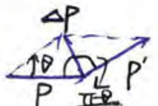
有: $2mEr^2+2m\alpha r-L^2=0$

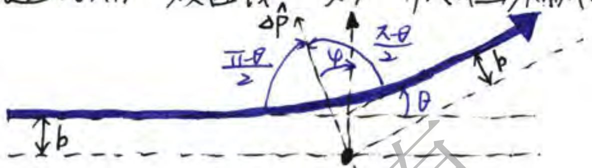
则: $r=\frac{\alpha}{2E}\left(1+\sqrt{1+\frac{2EL^2}{m\alpha^2}}\right)$

又: $L=mbv_\infty$; $E=\frac{1}{2}mv_\infty^2$　　b: 碰撞参数(瞄准距离)

$\Rightarrow$ $r_{min}=\frac{\alpha}{2E}\left(1+\sqrt{1+\frac{2mEb^2v_\infty^2}{k^2q^2Q^2}}\right)$

→仅一拐点，轨道无界，双曲线一支，力心在外侧.





● 计算电场(1,2)

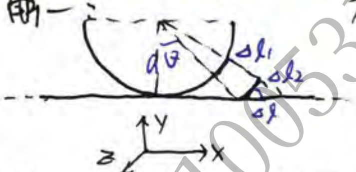
·叠加法: $\boxed{\vec{E}=\frac{1}{4\pi\varepsilon_0}\sum_i\frac{q_i}{r_i^2}\vec{e}_{r_i}=\frac{1}{4\pi\varepsilon_0}\iiint_V\frac{\rho dV}{r^2}\vec{e}_r}$

·高斯法: $\boxed{\sum_{闭合面}\vec{E}\cdot\Delta\vec{S}=\frac{1}{\varepsilon_0}\sum_{S内}q_i}$

·电势法: $\boxed{\vec{E}=-\left(\frac{\partial U}{\partial x}\vec{e}_x+\frac{\partial U}{\partial y}\vec{e}_y+\frac{\partial U}{\partial z}\vec{e}_z\right)=-\nabla U}$

☆[例]无限长均匀带电直导线外的电场强度.

解一:



有: $\Delta\theta=\Delta l_1/a=\Delta l_2/r=\Delta l\cos\theta/r$

→ $a/r=\cos\theta=\Delta l_1/\Delta l_2$

$\Delta E=k\Delta q/r^2=k\lambda\Delta l/r^2=k\frac{\lambda\Delta l_2}{r^2\cos\theta}=k\frac{\lambda\Delta l_1}{a^2}$

→ $f=f_0\cdot 2a=k\frac{\lambda}{a^2}\cdot 2a=k\frac{2\lambda}{a}\Rightarrow E=\frac{\lambda}{2\pi\varepsilon_0 a}$

解二: 取柱形高斯面，由对称性：电场必垂直于轴线.

注一定说明如何取及场方向!

有: $2\pi a\cdot hE=\lambda h/\varepsilon_0\Rightarrow E=\frac{\lambda}{2\pi\varepsilon_0 a}$

解三: (为避免积分无限大，这里使用不定积分)

有: $U_r=2\times\frac{1}{4\pi\varepsilon_0}\int\frac{\lambda\cdot dx}{\sqrt{x^2+y^2}}=\frac{\lambda}{2\pi\varepsilon_0}\ln(x+\sqrt{x^2+y^2})+C$

$\Rightarrow E_y=-\frac{\partial U}{\partial y}=\frac{1}{2\pi\varepsilon_0}\cdot\frac{\lambda}{a}$; $E_z=-\frac{\partial U}{\partial z}=0$; $E_x=-\frac{\partial U}{\partial x}=0$

注先求导，再带入上下限.

△常见电场计算:

·圆环轴线上电场: $E=\frac{\lambda}{2\varepsilon_0}\cdot\frac{R\cdot x}{(R^2+x^2)^{1.5}}$

·圆盘轴线上电场: 代换 $\lambda=\frac{dq}{2\pi r}=\frac{\sigma\cdot 2\pi r dr}{2\pi r}=\sigma\cdot dr$ 并积分

$E=\frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{|x|}-\frac{1}{\sqrt{R^2+x^2}}\right)$

