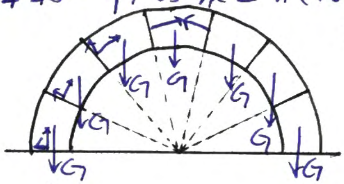
# 决赛专题

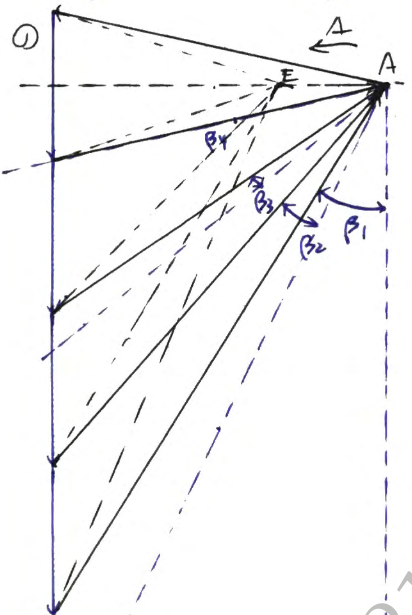
☆[例] $\alpha=\frac{\pi}{2}$，①最上块光滑，求维持的$\mu_{min}$ ②最上块也有$\mu$，如何.

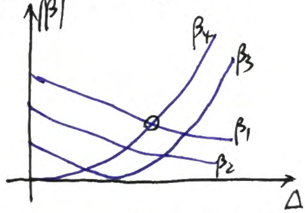



解：①作图：最大$\beta$：$\beta_1\approx32^\circ \Rightarrow \mu_{min}=0.63$

②假设受力对称，则A点水平向左平移.

分析各个角变化.



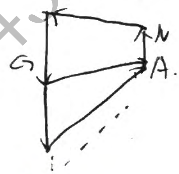




则$\beta_4=\beta_1$时对应有最小的$\mu$.

分析：(1) 实则应让E向外移动同时求$\beta_{max}$

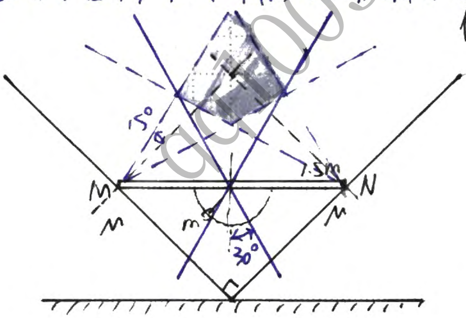
(2) 若问对顶块施最少多少力可使拱塌，可如下处理：

作图如：（$\mu$已知）




$\Rightarrow$ 然后再找$\beta_{max}$.

☆[例] 平面内水平释放m，使杆不滑，求$\mu_{min}$.



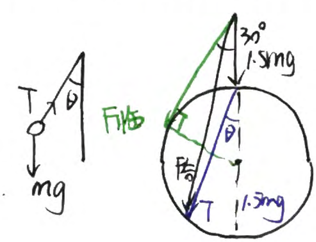

解：

$$\begin{cases} \frac{1}{2}mv^2=mgr\cos\theta \\ T-mg\cos\theta=mv^2/r \end{cases}$$

$\therefore T=3mg\cos\theta$

对杆受力分析.

$\Rightarrow \mu_{min}=\tan15^\circ=2-\sqrt{3}$.



有：

$$\begin{vmatrix} \lambda-2 & 1 & 0 & 0 & 0 & 1 \\ 1 & \lambda-2 & 1 & 0 & 0 & 0 \\ 0 & 1 & \lambda-2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda-2 & 1 & 0 \\ 0 & 0 & 0 & 1 & \lambda-2 & 1 \\ 1 & 0 & 0 & 0 & 1 & \lambda-2 \end{vmatrix} = \lambda \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \lambda-2 & 1 & 0 & 0 & 0 \\ 0 & 1 & \lambda-2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda-2 & 1 & 0 \\ 0 & 0 & 0 & 1 & \lambda-2 & 1 \\ 1 & 0 & 0 & 0 & 1 & \lambda-2 \end{vmatrix} = \lambda \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \lambda-3 & 0 & -1 & -1 & -1 \\ 0 & 1 & \lambda-2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda-2 & 1 & 0 \\ 0 & 0 & 0 & 1 & \lambda-2 & 1 \\ 0 & -1 & -1 & -1 & 0 & \lambda-3 \end{vmatrix}$$

$$= \lambda \begin{vmatrix} \lambda-3 & 0 & -1 & -1 & -1 \\ 1 & \lambda-2 & 1 & 0 & 0 \\ 0 & 1 & \lambda-2 & 1 & 0 \\ 0 & 0 & 1 & \lambda-2 & 1 \\ -1 & -1 & -1 & 0 & \lambda-3 \end{vmatrix} = \lambda(\lambda-3) \begin{vmatrix} \lambda-3 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & \lambda-2 & 1 & 0 \\ 0 & 0 & 1 & \lambda-2 & 1 \\ -1 & -1 & -1 & 0 & \lambda-3 \end{vmatrix} = \lambda(\lambda-3) \begin{vmatrix} \lambda-3 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & \lambda-2 & 1 & -1 \\ 0 & 0 & 1 & \lambda-2 & 1 \\ -1 & -1 & -1 & 0 & \lambda-2 \end{vmatrix}$$

$$= \lambda(\lambda-3) \begin{vmatrix} \lambda-3 & -1 & -1 & -1 \\ 0 & \lambda-2 & 1 & -1 \\ 0 & 1 & \lambda-2 & 1 \\ -1 & -1 & 0 & \lambda-2 \end{vmatrix} = \lambda(\lambda-3)^2 \begin{vmatrix} \lambda-3 & -1 & -1 & -1 \\ 0 & \lambda-2 & 1 & -1 \\ 1 & 0 & 1 & 0 \\ -1 & -1 & 0 & \lambda-2 \end{vmatrix} = \lambda(\lambda-3)^2 \begin{vmatrix} \lambda-2 & -1 & -1 & -1 \\ -1 & \lambda-2 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & \lambda-2 \end{vmatrix}$$

