

Towards Graph Foundation Models

WWW 2024 Tutorial

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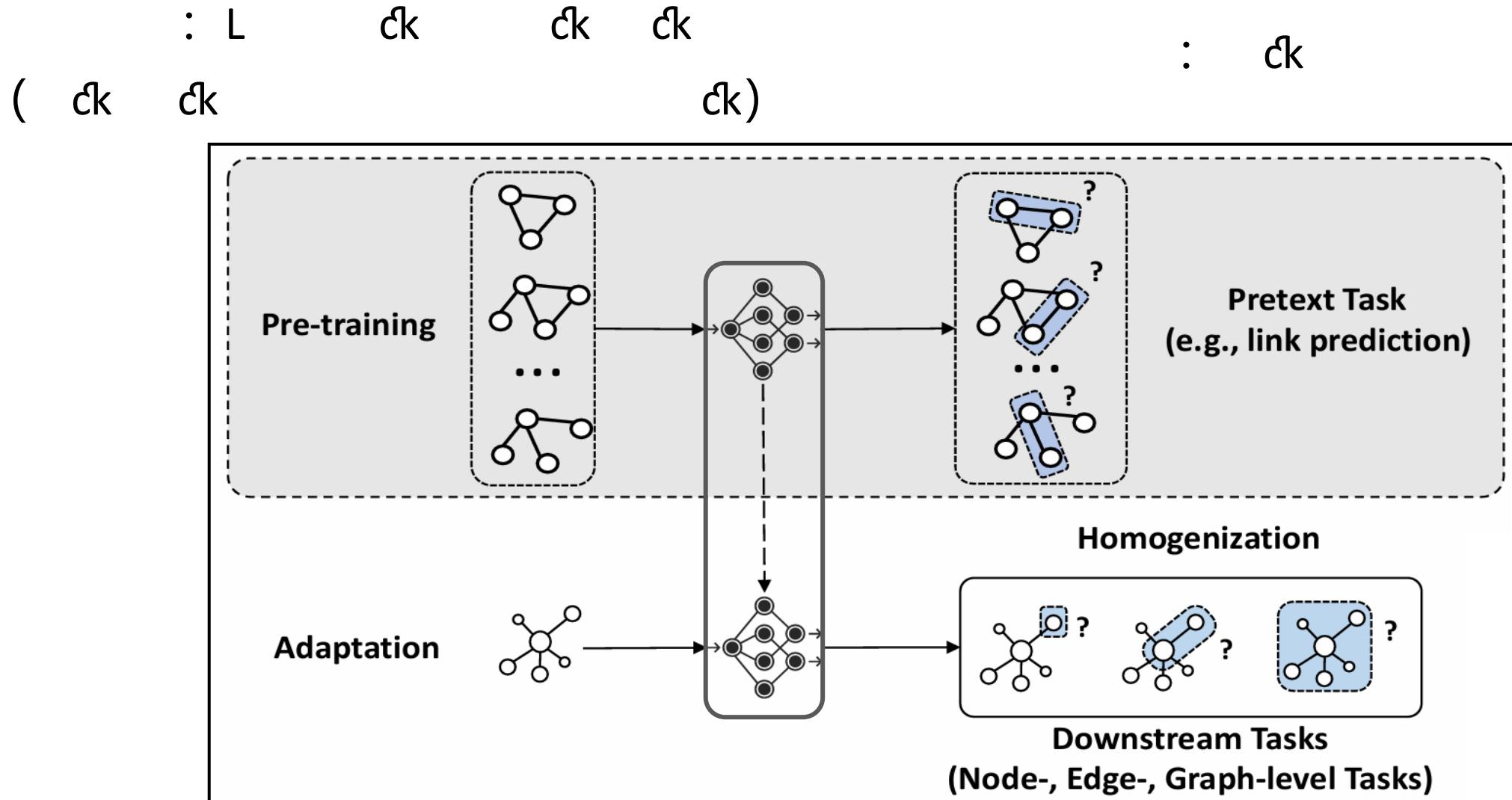
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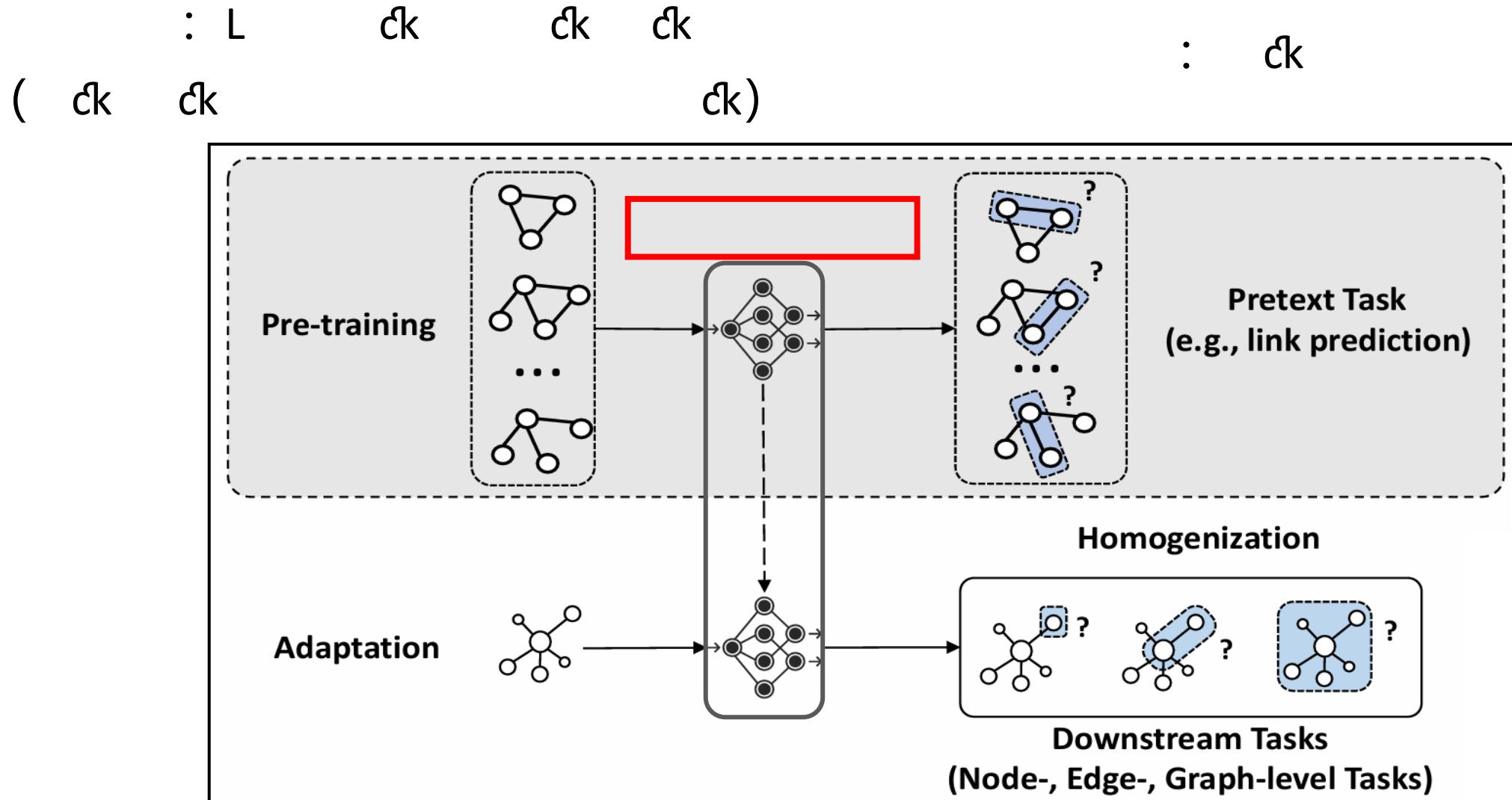
Cheng Yang

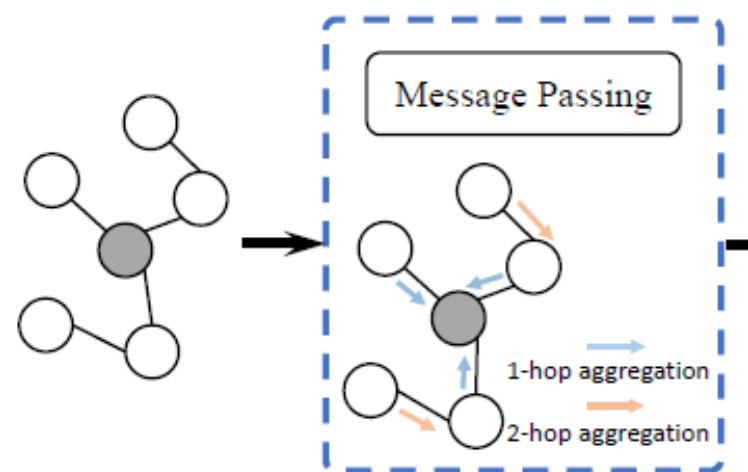
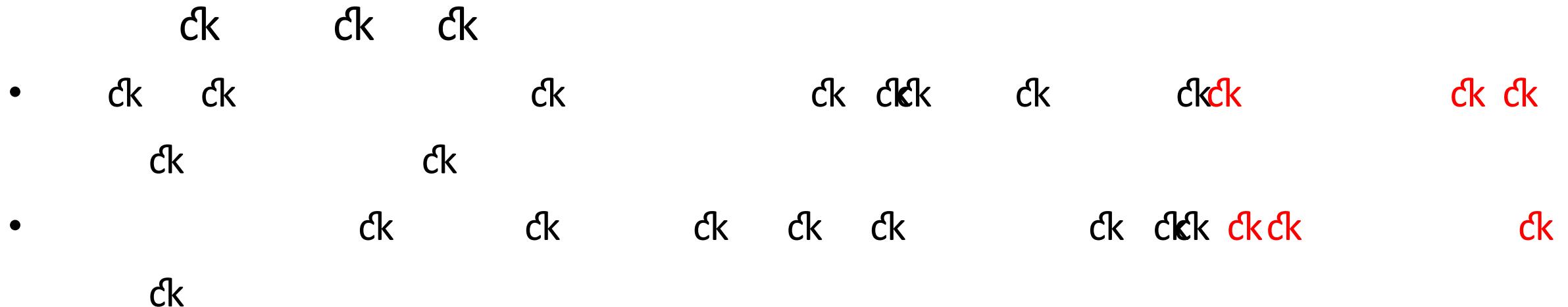
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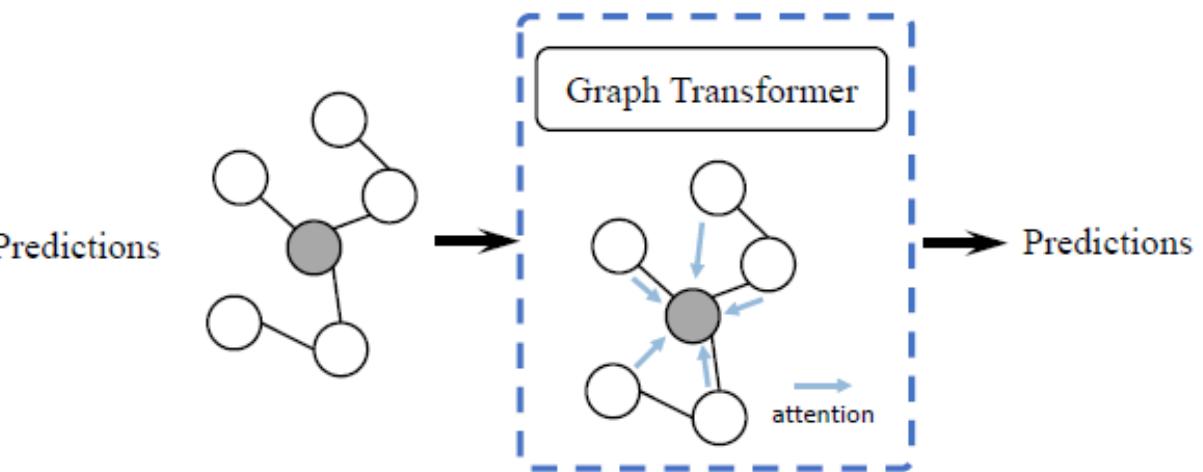




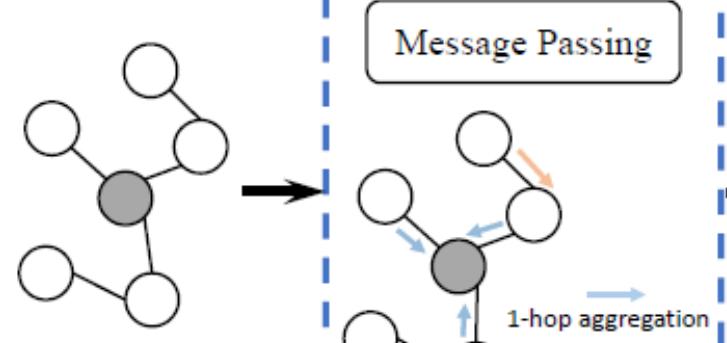
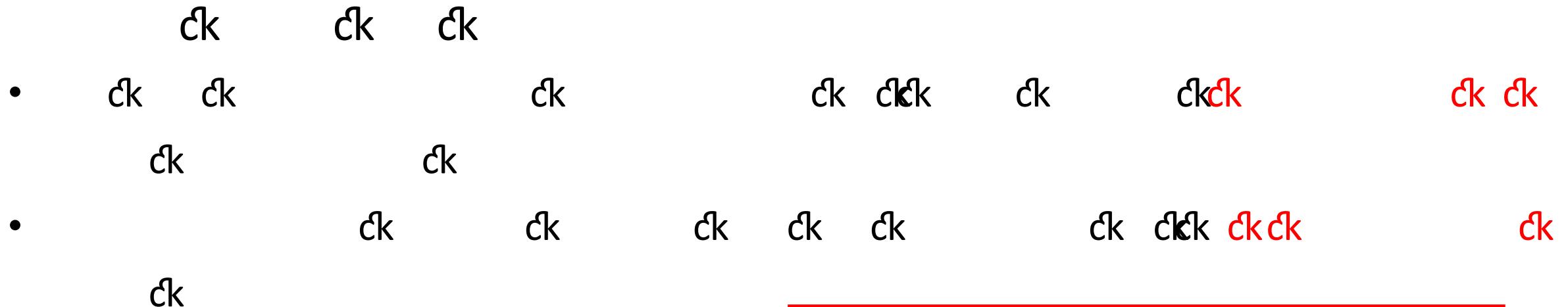




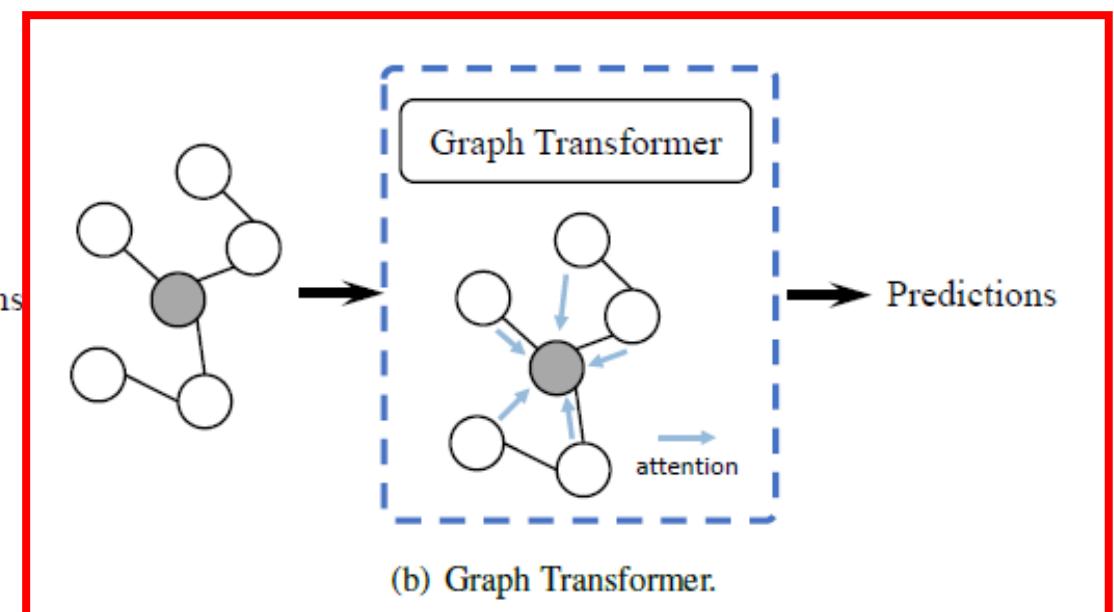
(a) Message Passing.



(b) Graph Transformer.



(a) Message Passing.



(b) Graph Transformer.

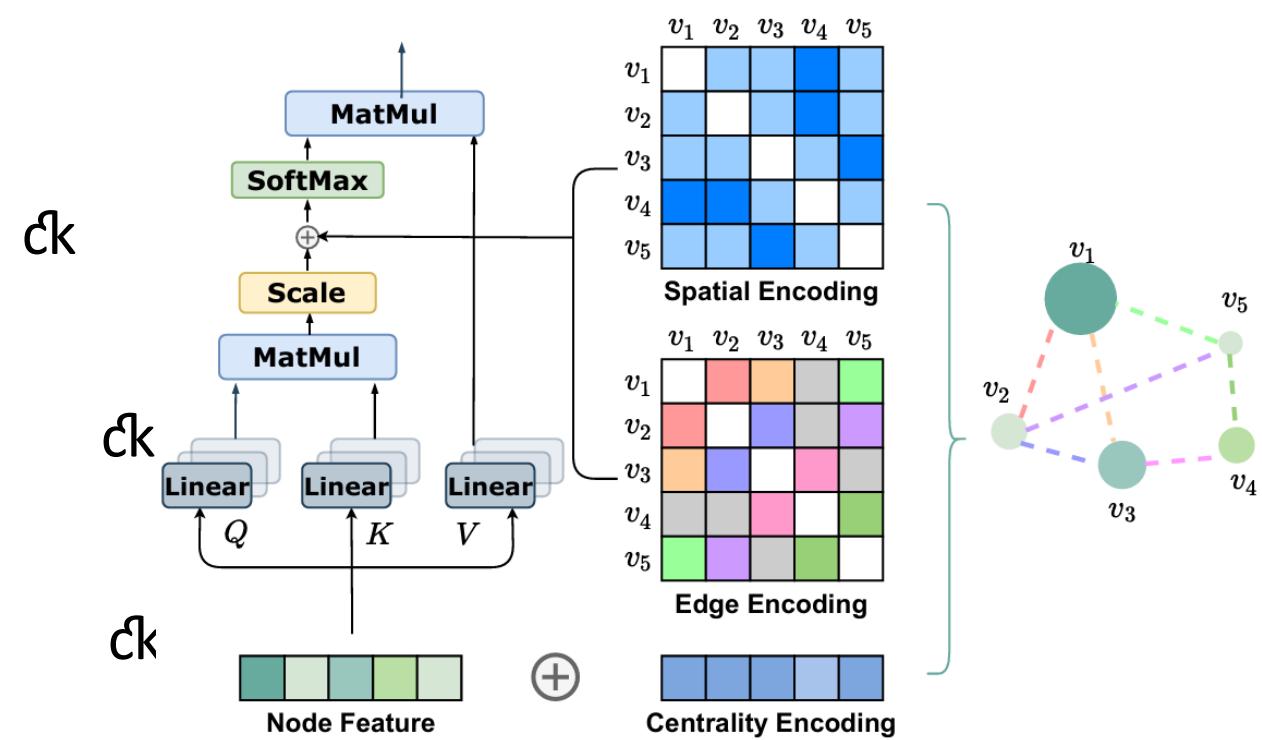
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Whether Transformer architecture is suitable to model graphs and how to make it work in graph representation learning?

