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Integrated Localization and Control for Accurate Multi-Agent Formation

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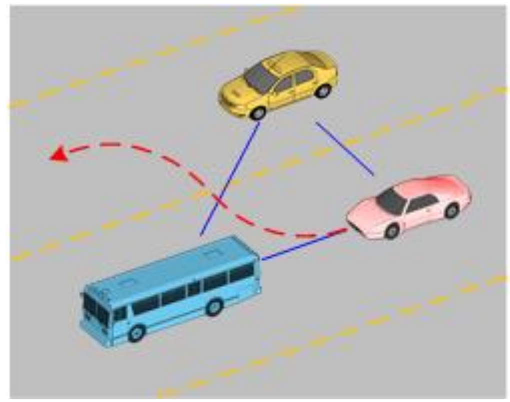
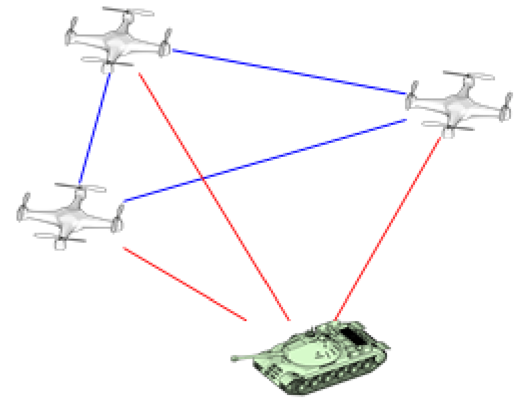
Outline

- Introduction
- Performance Metric
- Integrated Scheme
- Numerical Results
- Conclusions

Introduction

- Multi-Agent Systems
  - Applications
    - Internet of Vehicles
    - Target tracking
    - Cooperative SLAM
    - Communication relaying
    - Precise agriculture





High-Accuracy formation is essential for multi-agent systems to accomplish numerous missions!

How to realize high-accuracy formation in *wireless mobile networks*, which is threatened by the *limitation of spectrum resource*?

Introduction

- Existing Methods (Two *Separate* Steps)
  - Network Localization
    - *Analytical framework*: based on Fisher information[1]
    - *Relative localization*: shape estimation of the network[2]
    - *Network scheduling*: selection of the measurement links[3]
  - Formation Control

[1] Y. Shen, H. Wymeersch, and M. Z. Win, "Fundamental limits of wideband localization – Part II: Cooperative networks," IEEE Trans. Inf. Theory, vol. 56, no. 10, pp. 4981–5000, Oct. 2010.

[2] J. N. Ash and R. L. Moses, "On the relative and absolute positioning errors in self-localization systems," IEEE Trans. Signal Process., vol. 56, no. 11, pp. 5668 – 5679, Jun. 2008.

[3] T. Wang, Y. Shen, A. Conti, and M. Z. Win, "Network navigation with scheduling: Error evolution," IEEE Trans. Inf. Theory, vol. 63, no. 11, pp. 7509–7534, Nov. 2017.

Introduction

- Existing Methods (Two *Separate* Steps)
  - Network Localization
  - Formation Control
    - *Type of observations*: range only, no global coordinate[1]
    - *Graph-based control*: potential function based method[2]
    - *Performance evaluation*: stability, rather than accuracy[3]

[1] K. K. Oh, M. C. Park, and H. S. Ahn, "A survey of multi-agent formation control," Automatica., vol. 53, no. s, pp. 424–440, Mar. 2015.

[2] M. Cao, C. Yu, and B. D. Anderson, "Formation control using range-only measurements," Automatica., vol. 47, no. 4, pp. 776–781, Apr. 2011.

[3] Z. Lin, B. Francis, and M. Maggiore, "Necessary and sufficient graphical conditions for formation control of unicycles," IEEE Trans. Autom. Control, vol. 50, no. 1, pp. 121–127, Jan. 2005.

