General Physics Final Exam Reference Answer

1(a) $V = v_1 \times t \times A_1 = 10 \times 50 \times (\frac{2}{2})^2\pi = 500\pi\,(\mathrm{m}^3)$

1(b) $v_2 = v_1 \times \frac{A_1}{A_2} = v_1 \times (\frac{d_1}{d_2})^2 = 10 \times \frac{4}{5} = 8\,(\mathrm{m/s})$

1(c) $\Delta E = P_2(A_2v_2\Delta t) - P_1(A_1v_1\Delta t) = \Delta K + \Delta U = \frac{1}{2}\rho(A_1v_1\Delta t)v_1^2 - \frac{1}{2}\rho(A_2v_2\Delta t)v_2^2 + 0$

$P_2 - P_1 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2,\ P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 = const$

1(d) $P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 10^5 + \frac{1}{2} \times 10^3 \times (10^2 - 8^2) = 1.18 \times 10^5\,(\mathrm{Pa})$

2(a) $l = \sqrt{x^2 + d^2} - x,\ \lim\limits_{x\to\infty} \sqrt{x^2 + d^2} - x = 0$, there is no phase difference

2(b) $\frac{dl}{dx} = \frac{x}{\sqrt{x^2+d^2}} - 1 < 0$, the phase difference increase

2(c) maximum finite value for fully constructive interference $l = 1\lambda = 5\,(\mathrm{m})$

$x = \frac{d^2-l^2}{2l} = \frac{100-25}{10} = 7.5\,(\mathrm{m})$

2(d) maximum finite value for fully destructive interference $l = \frac{1}{2}\lambda = 2.5\,(\mathrm{m})$

$x = \frac{d^2-l^2}{2l} = \frac{100-6.25}{5} = 18.75\,(\mathrm{m})$

3(a) $\Delta E_{int} = \frac{3}{2}Nk_B\Delta T = \frac{3}{2}nR\Delta T = Q - W$

for constant V process, $Q = nC_v\Delta T$, $W = 0$, $C_v = \frac{3}{2}R$

for constant P process, $Q = nC_p\Delta T$, $W = P\Delta V = nR\Delta T$, $C_p = \frac{5}{2}R$

3(b) for adiabatic process, $Q = 0$, $dE_{int} = -dW = -PdV$

$nC_vdT = C_v\frac{PdV+VdP}{R} = C_v\frac{PdV+VdP}{C_p-C_v} = -PdV,\ C_vVdP + C_pPdV = 0$

$\int\frac{dP}{P} = -\frac{C_p}{C_v}\int\frac{dV}{V},\ \ln P = -\frac{C_p}{C_v}\ln V + \ln C,\ PV^{\frac{C_p}{C_v}} = const$

3(c) $P_1V_1 = P_2V_2,\ \frac{P_2}{P_1} = \frac{V_1}{V_2} = \frac{1}{2}$

$P_1V_1^{\frac{5}{3}} = P_3V_3^{\frac{5}{3}},\ \frac{P_3}{P_1} = (\frac{V_1}{V_3})^{\frac{5}{3}} = (\frac{1}{2})^{\frac{5}{3}}$

$T_1V_1^{\frac{2}{3}} = T_3V_3^{\frac{2}{3}},\ \frac{T_3}{T_1} = (\frac{V_1}{V_3})^{\frac{2}{3}} = (\frac{1}{2})^{\frac{2}{3}}$

3(d) $W = \int P\,dV = \int_{V_1}^{V_2} \frac{nRT_1}{V}\,dV = nRT_1 \ln\frac{V_2}{V_1},\ \frac{W}{nRT_1} = \ln\frac{V_2}{V_1} = \ln 2$

$Q = W,\ \frac{Q}{nRT_1} = \ln 2$

$\Delta E_{int} = 0,\ \frac{\Delta E_{int}}{nRT_1} = 0$

$\Delta S = \int \frac{dQ}{T} = nR\ln\frac{V_2}{V_1},\ \frac{\Delta S}{nR} = \ln 2$