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$$h_i^{(0)} = x_i + z_{\deg^-(v_i)}^- + z_{\deg^+(v_i)}^+$$

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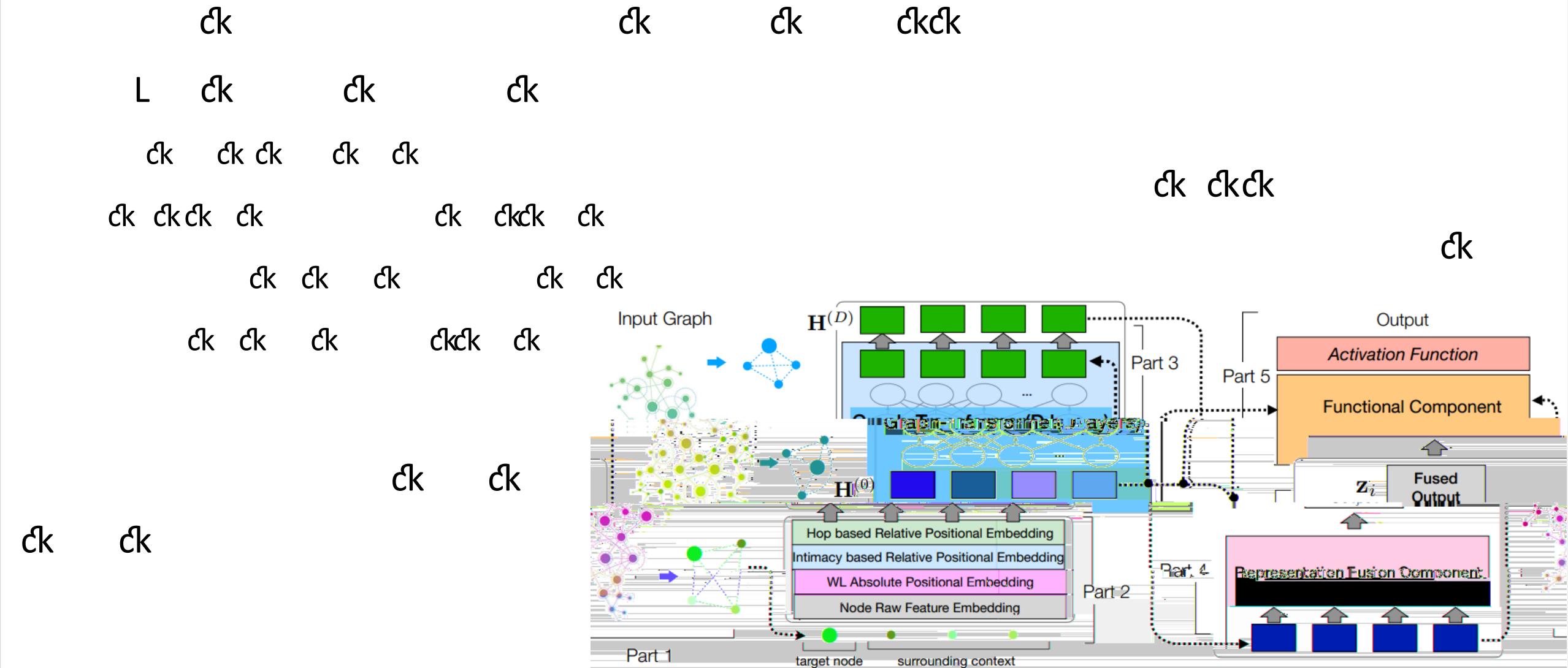
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$$A_{ij} = \frac{(h_i W_Q)(h_j W_K)^T}{\sqrt{d}} + b_{\phi(v_i, v_j)} + c_{ij}, \text{ where } c_{ij} = \frac{1}{N} \sum_{n=1}^N x_{e_n} (w_n^E)^T$$

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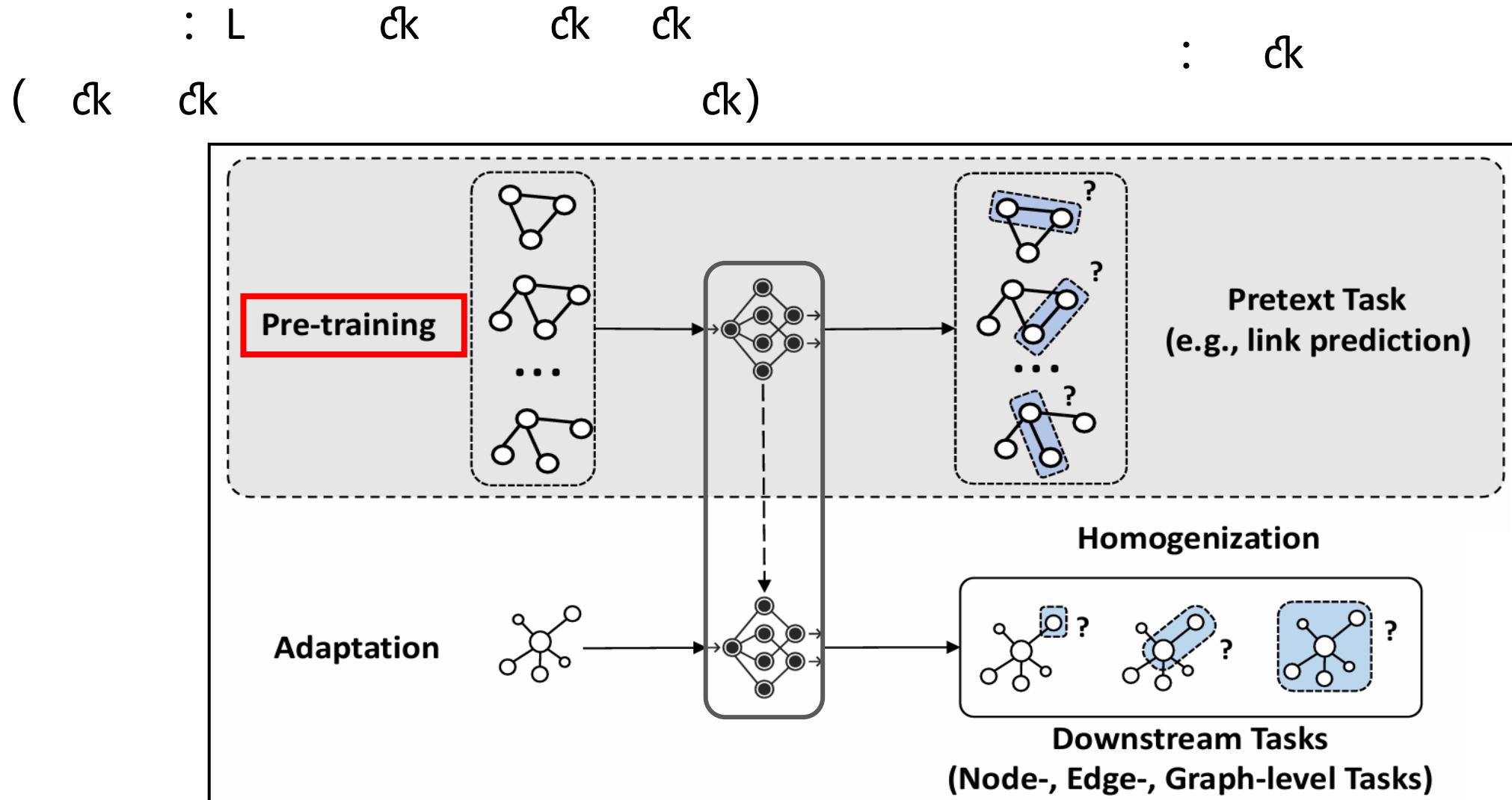
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$$\begin{aligned} \mathbf{e}_j^{(x)} &= \text{Embed}(\mathbf{x}_j) \in \mathbb{R}^{d_h \times 1} \\ \mathbf{e}_j^{(r)} &= \text{Position-Embed}(\text{WL}(v_j)) \\ &= \left[\sin\left(\frac{\text{WL}(v_j)}{10000^{\frac{2l}{d_h}}}\right), \cos\left(\frac{\text{WL}(v_j)}{10000^{\frac{2l+1}{d_h}}}\right) \right]_{l=0}^{\lfloor \frac{d_h}{2} \rfloor} \end{aligned}$$

$$\mathbf{e}_j^{(p)} = \text{Position-Embed}(\text{P}(v_j)) \in \mathbb{R}^{d_h \times 1}$$

$$\mathbf{e}_j^{(d)} = \text{Position-Embed}(\text{H}(v_j; v_i)) \in \mathbb{R}^{d_h \times 1}$$

$$\mathbf{h}_j^{(0)} = \text{Aggregate}(\mathbf{e}_j^{(x)}, \mathbf{e}_j^{(r)}, \mathbf{e}_j^{(p)}, \mathbf{e}_j^{(d)})$$



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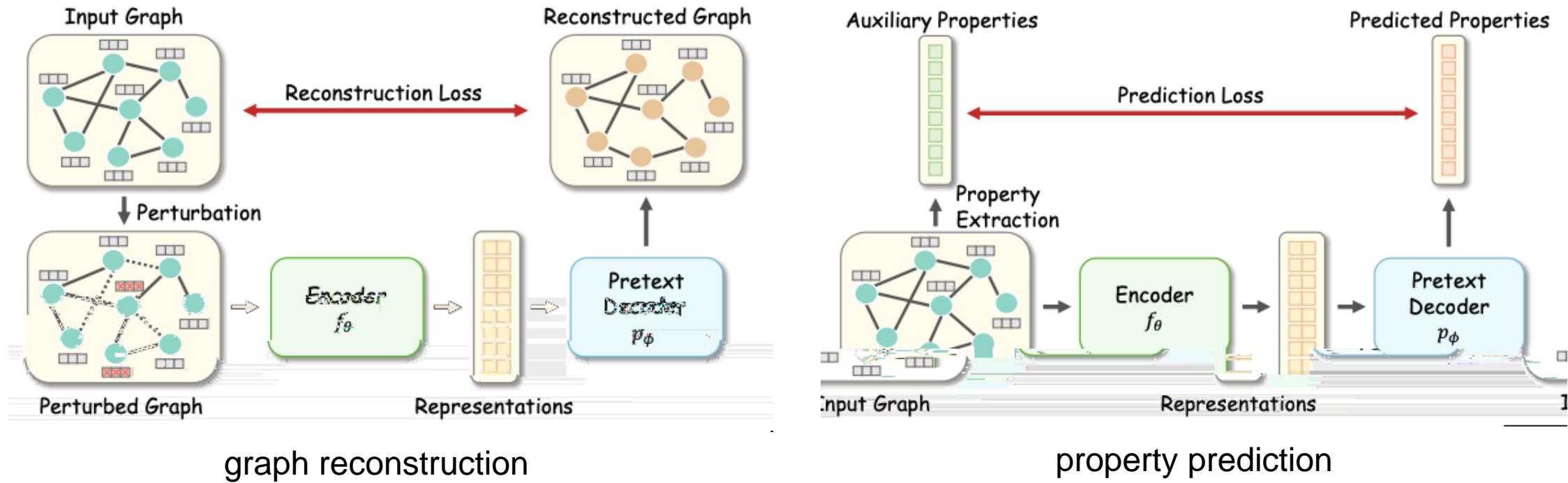
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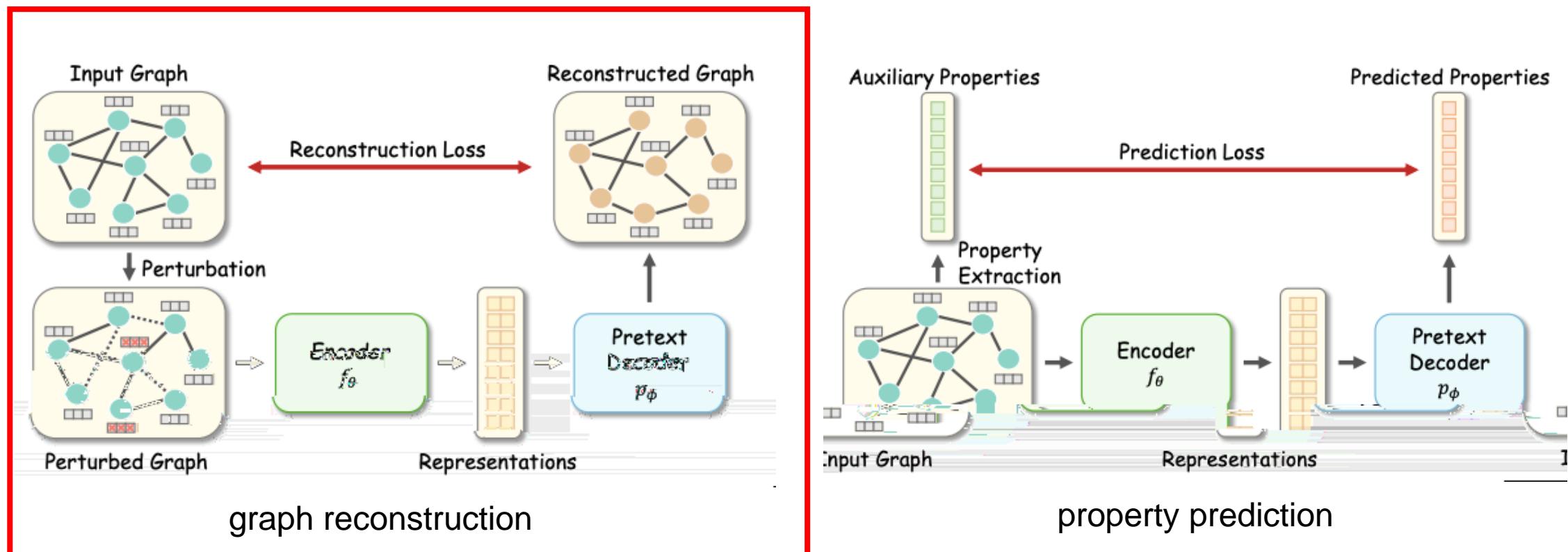


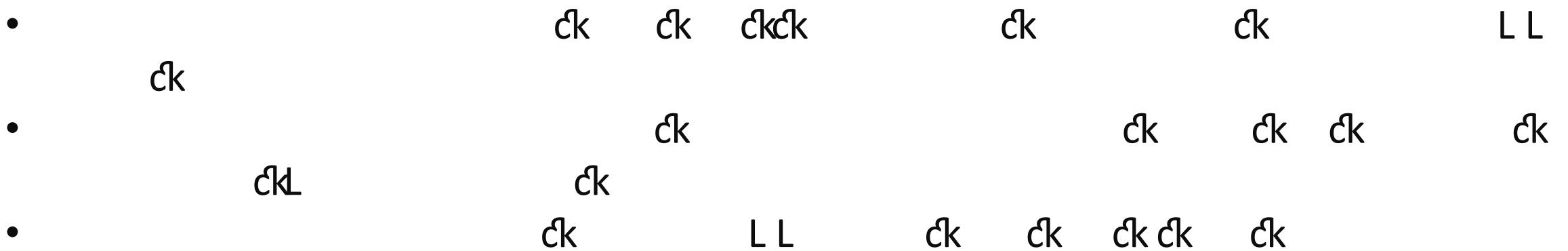
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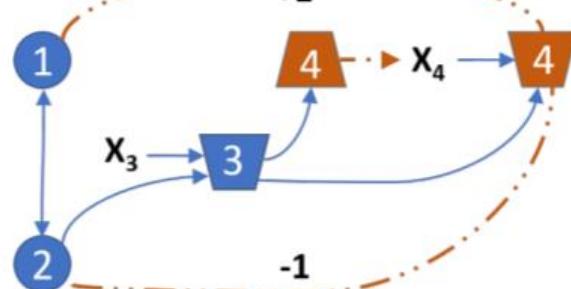
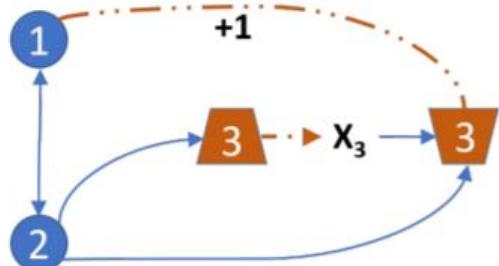
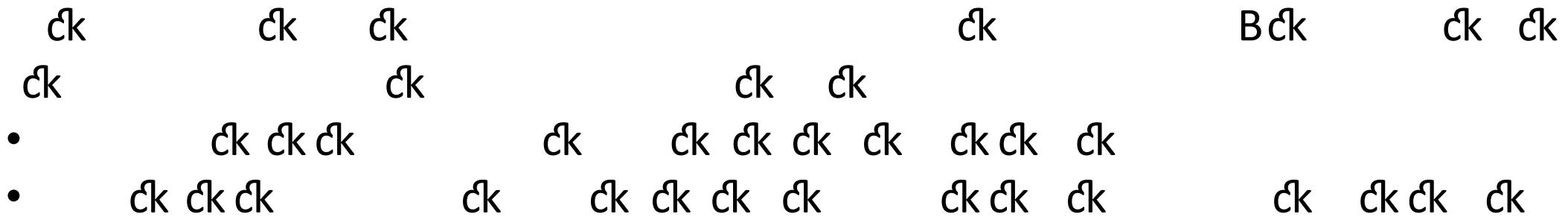
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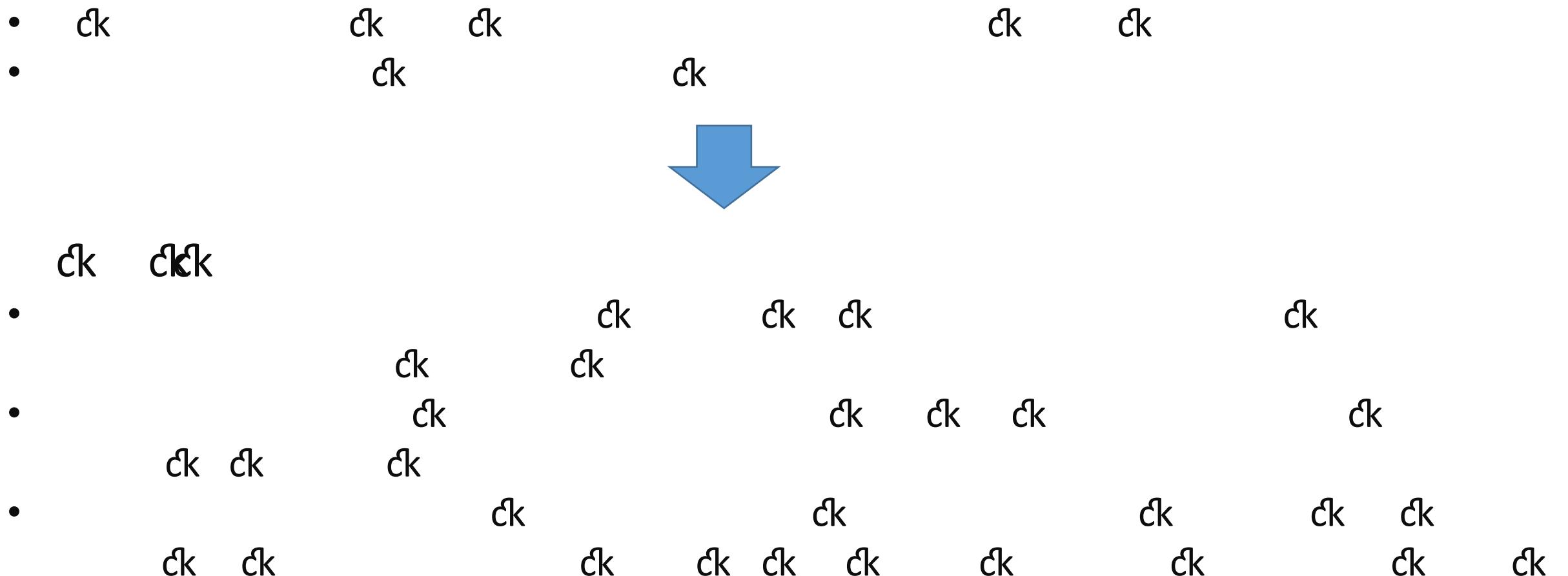
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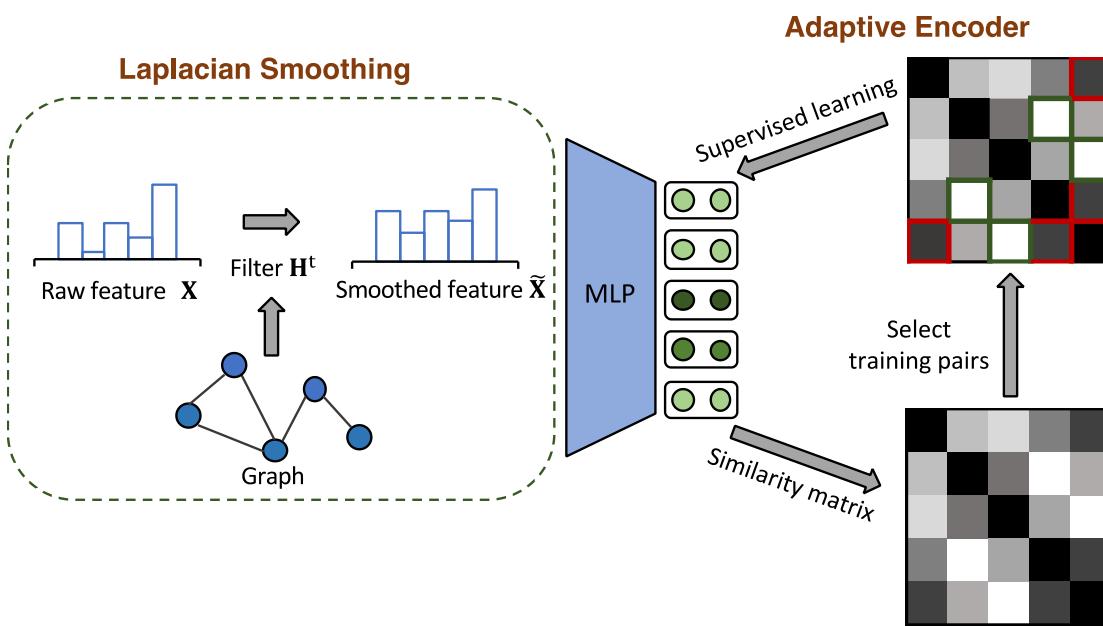