Introduction

- Drawbacks
  - Performance Metric
    - Either the probability of convergence to the target formation, or the deviation of the true trajectory from the planned one
    - The absence of a metric to characterize the accuracy of the formation
  - Separate Scheme
    - Stepwise optimization may lead to suboptimal performance, since
      1) the information dissipates between the procedures
      2) the localization-optimal scheduling strategy may not be formation-optimal, especially when the resource is very limited
    - Cross-step optimization can improve the overall performance

Introduction

- Motivation
  - Performance Metric
    - A new metric to quantize the difference between the true formation and the target formation
  - Integrated Scheme
    - The effect of localization on control: employ the estimated formation and the distribution information when determining the control vector *(Better utilization of the information in the measurements)*
    - The effect of control on localization: take the target formation and the control policy into consideration when designing scheduling strategy *(Better allocation of the limited spectrum resource)*

Introduction

- System Model
  - Control Model (Global & Local)
    - Suppose the agents can directly control their locations

$$\mathbf{x}_i^{(t)} = \mathbf{x}_i^{(t-1)} + \boldsymbol{c}_i^{(t)} + \mathbf{w}_i^{(t)}$$

Agent's location

Control error

  - Measurement Model (Range)
    - Distance measurements of neighboring agents

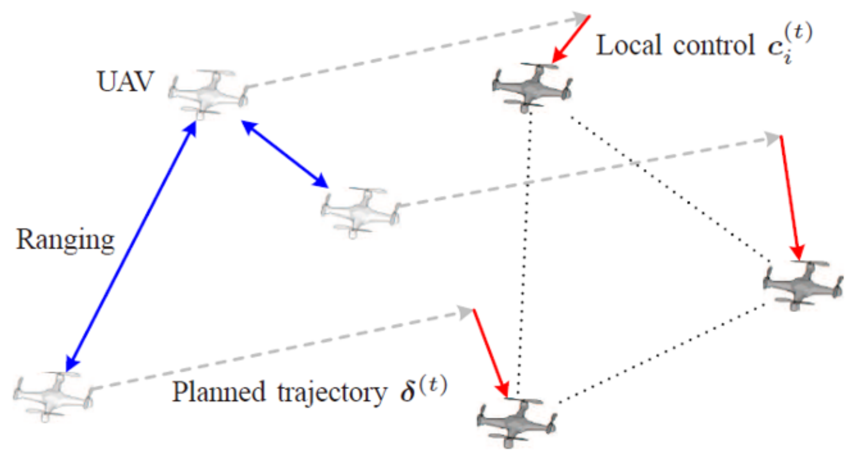
$$\mathrm{d}_{ij}^{(t)} = \left\| \mathbf{x}_i^{(t)} - \mathbf{x}_j^{(t)} \right\| + \mathrm{v}_{ij}^{(t)}$$

Measurement uncertainty

Without loss of generality, we can suppose $\boldsymbol{\delta}^{(t)} = \mathbf{0}$

Formation

$$\boldsymbol{x} = [\boldsymbol{x}_1^{\mathrm{T}} \quad \cdots \quad \boldsymbol{x}_N^{\mathrm{T}}]^{\mathrm{T}}$$



(a) Previous formation (b) New formation

Outline

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Performance Metric

- Difference Between Formations
  - Not related with the *position* and the *orientation* of the formation
  - Feasible Set
    - The feasible set $\mathcal{S}(\boldsymbol{\xi})$ of the formation $\boldsymbol{\xi}$ is defined as the collection of the formations that can be achieved by applying the translation and rotation operations on formation $\boldsymbol{\xi}$.
    - Mathematical description

$$\mathcal{S}(\boldsymbol{\xi}) = \{ \underbrace{[\boldsymbol{I}_N \otimes \boldsymbol{R}(\vartheta)]\boldsymbol{\xi}}_{\text{Rotation}} + \underbrace{\mathbf{1}_N \otimes \boldsymbol{t}}_{\text{Translation}} \ : \ \boldsymbol{t} \in \mathbb{R}^2, \ \vartheta \in [0, 2\pi) \}$$

$$\boldsymbol{R}(\vartheta) = \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix}$$

Kronecker product

Performance Metric

- Formation Error
  - Characterize the *minimum squared distance* between the true formation $\boldsymbol{x}$ and the target formation $\boldsymbol{\xi}$ *over arbitrary translation and rotation*, denoted by $\mathcal{F}(\boldsymbol{x}, \boldsymbol{\xi})$.
    - Optimization description

$$\mathcal{F}(\boldsymbol{x}, \boldsymbol{\xi}) = \min_{\boldsymbol{s} \in \mathcal{S}(\boldsymbol{\xi})} \|\boldsymbol{x} - \boldsymbol{s}\|^2$$

