Experiment
6

Oscillators

Objectives

Be familiar with the *Barkhausen criterion* and various kinds of oscillators.

Overview

1. The Wien-Bridge Oscillator

One of the simplest sinusoidal oscillator circuits is based on the Wien-bridge. Fig. 1 shows a Wien-bridge oscillator without the nonlinear gain-control network. The circuit consists of an op amp

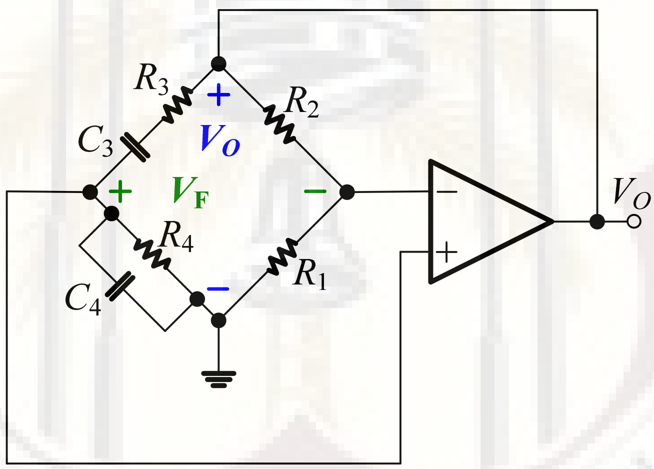


Fig. 1 A Wien-bridge oscillator without amplitude stabilization

$$V_F = V_O \times \frac{R_4 // \frac{1}{sC_4}}{\left(R_3 + \frac{1}{sC_3}\right) + \left(R_4 // \frac{1}{sC_4}\right)} - V_O \times \frac{R_1}{R_1 + R_2}$$

$$\beta = \frac{V_F}{V_O} = \frac{R_4 // \frac{1}{sC_4}}{\left(R_3 + \frac{1}{sC_3}\right) + \left(R_4 // \frac{1}{sC_4}\right)} - \frac{R_1}{R_1 + R_2}$$

$$\beta(s)=\frac{V_F}{V_O}=\frac{\dfrac{1}{\dfrac{1}{R_4}+sC_4}}{\left(R_3+\dfrac{1}{sC_3}\right)+\left(\dfrac{1}{\dfrac{1}{R_4}+sC_4}\right)}-\frac{R_1}{R_1+R_2}$$

$$=\frac{1}{1+\left(R_3+\dfrac{1}{sC_3}\right)\cdot\left(\dfrac{1}{R_4}+sC_4\right)}-\frac{R_1}{R_1+R_2}$$

$$\beta(s)=\frac{1}{1+\dfrac{R_3}{R_4}+\dfrac{C_4}{C_3}+s\cdot\left(R_3C_4+\dfrac{1}{R_4C_3}\right)}-\frac{R_1}{R_1+R_2}$$

$$\beta(j\omega)=\frac{1}{1+\dfrac{R_3}{R_4}+\dfrac{C_4}{C_3}+j\cdot\left(\omega R_3C_4-\dfrac{1}{\omega R_4C_3}\right)}-\frac{R_1}{R_1+R_2}$$

$$=\frac{1}{3+j\cdot\left(\omega R_3C_4-\dfrac{1}{\omega R_4C_3}\right)}-\frac{R_1}{R_1+R_2}$$

The loop gain will be a real number. Therefore, assume the imaginary part of denominator of $\beta(j\omega)$ equals 0, and $R_3 = R_4 = C_3 = C_4 =$ C, we have

$$\omega_0=\frac{1}{\sqrt{R_3R_4C_3C_4}}=\frac{1}{RC}$$

To oscillate the circuit, the **Barkhausen criterion** should be satisfied only at one frequency (ω_0), that is,

$$A(j\omega_0)\beta(j\omega_0)=1$$

$$\because A(j\omega_0)\approx\infty\rightarrow\beta(j\omega_0)=0=\frac{1}{3}-\frac{R_1}{R_1+R_2}$$