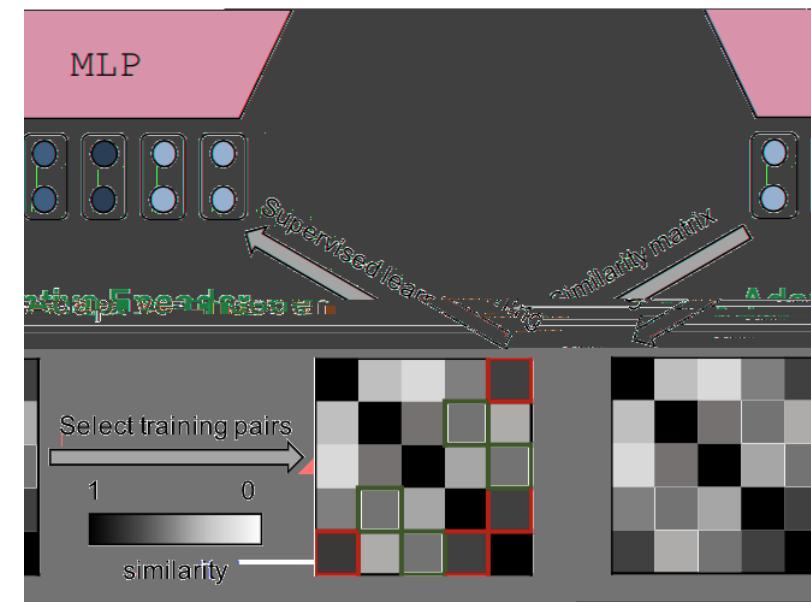


: Attribute generation node
 : Edge generation node



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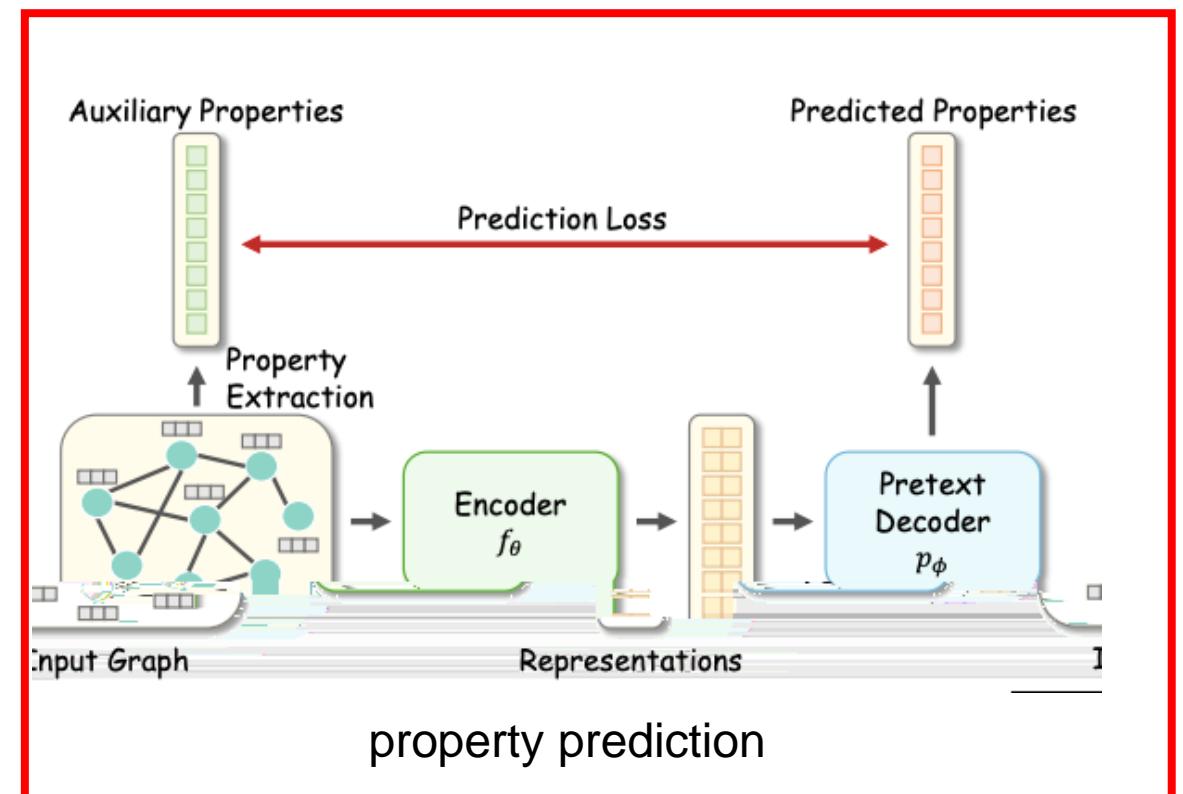
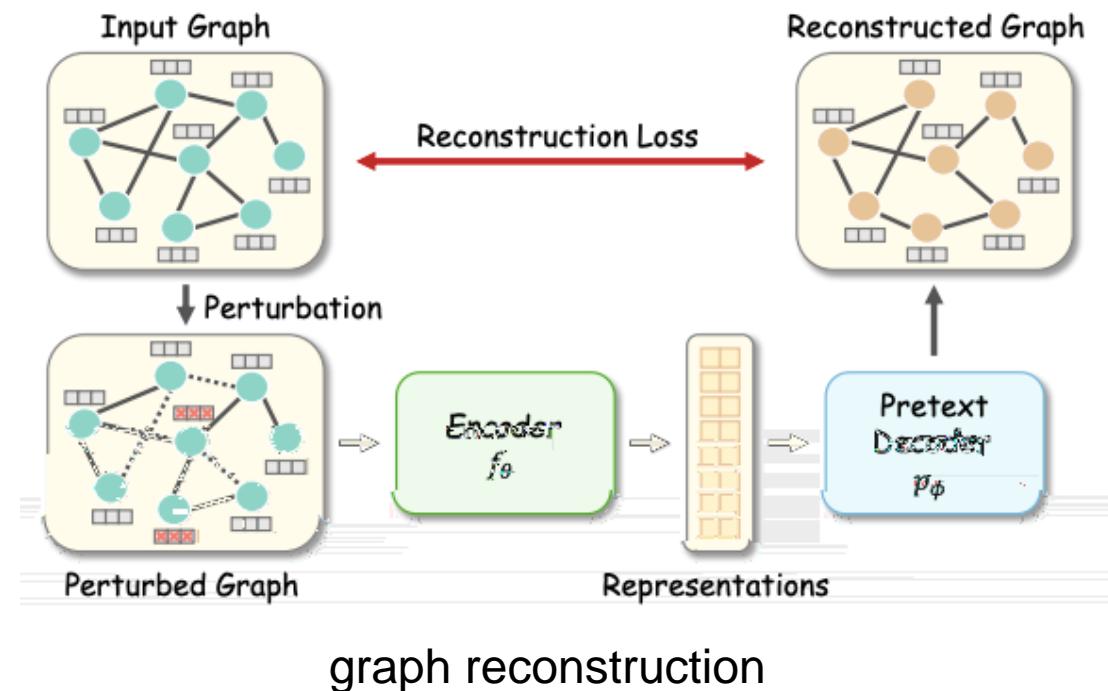


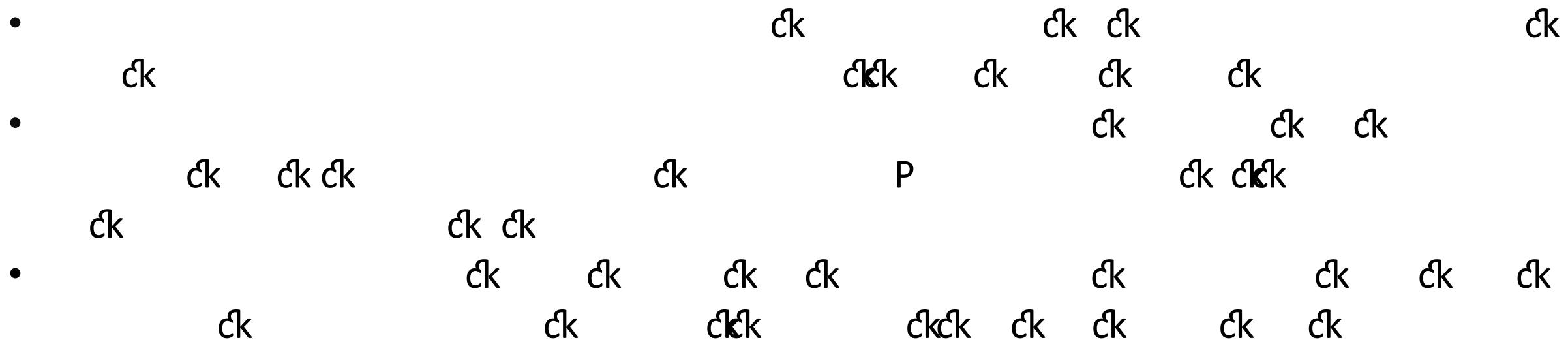
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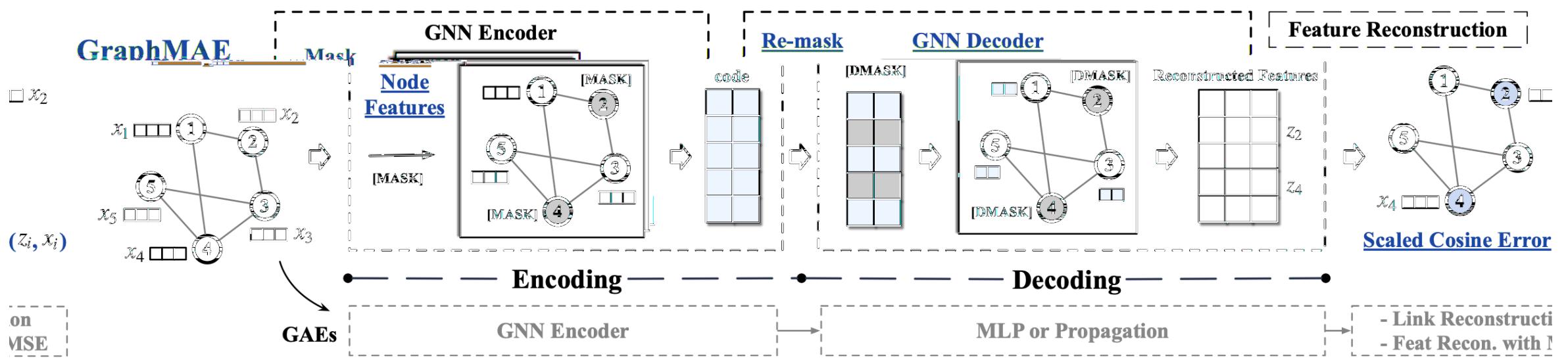
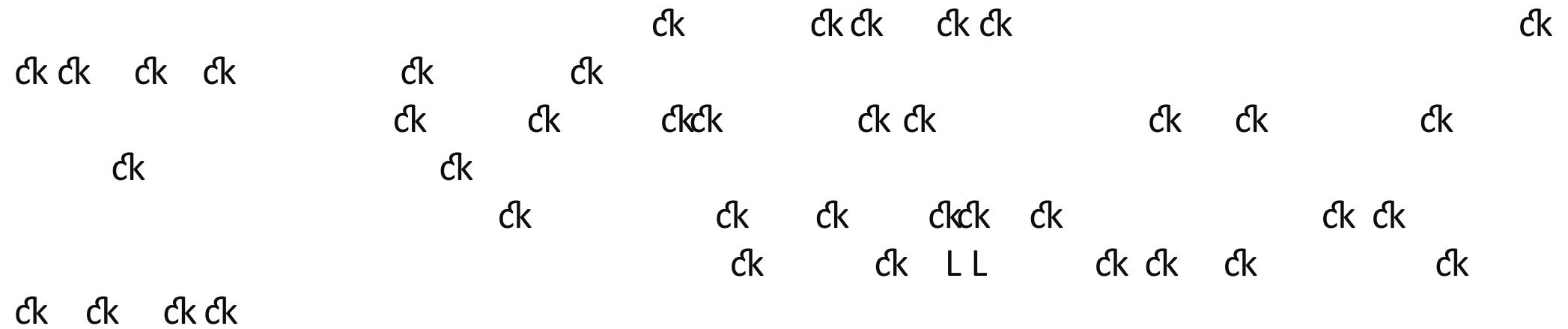
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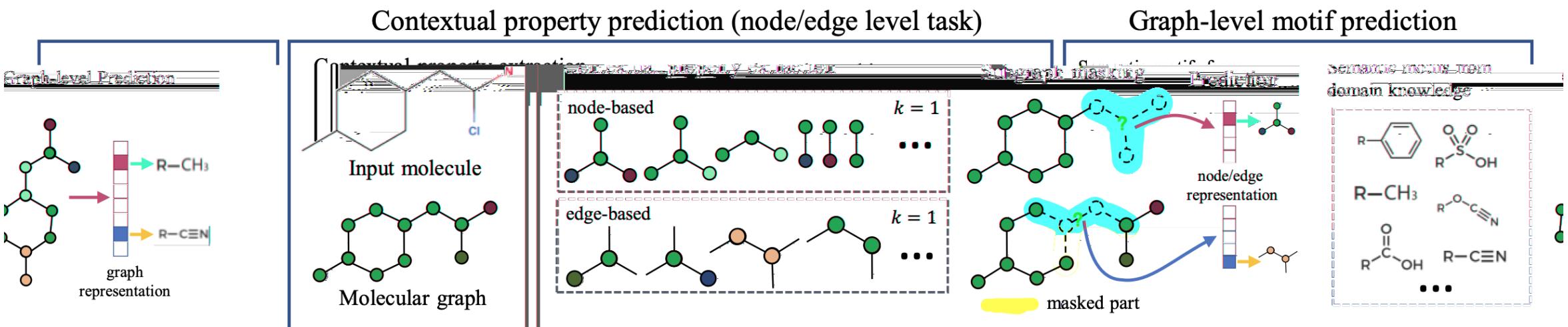






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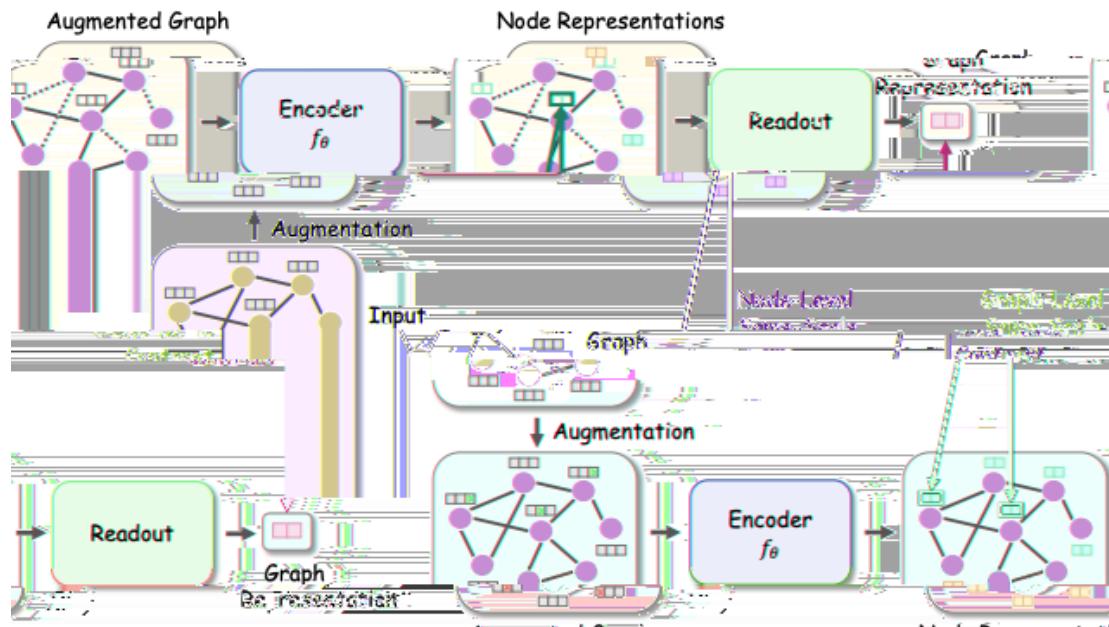
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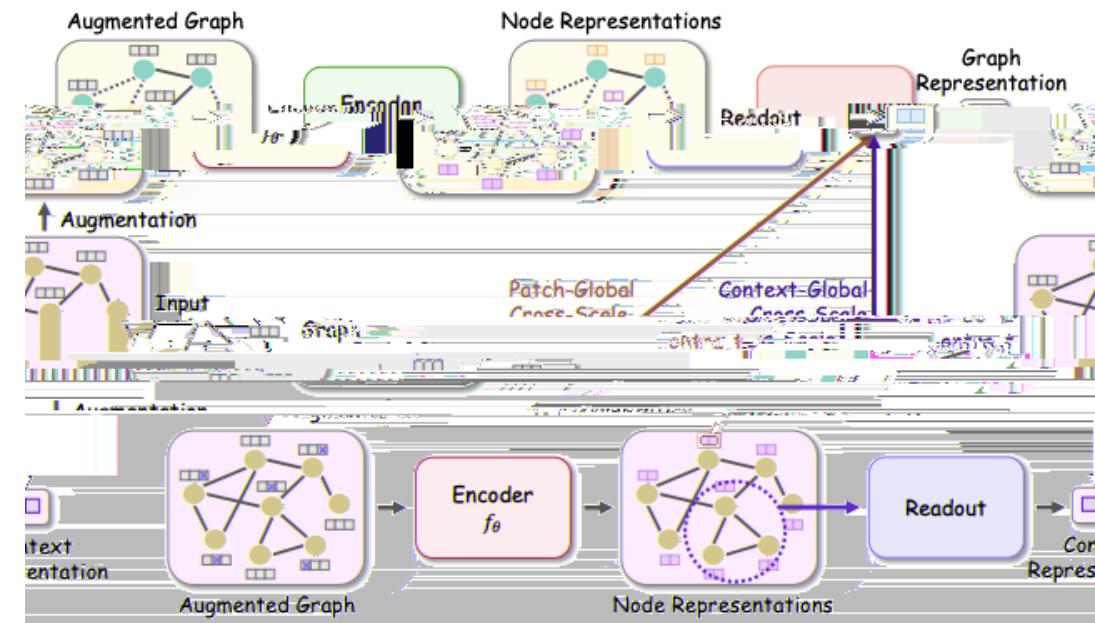
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same-scale contrastive learning