· 第二方程：

$\boxed{\sum U=\sum(\pm\varepsilon\pm I_r\pm IR)=0}$ ⇒ 对 P 支路，有 $P-N+1$ 方程

⇒ 共 $P-N+1+N-1=P$ 个方程 ⇒ 完备性

对称性网络电阻（5.5）

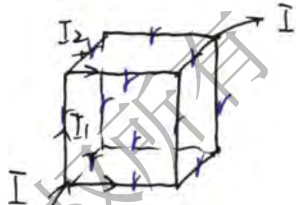
△ 方法总结：

· 电流叠加法：输入和输出电流叠加。

· 对称分析法：等势面；等势点，对称性。

· 基氏方程法：假设电流输入。

☆[例] 求对角线电阻。



解：$I=3I_1$

$I_1=2I_2$

$U_{AB}=I_1R+I_2R+I_1R$

$IR_{AB}=\frac{1}{3}IR+\frac{1}{6}IR+\frac{1}{3}IR=\frac{5}{6}IR$

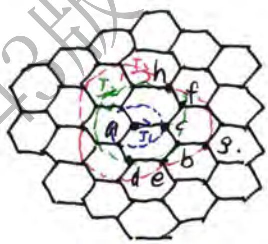
$\Rightarrow R_{AB}=\frac{5}{6}R$

☆[例] 每边电阻均为 r，求

(1) R_{ab}

(2) I_{de} (a入，g出)

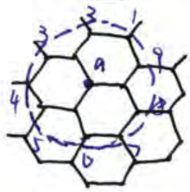
(3) R_{ah}



$I_1=\frac{1}{3}I$

$I_2=\frac{1}{6}I$

$I_3=\frac{1}{12}I$



解：(1) $I_{ac}=\frac{I}{3}+\frac{I}{6}=\frac{1}{2}I$

$I_{cb}=\frac{I}{3}+\frac{I}{6}=\frac{1}{2}I$

$R_{ab}=\frac{Ir/2+Ir/2}{I}=r$

(2) $I_1=I_4=I_7=I_A$

$I_2=I_3=I_5=I_6=I_8=I_9=I_B$.

$3I_A+6I_B=I$.

b.d 关于 a 对称：$I'_{de}=I_{be}=\frac{1}{2}I_A$

同理：从 g 流出时：$I''_{de}=I_B$

$\Rightarrow I_{de}=I'_{de}+I''_{de}=\frac{1}{2}I_A+I_B=\frac{1}{6}I$

(3) 从一个等势面到另一个等势面，若均以电阻 r 相接，不必对称，各电流相等：

$IR_{ab}=2\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{12}\right)=\frac{7}{6}Ir \Rightarrow R_{ah}=R_{ag}=\frac{7}{6}r$

理想气体的热力学过程总结

基本方程

$pV=\nu RT$； $E=\frac{i}{2}\nu RT$；

$Q=\Delta E+W$.

基本关系

$C_V=\frac{i}{2}R$；

$C_p=C_V+R$；

$\gamma=\frac{C_p}{C_V}=\frac{i+2}{i}$.

比较项目

吸收热量： $Q=nC\Delta T$ ；

内能增量： $\Delta E=nC_V\Delta T$；

对外做功： $W=\int PdV$.

基本过程

	过程方程	内能增量	对外做功	吸收热量
等容	$\frac{P}{T}=\frac{P_0}{T_0}=\frac{\nu R}{V}$	$\Delta E_V=\nu\cdot\underbrace{\frac{i}{2}R}_{C_V}\Delta T$	0	$Q_V=\nu\cdot\frac{i}{2}R\Delta T$
等压	$\frac{V}{T}=\frac{V_0}{T_0}=\frac{\nu R}{P}$	$\Delta E_p=\nu\frac{i}{2}R\Delta T$.	$W_V=P\cdot\Delta V$ $=nR\Delta T$	$Q_p=\nu\underbrace{(\frac{i}{2}+1)R}_{C_p}\Delta T$
等温	$PV=P_0V_0$ $=\nu RT$	0	③ $W_T=\nu RT\ln\frac{V}{V_0}$	$Q_T=\nu RT\ln\frac{V}{V_0}$
绝热	① $PV^{\gamma}=P_0V_0^{\gamma}$	$\Delta E_Q=\nu\cdot\frac{i}{2}R\Delta T$	② $W_Q=\frac{P_1V_1-P_2V_2}{\gamma-1}$	0

①： $Q=\Delta E+W\Rightarrow đQ=dE+đW=dE+PdV=0$

$\Rightarrow\frac{i}{2}\nu RdT+\nu RT\cdot\frac{dV}{V}=0$ 又 $\frac{i}{2}=\frac{1}{\gamma-1}\Rightarrow TV^{\gamma-1}=T_0V_0^{\gamma-1}\Rightarrow PV^{\gamma}=P_0V_0^{\gamma}$

②： $W_Q=-\Delta E_Q=\nu\cdot\frac{1}{\gamma-1}\cdot R\cdot(\frac{P_1V_1}{\nu R}-\frac{P_2V_2}{\nu R})=\frac{1}{\gamma-1}(P_1V_1-P_2V_2)$

或：$W_Q=\int PdV=\int P_1\frac{V_1^{\gamma}}{V^{\gamma}}dV=\frac{1}{\gamma-1}(P_1V_1-P_2V_2)$

③： $W_T=\int\nu RT\cdot\frac{dV}{V}=\nu RT\ln\frac{V}{V_0}$