    - Close-form solution

$$\mathcal{F}(\boldsymbol{x}, \boldsymbol{\xi}) = \|\boldsymbol{D}\boldsymbol{x}\|^2 + \|\boldsymbol{\xi}\|^2 - 2\|\boldsymbol{\xi}\| \left[\boldsymbol{x}^{\mathrm{T}}(\boldsymbol{D}\boldsymbol{P}\boldsymbol{D})\boldsymbol{x}\right]^{1/2}$$

$$\boldsymbol{D} = \boldsymbol{I}_{2N} - \frac{1}{N}(\mathbf{1}_N \mathbf{1}_N^{\mathrm{T}}) \otimes \boldsymbol{I}_2$$

Translate a formation until its mass center locates at the origin

$$\boldsymbol{P} = \frac{1}{\|\boldsymbol{\xi}\|^2}(\boldsymbol{\xi}\boldsymbol{\xi}^{\mathrm{T}} + \boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{T}})$$

Target formation $\boldsymbol{\eta}$ that can be achieved by rotating $\boldsymbol{\xi}$ for $\pi/2$

Performance Metric

- Formation Error
  - Equivalent Expression
    - Define the regularized formation $\boldsymbol{y} = \boldsymbol{D}\boldsymbol{x}$
    - Decomposed form of the formation error

$$\mathcal{F}(\boldsymbol{x}, \boldsymbol{\xi}) = \|\boldsymbol{y}_{\perp}\|^2 + \left(\|\boldsymbol{y}_{//}\| - \|\boldsymbol{\xi}\|\right)^2$$

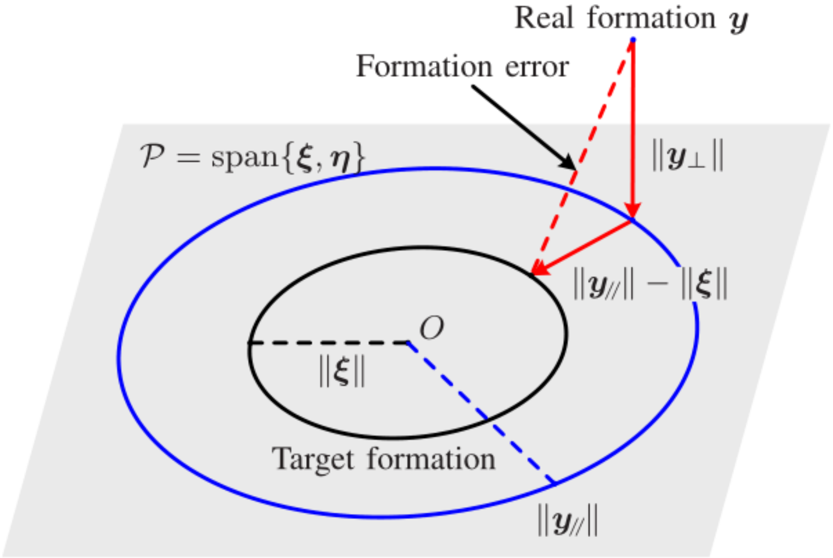
$$\boldsymbol{y}_{\perp} = (\boldsymbol{I} - \boldsymbol{P})\boldsymbol{y} \qquad \boldsymbol{y}_{//} = \boldsymbol{P}\boldsymbol{y}$$

    - This equivalent from is derived from the perspective of subspace theory, and yields a geometrical interpretation.

Performance Metric

- Formation Error
  - Geometrical Interpretation
    - *Proposition*: The subset of $\mathcal{S}(\boldsymbol{\xi})$, which includes all target formations with mass centers at the origin, is a circle of radius $\|\boldsymbol{\xi}\|$ centered at the origin in the hyperplane $\mathcal{P} = \mathrm{span}\{\boldsymbol{\xi}, \boldsymbol{\eta}\}$.
    - Formation error

$$\mathcal{F}(\boldsymbol{x}, \boldsymbol{\xi}) = \|\boldsymbol{y}_\perp\|^2 + \left(\|\boldsymbol{y}_{/\!/}\| - \|\boldsymbol{\xi}\|\right)^2$$



- ➢ Parallel Component $\boldsymbol{y}_{/\!/}$
- ➢ Parallel Error $\left(\|\boldsymbol{y}_{/\!/}\| - \|\boldsymbol{\xi}\|\right)^2$
- ➢ Orthogonal Component $\boldsymbol{y}_\perp$
- ➢ Orthogonal Error $\|\boldsymbol{y}_\perp\|^2$