cross-scale contrastive learning

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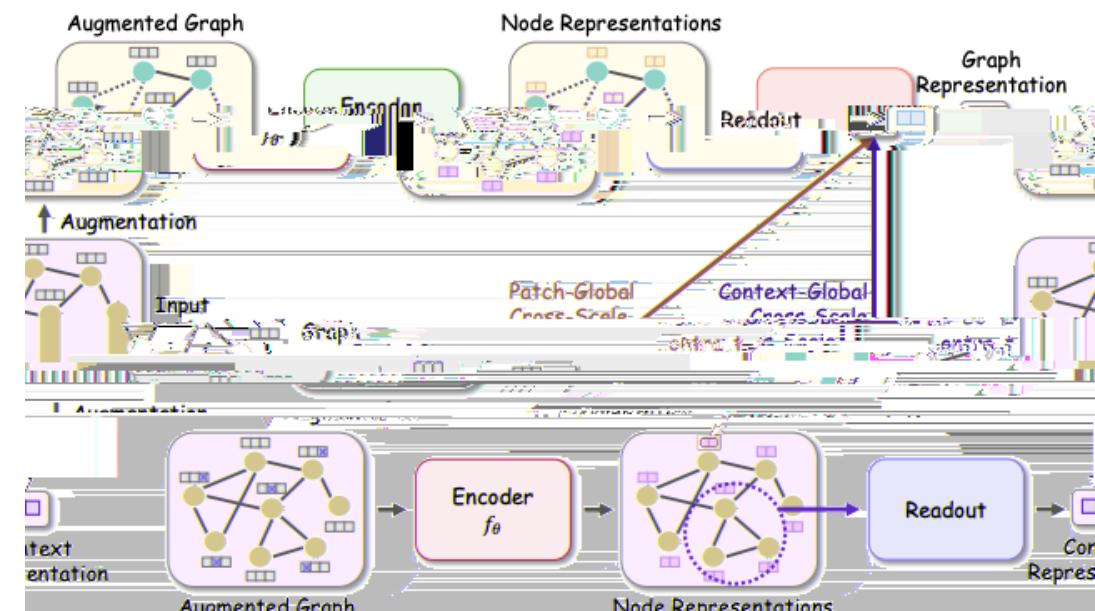
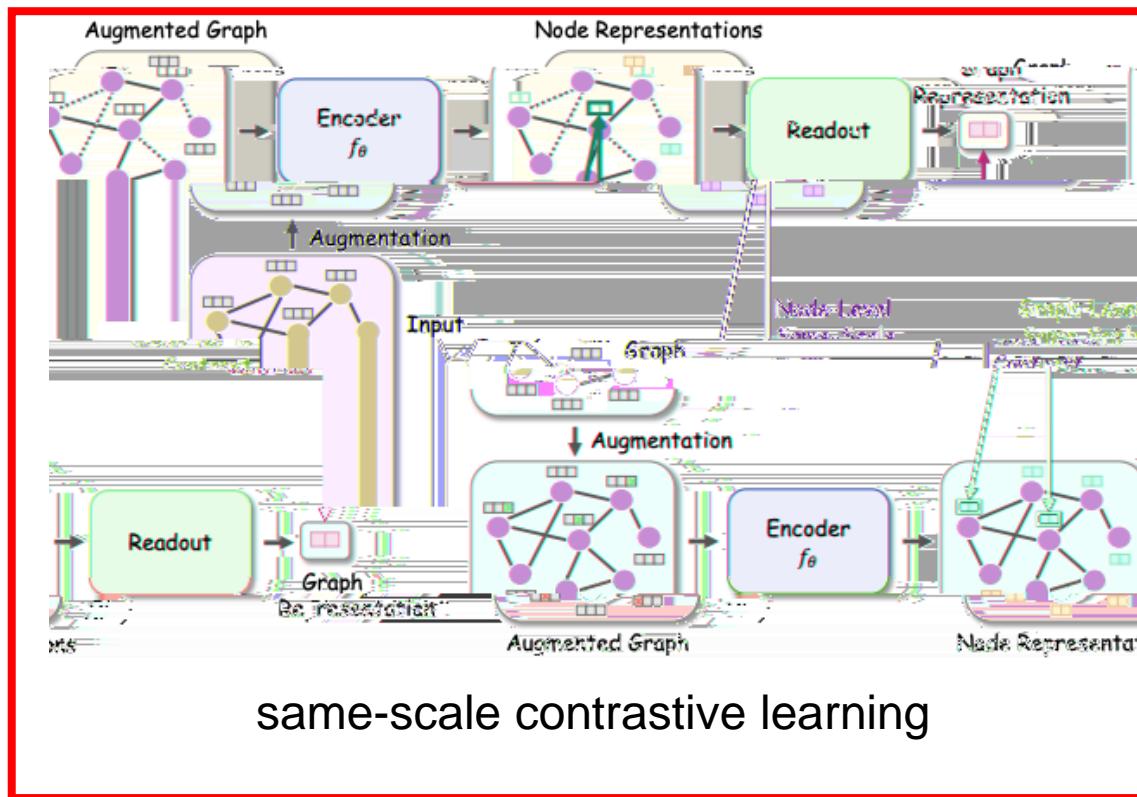
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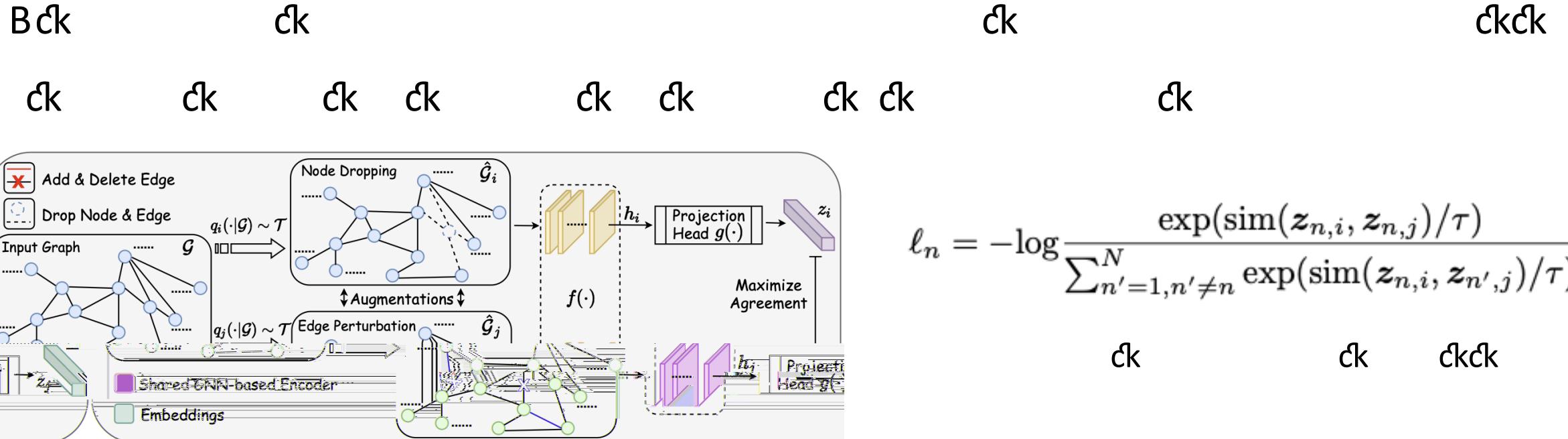
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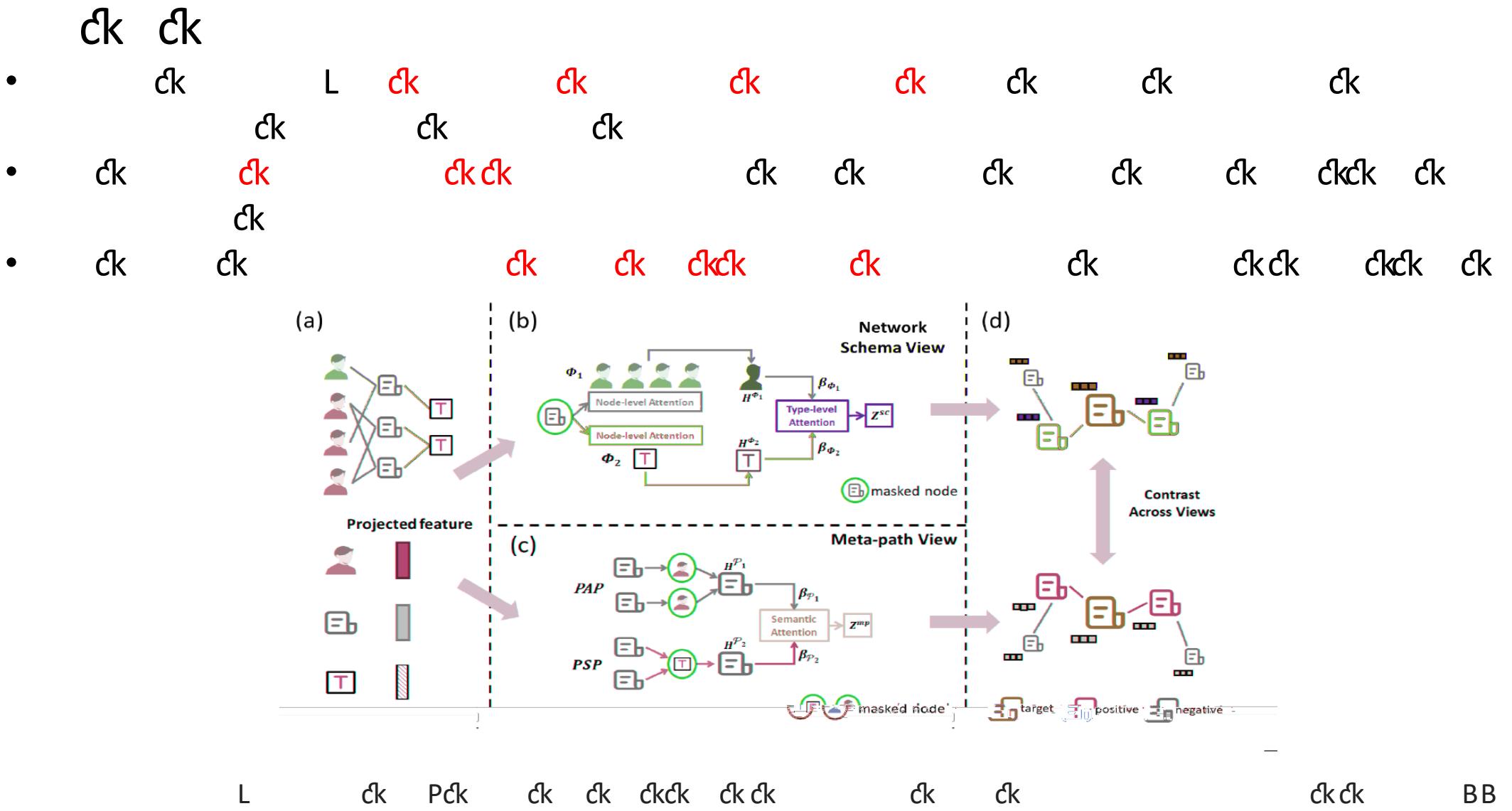
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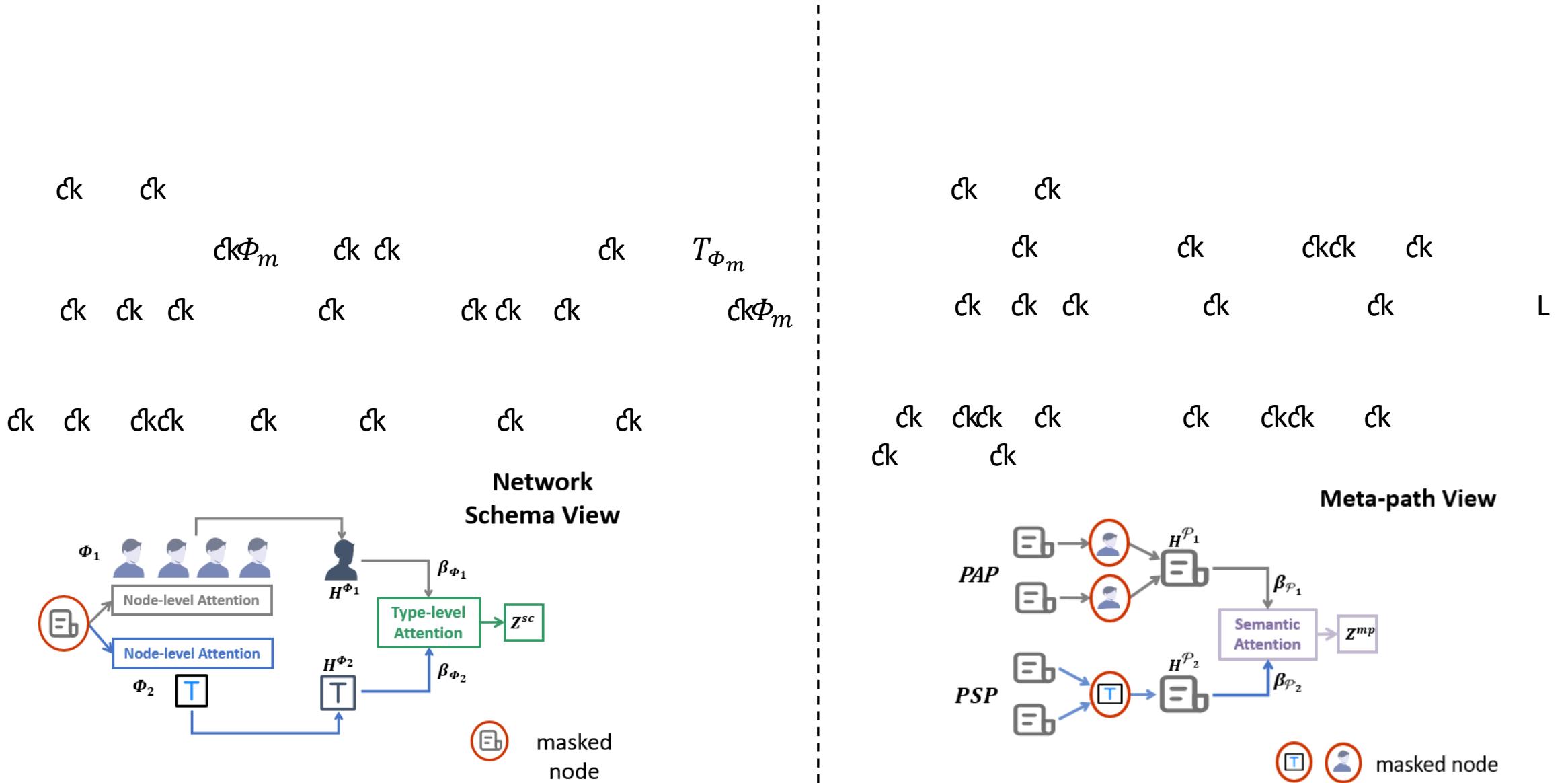
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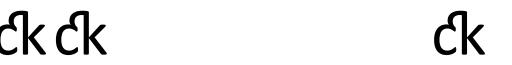
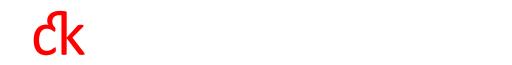


$$\ell_n = -\log \frac{\exp(\text{sim}(z_{n,i}, z_{n,j})/\tau)}{\sum_{n'=1, n' \neq n}^N \exp(\text{sim}(z_{n,i}, z_{n',j})/\tau)}$$

Data augmentation	Type	Underlying Prior
Node dropping	Nodes, edges	Vertex missing does not alter semantics.
Edge perturbation	Edges	Semantic robustness against connectivity variations.
Attribute masking	Nodes	Semantic robustness against losing partial attributes.
Subgraph	Nodes, edges	Local structure can hint the full semantics.





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- Top Right
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- Rotation
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- Jigsaw
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- Predict Relative Position

Can we generate better augmentations than typical random dropping-based methods?

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$$g(\mathbf{Z}; \mathbf{F}) = \mathbf{F}\mathbf{Z}, \quad h(\mathbf{Z}; \mathbf{W}) = \sigma(\mathbf{Z}\mathbf{W}), \quad \mathbf{F} = \mathbf{D}^{-\frac{1}{2}} \mathbf{A} \mathbf{D}^{-\frac{1}{2}}$$

$$GCN(\mathbf{X}) = h_L \circ g \circ h_{L-1} \circ g \circ \cdots \circ h_1 \circ g(\mathbf{X}),$$

$$SGC(\mathbf{X}) = h \circ g^{[L]}(\mathbf{X}),$$

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Datasets	Cora	CiteSeer	PubMed	Coauthor-CS	Amazon-C	Amazon-P	Avg. Acc.	Avg. Rank
GCN	82.5 ± 0.4	71.2 ± 0.3	79.2 ± 0.3	93.03 ± 0.3	86.51 ± 0.5	92.42 ± 0.2	-	-
GAT	83.0 ± 0.7	72.5 ± 0.7	79.0 ± 0.3	92.31 ± 0.2	86.93 ± 0.3	92.56 ± 0.4	-	-
InfoGCL	83.5 ± 0.3	73.5 ± 0.4	79.1 ± 0.2	-	-	-	-	-
DGI	82.3 ± 0.6	71.8 ± 0.7	76.8 ± 0.3	92.15 ± 0.6	83.95 ± 0.5	91.61 ± 0.2	83.10	8.5
GRACE	81.7 ± 0.4	71.5 ± 0.4	78.3 ± 0.3	92.95 ± 0.4	87.15 ± 0.2	92.18 ± 0.2	84.44	6.5
GCN	83.4 ± 0.3	73.4 ± 0.3	73.9 ± 0.3	93.01 ± 0.6	87.11 ± 0.1	92.53 ± 0.1	84.63	6.3
BGRL	81.7 ± 0.5	72.1 ± 0.5	80.2 ± 0.4	93.01 ± 0.2	88.23 ± 0.5	92.57 ± 0.3	84.63	6.5
GCA	83.4 ± 0.3	72.3 ± 0.1	80.2 ± 0.4	93.10 ± 0.0	87.85 ± 0.3	92.53 ± 0.2	84.89	4.9
SimGRACE	77.3 ± 0.3	71.4 ± 0.3	78.3 ± 0.3	93.45 ± 0.4	86.94 ± 0.2	91.39 ± 0.4	82.98	8.5
COLES	81.2 ± 0.2	71.5 ± 0.2	80.2 ± 0.7	92.65 ± 0.1	79.54 ± 0.9	89.93 ± 0.5	82.70	8.3
ARIEL	82.5 ± 0.1	72.2 ± 0.2	80.5 ± 0.3	93.36 ± 0.0	88.27 ± 0.2	91.43 ± 0.2	84.71	4.8
CCA-SSG	83.9 ± 0.4	73.1 ± 0.3	81.3 ± 0.4	93.37 ± 0.2	88.42 ± 0.3	92.44 ± 0.1	85.42	2.3
Base Model	81.1 ± 0.4	71.4 ± 0.1	79.1 ± 0.4	92.86 ± 0.3	87.65 ± 0.2	91.19 ± 0.3	83.88	9.0
MA-GCL	83.3 ± 0.4	73.6 ± 0.1	83.5 ± 0.4	94.19 ± 0.1	88.83 ± 0.3	93.80 ± 0.1	86.20	1.2

