General Physics Midterm Exam Reference Answer

1(a) $J = \int_0^{\Delta t} F(t)\,dt = \int_0^{10^{-3}} [(3\times10^6)t - (3\times10^9)t^2]\,dt = 0.5(\text{Ns})$

1(b) $F_{avg} = \frac{J}{\Delta t} = \frac{0.5}{10^{-3}} = 500(\text{N})$

1(c) $\frac{dF(t)}{dt} = (3\times10^6) - (6\times10^9)t = 0,\ t = 5\times10^{-4}(\text{s})$

$F_{max} = F(5\times10^{-4}) = (3\times10^6)(5\times10^{-4}) - (3\times10^9)(5\times10^{-4})^2 = 750(\text{N})$

1(d) $v = \frac{J}{m} = \frac{0.5}{0.25} = 2(\text{m/s})$

1(e) $J(t) = \int_0^t F(t')\,dt' = (\frac{3}{2}\times10^6)t^2 - 10^9t^3,\ v(t) = \frac{J(t)}{m} = (6\times10^6)t^2 - (4\times10^9)t^3$

$x = \int_0^{\Delta t} v(t)\,dt = \int_0^{10^{-3}} [(6\times10^6)t^2 - (4\times10^9)t^3]\,dt = 0.001(\text{m})$

2(a) $v' = \sqrt{v_0^2 - 2gh} = \sqrt{5^2 - 2\times10\times0.45} = 4(\text{m/s})$

2(b) $\frac{1}{2}mv_B^2 = \frac{1}{2}mv'^2 - (mg\sin\theta + \mu_k mg\cos\theta)L$

$v_B = \sqrt{v'^2 - 2gL(\sin\theta + \mu_k\cos\theta)} = \sqrt{4^2 - 2\times10\times0.3(\frac{1}{2} + \sqrt{3}\times\frac{\sqrt{3}}{2})} = 2(\text{m/s})$

2(c) $\Delta h = \frac{v_B^2}{2g} = \frac{4}{2\times10} = 0.2(\text{m}),\ h_{max} = h + L\sin\theta + \Delta h = 0.45 + 0.3\times\frac{1}{2} + 0.2 = 0.8(\text{m})$

2(d) The increase in block mass does not change the result of calculation $2(a)$ and $2(b)$, so the greatest height is still 0.8(m)

2(e) $v'^2 = 2g(\sin\theta + \mu_k'\cos\theta)d = 100d = 16,\ d = 0.16(\text{m})$

$h_{max} = h + d\sin\theta = 0.45 + 0.16\times\frac{1}{2} = 0.53(\text{m})$

3(a) $U = -\int_\infty^{|\vec{r}_1-\vec{r}_2|}(-\frac{Gm_1m_2}{r^2})\,dr = -\frac{Gm_1m_2}{|\vec{r}_1-\vec{r}_2|}$

3(b) $\vec{R} = \frac{m_1}{m_1+m_2}\vec{r}_1 + \frac{m_2}{m_1+m_2}\vec{r}_2,\ \vec{r} = \vec{r}_1 - \vec{r}_2$

$\vec{r}_1 = \vec{R} + \frac{m_2}{m_1+m_2}\vec{r},\ \vec{r}_2 = \vec{R} - \frac{m_1}{m_1+m_2}\vec{r}$

3(c) $\vec{V} = \frac{d\vec{R}}{dt} = \frac{m_1}{m_1+m_2}\vec{v}_1 + \frac{m_2}{m_1+m_2}\vec{v}_2,\ \vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_1 - \vec{v}_2$

$\vec{v}_1 = \vec{V} + \frac{m_2}{m_1+m_2}\vec{v},\ \vec{v}_2 = \vec{V} - \frac{m_1}{m_1+m_2}\vec{v}$

$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1(V^2 + \frac{m_2^2}{(m_1+m_2)^2}v^2) + \frac{1}{2}m_2(V^2 + \frac{m_1^2}{(m_1+m_2)^2}v^2)$

$K = \frac{1}{2}(m_1+m_2)V^2 + \frac{1}{2}\frac{m_1 m_2^2 + m_2 m_1^2}{(m_1+m_2)^2}v^2 = \frac{1}{2}MV^2 + \frac{1}{2}\mu v^2$

3(d) $E = K + U = \frac{1}{2}\mu v_{esc}^2 - \frac{Gm_1 m_2}{r} = 0,\ v_{esc} = \sqrt{\frac{2G(m_1+m_2)}{r}}$

3(e) $\frac{Gm_1 m_2}{r^2} = \frac{m_1 v_1^2}{r_1} = \frac{m_1}{r_1}(\frac{2\pi r_1}{T})^2 = \frac{4\pi^2 m_1 r_1}{T^2} = \frac{4\pi^2}{T^2}\frac{m_1 m_2}{m_1+m_2}r,\ T = 2\pi\sqrt{\frac{r^3}{G(m_1+m_2)}}$

4(a) $U = mgh = mgL(1-\cos\theta)$

4(b) $U = mgL(1-(1-\frac{\theta^2}{2})) = \frac{1}{2}mgL\theta^2 = \frac{1}{2}mg\frac{x^2}{L} = \frac{1}{2}kx^2 \propto x^2,\ k = \frac{mg}{L}$

4(c) $F_x = -mg\sin\theta = -mg\theta = -mg\frac{x}{L} = -kx$

4(d) $m\frac{d^2x}{dt^2} = -kx,\ \frac{d^2x}{dt^2} = -\frac{k}{m}x,\ x(t) = A\sin(\sqrt{\frac{k}{m}}t + B)$

$x(0) = A\sin B = 0,\ x'(0) = A\sqrt{\frac{k}{m}} = v_0,\ x(t) = \sqrt{\frac{mv_0^2}{k}}\sin(\sqrt{\frac{k}{m}}t)$