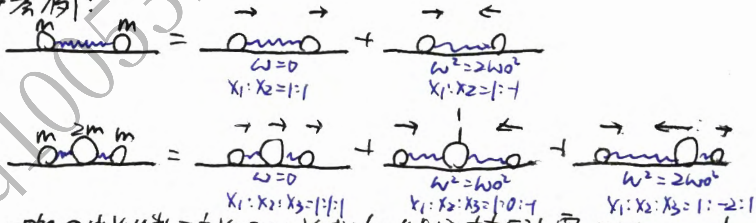
$$= \lambda(\lambda-3)^2 \begin{vmatrix} \lambda-2 & -1 & -1 \\ -1 & \lambda-2 & -1 \\ -1 & -1 & \lambda-2 \end{vmatrix} = \lambda(\lambda-3)^2(\lambda-4) \begin{vmatrix} 1 & 1 & 1 \\ -1 & \lambda-2 & -1 \\ -1 & -1 & \lambda-2 \end{vmatrix} = \lambda(\lambda-3)^2(\lambda-4) \begin{vmatrix} 1 & 1 & 1 \\ 0 & \lambda-1 & 0 \\ 0 & 0 & \lambda-1 \end{vmatrix}$$

$$= \lambda(\lambda-3)^2(\lambda-4) \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-1 \end{vmatrix} = \lambda(\lambda-3)^2(\lambda-4)(\lambda-1)^2$$

分析：①这是循环行列式，有特别解法。亦可用特征值处理。

②更重要的是"猜"出简正模！

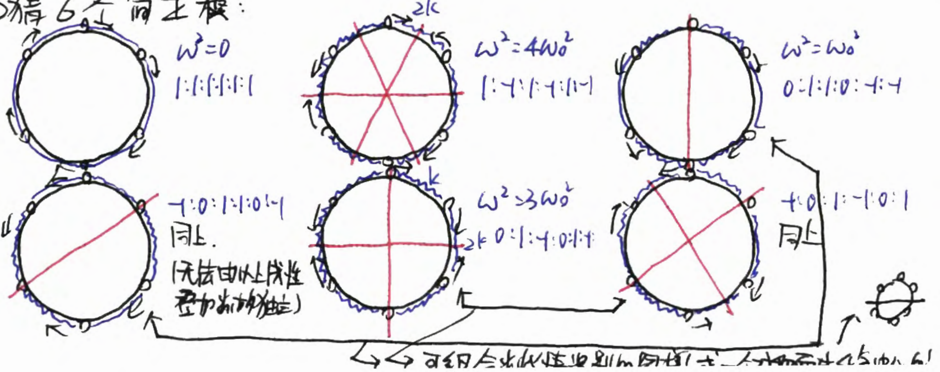
→举例：




理解：①对称性 ②2m拆成2个m分别和左右运动，而$-1+(-1)=-2$. ←同样可知$\omega^2=2\omega_0^2$！

法二：

⇒猜6个简正模：

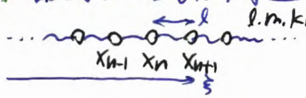



可组合出此情况即m同样[illegible]

## △振子波:

核心: 由平移对称性进行分析.

### ☆[例]弹簧绳波,求运动.



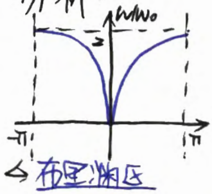

解: 猜: $x_n = Ae^{-i\omega t+in\varphi} \Rightarrow x(\xi,t) = Ae^{-i\omega t+i\xi\frac{\varphi}{l}}$ (波矢)

牛二: $m\ddot{x}_n = -k(2x_n - x_{n+1} - x_{n-1})$, 令 $\frac{k}{m} = \omega_0^2$

代回: $-\frac{\omega^2}{\omega_0^2} = -k(2 - e^{i\varphi} - e^{-i\varphi}) = -k\,4\sin^2\frac{\varphi}{2}$

$\Rightarrow \omega = \pm 2\omega_0 \sin\frac{\varphi}{2} = \pm 2\omega_0 \sin\frac{kl}{2}$ (左右传播) $\Rightarrow x_n = x_n(t)$

分析:




有: 一个波矢对应一个频率, 周期: $T = \frac{2\pi}{l}$

① $\varphi$较小时($k$较小), $\omega = kl\,\omega_0$, 相速度 $v_p = \frac{\omega}{k} = \omega_0 l$ 不变

——仍然无色散. ——对应实际: $m.k$ 极密.

$\lambda \gg l$ 的绳波, 无色散存在.

② 正常的$\omega$(包括复数):

由: $\frac{\omega^2}{\omega_0^2} = 2 - 2\cos\varphi$

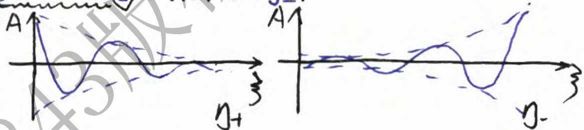
1° $\omega \in R$. $\omega \in [0, 2\omega_0] \Rightarrow \varphi = \pm \arcsin\frac{\omega}{2\omega_0}$, 是有色散的行波

2° $\omega \in R$. $\omega \in [2\omega_0, +\infty)$, 代 $\cos\varphi = (e^{i\varphi} + e^{-i\varphi})/2$

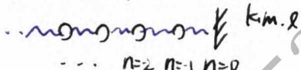
$\Rightarrow \varphi = -i\ln\left[\underbrace{\frac{2 - \omega^2/\omega_0^2 \pm \sqrt{(\omega^2/\omega_0^2)^2 - 4}}{2}}\right]$, 结果记为: $\eta_\pm$.

