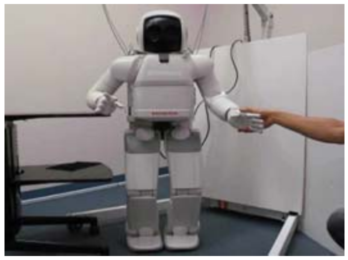
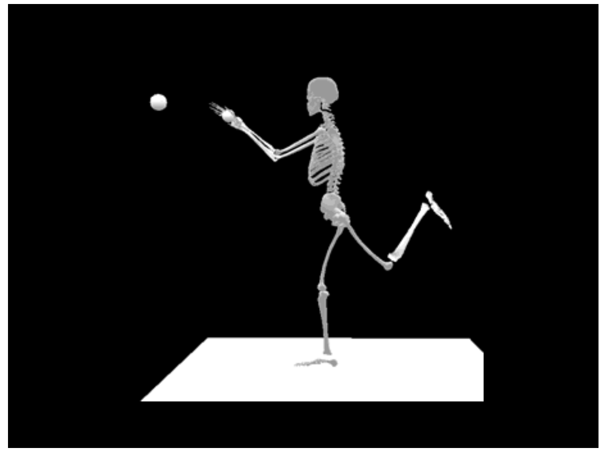


Redundancy

1



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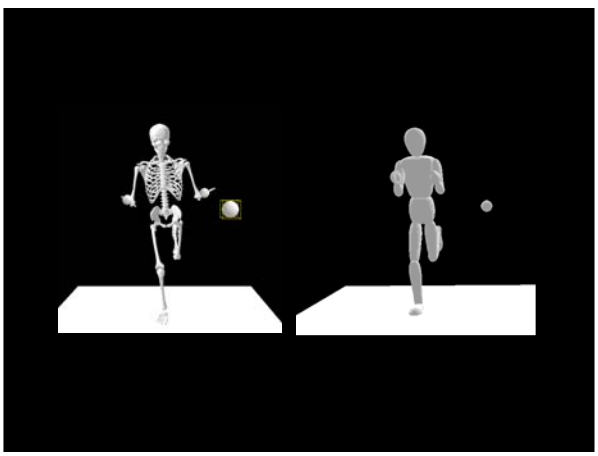


3

Equations of Motion

Joint Space

$$A(q)\ddot{q} + b(q,\dot{q}) + g(q) = \Gamma$$

Operational Space

$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F$$

Relationships

$$\Gamma = J^T F$$

$$(A\ddot{q} + b + g) = J^T(\Lambda\ddot{x} + \mu + p)$$

$$(A\ddot{q} + b) = J^T(\Lambda\ddot{x} + \mu) \quad \text{Inertial forces}$$

4

Non Redundancy

$$\begin{tikzcd} \boxed{A\ddot{q}+b+g=\Gamma} \arrow[d, shift right, "J^{-T}"'] & \text{(joint dynamics)} \\ \boxed{\Lambda\ddot{x}+\mu+p=F} \arrow[u, shift right, "J^{T}"'] & \text{(Task dynamics)} \end{tikzcd}$$

5

Redundancy

projection

$$\begin{tikzcd} \boxed{A\ddot{q}+b+g=\Gamma} \arrow[d, shift right, "\bar{J}^{T}"'] & \text{(joint dynamics)} \\ \boxed{\Lambda\ddot{x}+\mu+p=F} \arrow[u, shift right, "J^{T}"'] & \text{(Task dynamics)} \end{tikzcd}$$

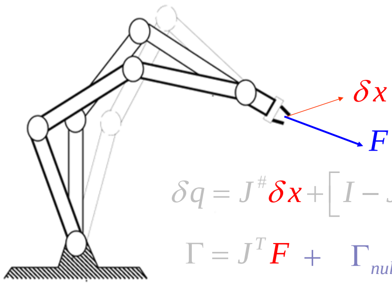
where

$$\bar{J} = A^{-1}J^T\Lambda \quad \text{and} \quad \Lambda^{-1} = J A^{-1} J^T$$