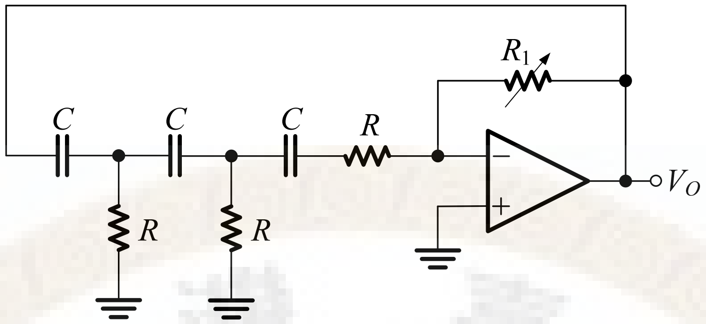
$$\rightarrow\frac{R_1}{R_1+R_2}=\frac{1}{3}\rightarrow 3R_1=R_1+R_2\rightarrow 2R_1=R_2$$

The condition of oscillations at ω_0 is,

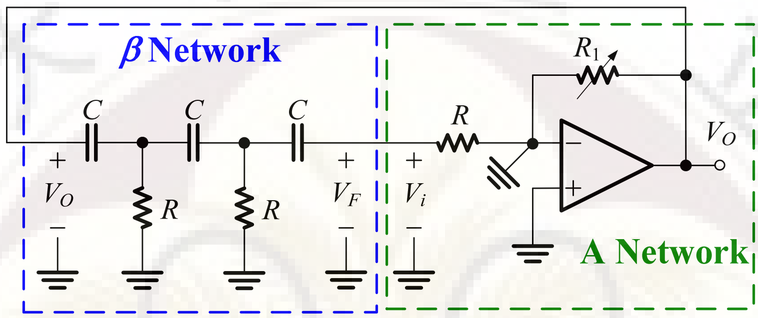
$$\frac{R_2}{R_1}=\frac{\text{橋上電阻}}{\text{橋下電阻}}=2$$

To ensure that oscillations will start, one chooses R_2/R_1 slightly greater than 2.

2. Phase-Shift Oscillator



The A-network and β-network is shown as follow



$$A = \frac{V_O}{V_i} = -\frac{R_1}{R}$$

$$V_F = V_O \times \frac{\left[\left(\left(R+Z_C\right)//R\right)+Z_C\right]//R}{\left\{\left[\left(\left(R+Z_C\right)//R\right)+Z_C\right]//R\right\}+Z_C} \times \frac{\left(R+Z_C\right)//R}{\left(\left(R+Z_C\right)//R\right)+Z_C} \times \frac{R}{R+Z_C}$$

$$\beta = \frac{V_F}{V_O} = \frac{\left[\left(\left(R+\frac{1}{sC}\right)//R\right)+\frac{1}{sC}\right]//R}{\left\{\left[\left(R+\frac{1}{sC}\right)//R+\frac{1}{sC}\right]//R\right\}+\frac{1}{sC}} \times \frac{\left(R+\frac{1}{sC}\right)//R}{\left[\left(R+\frac{1}{sC}\right)//R\right]+\frac{1}{sC}} \times \frac{sCR}{sCR+1} = (1)\times(2)\times(3)$$

$where:$

$$R+\frac{1}{sC} = \frac{1+sCR}{sC},\ \left(R+\frac{1}{sC}\right)//R = \frac{1}{\frac{sC}{1+sCR}+\frac{1}{R}} = \frac{(1+sCR)R}{sCR+1+sCR} = \frac{R+sCR^2}{1+s2CR}$$

$$\left(R+\frac{1}{sC}\right)//R+\frac{1}{sC} = \frac{R+sCR^2}{1+s2CR}+\frac{1}{sC} = \frac{sC\left(R+sCR^2\right)+1+s2CR}{sC\left(1+s2CR\right)} = \frac{s^2C^2R^2+s3CR+1}{sC+s^2 2C^2R}$$