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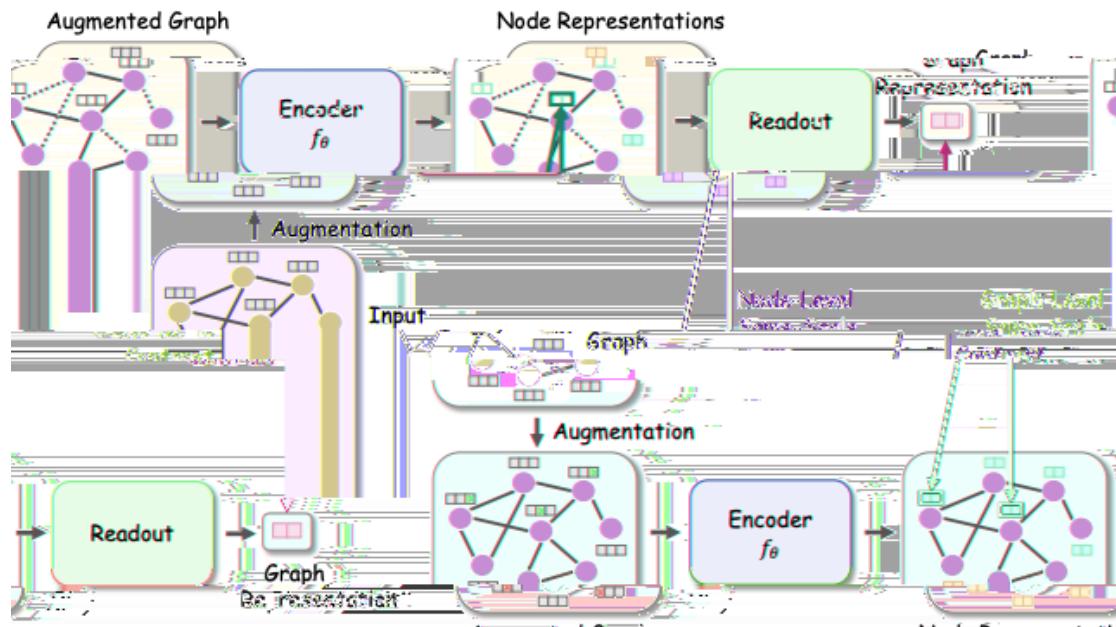
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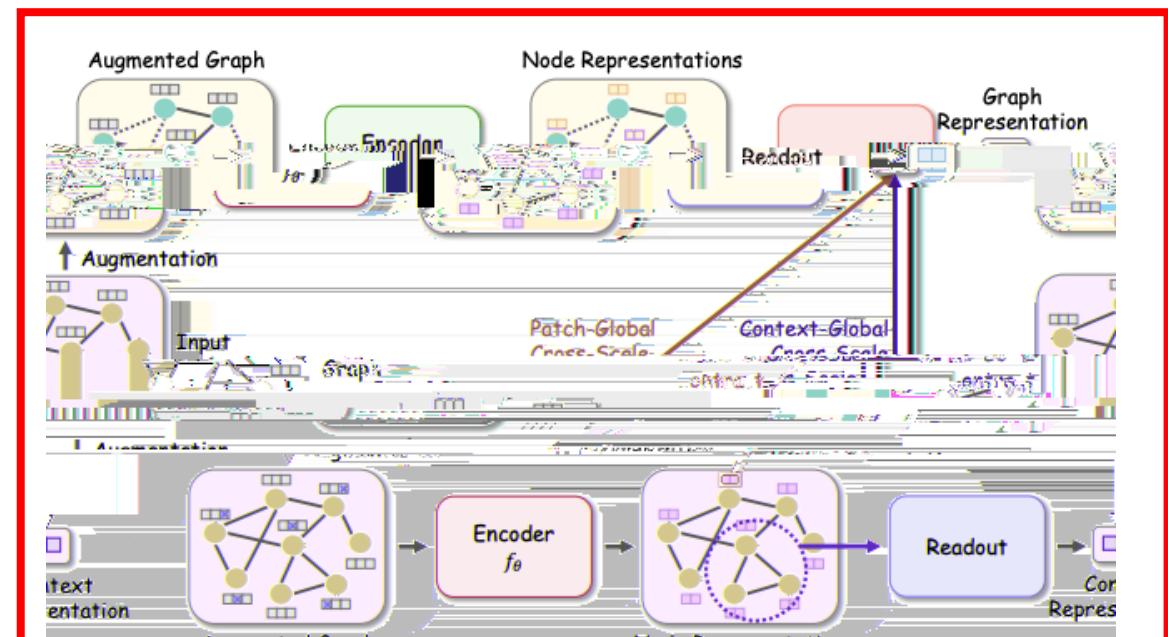
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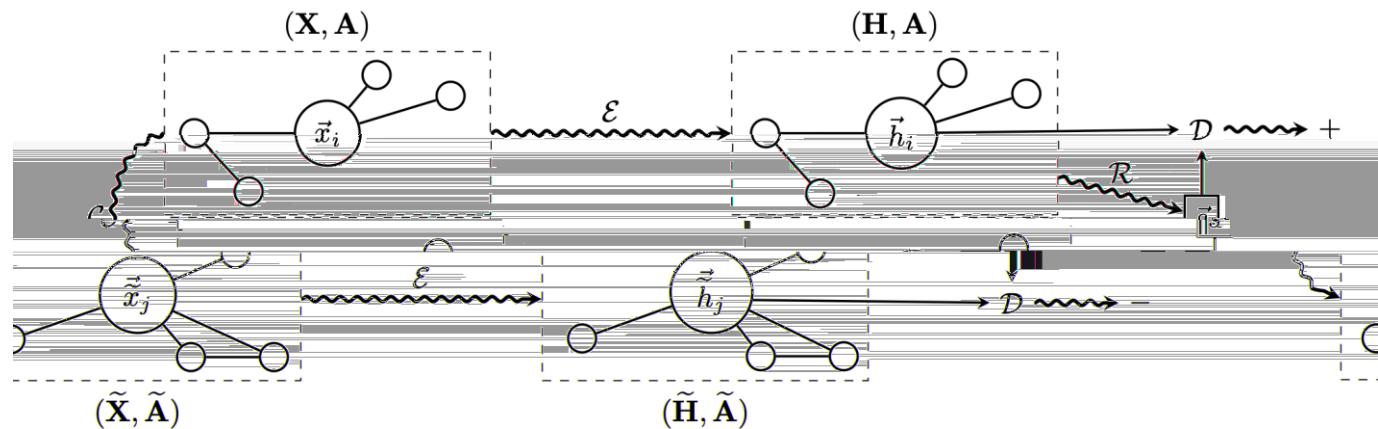
same-scale contrastive learning

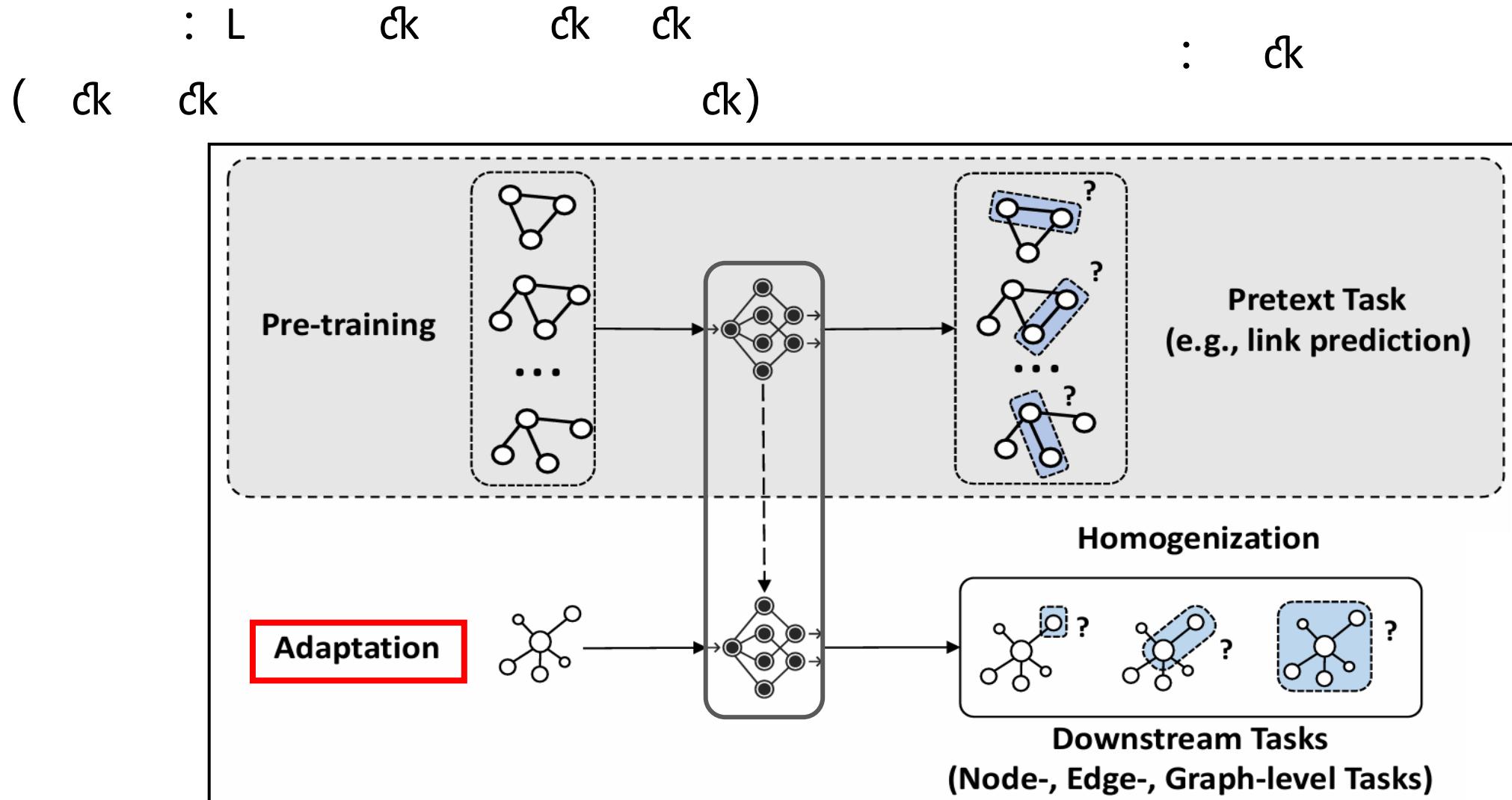


cross-scale contrastive learning

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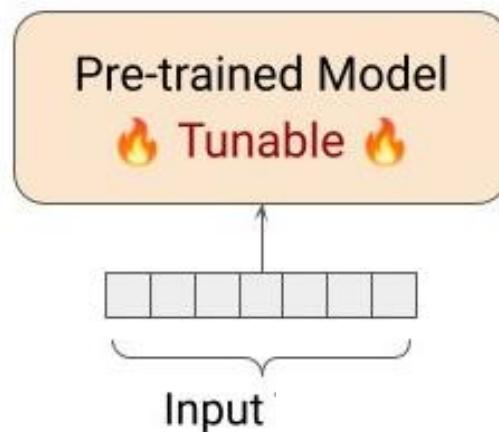
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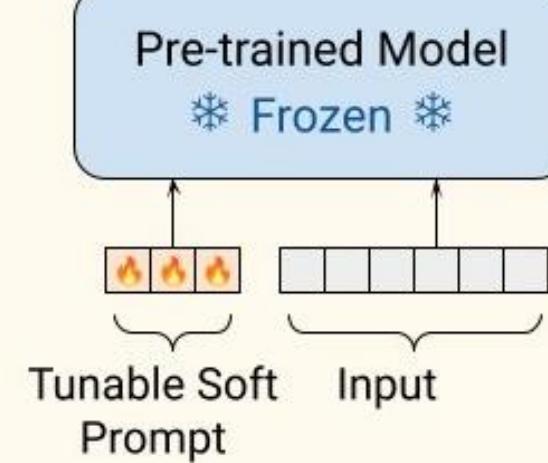


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Model Tuning (a.k.a. "Fine-Tuning")



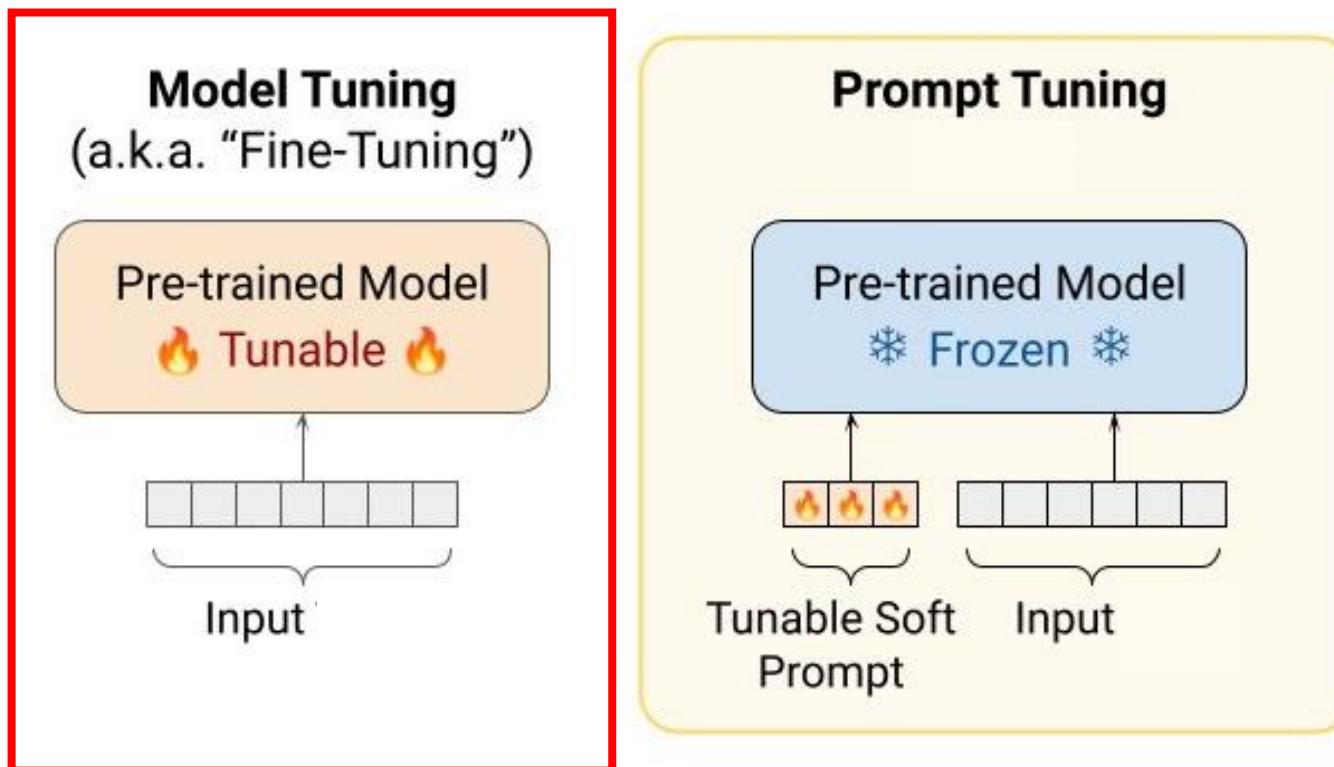
Prompt Tuning



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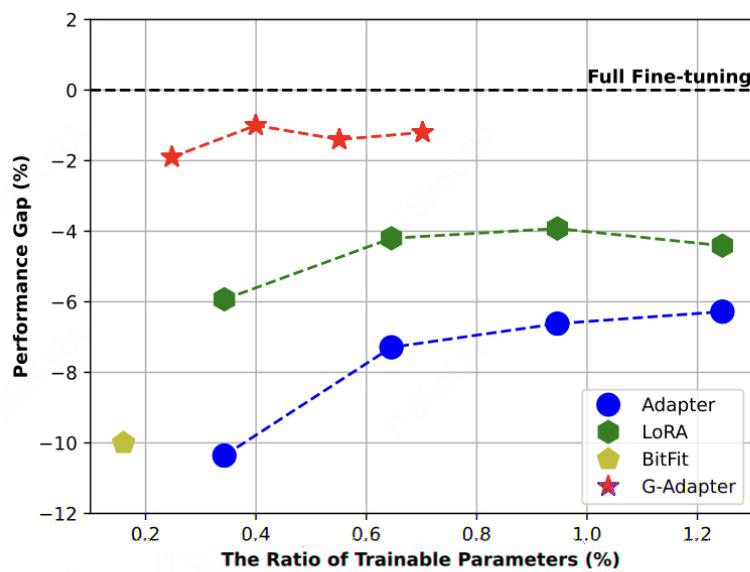
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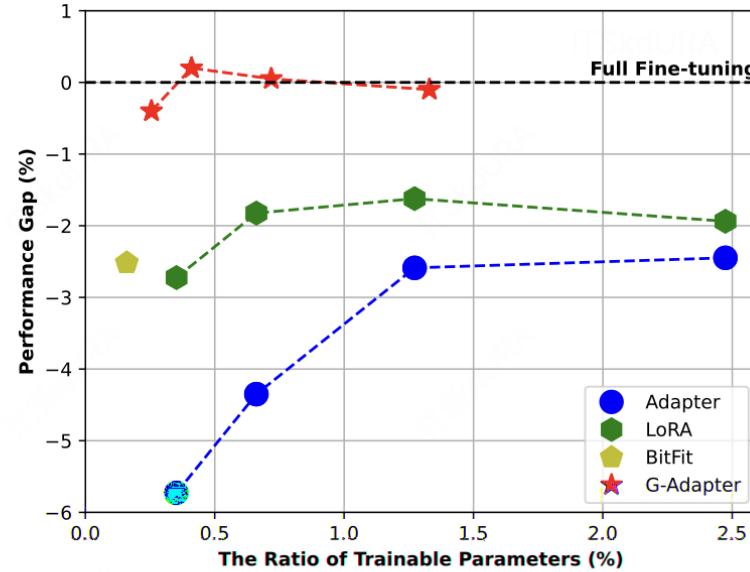
Can PEFTs from the language domain be transferred directly to graph-based tasks?

- There is a significant gap between traditional PEFTs and full fine-tuning, especially on large-scale datasets.

How to design a graph-specific PEFT method?



(a) On large-scale datasets.



(b) On small-scale datasets.

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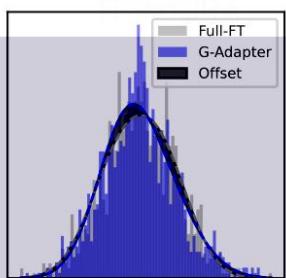
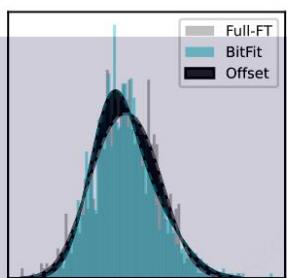
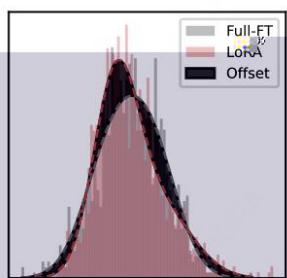
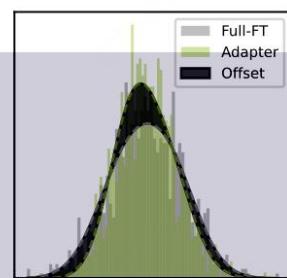
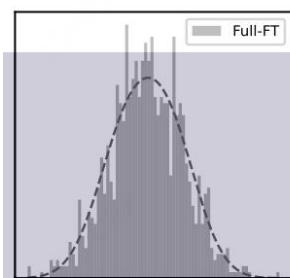
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Adapter: 11.12%

LoRA: 10.01%

BitFit: 8.22%

G-Adapter: 1.98%

- Delta tuning improves the traditional fine-tuning in the catastrophic forgetting of pre-trained knowledge problem and overfitting problem.