→ 对$\eta_+$, 策动力在原点:

→ 对$\eta_-$, 策动力在∞处:




### ☆[例]反射波.求运动.




解: 方程线性,故可叠加.

入射: $x_{n_1} = \tilde{A}\cdot e^{i(-\omega t + n\varphi)}$

反射: $x_{n_2} = \tilde{r}\cdot\tilde{A}\cdot e^{i(-\omega t - n\varphi)}$

$\Rightarrow x_n = \tilde{A}(e^{i(-\omega t+n\varphi)} + \tilde{r}\cdot e^{i(-\omega t - n\varphi)})$; 牛二: $m\ddot{x}_n = -k(2x_n - x_{n+1} - x_{n-1})$, $n \le -1$

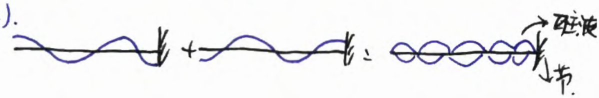
扩展定义域: 对$n=-1$ 也成立. 边界条件为 $x_0 = 0$, 有: $m\ddot{x}_{-1} = -k(2x_{-1} - x_{-2} - \underbrace{x_0}_{=0})$.

故有: $x_0 = \tilde{A}(e^{i(-\omega t)} + \tilde{r}\cdot e^{i(-\omega t)}) = 0 \Rightarrow \tilde{r} = -1 = e^{i\pi}$ ——半波损失.

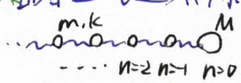
再代入一般式: $m\cdot(-\omega^2)\cdot\tilde{A}(e^{in\varphi} + \tilde{r}\cdot e^{-in\varphi}) = -k[2\cdot\tilde{A}(e^{in\varphi} + \tilde{r}\cdot e^{-in\varphi})$
$- \tilde{A}(e^{i(n+1)\varphi} + \tilde{r}\cdot e^{-i(n+1)\varphi}) - \tilde{A}(e^{i(n-1)\varphi} + \tilde{r}\cdot e^{-i(n-1)\varphi})]$

$\Rightarrow \omega = f(\varphi) \Rightarrow x_n = x_n(t)$.

分析: 半波损失实质为:




### ☆[例]空气边界,求运动.



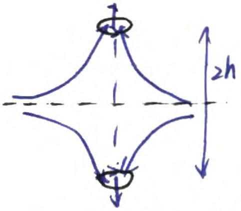

解: 入+反: $x_n = \tilde{A}e^{i(n\varphi - \omega t)} + \tilde{A}\tilde{r}\cdot e^{i(-n\varphi - \omega t)}$ 令定义域延拓

牛二: $m\ddot{x}_n = -k(2x_n - x_{n+1} - x_{n-1})$ 令 $\frac{M}{m} = \eta$. $\omega_0^2 = \frac{k}{m}$

对$n=0$: $-\eta\frac{\omega^2}{\omega_0^2}(1+\tilde{r}) = -(1 + \tilde{r} - e^{-i\varphi} - \tilde{r}e^{i\varphi})$

$$\Rightarrow \tilde{r} = -\frac{\eta\frac{\omega^2}{\omega_0^2} - 1 + e^{-i\varphi}}{\eta\frac{\omega^2}{\omega_0^2} - 1 + e^{i\varphi}} = -\frac{\eta\frac{\omega^2}{\omega_0^2} - 1 + \cos\varphi - i\sin\varphi}{\eta\frac{\omega^2}{\omega_0^2} - 1 + \cos\varphi + i\sin\varphi} = e^{i(\pi+\alpha)}$$

$\hookrightarrow \alpha = 2\arctan\frac{\sin\varphi}{\eta\frac{\omega^2}{\omega_0^2} - 1 + \cos\varphi}$




(2) $\vec{B}=\frac{\mu_0}{4\pi}\cdot\frac{2m}{z^3}\hat{z}$

有：$|F|=\left|u\cdot\frac{dB}{dz}\Big|_{z=2h}\right|=\left|I\cdot\pi r^2\frac{\mu_0}{4\pi}\cdot\frac{2m}{(2h)^4}(-3)\right|=mg$

$\Rightarrow h=\sqrt[4]{\frac{3I^2\pi r^4\mu_0}{32mg}}$.

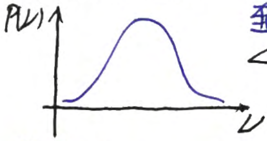
故先对B求导后再代入.

分析：再求振动：$\ddot{x}\cdot m=-\frac{4\cdot 6\cdot\mu_0\cdot u^2}{4\pi\cdot 16\cdot x_0^5}\cdot x\Rightarrow \omega$

# 量子力学初步(83)

△黑体辐射：

·全频段叠加：




辐射本领 $P(\nu)=\frac{dE}{dSdt\,d\nu}$ $\xrightarrow[\text{分布}]{\text{不同}\nu}$ 黑体: $J=\frac{dE}{dSdt}=\sigma T^4$

↳ 反射下: $J'=(1-r)\sigma T^4$ (r: 反射率)

☆[例] 已知 $r_S$. $r_{ES}$. $T_S$. (1) 求 $T_E$ (2) 地表包一层黑体壳，求 $T_E$.