$$\left[\left(R+\frac{1}{sC}\right)//R+\frac{1}{sC}\right]//R = \left(\frac{1}{R}+\frac{sC+s^2 2C^2R}{s^2C^2R^2+s3CR+1}\right)^{-1}$$

$$= \left(\frac{s^2C^2R^2+s3CR+1+R\left(sC+s^2 2C^2R\right)}{R\left(s^2C^2R^2+s3CR+1\right)}\right)^{-1} = \frac{R\left(s^2C^2R^2+s3CR+1\right)}{s^2C^2R^2+s3CR+1+R\left(sC+s^2 2C^2R\right)}$$

$$= \frac{R\left(s^2C^2R^2+s3CR+1\right)}{s^2C^2R^2+s3CR+1+sCR+s^2 2C^2R^2} = \frac{s^2C^2R^3+s3CR^2+R}{s^2 3C^2R^2+s4CR+1}$$

$$(1)\ \frac{\left[\left(\left(R+\frac{1}{sC}\right)//R\right)+\frac{1}{sC}\right]//R}{\left\{\left[\left(R+\frac{1}{sC}\right)//R+\frac{1}{sC}\right]//R\right\}+\frac{1}{sC}}=\frac{\dfrac{s^2C^2R^3+s3CR^2+R}{s^23C^2R^2+s4CR+1}}{\dfrac{s^2C^2R^3+s3CR^2+R}{s^23C^2R^2+s4CR+1}+\dfrac{1}{sC}}$$

$$=\frac{\dfrac{s^2C^2R^3+s3CR^2+R}{s^23C^2R^2+s4CR+1}}{\dfrac{sC\left(s^2C^2R^3+s3CR^2+R\right)+s^23C^2R^2+s4CR+1}{sC\left(s^23C^2R^2+s4CR+1\right)}}$$

$$=\frac{sC\left(s^2C^2R^3+s3CR^2+R\right)}{sC\left(s^2C^2R^3+s3CR^2+R\right)+s^23C^2R^2+s4CR+1}$$

$$=\frac{s^3C^3R^3+s^23C^2R^2+sCR}{s^3C^3R^3+s^23C^2R^2+sCR+s^23C^2R^2+s4CR+1}$$

$$=\frac{s^3C^3R^3+s^23C^2R^2+sCR}{s^3C^3R^3+s^26C^2R^2+s5CR+1}=\frac{s^3+s^2\left(\frac{3}{CR}\right)+s\left(\frac{1}{C^2R^2}\right)}{s^3+s^2\left(\frac{6}{CR}\right)+s\left(\frac{5}{C^2R^2}\right)+\left(\frac{1}{C^3R^3}\right)}$$

$$(2)\ \frac{\left(R+\frac{1}{sC}\right)//R}{\left[\left(R+\frac{1}{sC}\right)//R\right]+\frac{1}{sC}}=\frac{\dfrac{R+sCR^2}{1+s2CR}}{\dfrac{R+sCR^2}{1+s2CR}+\dfrac{1}{sC}}=\frac{\dfrac{R+sCR^2}{1+s2CR}}{\dfrac{sC\left(R+sCR^2\right)+1+s2CR}{sC\left(1+s2CR\right)}}$$

$$=\frac{sC\left(R+sCR^2\right)}{sC\left(R+sCR^2\right)+1+s2CR}=\frac{sCR+s^2C^2R^2}{\left(sCR+s^2C^2R^2\right)+1+s2CR}=\frac{s^2C^2R^2+sCR}{s^2C^2R^2+s3CR+1}$$

$$=\frac{s^2+s\left(\frac{1}{CR}\right)}{s^2+s\left(\frac{3}{CR}\right)+\left(\frac{1}{C^2R^2}\right)};\qquad (3)\ \frac{R}{R+Z_C}=\frac{R}{R+\frac{1}{sC}}=\frac{sCR}{sCR+1}$$