How to effectively utilize the advantages of delta tuning while preserving the expressivity of GNNs?

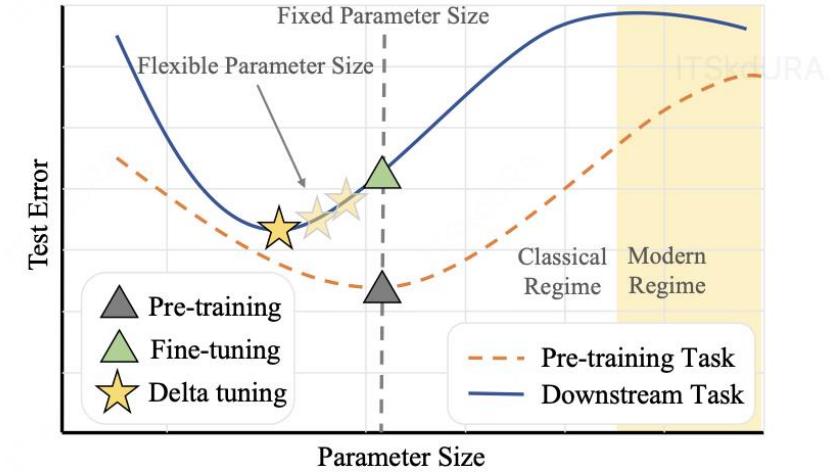
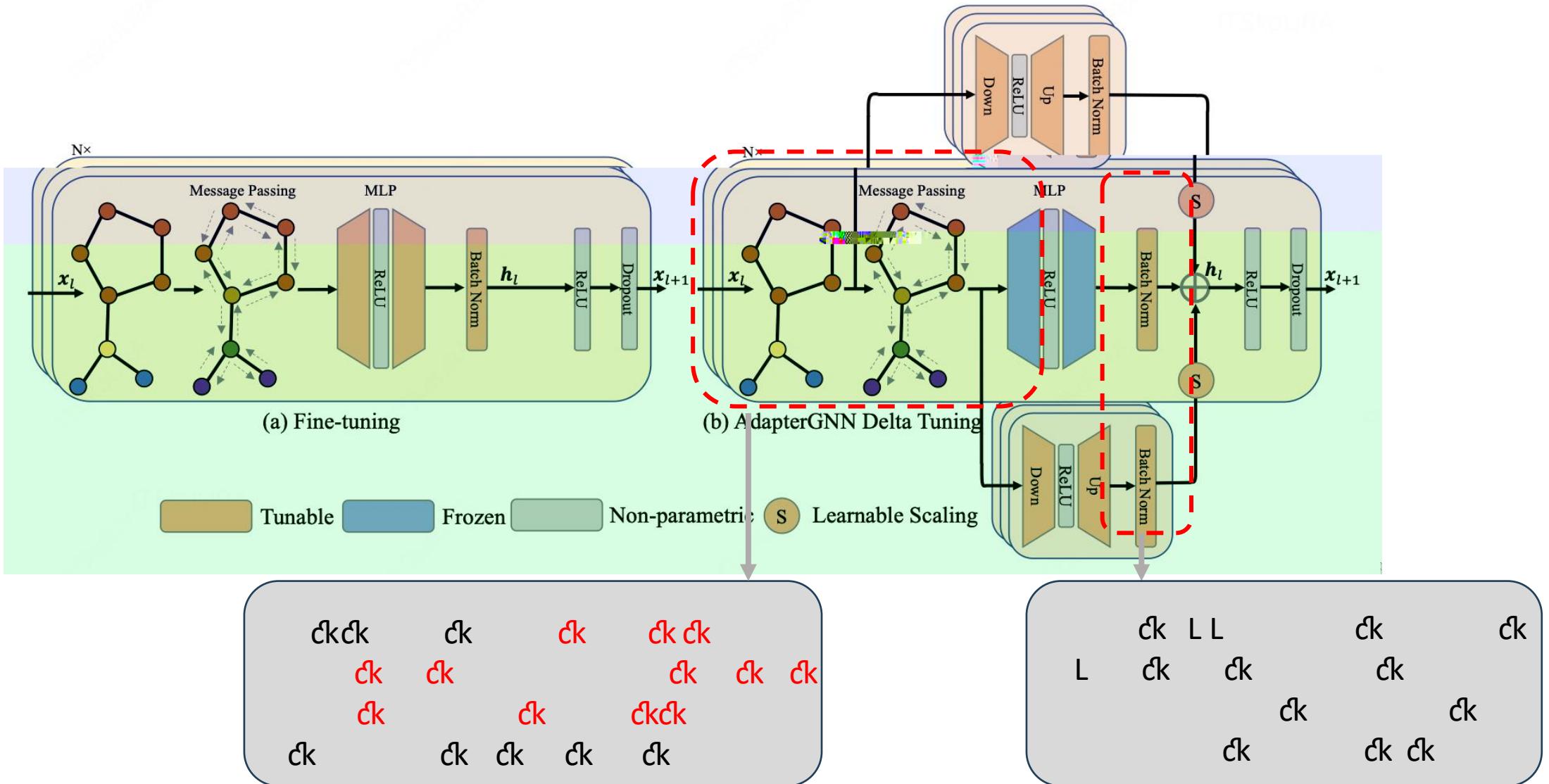
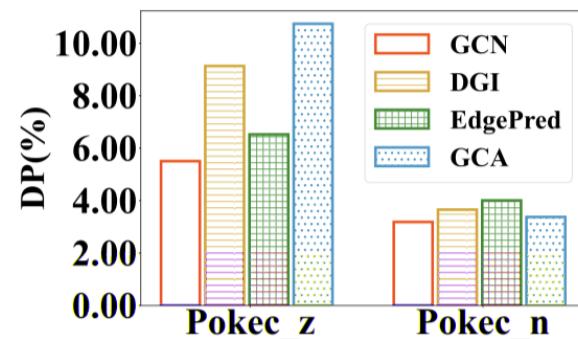


Figure 1: A large model is often employed for pre-training \blacktriangle when sufficient data is available. However, for downstream tasks with limited data, a smaller model is optimal in the classical regime. Compared with fine-tuning \blacktriangle , delta tuning \star preserves expressivity while reducing the size of parameter space, leading to lower test error.

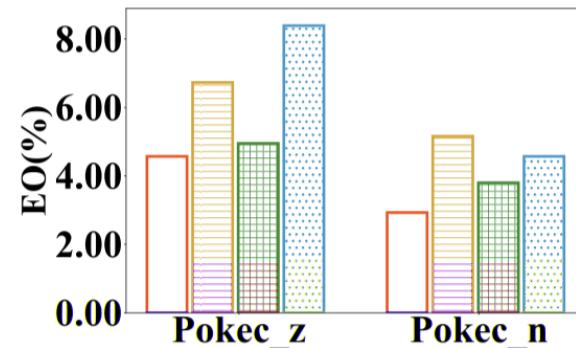


- Recent works have demonstrated that pre-trained language models tend to inherit bias from pre-training corpora.
- Pre-trained Graph Models(PGMs) can well capture semantic information on graphs during the pre-training phase, which inevitably contains sensitive attribute semantics.

How to improve the fairness of PGMs?



(a) Demographic Parity (DP).



(b) Equality Opportunity (EO).

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- Existing works generally train a fair GNN for a specific task.
- Debiasing for a specific task in the pre-training phase is inflexible, and maintaining a specific PGM for each task is inefficient.

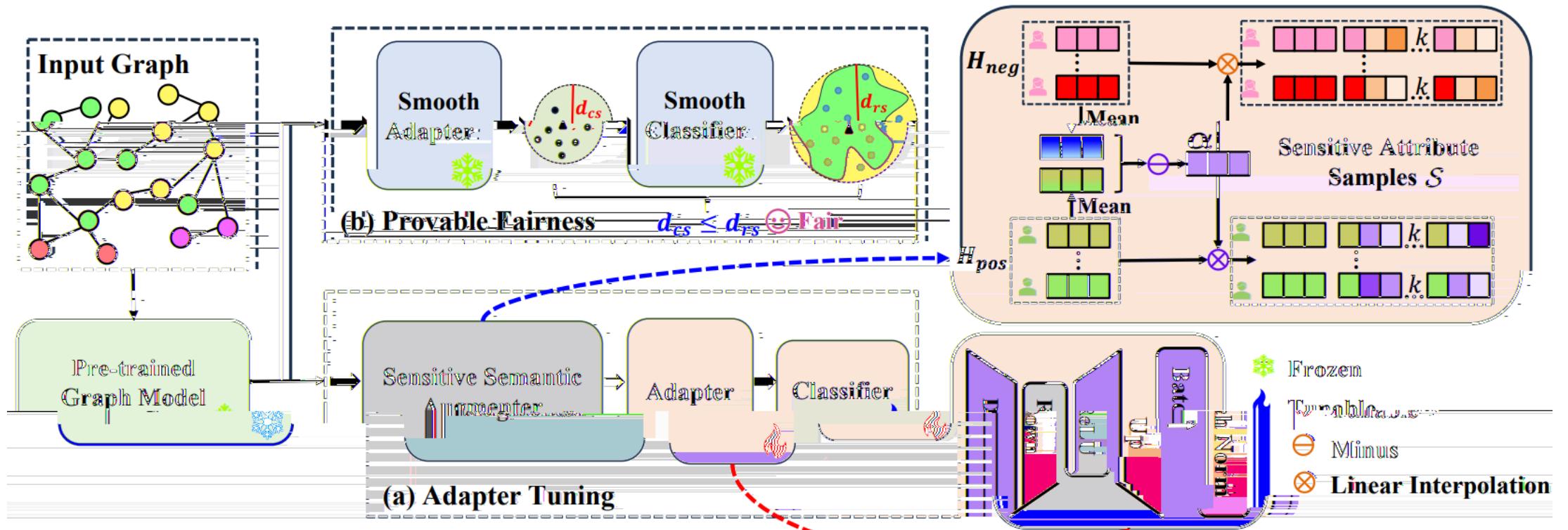
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- Provable lower bounds on the fairness of model prediction.

How to efficiently and flexibly endow PGMs fairness with practical guarantee?



$$\boldsymbol{\alpha} = \mathbf{h}_{pos} - \mathbf{h}_{neg},$$

$$\mathbf{h}_{pos} = \frac{1}{n_{pos}} \sum_{i=1}^{n_{pos}} \mathbf{H}_{pos,i}, \mathbf{h}_{neg} = \frac{1}{n_{neg}} \sum_{i=1}^{n_{neg}} \mathbf{H}_{neg,i}$$

$$\mathcal{S}_i := \{\mathbf{h}_i + t \cdot \boldsymbol{\alpha} \mid |t| \leq \epsilon\} \subseteq \mathbb{R}^p,$$

$$\mathcal{L}_{\text{RandAT}} = \mathbb{E}_{i \in \mathcal{V}_L} \left[\mathbb{E}_{\mathbf{h}'_i \in \hat{\mathcal{S}}_i} [\ell(d \circ g(\mathbf{h}'_i), y_i)] \right],$$

$$\mathcal{L}_{\text{MinMax}} (\mathbf{h}_i) \approx \max_{\mathbf{h}'_i \in \hat{\mathcal{S}}_i} \|g(\mathbf{h}_i) - g(\mathbf{h}'_i)\|_2.$$

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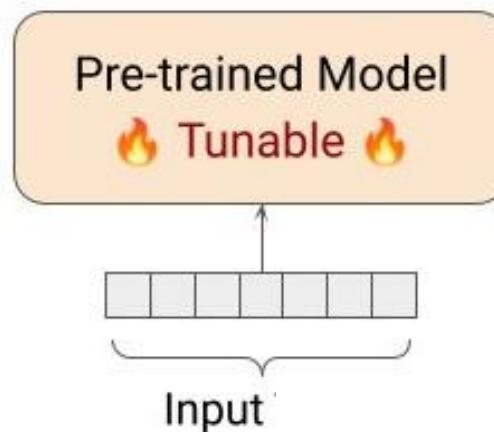
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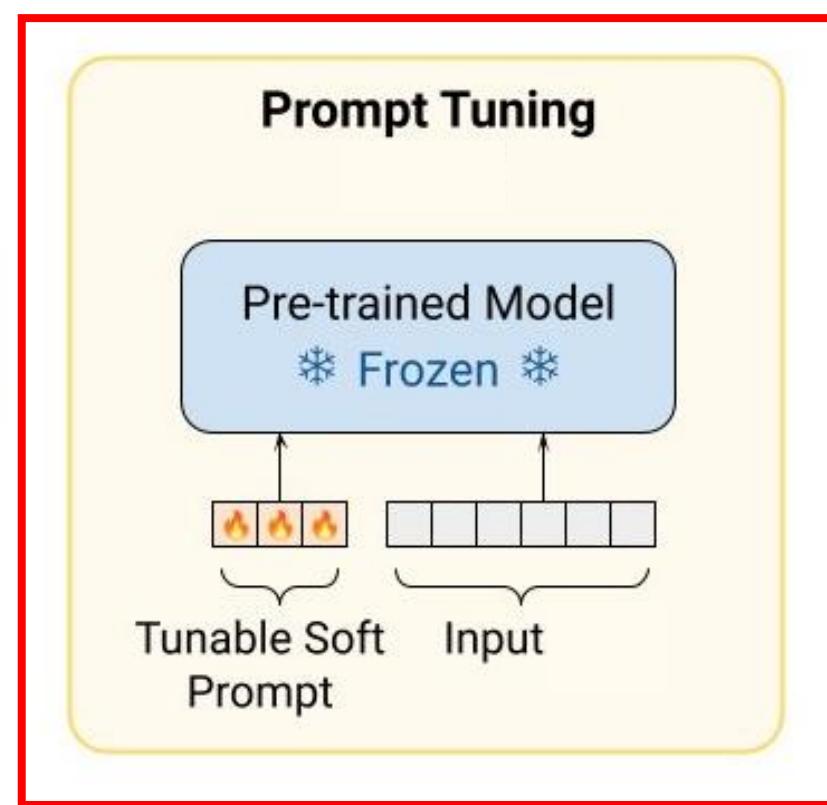
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Model Tuning (a.k.a. "Fine-Tuning")

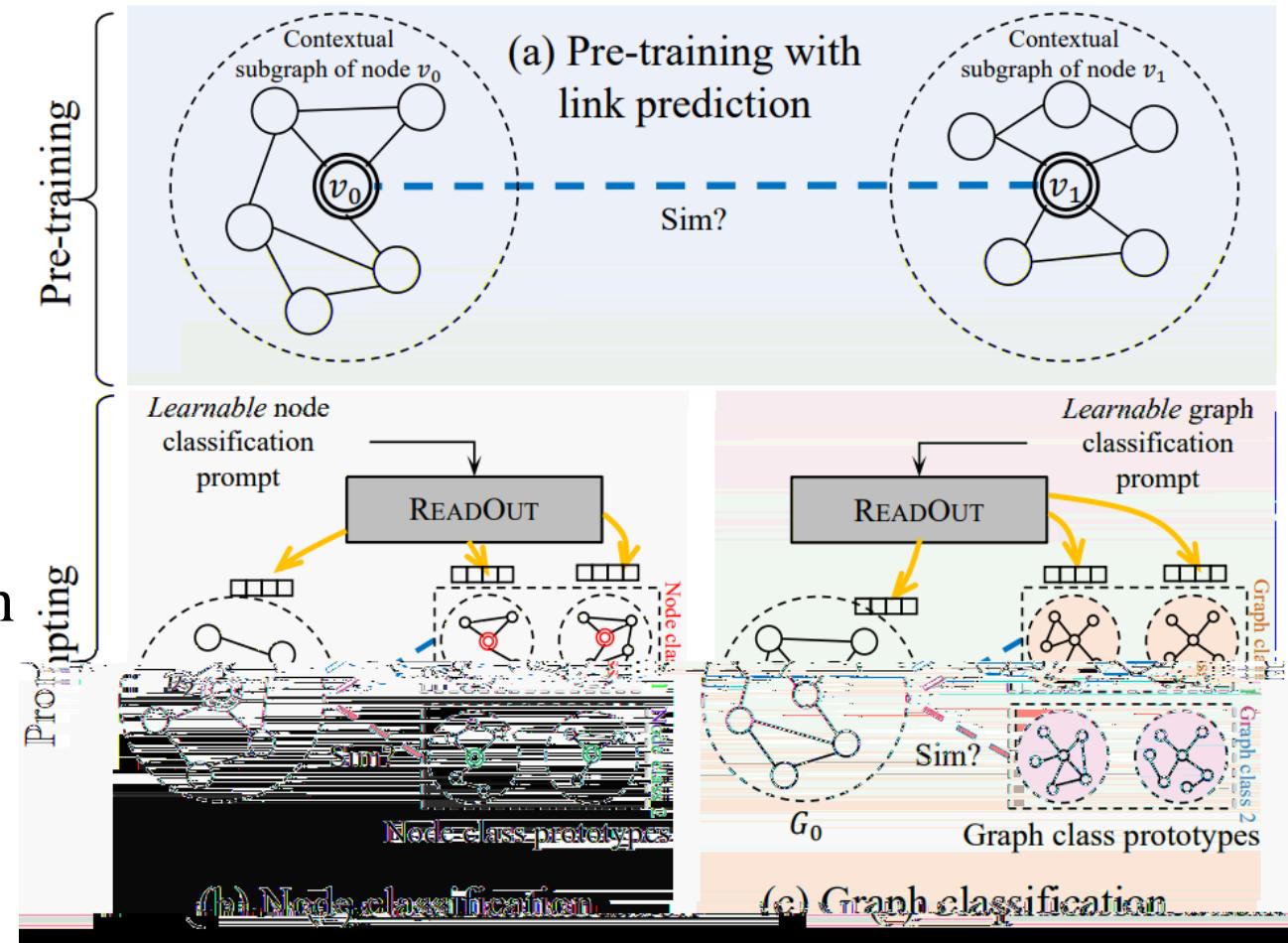


Prompt Tuning



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- How to unify various pre-training and downstream tasks on graph?
- How to design prompts on graph?
⋮
- A unified task template based on subgraph similarity computation
- Use a learnable prompt to guide graph readout for different tasks



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Different downstream tasks require
different subgraph readout
→ Use task-specific learnable prompts