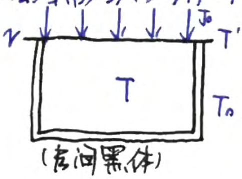
解: (1) $\sigma T_S^4\cdot 4\pi r_S^2\cdot\frac{\pi r_E^2}{4\pi r_{ES}^2}=\sigma T_E^4\cdot 4\pi r_E^2 \Rightarrow T_E=\left(\frac{r_S^2}{4r_{ES}^2}\right)^{\frac14}T_S$

(2) 地球平衡: 地→壳 = 壳→地

壳+地球平衡: 太阳→壳 = 壳→外 $\Rightarrow J'=J_0$, $T_E'=T_E$

薄壳: 壳→地 = 壳→外

☆[例] 高频反射率 $r_H$, 低频反射率 $r_L$, 求T. (散热 $\frac{dQ}{dt}=K(T-T_0)$)




解: 房顶: $J_0(1-r_H)S+\sigma T'^4 S(1-r_L)=\sigma T'^4(1-r_L)\cdot 2S$

房间: $\sigma T^4(1-r_L)S=\sigma T'^4(1-r_L)S+K(T-T_0)$

$\Rightarrow T, T'$

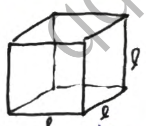
·频率分布细化:

单位频率能量密度: $u(\nu,T)=\frac{dE}{dV\cdot d\nu}=G(\nu)\cdot\varepsilon(\nu,T)$ ← 一个模式下的平均能

↳ 模式密度: $G(\nu)=\frac{dM(\nu)}{d\nu\cdot dV}$ 模式个数, 与T无关

单位频率能流密度: $\sigma=\frac{dE}{dSdtd\nu}\Rightarrow P=\frac{dE}{dSdt}$

☆[例] 使用驻波条件，使用离散的 $k_x$. $k_y$. $k_z$. 由最大频率 $\nu_0$, c, l 求驻波模式个数 M, 模式密度 G.



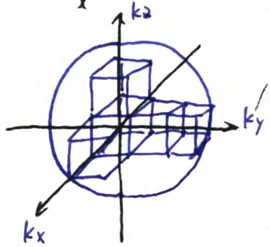

解: 一个方向上有2个驻波!

$E_y(x,t)=E_0\cos(k_x x-\omega t)$ c.c. 这里已设 $k_i>0$.

$E_z(x,t)=E_0\cos(k_x x-\omega t)$

驻波条件: $k_x l=n_x\pi$; $k_y l=n_y\pi$; $k_z l=n_z\pi$

作波矢空间: $k^2=k_x^2+k_y^2+k_z^2$ $k_{max}=\frac{\omega_{max}}{c}=\frac{2\pi\nu_0}{c}$




模式个数: $\frac18$球中有N个顶点⇔含N个正方体.

有: $M=\underbrace{2}_{\text{一方向2个驻波}}\times\frac{\frac18\cdot\frac43\pi\left(\frac{2\pi\nu_0}{c}\right)^3}{\left(\frac{\pi}{l}\right)^3}=\frac{8}{3}\pi\nu_0^3\frac{l^3}{c^3}$

$\Rightarrow G=\frac{dM}{d\nu}\cdot\frac{1}{l^3}=\frac{8\pi\nu_0^2}{c^3}$

在 $\Delta t$ 一个周期内计算它：

$$\Delta\vec{B}=\int_0^T\left(\frac{d\vec{B}}{dt}\right)dt=\int_0^{2\pi}\hat{\theta}\cdot\frac{GMm}{r_0^3}\alpha r^3 d\theta=\int_0^{2\pi}(-\sin\theta,\cos\theta)\cdot\frac{GMm\alpha}{r_0^3}\frac{r_0^3}{(1+e\cos\theta)^3}d\theta$$

$$=\int_0^{2\pi}(-\sin\theta,\cos\theta)\cdot GMm\alpha\cdot(1-3e\cos\theta)d\theta=-\frac{3}{2}\cdot eGMm\cdot 2\pi\,\hat{y}$$

$$\Rightarrow\overline{\left(\frac{d\vec{B}}{dt}\right)}=-\frac{3}{2}eMmG\cdot\alpha w_0\hat{y}=\left(-\frac{3}{2}\alpha\cdot w_0\hat{z}\right)\times\vec{B}$$

$\vec{B}$ 进动（与行星进动反向）.

☆[例] 在 $e\ll 1$ 时，求轨迹变化.（有 $W_{电子}=W_{E}$）

$\vec{E}=E_0\cos wt$

解：$\frac{d\vec{B}}{dt}=\frac{d\vec{v}}{dt}\times\vec{L}-GMm\dot{\hat{r}}=qE_0\cos wt\cdot mr^2\dot{\theta}\hat{y}=qE_0\cos wt\, r^2\dot{\theta}\hat{x}$

$$\therefore\Delta\vec{B}=\int_0^T\left(\frac{d\vec{B}}{dt}\right)\cdot dt=\int_0^T qE_0\cos wt\cdot\frac{e^2p^2}{(1+e\cos wt)^2}d wt\,\hat{x}$$

$$=\int_0^{2\pi}qE_0\cdot e^2p^2\times\cos\theta(1-2e\cos\theta)d\theta\,\hat{x}=-qE_0\cdot e^2p^2\times 2e\pi=-2qE_0e^3p^2\pi\hat{x}$$

$$\Rightarrow\overline{\left(\frac{d\vec{B}}{dt}\right)}=2qE_0e^3p^2\omega\hat{x}=2qE_0W\times\frac{L^2}{kq^2m}\times\frac{1}{kq}\cdot\vec{B}=\frac{2E_0wL^2}{k^2q^3m}\vec{B}$$

$\therefore$ 有大致关系：$B=B_0\exp\left(\frac{2E_0wL^2}{k^2q^3m}t\right)$ 含义：量子下能级跃迁.

☆[例] 用方向向量处理轨迹.