$\bar{J}$: dynamically consistent generalized inverse

6

Redundancy



$$\delta q = J^{\#}\delta x + \left[I - J^{\#} J \right] \delta q_0$$

$$\Gamma = J^T F \; + \; \Gamma_{nullspace}$$

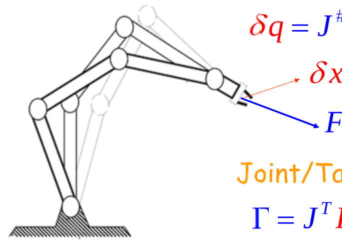
7

Redundancy

Joint/Task Displacements

$$\delta x = J \delta q$$

$$\delta q = J^{\#}\delta x + \left[I - J^{\#} J \right] \delta q_0$$



Joint/Task Forces

$$\Gamma = J^T F$$

$$F = ?$$

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$$\Gamma = J^T F$$

Given F , Γ is $\left(J^T F \right)$

Given Γ , what is F

$$F = J^{\#^T} \Gamma \; ?$$

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However

different selections of $J^{\#}$ $\left(J = J\, J^{\#} J \right)$

would lead

to different solutions

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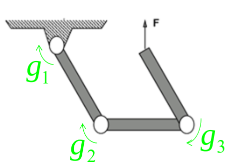
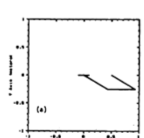
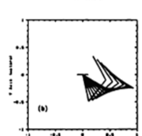
Gravity Example





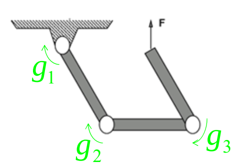


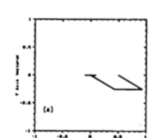
$\boxed{p = ?}$ $\quad p = J^{+^T} g$

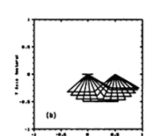
$$J^+ = J^T \left(J\, J^T \right)^{-1}$$

11

Gravity Example







$\boxed{p = ?}$ $\quad p = \bar{J}^T g$

$$\bar{J} = A^{-1} J^T \Lambda$$

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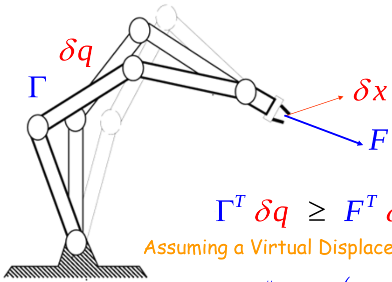
Redundancy



$$\Gamma^T\,\delta q \;\geq\; F^T\,\delta x$$

Assuming a Virtual Displacement

$$\delta q = J^{\#}\,\delta x + \left(I - J^{\#}J\right)\,\delta q_0$$

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Virtual Displacement
$$\delta q = J^{\#}\,\delta x + \left(I - J^{\#}J\right)\,\delta q_0$$

Virtual Work
$$\delta w = \Gamma^T\,\delta q$$

$$\delta w = \delta w_1 + \delta w_2$$

$$\left(J^{\#^T}\Gamma\right)^T \delta x \qquad \left[\left(I - J^{\#}J\right)^T \Gamma\right]^T \delta q_0$$

$$\Downarrow$$

$$\Gamma = J^T\,(J^{\#^T}\Gamma) + \left(I - J^T J^{\#^T}\right)\Gamma$$

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Decomposition

$$\Gamma \;=\; J^T\left[J^{\#^T}\Gamma\right] \;+\; \left[I - J^T J^{\#^T}\right]\Gamma$$

Task Space Forces (F)

Joint Torques acting in the null space

$$\Gamma = J^T F + \left[I - J^T J^{\#^T}\right]\Gamma_0$$

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Dynamic Constraints

$$A(q)\,\ddot{q} + b(q,\dot{q}) + g(q) \;=\; \Gamma$$

$$\Gamma = J^T F + \left[I - J^T J^{\#^T}\right]\Gamma_0$$

$$A\ddot{q} + \left(b + g\right) = J^T F + \left[I - J^T J^{\#T}\right]\Gamma_0$$

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$$\ddot{q} + A^{-1}\left(b + g\right) = A^{-1}J^T F + A^{-1}\left[I - J^T J^{\#^T}\right]\Gamma_0$$

$$J \;\; \Downarrow$$

$$J\ddot{q} + JA^{-1}\left(b + g\right) = JA^{-1}J^T F + JA^{-1}\left[I - J^T J^{\#^T}\right]\Gamma_0$$

$$J\ddot{q} \;=\; \ddot{x} - \dot{J}\dot{q} \quad \Downarrow$$

$$\ddot{x} + \left[JA^{-1}\left(b + g\right) - \dot{J}\dot{q}\right] = \left(JA^{-1}J^T\right)F + JA^{-1}\left[I - J^T J^{\#T}\right]\Gamma_0$$

$$\Lambda^{-1} \qquad \ddot{x}_n = 0$$

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Dynamic Consistency

$$\Gamma \longrightarrow J\,A^{-1}\Gamma$$

joint torques → task acceleration

Relationship

$$\Gamma = J^T F + \left(I - J^T J^{\#^T}\right)\Gamma_0$$

Dynamic Constraint

$$JA^{-1}\left(I - J^T J^{\#^T}\right)\Gamma_0 \equiv 0$$

$$JA^{-1} = \left(JA^{-1}J^T\right)J^{\#^T}$$

$$\Lambda \quad \left(JA^{-1}J^T\right)^{-1} JA^{-1} = J^{\#^T}$$

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Dynamic Consistency

$\bar{J}(q)$ is the Dynamically Consistent Generalized Inverse

Theorem (Consistency)