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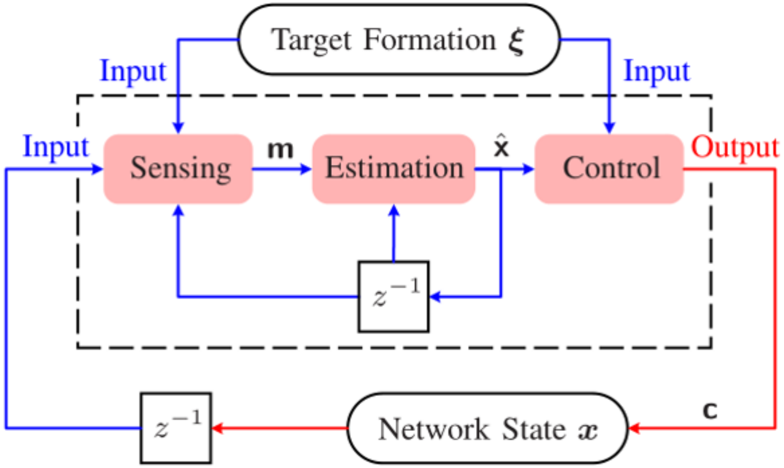
Integrated Scheme

- General Framework
  - A typical division of steps in cyber-physical systems
    - Environment sensing
      make measurements **m** according to certain scheduling strategy
    - State estimation
      estimate the true formation based on the observations
    - Formation control
      adjust the formation **c** according to certain control policy



Integrated Scheme

- Integrated Localization and Control
  - Formation after control

$$\mathbf{x}^{(t)} = \mathbf{x}^{(t-1)} + \mathbf{c}^{(t)}\left(\hat{\mathbf{x}}^{(t)}\left(\mathbf{m}^{(t)}\right)\right) + \mathbf{w}^{(t)}$$

    - The measurement is determined by the scheduling strategy $\mathscr{A}$
    - The control vector is determined by the control policy $\mathscr{C}$
  - Performance metric *mean formation error*

$$F(\{\mathscr{A}, \mathscr{C}\}) = \mathbb{E}\{\mathcal{F}(\mathbf{x}^{(t)}, \boldsymbol{\xi})\}$$

  - *Optimal combination* of the scheduling strategy and the control policy

$$\{\mathscr{A}, \mathscr{C}\}^{\star} = \arg\min_{\mathscr{A}, \mathscr{C}} F(\{\mathscr{A}, \mathscr{C}\})$$

Integrated Scheme

- Integrated Localization and Control
  - Optimization in a simultaneous manner
    - Manipulate the scheduling strategy $\mathscr{A}$ and the control policy $\mathscr{C}$ at the same time to solve the problem
    - Not tractable due to the intricate relationship of the two steps
  - Optimization in an iterative manner
    - Given control policy $\mathscr{C}_0$, the optimal scheduling strategy $\mathscr{A}^\star$ refers to

$$\mathscr{A}^\star(\mathscr{C}_0) = \arg\min_{\mathscr{A}} F(\{\mathscr{A}, \mathscr{C}_0\})$$

    - Given scheduling strategy $\mathscr{A}_0$, the optimal control policy $\mathscr{C}^\star$ refers to

$$\mathscr{C}^\star(\mathscr{A}_0) = \arg\min_{\mathscr{C}} F(\{\mathscr{A}_0, \mathscr{C}\})$$

Integrated Scheme

- Case Study
  - Design the scheduling strategy under a given control policy
  - *MMFE control*
    - Minimize the mean formation error when the *true formation* $\boldsymbol{x}$ is known, in the presence of control error $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma_{\mathrm{c}}^2 \boldsymbol{I})$

$$F = \|\boldsymbol{y}_\perp + \boldsymbol{c}_\perp\|^2 + h(\|\boldsymbol{y}_{/\!/} + \boldsymbol{c}_{/\!/}\|^2) + (2N-2)\sigma_{\mathrm{c}}^2$$

    - The control vectors that minimize the above expression

$$C^\star(\boldsymbol{x}) = \rho \frac{\boldsymbol{PDx}}{\|\boldsymbol{PDx}\|} - \boldsymbol{Dx}$$

    - When the true formation is not available, the policy use the estimated formation instead of the true formation to calculate the vector