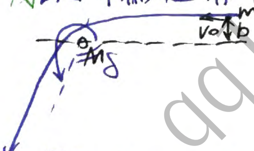
解：$mr^2\dot{\theta}=L\Rightarrow r^2d\theta=\frac{L}{m}dt$ $\begin{cases}d\hat{r}=d\theta\hat{\theta}\\ d\hat{\theta}=-d\theta\hat{r}\end{cases}$

由 $d\vec{v}=-\frac{\vec{F}}{m}dt=-\frac{GMm}{L}d\theta\cdot\hat{r}$

$\Rightarrow d\vec{v}=+\frac{GMm}{L}d\hat{\theta}$ 积分 $\Rightarrow\vec{v}-v_0\hat{y}=+\frac{GMm}{L}(\hat{\theta}-\hat{y})$，点乘 $\hat{\theta}$

$$\Rightarrow\frac{L}{mr}=\frac{GMm}{L}+\left(v_0-\frac{GMm}{L}\right)\cos\theta\Rightarrow r=\frac{ep}{1+e\cos\theta}$$

☆[例] 求偏转角.



解：$d\vec{v}=\frac{GMm}{L}d\hat{\theta}$ 有 $v_0\cdot 2\cdot\cos\frac{\delta}{2}=\frac{GMm}{L}\cdot 2\sin\frac{\delta}{2}$

$$\Rightarrow\tan\frac{\delta}{2}=\frac{v_0^2b}{GM}$$

$\boxed{\text{或}}$ $\vec{B}=\vec{v}\times\vec{L}+\alpha\hat{r}=v_0bmv_0\hat{y}+(-\alpha)\hat{x}\propto[\sin\frac{\delta}{2},\cos\frac{\delta}{2}]$

$$\Rightarrow\tan\frac{\delta}{2}=-\frac{mbv_0^2}{\alpha}=\frac{v_0^2b}{GM}$$

☆后者更推荐.

☆[例] 用LRL矢量证明动量空间为一个圆.

证：$GMm\hat{r}=\vec{v}\times\vec{L}-\vec{B}\Rightarrow G^2M^2m^2=v^2L^2-(\vec{v}\cdot\vec{L})^2+B^2-2\vec{B}\cdot\vec{v}\times\vec{L}$

$$\therefore\left(\frac{GMm}{L}\right)^2=P_x^2+\left(P_y+\frac{Mm}{L}\right)^2$$

有：$\operatorname{sh}k = \frac{at}{c} \Rightarrow \operatorname{sh}\frac{a\tau}{c} = \frac{at}{c} \Rightarrow$ 世界线：$\begin{cases} ct = \frac{c^2}{a}\cdot\operatorname{sh}\frac{a\tau}{c} \\ x = \frac{c^2}{a}(\operatorname{ch}\frac{a\tau}{c}-1) \end{cases}$

分析：观察同方向LT：$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \gamma_1 & -\gamma_1\beta_1 \\ -\gamma_1\beta_1 & \gamma_1 \end{bmatrix}\begin{bmatrix} ct \\ x \end{bmatrix} \overset{记}{=} \begin{bmatrix} \operatorname{ch}k_1 & -\operatorname{sh}k_1 \\ -\operatorname{sh}k_1 & \operatorname{ch}k_1 \end{bmatrix}\begin{bmatrix} ct \\ x \end{bmatrix}$

$(\operatorname{ch}k = \gamma,\ \operatorname{sh}k = \gamma\beta)$ $\quad\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \gamma_2 & -\gamma_2\beta_2 \\ -\gamma_2\beta_2 & \gamma_2 \end{bmatrix}\cdot\begin{bmatrix} ct' \\ x' \end{bmatrix} \overset{记}{=} \begin{bmatrix} \operatorname{ch}k_2 & -\operatorname{sh}k_2 \\ -\operatorname{sh}k_2 & \operatorname{ch}k_2 \end{bmatrix}\begin{bmatrix} ct' \\ x' \end{bmatrix}$

$\Rightarrow \begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \operatorname{ch}(k_1+k_2) & -\operatorname{sh}(k_1+k_2) \\ -\operatorname{sh}(k_1+k_2) & \operatorname{ch}(k_1+k_2) \end{bmatrix}\begin{bmatrix} ct \\ x \end{bmatrix}$ —— $K = K_1 + K_2$：快度.

今用 $ict$ 表达：

$\begin{bmatrix} ict' \\ x' \end{bmatrix} = \begin{bmatrix} \operatorname{ch}k_1 & -i\operatorname{sh}k_1 \\ +i\operatorname{sh}k_1 & \operatorname{ch}k_1 \end{bmatrix}\begin{bmatrix} ict \\ x \end{bmatrix}$ $\quad\begin{cases} \operatorname{ch}ik_1 = \cos k_1 \\ -i\operatorname{sh}ik_1 = \sin k_1 \end{cases}$ ⟶ 复数意义下转动叠加.

今本例：$\begin{cases} dk = \frac{a}{c}d\tau \\ S_{\tau_1} \to S_{\tau_2} \to \cdots \to S_{\tau_n} \end{cases}$ $\therefore K = \int dk = \frac{a}{c}\tau$.

$\frac{a\,d\tau}{c} = \operatorname{th}dk \approx dk$ —— 解决火箭等问题

· 增加问题：(5) 求 $S_\tau \to S_0$ 换系.

(6) S看T距离.