$$\beta(s)=\frac{V_F}{V_O}=(1)\times(2)\times(3)$$

$$=\frac{s^3+s^2\left(\frac{3}{CR}\right)+s\left(\frac{1}{C^2R^2}\right)}{s^3+s^2\left(\frac{6}{CR}\right)+s\left(\frac{5}{C^2R^2}\right)+\left(\frac{1}{C^3R^3}\right)}\times\frac{s^2+s\left(\frac{1}{CR}\right)}{s^2+s\left(\frac{3}{CR}\right)+\left(\frac{1}{C^2R^2}\right)}\times\frac{sCR}{sCR+1}$$

$$=\frac{s\left[s^2+s\left(\frac{3}{CR}\right)+\left(\frac{1}{C^2R^2}\right)\right]}{s^3+s^2\left(\frac{6}{CR}\right)+s\left(\frac{5}{C^2R^2}\right)+\left(\frac{1}{C^3R^3}\right)}\times\frac{s^2+s\left(\frac{1}{CR}\right)}{s^2+s\left(\frac{3}{CR}\right)+\left(\frac{1}{C^2R^2}\right)}\times\frac{sCR}{sCR+1}$$

$$\rightarrow \beta(s)=\frac{V_F}{V_O}=\frac{s^2\left(s+\frac{1}{CR}\right)}{s^3+s^2\left(\frac{6}{CR}\right)+s\left(\frac{5}{C^2R^2}\right)+\left(\frac{1}{C^3R^3}\right)}\times\frac{s}{s+\frac{1}{CR}}$$

$$=\frac{s^3}{s^3+s^2\left(\frac{6}{CR}\right)+s\left(\frac{5}{C^2R^2}\right)+\left(\frac{1}{C^3R^3}\right)}=\frac{1}{1+\left(\frac{6}{sCR}\right)+\frac{5}{s^2C^2R^2}+\frac{1}{s^3C^3R^3}}$$

$$\rightarrow \beta(j\omega)=\frac{1}{1+\left(\frac{6}{sCR}\right)+\frac{5}{s^2C^2R^2}+\frac{1}{s^3C^3R^3}}=\frac{1}{1+\left(\frac{6}{j\omega CR}\right)-\frac{5}{\omega^2C^2R^2}-\frac{1}{j\omega^3C^3R^3}}$$

$$\rightarrow \beta(j\omega)=\frac{1}{1-\frac{5}{\omega^2C^2R^2}+j\left(\frac{1}{\omega^3C^3R^3}-\frac{6}{\omega CR}\right)}$$

Physically, $\beta(j\omega)=$ real number. Thus, we have

$$\frac{1}{\omega^3C^3R^3}-\frac{6}{\omega CR}=0,\rightarrow \frac{1}{\omega^3C^3R^3}=\frac{6}{\omega CR},\rightarrow \omega_0^2=\frac{1}{6R^2C^2},\left(\text{or } \omega_0^2R^2C^2=\frac{1}{6}\right)$$

$$\rightarrow \beta(j\omega_0)=\frac{1}{1-\frac{5}{\omega_0^2C^2R^2}}=\frac{1}{1-30}=-\frac{1}{29},\because A=\frac{V_O}{V_i}=-\frac{R_1}{R}$$

Apply Barkhausen stability criterion, we have

$$A\beta(j\omega_0)=\left(-\frac{R_1}{R}\right)\times\left(-\frac{1}{29}\right)=1,\rightarrow \boxed{\frac{R_1}{R}=29}$$