Integrated Scheme

- Case Study
  - Design the scheduling strategy under a given control policy
  - *Scheduling strategy that suits the MMFE control*
    - Formation after control is

$$\mathbf{x}^{+} = \boldsymbol{x} + C^{\star}(\hat{\mathbf{x}}) + \mathbf{w}$$

    - The approximate theoretical bound of the MFE

$$\begin{aligned} F &= \mathbb{E}\{\mathcal{F}(\boldsymbol{x} + C^{\star}(\hat{\mathbf{x}}) + \mathbf{w}, \boldsymbol{\xi})\} \\ &\approx \mathbb{E}\{(\boldsymbol{y} - \hat{\mathbf{y}})^{\mathrm{T}} \boldsymbol{B} (\boldsymbol{y} - \hat{\mathbf{y}})\} + (\rho - \|\boldsymbol{\xi}\|)^2 + \left(2N - 2 - \frac{\|\boldsymbol{\xi}\|}{\rho}\right)\sigma_{\mathrm{c}}^2 \\ &\geq \mathrm{tr}\{\boldsymbol{B}\boldsymbol{D}\boldsymbol{J}(\boldsymbol{x})^{-1}\} + (\rho - \|\boldsymbol{\xi}\|)^2 + \left(2N - 2 - \frac{\|\boldsymbol{\xi}\|}{\rho}\right)\sigma_{\mathrm{c}}^2 \end{aligned}$$

Error resulted from the localization step

Error resulted from the control step

Integrated Scheme

- Case Study
  - Design the scheduling strategy under a given control policy
  - *Scheduling strategy that suits the MMFE control*
    - Minimize the approximate theoretical bound of the MFE, under the given number L of the measurement links

$$\mathscr{A}^\star = \arg\min_{\mathscr{A}} \mathrm{tr}\{\boldsymbol{B}\boldsymbol{D}\boldsymbol{J}(\boldsymbol{x})^{-1}\}$$

    - A greedy realization of this algorithm: choose L links one by one, and in each resource unit, select the link with the largest weight

$$v_{i,j} = \mathrm{tr}\{\boldsymbol{B}\boldsymbol{D}[\boldsymbol{J}_0(\boldsymbol{x})^{-1} - \boldsymbol{J}_{i,j}(\boldsymbol{x})^{-1}]\}$$

which is the reduction of the approximated MFE bound if the link is selected

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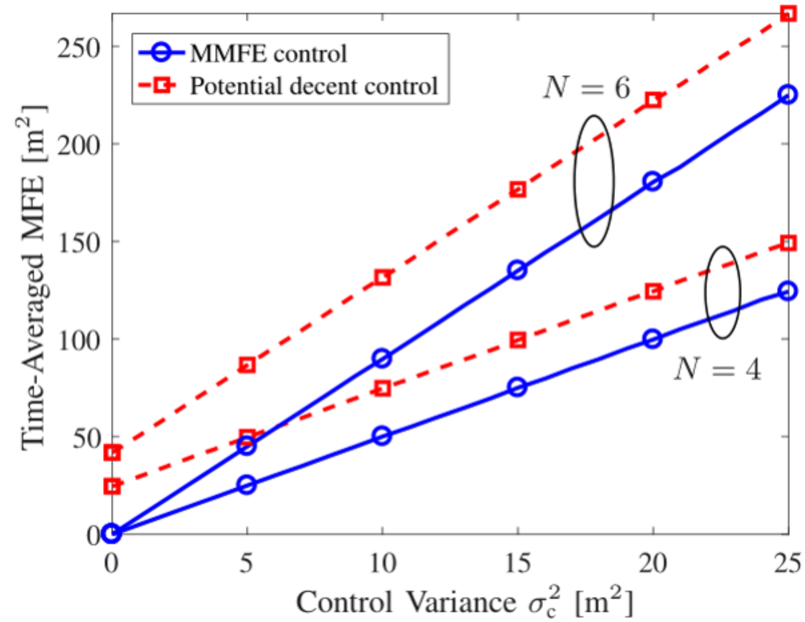
Numerical Results

- General Configuration
  - A network of N UAVs
    - The target formation is a uniform line
- Control policy
  - Comparison
    - The MMFE control policy
    - The potential decent control
  - Gain of the proposed method
    - Higher accuracy
    - Smaller control cost
    - Lower computational complexity