解：(5) 对于点 $(ct', x) = (c\tau, 0)$ 时满足：$\begin{cases} ct = \frac{c^2}{a}\operatorname{sh}\frac{a\tau}{c} \\ x = \frac{c^2}{a}(\operatorname{ch}\frac{a\tau}{c}-1) \end{cases}$

代入含 $x_0, t_0$ 的变换：$\begin{cases} c\tau = \operatorname{ch}k\cdot\frac{c^2}{a}\operatorname{sh}\frac{a\tau}{c} - \operatorname{sh}k\cdot\frac{c^2}{a}(\operatorname{ch}\frac{a\tau}{c}-1) + ct_0 \\ 0 = -\operatorname{sh}k\cdot\frac{c^2}{a}\operatorname{sh}\frac{a\tau}{c} + \operatorname{ch}\cdot\frac{c^2}{a}(\operatorname{ch}\frac{a\tau}{c}-1) + x_0 \end{cases}$

$\therefore \begin{cases} ct_0 = c\tau - \frac{c^2}{a}\operatorname{sh}\frac{a\tau}{c} \\ x_0 = -\frac{c^2}{a} + \frac{c^2}{a}\operatorname{ch}\frac{a\tau}{c} \end{cases}$ ⟶ 用 $\tau\to0$ 检验. $\begin{cases} t_0 \to 0 \\ x_0 \to \frac{1}{2}a\tau^2 \end{cases}$

$\Rightarrow$ L.T.：$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \operatorname{ch}k & -\operatorname{sh}k \\ -\operatorname{sh}k & \operatorname{ch}k \end{bmatrix}\begin{bmatrix} ct \\ x \end{bmatrix} + \begin{bmatrix} ct_0 \\ x_0 \end{bmatrix}$

(6). 在 $S_\tau$ 系中，时间为 $\tau$ 时 找T的 $\tau$.

处理：写T的世界线：$\begin{cases} ct = \frac{c^2}{a}\cdot\operatorname{sh}\frac{a\tau}{c} \\ x = \frac{c^2}{a}\cdot(\operatorname{ch}\frac{a\tau}{c}-1) + l \end{cases}$


表明同时

$\Rightarrow$ 换系：$\begin{cases} c\tau_s = \operatorname{ch}\frac{a\tau_s}{c}\cdot\frac{c^2}{a}\operatorname{sh}\frac{a\tau_T}{c} - \operatorname{sh}\frac{a\tau_s}{c}[\frac{c^2}{a}(\operatorname{ch}\frac{a\tau_T}{c}-1)+l] + c\tau_s - \frac{c^2}{a}\operatorname{sh}\frac{a\tau_s}{c} \\ l_T = -\operatorname{sh}\frac{a\tau_s}{c}\cdot\frac{c^2}{a}\operatorname{sh}\frac{a\tau_T}{c} + \operatorname{ch}\frac{a\tau_s}{c}[\frac{c^2}{a}(\operatorname{ch}\frac{a\tau_T}{c}-1)+l] + \frac{c^2}{a}[\operatorname{ch}\frac{a\tau_s}{c}-1] \end{cases}$

看到的距离

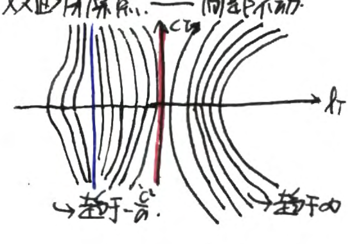
$\Rightarrow \begin{cases} \operatorname{sh}\frac{a(\tau_T-\tau_s)}{c} = \frac{al}{c^2}\operatorname{sh}\frac{a\tau_s}{c} \\ l_T = \frac{c^2}{a}\operatorname{ch}\frac{a(\tau_T-\tau_s)}{c} - \frac{c^2}{a} + l\cdot\operatorname{ch}\frac{a\tau_s}{c} \end{cases}$ ⟶ 当 $\frac{a\tau_s}{c} \ll 1$, $\frac{al}{c^2} \ll 1$

$\Rightarrow l_T = \frac{c^2}{a}\sqrt{1+(\frac{al}{c^2}\operatorname{sh}\frac{a\tau_s}{c})^2} - \frac{c^2}{a} + l\operatorname{ch}\frac{a\tau_s}{c}$

分析：当 $l = -\frac{c^2}{a} \Rightarrow l_T = -l$ —— S中双曲线原点 —— 间距不动.

当 $l = 0 \Rightarrow l_T = 0$ —— 自己不动

当 $l > 0 \Rightarrow l_T \nearrow$

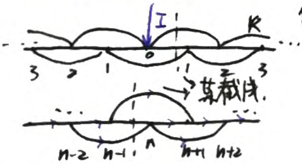


# (六)电路

## 基尔霍夫方程(6.1)

处理：节点多，回路少：设电流，列回路。
　　　节点少，回路多：设电压，列节点。

### ☆[例]求电流分布。




解：支路多节点少，设电压。设电势如图。
→列基尔霍夫第一定律：对点n：

$$\frac{U_{n-1}-U_n}{R}+\frac{U_{n-2}-U_n}{R}-\frac{U_n-U_{n+1}}{R}-\frac{U_n-U_{n+2}}{R}=0$$

$\therefore U_{n-2}+U_{n-1}-4U_n+U_{n+1}+U_{n+2}=0$　　看不出重根

猜：$U_n=A\cdot\lambda^n \Rightarrow 1+\lambda-4\lambda^2+\lambda^3+\lambda^4=0$ → 下用CASIO. — $\boxed{处理一}$

→也可以利用对称性：$\frac{1}{\lambda^2}+\frac{1}{\lambda}-4+\lambda+\lambda^2=0$.（$\lambda$为解，$\frac{1}{\lambda}$也为解）

$\boxed{处理二}$：$I=0$也为解 ⇒ $\lambda=1$为解，$\lambda=\frac{1}{\lambda}$也为解.（是重根）

∴短降 ⇒ $(\lambda-1)^2(\lambda^2+3\lambda-1)=0 \Rightarrow \lambda$

$\boxed{处理三}$：令 $\lambda+\frac{1}{\lambda}=a$ ∴ $a+a^2=6 \Rightarrow a_1=2\quad a_2=-3 \Rightarrow \lambda$.