3. Triangular waveform using bistable multivibrator

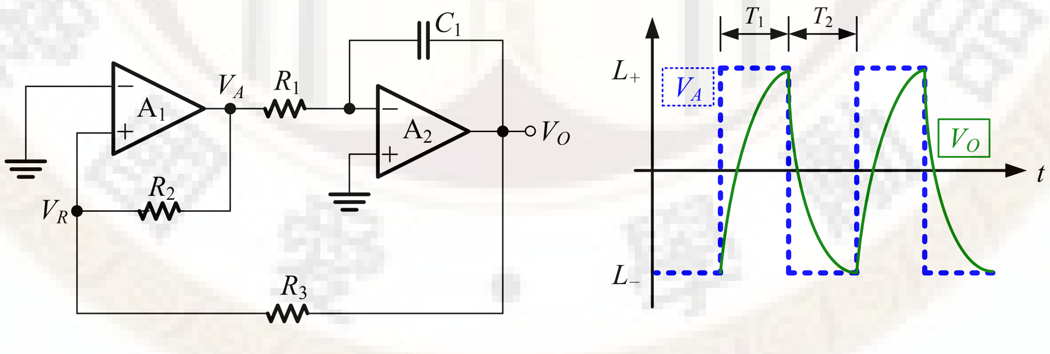


Fig. 2 Triangular waveform using bistable multivibrator

$$V_R\times\left(\frac{1}{R_2}+\frac{1}{R_3}\right)=\frac{V_A}{R_2}+\frac{V_O}{R_3}\rightarrow V_R\times\left(\frac{R_2+R_3}{R_2R_3}\right)=\frac{V_A}{R_2}+\frac{V_O}{R_3}\rightarrow V_R=V_A\frac{R_3}{R_2+R_3}+V_O\frac{R_2}{R_2+R_3}$$

1. *Assume* $t=T_1^-,\ V_A=L_-,\ V_R=0^-,\ V_{C_1}\uparrow,$

2. $t=T_1,\ V_A=L_+, V_R=0\rightarrow 0=V_A\frac{R_3}{R_2+R_3}+V_O\frac{R_2}{R_2+R_3}\rightarrow V_O=-V_A\frac{R_3}{R_2}=L\frac{R_3}{R_2}$

3. $Assume\ t = T_2^-,\ V_A = L_+,\ \ V_R = 0^+,\ V_{C_1}\ \downarrow,$

4. $t = T_2,\ V_A = L_-,, V_R = 0, \rightarrow 0 = V_A \dfrac{R_3}{R_2 + R_3} + V_O \dfrac{R_2}{R_2 + R_3} \rightarrow V_O = -V_A \dfrac{R_3}{R_2} = -L \dfrac{R_3}{R_2}$

Oscillation condition: $\boxed{R_2 > R_3}$.

Otherwise if $V_R = 0^-$, $V_A = -L$, $V_O = -V_A \dfrac{R_3}{R_2} = -V_A = L$, and next moment $V_R = 0^+$,

$V_A = L = V_O$, the capacitor C will not be able to charged.

Components and Instrumentation

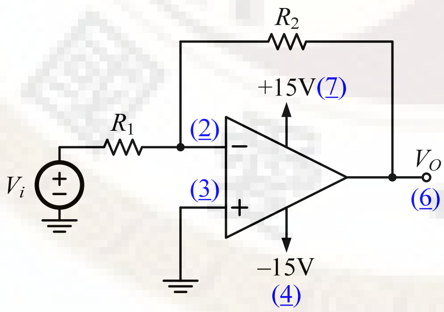
Instrument	Quantity	Components	Quantity
Oscilloscope	1	μA 741 (OP-Amp)	2
Multi-meter	1	1N 4001 (Diode)	2
Power supplier	2	Resistance	Designed value
Function Gen.	1	Capacitance	Designed value

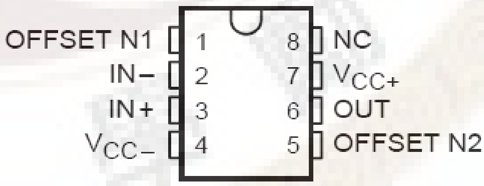
Instrument confirmation

Before you proceed to any part of the experiment, please remember to do the **Instrument Examinations** to the instruments before performing any experiment. The examining procedures are shown in experiment 1.