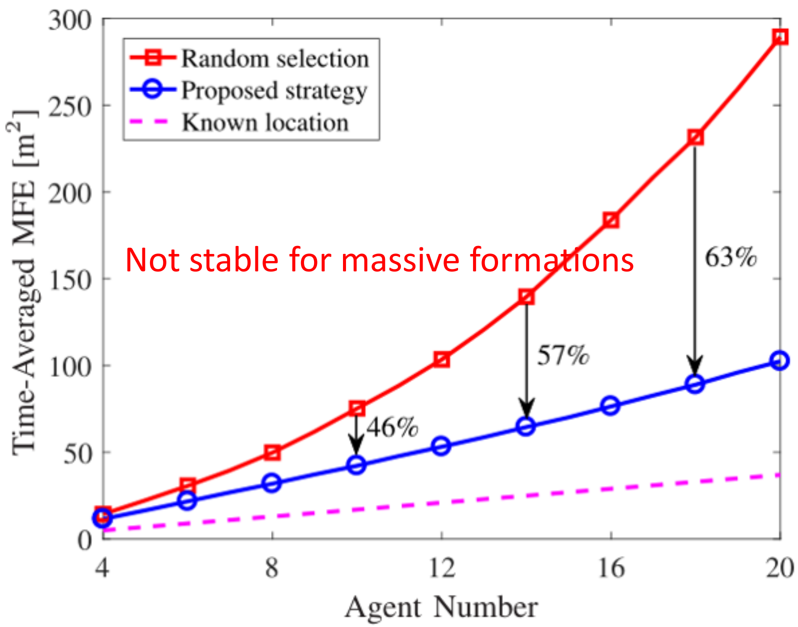
The true formation $\mathbf{x}$ is generated from $\mathcal{N}(\boldsymbol{\xi}, \boldsymbol{I})$



Numerical Results

- General Configuration
  - A network of N UAVs
    - The target formation is a uniform line
- Scheduling Strategy
  - Comparison
    - The proposed scheduling strategy
    - Random scheduling
    - Known position (baseline)
  - Gain of the proposed method
    - Linear with the agent number
    - The gap grows with agent number

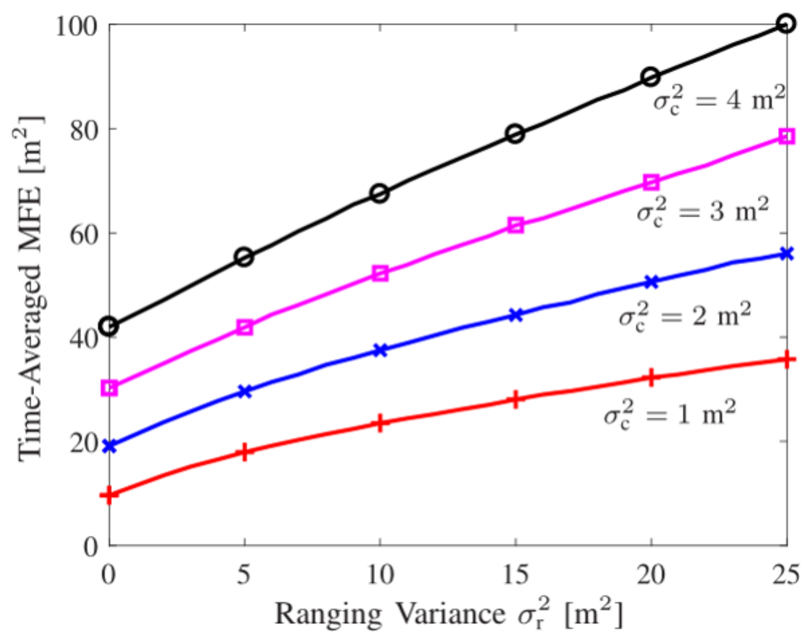
The MMFE control policy is adopted



Numerical Results

- General Configuration
  - A network of 4 UAVs
    - The target formation is a uniform line
- System Parameter
  - Insights
    - Intricate relationship between the MFE and both parameters
    - Analyze the bound of the MFE
  - A special case $\sigma_{\mathrm{c}} \gg \sigma_{\mathrm{r}}$
    - The MFE (or weighted CRB) is approximately linear with both parameters σ_{c}^2 and σ_{r}^2

The variance of the ranging noise & the control error



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Conclusions

- A new performance metric
  - Formation error
    - Optimization formulation and a closed-form solution
    - An equivalent expression and the geometrical interpretation
- The integrated localization and control scheme
  - Advantages
    - Better allocation of the resource
    - Better utilization of the information
  - Case study
    - Optimize the scheduling strategy and the control policy iteratively
    - The MMFE control and the scheduling strategy that suits it

Q & A

Thanks for your attention!

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