⇒得 $\lambda_{1,2}=1$.　$\lambda_{3,4}=\frac{-3\pm\sqrt{13}}{2}$

$\therefore U_n=(A_1+nA_2)+A_3\cdot\lambda_3^n+A_4\cdot\lambda_4^n$

→边界定系数：①当$n\to\infty$，$U_n$应比最大 $n'\to\infty$. ∴ $A_4=0$.（$\lambda_4$为+）　实际：左右对称.
②差值有意义：令$A_1=0$

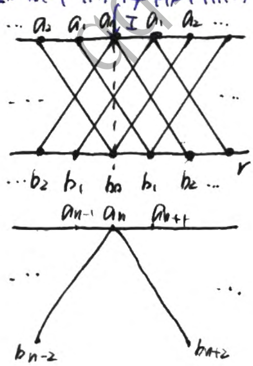
⇒ $U_n=n\cdot A_2+A_3\cdot\left(\frac{-3+\sqrt{13}}{2}\right)^n$

③找一截线：$\frac{U_{n-1}-U_{n+1}}{R}+\frac{U_{n-1}-U_n}{R}+\frac{U_{n-1}-U_n}{R}=I/2$

（或直接看1处）$(U_1-U_0)+(U_1-U_2)+(U_1-U_{-1})+(U_1-U_3)=0$

④端点：$2(U_0-U_1)+2(U_0-U_2)=I$　或边界从端点和对称考虑.

### ☆[例](中间接触点不连通)求电流分布.




解：基一：$(a_n-a_{n-1})+(a_n-a_{n+1})+(a_n-b_{n-2})+(a_n-b_{n+2})=0$

$$\Rightarrow\begin{cases}a_{n-1}-4a_n+a_{n+1}+b_{n-2}+b_{n+2}=0\\ a_{n-2}+a_{n+2}+b_{n-1}-4b_n+b_{n+1}=0\end{cases}$$

猜：$\begin{pmatrix}a_n\\b_n\end{pmatrix}=\begin{pmatrix}a\\b\end{pmatrix}\lambda^n \Rightarrow \begin{cases}(\frac{1}{\lambda}-4+\lambda)A+(\lambda^2+\frac{1}{\lambda^2})B=0\\(\lambda^2+\frac{1}{\lambda^2})A+(\frac{1}{\lambda}-4+\lambda)B=0\end{cases}$

要求 $\det=0$ ⇒ 用 $x=\lambda+\frac{1}{\lambda}$ 换元 ⇒ $\lambda_{1,2,\dots,8}$.

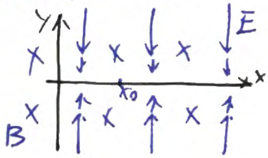
⇒由端点值和对称性待定出系数

注：一般极限分析⇒都只剩 $nA+B\cdot\lambda^n$ ≥2次.

△连续对称性:

找守恒量. 消自由度得有效势.

☆[例] $\vec{B}=B_0(-\hat{z})$. $\vec{E}=-\alpha y\hat{y}$. 初 $\vec{v}_0=v_0\hat{y}$. 求解运动.




解: x方向平移不变: $\frac{\partial V}{\partial x}=E_x=0$; $\frac{\partial \vec{B}}{\partial x}=0$

有: $P_x=p_x+qBy=mv_x+qBy=0$.

$\Rightarrow \frac{1}{2}mv_y^2+\frac{1}{2}mv_x^2+\frac{1}{2}\alpha q y^2=\frac{1}{2}mv_0^2$

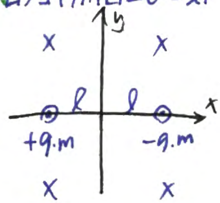
代入 $\Rightarrow \frac{1}{2}mv_y^2+\underbrace{\frac{1}{2}\frac{q^2B^2}{m}y^2+\frac{1}{2}\alpha q y^2}_{V_{eff}}=\frac{1}{2}mv_0^2$

⇒有效势得y方向振动: $\omega=\sqrt{\left(\frac{qB}{m}\right)^2+\frac{\alpha q}{m}}$

∴ $v_y=v_0\cos\omega t$. $y=\frac{v_0}{\omega}\sin(\omega t)$

∴ $v_x=-\frac{qB}{m}\frac{v_0}{\omega}\sin\omega t$. $x=x_0+\frac{qBv_0}{m\omega^2}(\cos(\omega t)-1)$ —— 正椭圆.

☆[例] 初相距2l. 静止释放. 弹性碰撞. $v\ll c$. 求解运动.




解: 只看一边: $\frac{d\vec{P}}{dt}=-q\vec{v}\times\vec{B}$

$\Rightarrow P_y=qBl-qBx$

能: $2\cdot\frac{m}{2}(\dot{x}^2+\dot{y}^2)-\frac{1}{2x}kq^2=-\frac{1}{2l}kq^2$

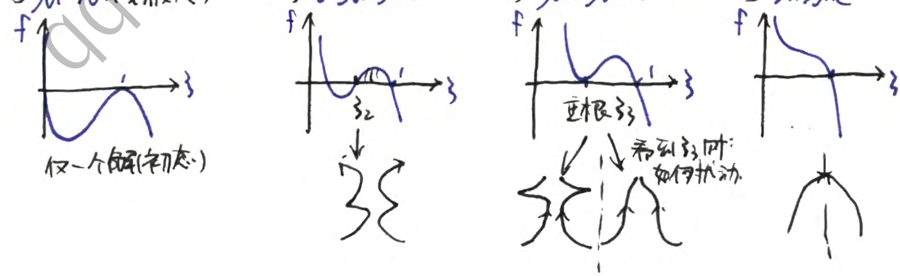
$\Rightarrow \frac{1}{2}m\dot{x}^2+\underbrace{\frac{1}{2}\frac{q^2B^2}{m}(l-x)^2-\frac{kq^2}{4x}}_{V_{eff}}=-\frac{kq^2}{4l}$

→无量纲化. 令 $\frac{x}{l}=\xi$. $\frac{kq^2}{4l}\cdot\frac{2m}{q^2B^2l^2}=\mu$.