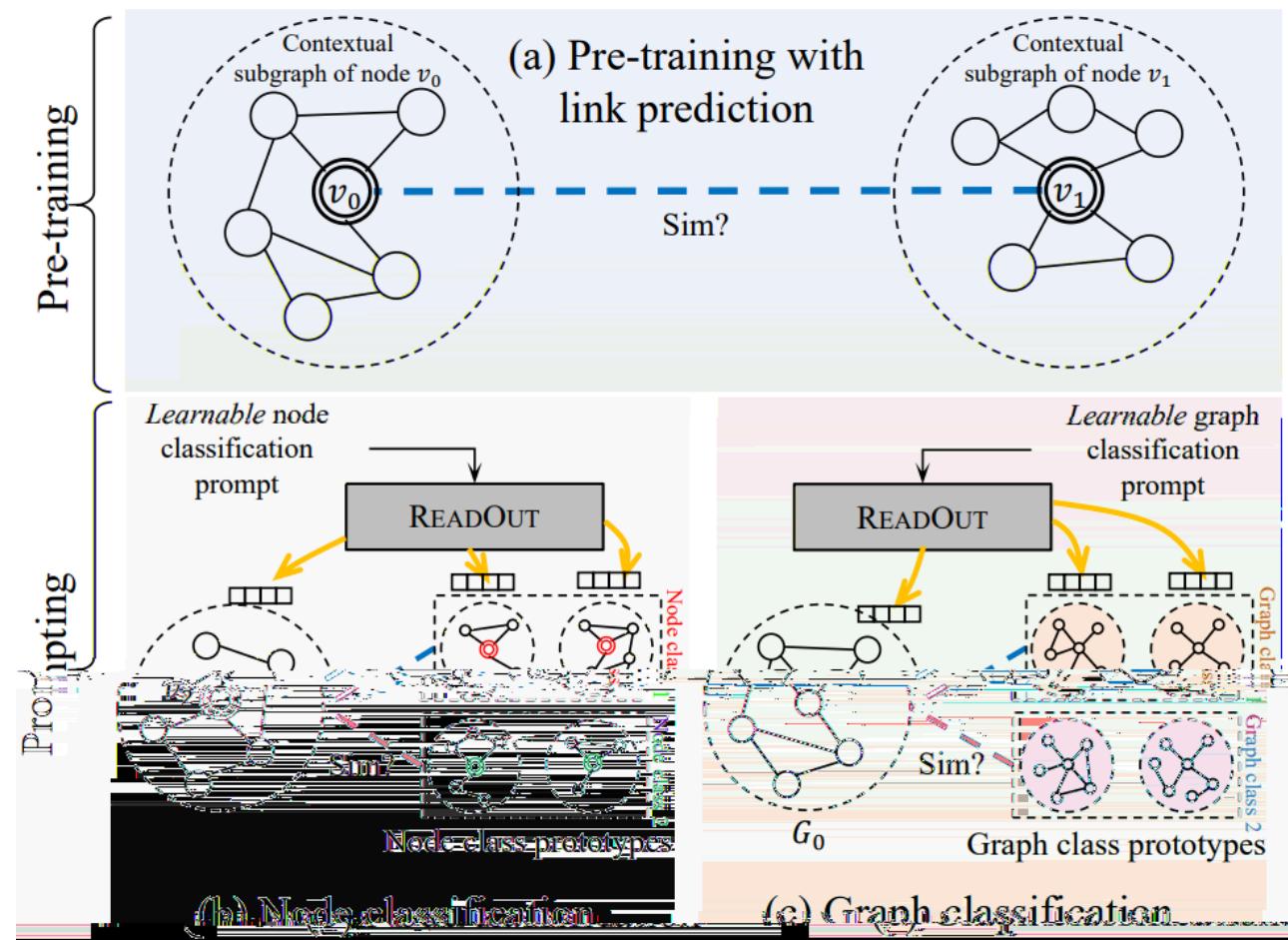
**Prompt vector added to the readout
layer of the pre-trained GNN**

$$\mathbf{s}_{t,x} = \text{READOUT}(\mathbf{p}_t \odot \mathbf{h}_v : v \in V(S_x))$$

$\mathbf{s}_{t,x}$: (sub)graph embedding of x for a task t

\mathbf{h}_v : node v 's embedding vector

\mathbf{p}_t or \mathbf{P}_t : learnable prompt vector or matrix for task t



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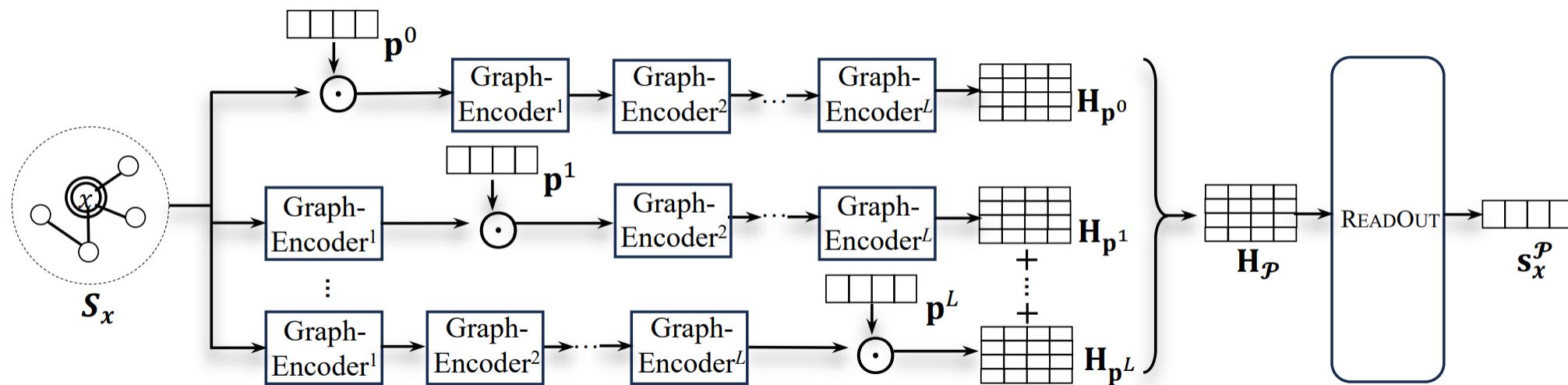
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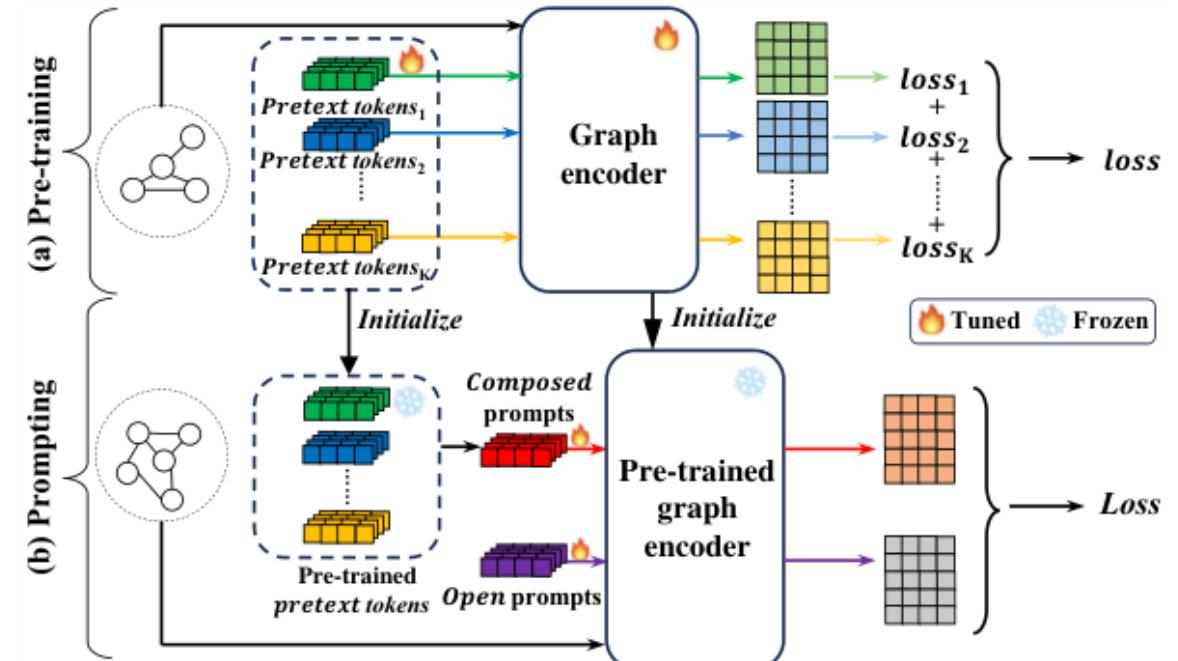


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- To cater to diverse downstream tasks, pre-training should broadly extract knowledge from various aspects.

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- Different pretext tasks often have different objectives, directly combining them lead to task interference.
- Multiple pretext tasks further complicates the alignment of downstream objectives with the pre-trained model.



- C1: How can we leverage diverse pre-text tasks for graph models in a synergistic manner?*
- C2: How can we transfer both task-specific and global pre-trained knowledge*

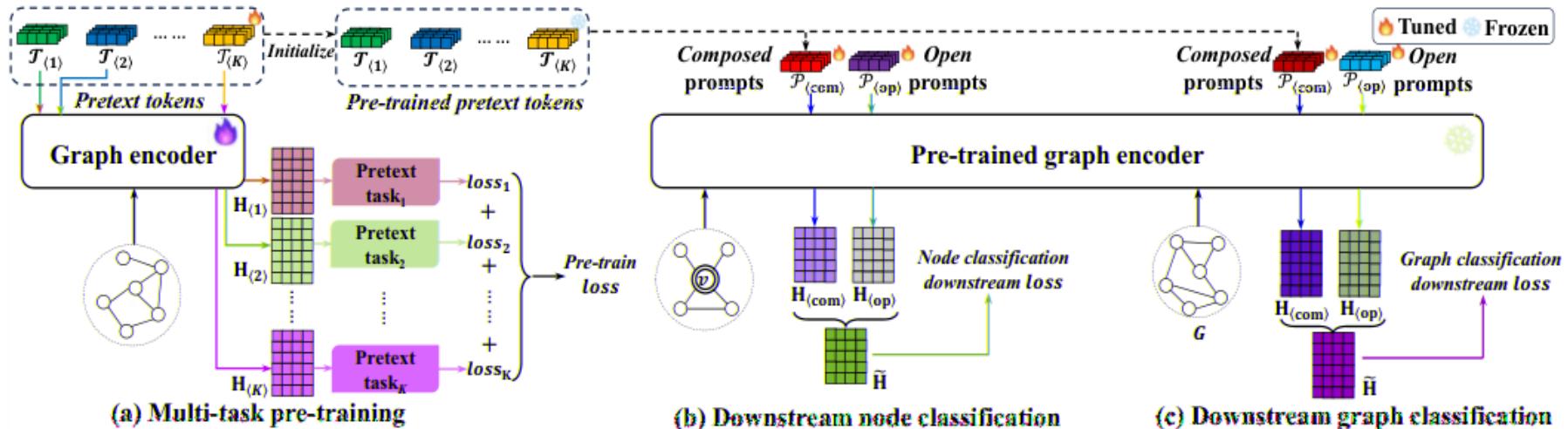
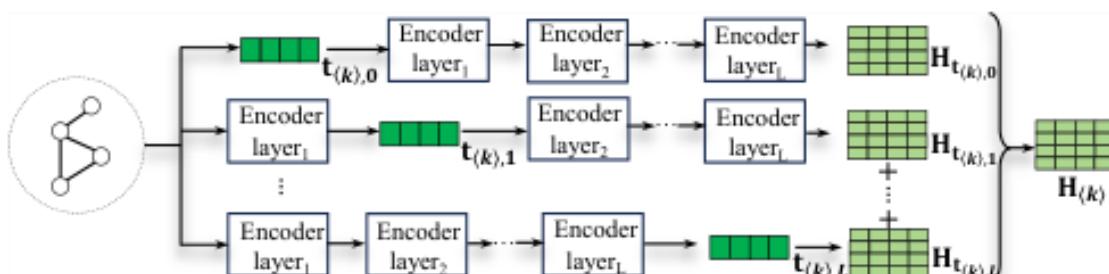
Multi-task pre-training

$$\mathcal{T}_{\langle k \rangle} = \{t_{\langle k \rangle,0}, t_{\langle k \rangle,1}, \dots, t_{\langle k \rangle,L}\}$$

$$\mathbf{H}^{l+1} = \text{MP}(\mathbf{t}_{\langle k \rangle,l} \odot \mathbf{H}^l, \mathbf{A}; \theta^l)$$

$$\mathbf{H}_t = \text{GRAPHENCODER}_t(\mathbf{X}, \mathbf{A}; \Theta)$$

$$\mathbf{H}_{\langle k \rangle} = \sum_{l=0}^L \alpha_l \mathbf{H}_{t_{\langle k \rangle,l}} \quad \mathcal{L}_{\text{pre}}(\mathcal{H}; \mathcal{T}, \Theta) = \sum_{k=1}^K \beta_k \mathcal{L}_{\text{pre}_{\langle k \rangle}}(\mathbf{H}_{\langle k \rangle}; \mathcal{T}_{\langle k \rangle}, \Theta),$$



Prompt tuning

$$\mathcal{P}_{\langle \text{com} \rangle} = \{p_{\langle \text{com} \rangle,0}, p_{\langle \text{com} \rangle,1}, \dots, p_{\langle \text{com} \rangle,L}\}$$

$$p_{\langle \text{com} \rangle, l} = \text{COMPOSE}(t_{\langle \text{com} \rangle,0}, t_{\langle \text{com} \rangle,1}, \dots, t_{\langle \text{com} \rangle,l}; \Gamma)$$

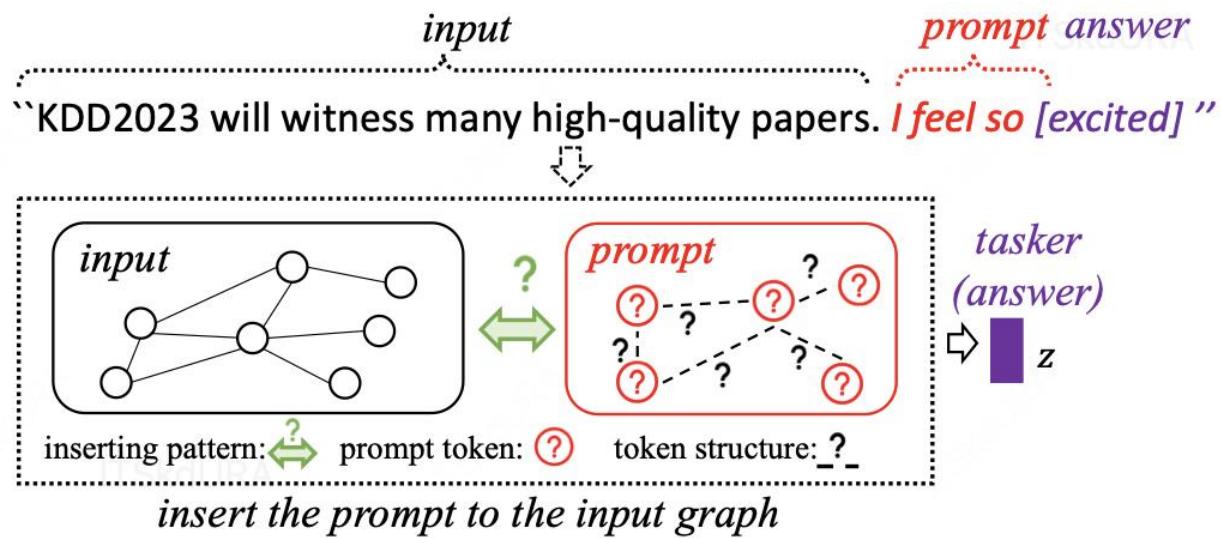
$$\mathcal{P}_{\langle \text{op} \rangle} = \{p_{\langle \text{op} \rangle,0}, p_{\langle \text{op} \rangle,1}, \dots, p_{\langle \text{op} \rangle,L}\}$$

$$\mathbf{H}_p = \text{GRAPHENCODER}_p(\mathbf{X}, \mathbf{A}; \Theta_{\text{pre}})$$

$$\tilde{\mathbf{H}} = \text{AGGR}(\mathbf{H}_{\langle \text{com} \rangle}, \mathbf{H}_{\langle \text{op} \rangle}; \Delta)$$

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- Graph prompt not only requires the prompt “content” but also needs to know how to organize these tokens and how to insert the prompt into the original graph.
- There is a huge difficulty in reconciling downstream problems to the pre-training task.
- Learning a reliable prompt needs huge manpower and is more sensitive in multi-task setting.

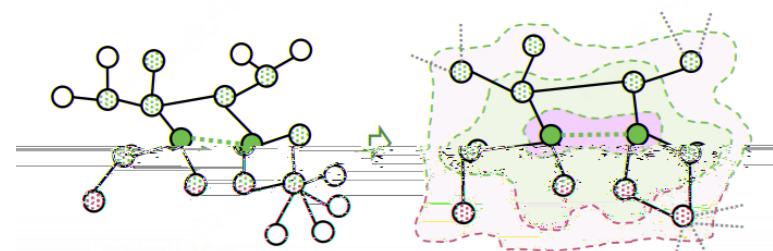
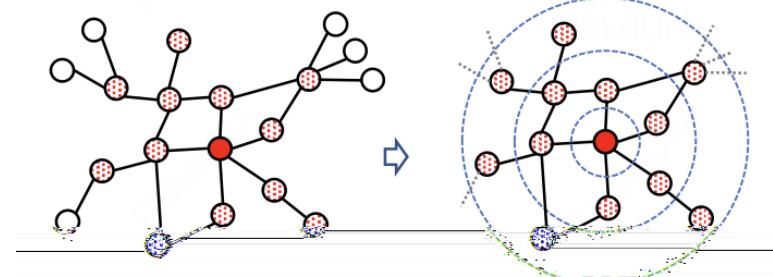


\mathcal{Ck} \mathcal{CkB} \mathcal{Ck} :

- This work reformulates node-level and edge-level tasks to graph-level tasks by building induced graphs for nodes and edges, respectively.

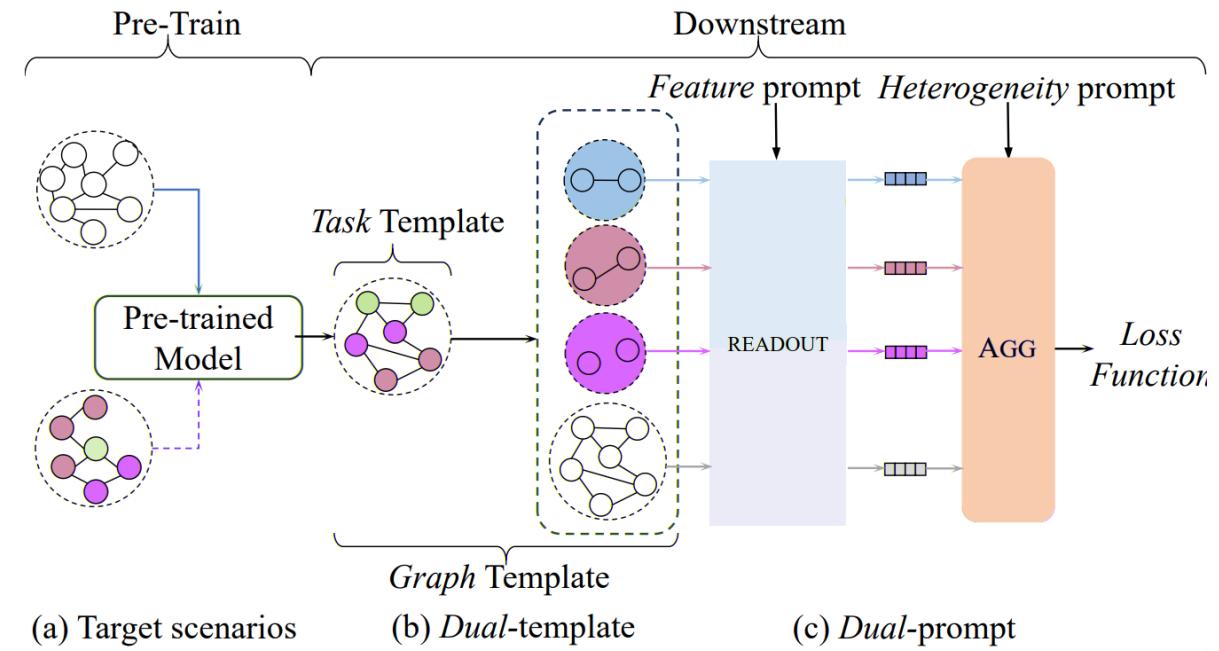
$B\mathcal{Ck}$:

- This work introduces some prompt nodes with unique connection relationships between them and adaptively insert them into the original input graph, in order to obtain a prompt graph.