Components confirmation

μA741 — PINOUT & Functional confirmation





µA741M . . . U PACKAGE
(TOP VIEW)

$Check\ 1.\ A_V = \dfrac{V_o}{V_i} \overset{?}{=} -\left(\dfrac{R_2}{R_1}\right)$

$Check\ 2.\ f_{3dB} \overset{?}{=} 100kHz \sim 500kHz$

Note: Recall the expression of gain—bandwidth product: $A_V \times f_{3dB}$=constant, that is, f_{3dB} will reduce as A_V increases. If the results of confirmation appear the same as those shown in check 1 and 2, the μA741 chip of OP amp is workable.

Lab Work

1. Sinusoidal oscillators: Wien-bridge oscillator

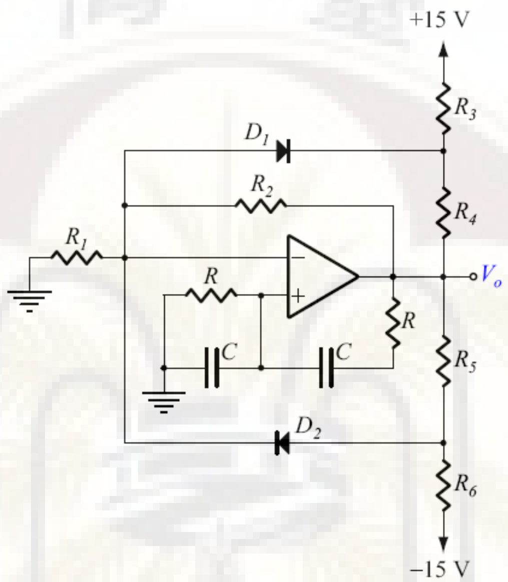


Fig. 3 Wien-bridge oscillator sinusoidal generator

Step 1. Without amplitude stabilization

(1) In Fig. 3, disconnect D_1, D_2, R_3, R_4, R_5, and R_6.
(2) Use R_1 = 3.3 $k\Omega$, R_2 = 10 $k\Omega$,
(3) Oscilloscope ► Press the **DISPLAY** button ► **Format** ► **YT mode.**
(4) Oscilloscope ► Press the **Measure** button ► Observe $V_{out\ (p\text{-}p)}$ in CH1.
(5) Record $V_{out\ (p\text{-}p)}$ = _______V, f = _______Hz.

Step 2. With amplitude stabilization

(1) In Fig. 3, connect D_1, D_2, R_3, R_4, R_5, and R_6.
(2) Use R_1 = 3.3 $k\Omega$, VR(R_2) = 10 $k\Omega$,
(3) Oscilloscope ► Press the **DISPLAY** button ► **Format** ► **YT mode.**
(4) Oscilloscope ► Press the **Measure** button ► Observe $V_{out\ (p\text{-}p)}$ in CH1.
(5) Adjust VR(R_2) until the sinusoidal vibration with the *minimum amplitude*

occurs in V_{out}.

(6) Record $V_{out\ (p\text{-}p)}$ = _______V, f = _______Hz, R_2 = _______ $k\Omega$.

(7) R_2 / R_1 = _______ .

(8) Adjust VR(R_2) until the sinusoidal vibration with the *maximum amplitude* occurs in V_{out}.

(9) Record $V_{out\ (p\text{-}p)}$ = _______V, f = _______Hz, R_2 = _______ $k\Omega$.

(10) R_2 / R_1 = _______ .

2. Sinusoidal oscillators: Phase-shift oscillator

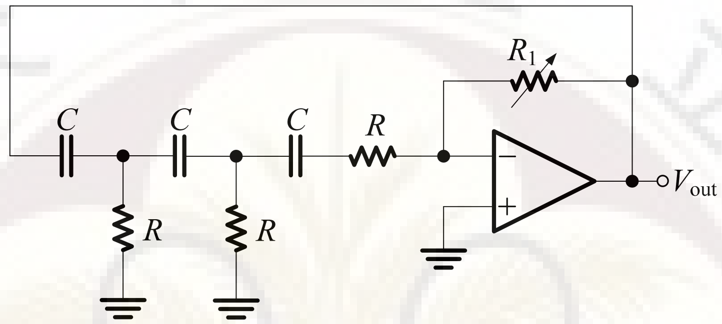


Fig. 4 Phase-shift oscillator sinusoidal generator

(1) In Fig. 4, use the designed value from Pre-Lab Work for components.