则: 令: $f(x)=-\frac{kq^2}{4l}+\frac{kq^2}{4x}-\frac{q^2B^2}{2m}(l-x)^2$

$=\frac{q^2B^2l^2}{2m}\left(-\mu+\frac{\mu}{\xi}-(\xi-1)^2\right)=\frac{1}{2}m\dot{x}^2>0$

① $\mu\to 0$.(l极大)　② $0<\mu<\mu_c$　③ $\mu=\mu_c=0.25$　④ $\mu>\mu_c$



☆[例] 不同理气混合. $C_V=\frac{3}{2}R$. $V_i=\frac{V}{N}$. $P_i=\alpha_i P_0$ $T_i=t_iT_0$. 求$\Delta S$.

解: $\Delta E=0$: $\sum(\nu_i C_V T_i)=(\sum\nu_i)C_V T_f$

$$\Rightarrow T_f=\frac{\sum\nu_iT_i}{\sum\nu_i}=T_0\frac{\sum\alpha_i}{\sum\frac{\alpha_i}{t_i}}$$

看某一组分: $\Delta S(V,T)=nC_V\ln\frac{PV^\gamma}{P_0V_0^\gamma}=nR\ln\left(\left(\frac{T}{T_0}\right)^{\frac{C_V}{R}}\frac{V}{V_0}\right)$

∴ $\Delta S_i=\nu_iR\ln\left(\left(\frac{T_f}{t_iT_0}\right)^{\frac{C_V}{R}}\cdot N\right)$

$\Rightarrow \Delta S=\sum_i\nu_iR\ln\left(\left(\frac{T_f}{t_iT_0}\right)^{\frac{C_V}{R}}N\right)$

分析: 取 $n=2$: 令 $\alpha_1=\frac{3}{2}$. $\alpha_2=\frac{1}{2}$. $t_1=\frac{3}{2}$. $t_2=\frac{1}{2}$.

则: $T_f=T_0$. $\Delta S_1=\frac{1}{2}\nu_0R\ln\left(\left(\frac{1}{3/2}\right)^{\frac{C_V}{R}}\cdot2\right)$ $\Delta S_2=\frac{1}{2}\nu_0R\ln\left(\left(\frac{1}{1/2}\right)^{\frac{C_V}{R}}\cdot2\right)$

$\Rightarrow \Delta S=\underbrace{\nu_0R\ln2}_{混合}+\underbrace{\frac{1}{2}\nu_0C_V\ln4/3}_{传热}$.

$\boxed{或}$: 先传热, 再抽板:

$\Delta S=\int\frac{dQ}{T}=\nu C_V\ln\frac{T}{T_0}$ $\Delta S_左=\frac{\nu_0}{2}C_V\ln\frac{2}{3}$ $\Delta S_右=\frac{\nu_0}{2}C_V\ln2$

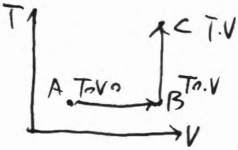
$\Rightarrow$ 混前: $\Delta S_1=\frac{1}{2}\nu_0C_V\ln4/3$

再抽板: ① A.B不同种: $\Delta S_2=\frac{\nu_0}{2}R\ln2+\frac{\nu_0}{2}R\ln2=\nu_0R\ln2$

② A.B同种气体: $\Delta S_2=0$.

↳ "全同粒子" 不能区分同种气体的2个分子.

☆[例] 范氏气体: $(P+\frac{a}{V^2})(V-b)=RT$ $U=C_VT-\frac{a}{V}$. 求绝热方程与熵表达式.




解: $\boxed{法一}$: $dQ=PdV+dU$ 积分

$\boxed{法二}$: $S_B-S_A=\int\frac{dQ}{T}=\int\frac{PdV+(dU)_T}{T_0}$

$$=\frac{1}{T_0}\int_{V_0}^{V}\left(\frac{RT_0}{V-b}-\frac{a}{V^2}\right)dV-d\left(-\frac{a}{V}\right)$$

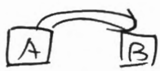
$$=R\ln\frac{V-b}{V_0-b}.$$

$S_C-S_B=\int\frac{dQ}{T}=\int\frac{(dU)_V}{T}=\int_{T_0}^{T}\frac{C_VdT}{T}=C_V\ln\frac{T}{T_0}$.

$$\Rightarrow S_C-S_A=R\ln\frac{(V-b)T^{C_V/R}}{(V_0-b)T_0^{C_V/R}}$$

∴ 绝热线: $(V-b)T^{C_V/R}=Const.$

☆[例] A: m. 100°C 的水; B: m. 0°C 的水. ① 若只能热交换(取水靠上) 能否将0°C水升到100°C? ② 不限制传热, 可用热机, 可否?




解: ① 尽量让A的水传更多热: 大温差.

∴ 将A $\infty$分. $m/N$. 则 n 次后: $T_B=T_n$

则: $T_{n+1}(m+\frac{m}{N})=\frac{m}{N}\cdot100+m\cdot T_n$

$\Rightarrow 100-T_{n+1}=\frac{N}{N+1}(100-T_n)$

$\Rightarrow 100-T_N=\left(\frac{N}{N+1}\right)^N(100-T_0)$ ∴ 极限大: $T_N=100-\frac{1}{e}\cdot100\approx63.2°C$