3(e) $W = 0,\ \frac{W}{nRT_1} = 0$

$Q = nC_v(T_3 - T_2) = \frac{3}{2}nRT_1((\frac{V_1}{V_3})^{\frac{2}{3}} - 1),\ \frac{Q}{nRT_1} = \frac{3}{2}((\frac{1}{2})^{\frac{2}{3}} - 1)$

$\Delta E_{int} = Q,\ \frac{\Delta E_{int}}{nRT_1} = \frac{3}{2}((\frac{1}{2})^{\frac{2}{3}} - 1)$

$\Delta S = \int \frac{dQ}{T} = \int_{T_2}^{T_3} \frac{nC_v dT}{T} = \frac{3}{2}nR\ln\frac{T_3}{T_1},\ \frac{\Delta S}{nR} = -ln2$

3(f) $W = -\Delta E_{int} = \frac{3}{2}nRT_1((\frac{V_1}{V_3})^{\frac{2}{3}} - 1),\ \frac{W}{nRT_1} = \frac{3}{2}((\frac{1}{2})^{\frac{2}{3}} - 1)$

$Q = 0,\ \frac{Q}{nRT_1} = 0$

$\Delta E_{int} = -\frac{3}{2}nRT_1((\frac{V_1}{V_3})^{\frac{2}{3}} - 1),\ \frac{\Delta E_{int}}{nRT_1} = -\frac{3}{2}((\frac{1}{2})^{\frac{2}{3}} - 1)$

$\Delta S = \int \frac{dQ}{T} = 0,\ \frac{\Delta S}{nR} = 0$

4(a) $M = \frac{4}{3}\pi\rho y^2,\ F = -\frac{GMm}{y^2} = -\frac{4}{3}\pi G\rho m y = ma$ is simple harmonic motion

4(b) $a = -\frac{4}{3}\pi G\rho y = -\omega^2 y,\ \omega = \sqrt{\frac{4}{3}\pi G\rho},\ T = \frac{2\pi}{\omega} = \sqrt{\frac{3\pi}{G\rho}}$

4(c) $(\frac{y}{y_{max}})^2 + (\frac{v}{\omega y_{max}})^2 = 1 = (\frac{y_0}{y_{max}})^2 + (\frac{y_0}{y_{max}})^2,\ y_{max} = \sqrt{2}y_0$

$y(t) = y_{max}\cos(\omega t + \phi),\ y(0) = y_{max}\cos\phi = y_0,\ \phi = \frac{\pi}{4}$

$y(t) = \sqrt{2}y_0\cos(\sqrt{\frac{4}{3}\pi G\rho}t + \frac{\pi}{4})$

4(d) $t = \frac{T}{8} = \sqrt{\frac{3\pi}{64G\rho}}$

4(e) $v_{max} = \omega y_{max} = \sqrt{\frac{4}{3}\pi G\rho} \times \sqrt{2}y_0 = \sqrt{\frac{8}{3}\pi G\rho}y_0$

5(a) $Mg\sin\theta - f_s = Ma_{com},\ f_sR = I_{com}\alpha = \frac{2}{3}MR^2 \times \frac{a_{com}}{R} = \frac{2}{3}MRa_{com}$

$Mg\sin\theta = f_s + Ma_{com} = \frac{5}{3}Ma_{com},\ a_{com} = \frac{3}{5}g\sin\theta$

5(b) suppose the COM axis is at $(0,0)$ and the parallel axis ia at (a,b)

$I = \int (x-a)^2 + (y-b)^2 \, dm = \int (x^2+y^2) \, dm + \int (a^2+b^2) \, dm - 2a \int x \, dm - 2b \int y \, dm$

$I = I_{com} + \int h^2 \, dm + 0 + 0 = I_{com} + Mh^2$

5(c) without sliding, the contact point is momentarily stationary

the motion of the entire sphere is equivalent to a rotation around the contact point

5(d) $I = I_{com} + MR^2 = \frac{5}{3}MR^2$

$\tau = Mg \sin\theta R = I\alpha$, $\alpha = \frac{3g\sin\theta}{5R} = \frac{a_{com}}{R}$, $a_{com} = \frac{3}{5} g \sin\theta$