General Physics Midterm Exam

2024.10.24

1. (25%) A soccer player kicks a soccer ball of mass $0.25\,kg$ that is initially at rest. The foot of the player is in contact with the ball for $10^{-3}s$, and the force of the kick is given by $F(t) = (3 \times 10^{6}t - 3 \times 10^{9}t^{2})N$ for $0 \le t \le 10^{-3}s$, where t is in seconds. Find the magnitudes of the following quantities.

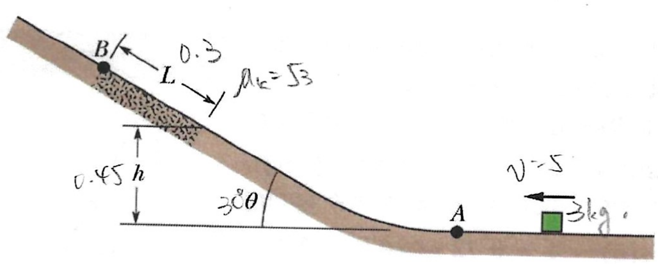
a. (5%) The impulse on the ball due to the kick.

b. (5%) The average force on the ball from the player's foot during the period of contact.

c. (5%) The maximum force on the ball from the player's foot during the period of contact.

d. (5%) The ball's velocity at $t = 10^{-3}s$.

e. (5%) The ball's displacement during $0 \le t \le 10^{-3}s$.



2. (25%). In the above figure, a block of mass $3\,kg$ slides along a path that is without friction until the block reaches the rough region of length $L = 0.3\,m$, which begins at height $h = 0.45\,m$ on a ramp of angle $\theta = 30^{\circ}$. In that rough region, the coefficient of kinetic friction is $\sqrt{3}$. The block passes through point A with a speed of $5\,m/s$. Assume the gravitational acceleration $g = 10\,m/s^{2}$.

a. (5%) What is the velocity of the block when it just touches the rough region?

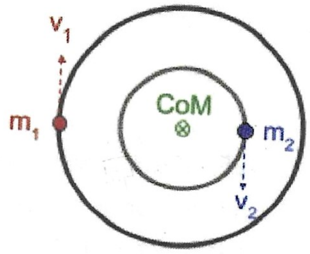
b. (5%) What is the velocity of the block when it reaches point B?

c. (5%) What is the greatest height the block can reach?

d. (5%) Assuming that the block mass increases from $3\,kg$ to $5\,kg$, what is the greatest height the block can reach?

e. (5%) Assuming that the coefficient of kinetic friction increases from $\sqrt{3}$ to $3\sqrt{3}$, what is the greatest height the block can reach?

3. (25%) Consider two point masses, m_1 and m_2, located at position vectors $\vec{r}_1$ and $\vec{r}_2$, respectively.



a. (5%) Derive the gravitational potential energy of this system.

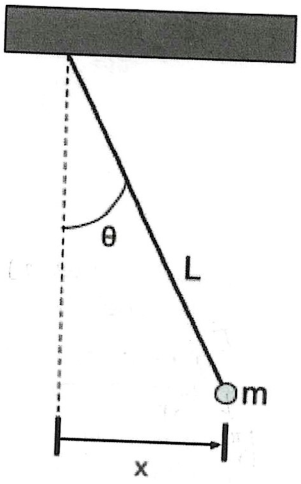
b. (5%) Express the position vectors $\vec{r}_1$ and $\vec{r}_2$ in terms of the center-of-mass position $\vec{R}$ and the relative position $\vec{r} \equiv \vec{r}_1 - \vec{r}_2$.

c. (5%) Demonstrate that the total kinetic energy can be written as $\frac{1}{2}MV^2 + \frac{1}{2}\mu v^2$, where $M \equiv m_1 + m_2$ is the total mass, $\mu \equiv m_1 m_2/(m_1 + m_2)$ is the reduced mass, $\vec{V} \equiv d\vec{R}/dt$ is the center-of-mass velocity, and $\vec{v} = d\vec{r}/dt$ is the relative velocity.

d. (5%) What is the criterion for the two point masses to escape from each other (i.e., the escape velocity)? [Hint: Assume m_1 and m_2 are comparable in magnitude.]

e. (5%) When the two point masses are in circular orbits around their common center of mass (see the figure on the right), what is their common orbital period?

4. (25%) Consider the simple pendulum shown in the figure on the right, where a ball of mass m is attached to a string of length L.



a. (5%) What is the potential energy U of this system as a function of θ?

b. (7%) For small-angle oscillations, show that $U \propto x^2$ (like a spring), where x is the horizontal displacement. What is the effective spring constant k? [Hint: $\sin(\theta) \sim \theta$ and $\cos(\theta) \sim 1 - \theta^2/2$ when $\theta \ll 1$.]

c. (7%) Show that for $\theta \ll 1$, the horizontal force on the ball satisfies Hooke's law, $F_x = -kx$, where k is the spring constant in part (b).

d. (6%) From part (c) and assuming $x = 0$ and $dx/dt = v_0$ at $t = 0$, find the analytical expression for $x(t)$ (i.e., the horizontal displacement as a function of time).