$\bar{J}$ is unique and $\bar{J} = A^{-1} J^T \Lambda$

Non-redundant

$$\bar{J} = J^{-1}$$

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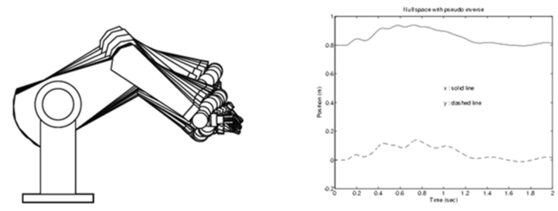

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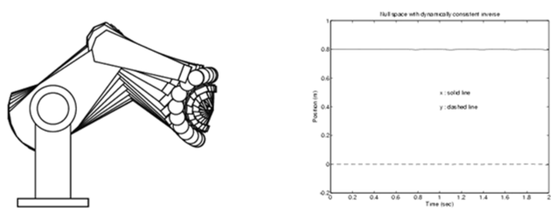

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Velocity Force Duality

	Velocity	Force
Non Red.	$\delta q = J^{-1} \delta x$	$\Gamma = J^T F$
Redundant	$\delta q = \bar{J} \delta x + \left[I - \bar{J} J \right] \delta q_0$	$\Gamma = J^T F + \left[I - J^T \bar{J}^T \right] \Gamma_0$

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Task dynamics

$$\Lambda(q)\ddot{x} + \mu(q,\dot{q}) + p(q) = F$$

$$\Lambda = \left(J A^{-1} J^T \right)^{-1}$$

$$\mu(q,\dot{q}) = \bar{J}^T b(q,\dot{q}) - \Lambda(q) \dot{J}(q) \dot{q}$$

$$p(q) = \bar{J}^T g(q)$$

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Redundant Robot Control

Task Space: J^T

Null Space: N^T where $N = I - \bar{J} J$

Robot Control

$$\Gamma = \underbrace{J^T F}_{\Gamma_1} + \underbrace{N^T \Gamma_0}_{\Gamma_2}$$

dynamically decoupled

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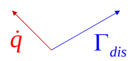
Stability

$\Gamma_{dis}^{T}\,\dot{q} \le 0$; for $\dot{q} \neq 0$



$\Gamma_{dis} = -k_v J^T \dot{x} = -k_v J^T J \dot{q}$

$\dot{q}^T D(q)\,\dot{q} \ge 0$; $\dot{q} \neq 0$

$$D(q) = k_v \left(J^T J \right)$$

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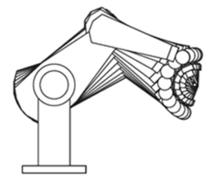
$J^T J$: is a n x n matrix of rank m_0
it is Positive Semi-definite

The System is Stable, but not asymptotically stable

$\dot{q}^T D(q)\,\dot{q} = 0$



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Asymptotic Stability

$\Gamma_{dis}^{T}\,\dot{q} < 0$; for $\dot{q} \neq 0$

$\Gamma_{dis} = -k_v J^T J \dot{q} - k_v N^T \dot{q}$

$\Downarrow$

$$D(q) = k_v\,(J^T J + N^T)$$

Positive definite

$\dot{q}^T D(q)\,\dot{q} > 0$ for $\dot{q} \neq 0$

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Asymptotic Stability

$\Gamma_{dis}^{T}\,\dot{q} < 0$; for $\dot{q} \neq 0$

$\Gamma_{dis} = -k_v J^T J \dot{q} - k_{vq} \dot{q}$

$\Downarrow$

$$D(q) = k_v\, J^T J + k_{vq}\, I_n$$

Positive definite

$\dot{q}^T D(q)\,\dot{q} > 0$ for $\dot{q} \neq 0$

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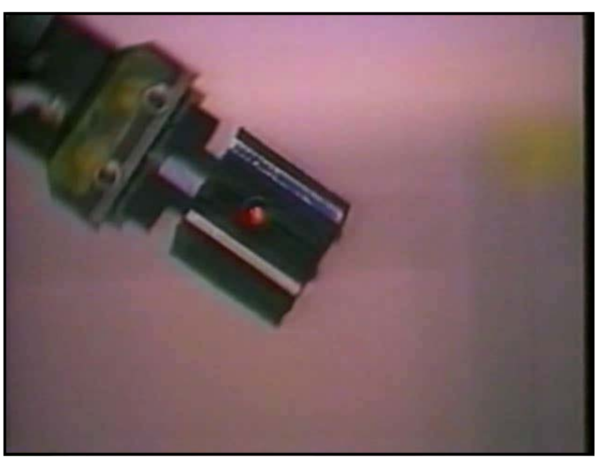