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- Gap between homogeneous and heterogeneous graph.
- Different downstream tasks focus on heterogeneous aspect.
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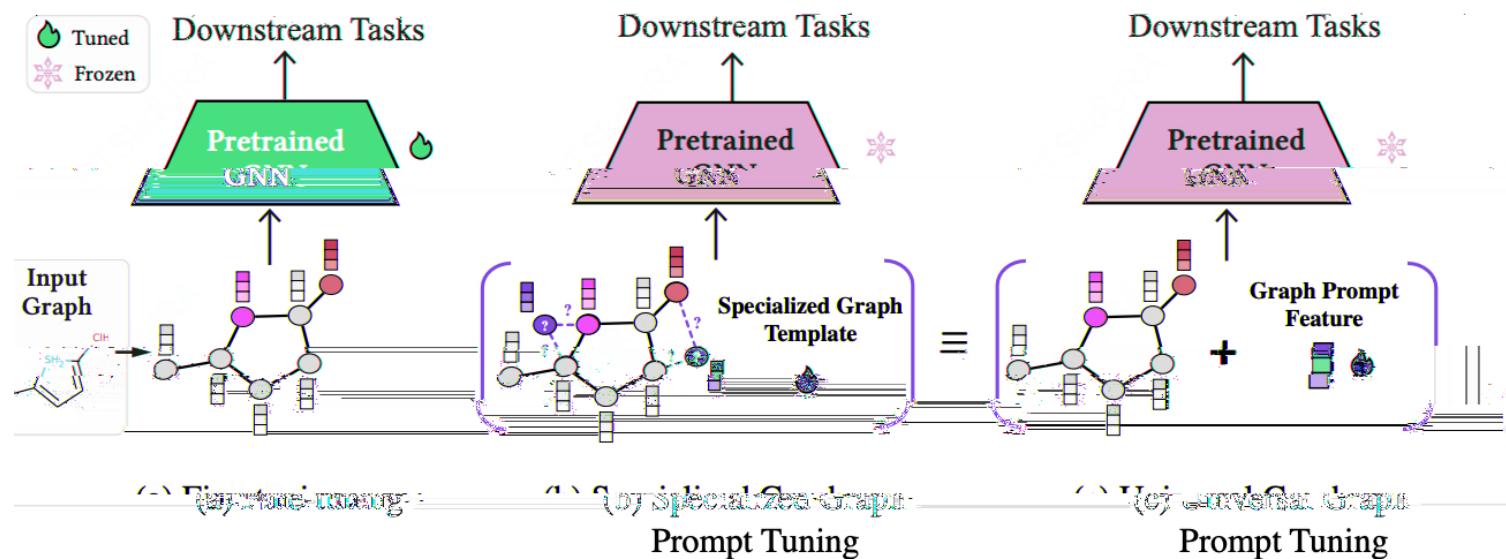


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- Diverse pre-training strategies employed on graphs make it difficult to design suitable prompting functions.
- Existing prompt-based tuning methods for GNN models are predominantly designed based on intuition, lacking theoretical guarantees for their effectiveness.

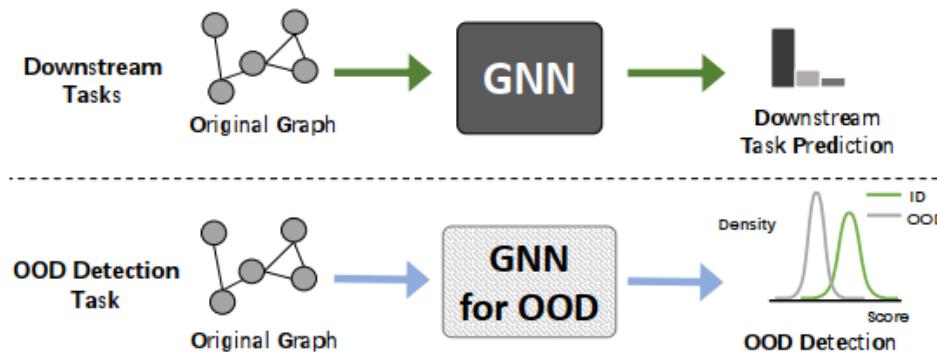
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- This work proposes a universal prompt-based tuning method that can be applied to the pre-trained GNN models that employ any pre-training strategy.
- **GPF operates on the input graph's feature space** and involves adding a shared learnable vector to all node features in the graph.
- GPF-plus is a theoretically stronger variant of GPF, for practical application, which incorporates different prompted features for different nodes in the graph.

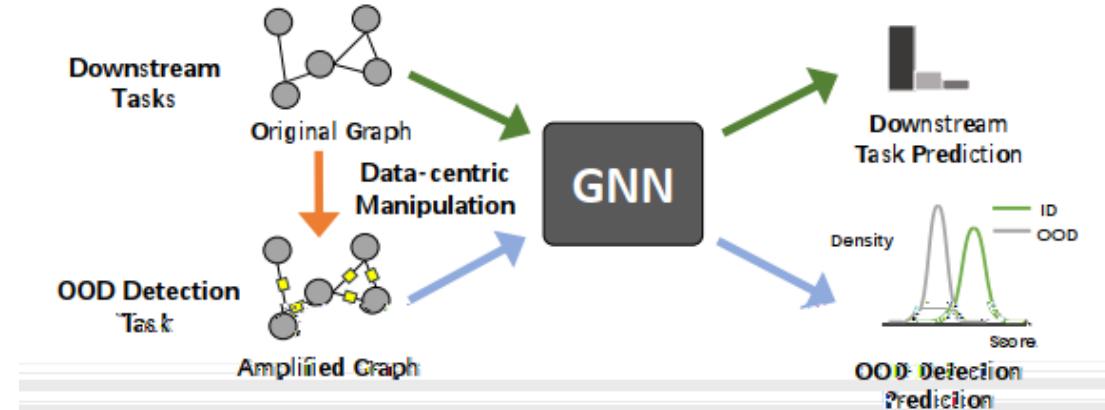


- A reliable GNN should not only perform well on know samples (ID) but also identify graphs it has not been exposed to before (OOD) .
- Existing works proposes to train a neural network specialized for the OOD detection task.

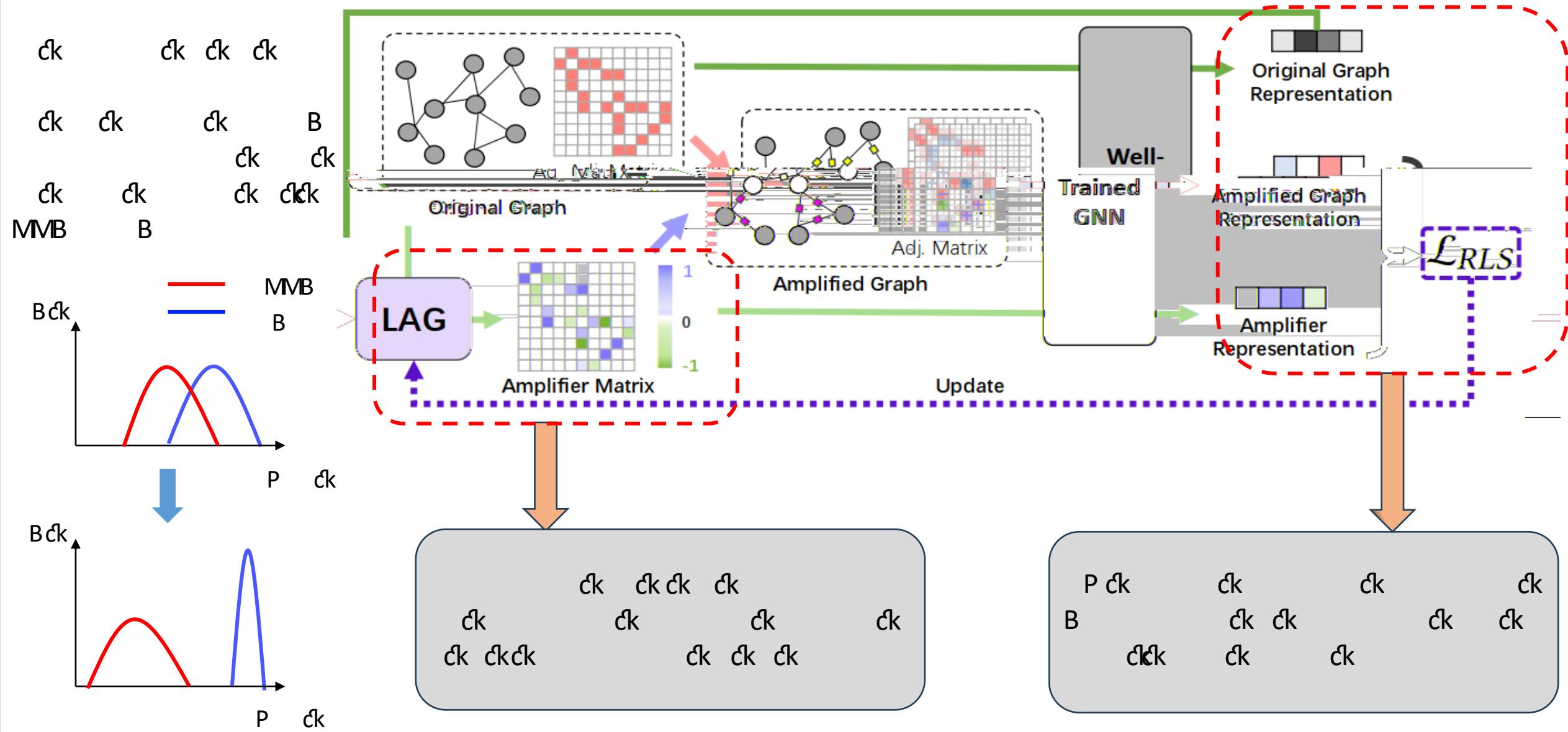
Can we build a graph prompt that can solve OOD detection given a well-trained GNN?



(1) Traditional works



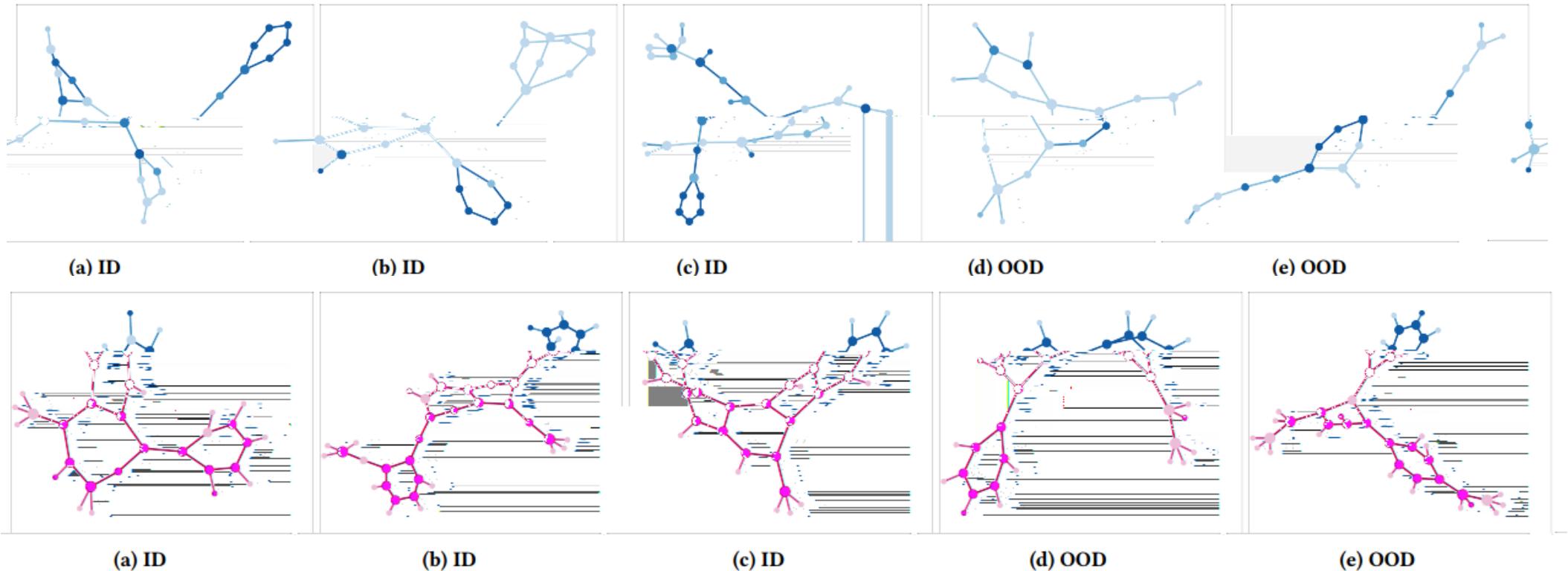
(2) Our proposed framework



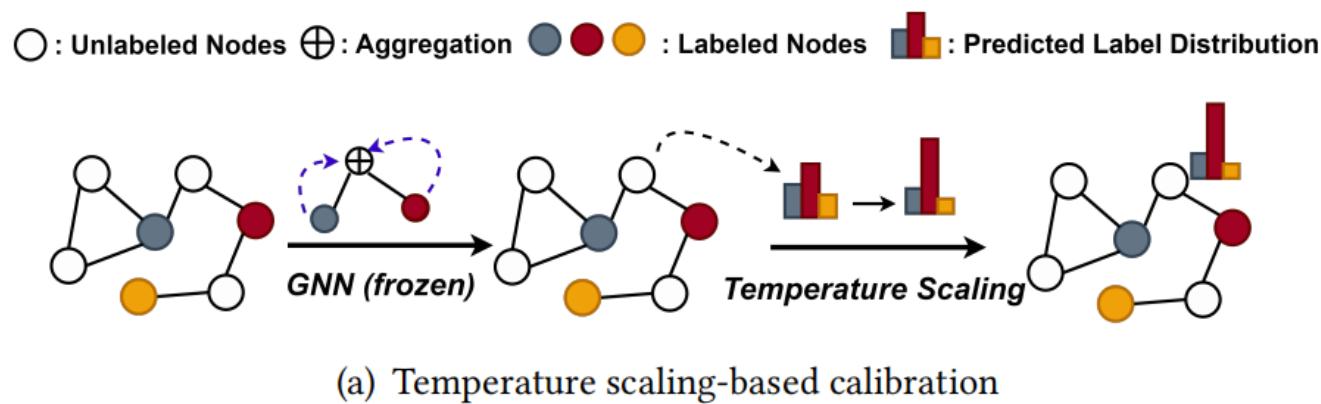
We conducted experiments on five dataset pairs over four GNNs to verify performance.

ID	OOD	Metric	GCL _S	GCL _{S+}	Improv.	GCL _L	GCL _{L+}	Improv.	JVAUS	JVAUS+	improv.	JVAUL	JVAUL+	improv.
ENZYMES	PROTEIN	AUC ↑	62.97	73.76	+17.14%	62.56	67.15	+7.34%	61.20	74.19	+21.23%	59.68	65.11	+9.10%
		AUPR ↑	62.47	75.27	+20.49%	65.45	65.18	-0.41%	61.30	77.10	+25.77%	64.16	64.49	+0.51%
		FPR95 ↓	93.33	88.33	-5.36%	93.30	85.00	-8.90%	90.00	81.67	-9.26%	96.67	85.00	-12.07%
IMDBM	IMDBB	AUC ↑	80.52	83.84	+4.12%	61.08	68.64	+12.38%	80.40	82.80	+2.99%	48.25	64.32	+33.31%
		AUPR ↑	74.43	80.16	+7.70%	59.52	68.03	+14.30%	74.70	77.77	+4.11%	47.88	61.62	+28.70%
		FPR95 ↓	38.67	38.33	-0.88%	96.67	91.33	-5.52%	44.70	42.00	-6.04%	98.00	94.00	-4.08%
BZR	COX2	AUC ↑	75.00	97.31	+29.75%	34.69	65.00	+87.37%	80.00	95.25	+19.06%	41.80	65.62	+56.99%
		AUPR ↑	62.41	97.17	+55.70%	39.07	62.89	+60.97%	67.10	94.34	+40.60%	56.70	67.22	+18.55%
		FPR95 ↓	47.50	15.00	-%	47.50	15.00	-%	47.50	15.00	-%	47.50	15.00	-%

Case study: We visualize the learned graph prompts (i.e., amplifiers) for interpretability analysis.



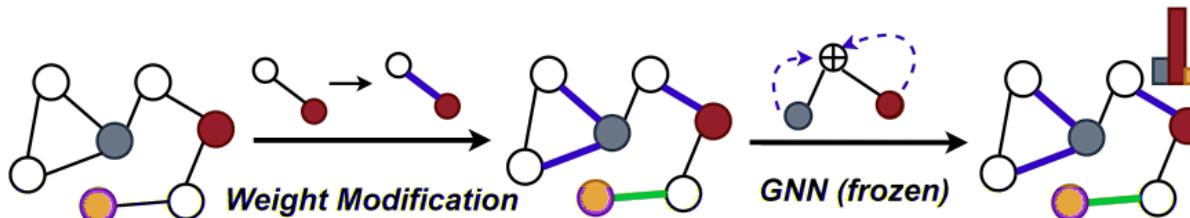
- Existing calibration methods focus on improving GNN models. Recent work has shown that the post-hoc methods, such as temperature scaling-based calibration, can achieve a better trade-off between accuracy and calibration.



- Through evaluating the expected calibration error (ECE) on Cora and Photo datasets with five different GNNs, we find that the ECEs on Cora (10.25%-18.02%) are always larger than those on Photo (4.38%-8.27%), indicating that **the calibration performance depends more on the datasets instead of GNN model**.

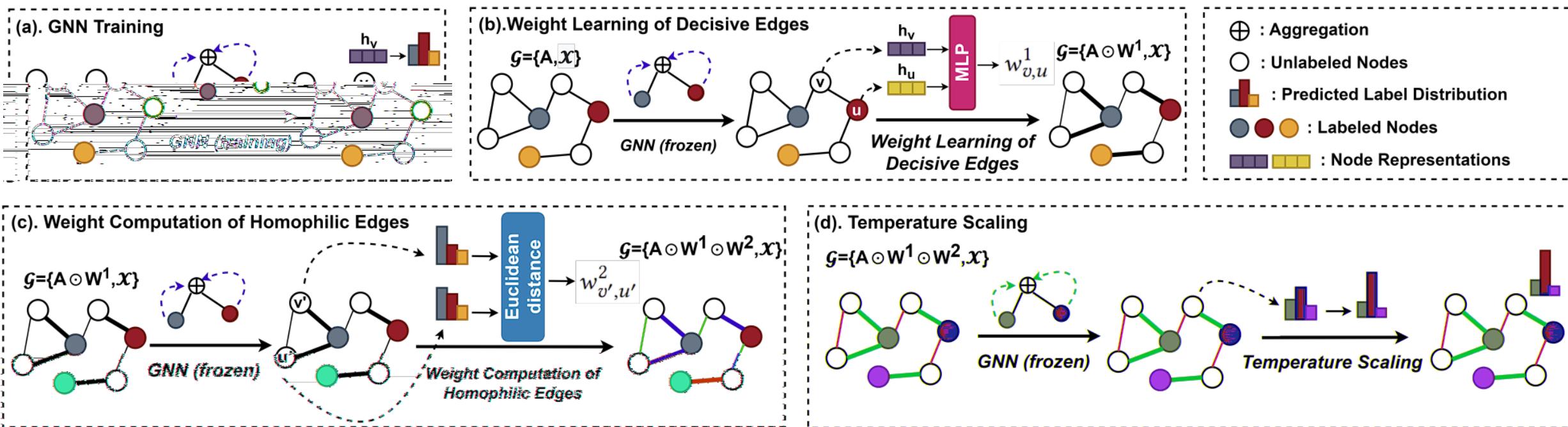
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Can we modify the graph data instead for better calibration performance without losing accuracy?



(b) Data-centric calibration

- We propose Data-centric Graph Calibration (DCGC) with two edge weighting modules to adjust the input graph.



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