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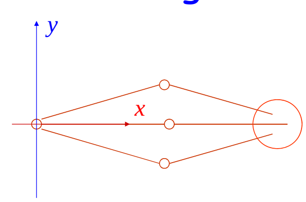
Kinematic Singularities



Joint Space Formulation

Find a pseudo-inverse J^+

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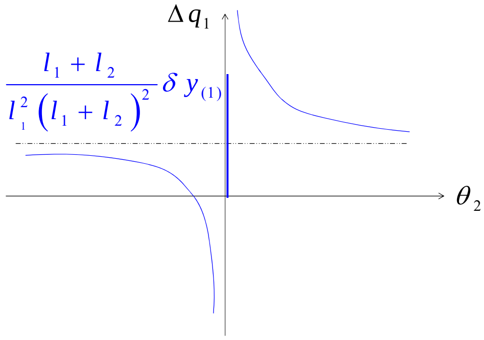
Pseudo Inverse Solution



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Kinematic Singularities

The end-effector mobility locally decreases

Singularities

$$S(q) = \det[J(q)] = S_1(q).S_2(q)\cdots S_{n_s}(q)$$

Singular direction

$S_i = 0$; ζ_i → Infinite effective mass → infinite effective inertia

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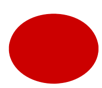
Kinematic Singularities

Singularity Neighborhood

$$S(q) = S_1(q).S_2(q).S_3(q)\cdots S_{n_s}(q)$$

Singularity S_i

$$D_{S_i} = \{ q \mid |S_i(q)| \le S_{0_i} \}$$



$|S_i| \le S_{0_i}$

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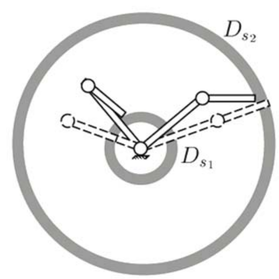
Singularity Neighborhood



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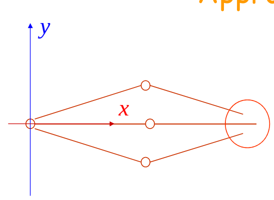
Approach



In D_S, the robot is treated as redundant w.r.t. motions in the subspace $\perp$ to the singular direction

- Along Singular Directions:
  Control in Null Space $\Gamma_{null-space}$
- In subspace $\perp$ to singular direction
  Control in sub-O-Space F_{sub-os}

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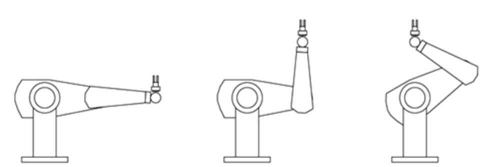
Types of Singularities



Elbow Lock — Wrist Lock — Overhead Lock

Type 1 — Type 2

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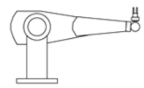
Types of Singularities

Control Strategy

Type 1

Motion in Null Space



$\Rightarrow$ Motion along/about ζ_i

Control S_i

Type 2

Motion in Null Space

$\Rightarrow$ Only changes of ζ_i

Control ζ_i

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Singularity Control

$$\boxed{\Gamma = J^T_{sub} F_{sub} + N^T_{sub} \Gamma_{s_i}}$$

where

$N_{sub} = I - \bar{J}_{sub} J_{sub}$ and $\Gamma_{S_i} = -\nabla V_i(S_i)$

Moving to a singularity

Control $S_i(q)$ to reach $S_i = 0$

Moving out of a singularity

Control $\dot{S}_i$ from zero to the desired Velocity at the singularity boundary

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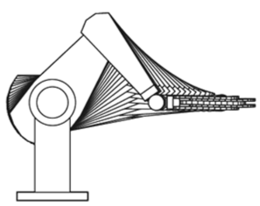

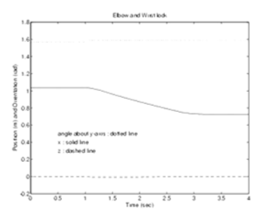

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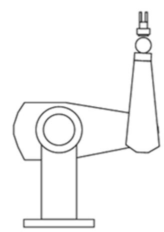

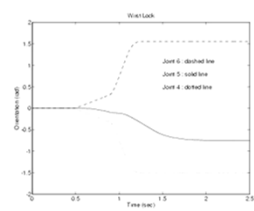

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