(2) Oscilloscope ► Press the **DISPLAY** button ► **Format** ► **YT mode.**

(3) Oscilloscope ► Press the **Measure** button ► Observe $V_{out\ (p\text{-}p)}$ in CH1.

(4) Adjust VR (R_1) until the sinusoidal vibration with the *minimum amplitude* occurs in V_O,

(5) Record $V_{out\ (p\text{-}p)}$ = _______V, f = _______Hz, R_1 = _______ $k\Omega$.

(6) R_1 / R = _______ .

(7) Adjust VR (R_1) until the sinusoidal vibration with the *maximum amplitude* occurs in V_O,

(8) Record $V_{out\ (p\text{-}p)}$ = _______V, f = _______Hz, R_1 = _______ $k\Omega$.

(9) R_1 / R = _______ .

3. Triangular waveform generator: RC−circuit bistable multivibrator

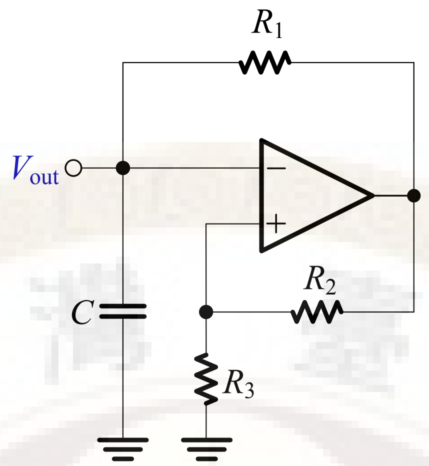


Fig. 5 RC−circuit Triangular waveform generator

(1) In Fig. 5, use the designed value from PR (預報) for components.

(2) Oscilloscope ▶ Press the **DISPLAY** button ▶ **Format** ▶ **YT mode.**

(3) Oscilloscope ▶ Press the **Measure** button ▶ Observe $V_{out\,(p\text{-}p)}$ in CH1.

(4) Record $V_{out\,(p\text{-}p)}$ = _______V, f = _______Hz.

(5) R_1 = _______ $k\Omega$, R_2 = _______ $k\Omega$, R_3 = _______ $k\Omega$, C = ______F.

4. Triangular waveform generator: Integrator bistable multivibrator

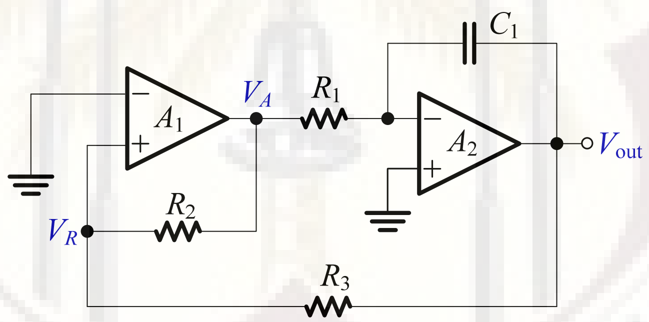


Fig. 6 Integrator bistable multivibrator

(1) In Fig. 6, use the designed value from PR (預報) for components.

Note: Choose R_2 slightly greater R_3 rather than identity.

(2) Oscilloscope ▶ Press the **DISPLAY** button ▶ **Format** ▶ **YT mode.**

(3) Oscilloscope ▶ Press the **Measure** button ▶ Observe $V_{out\,(p\text{-}p)}$ in CH1.

(4) Record $V_{out\,(p\text{-}p)}$ = _______V, f = _______Hz,

(5) Record R_2 = _______ $k\Omega$, R_3 = _______ $k\Omega$.

Reference

1. A.S. Sedra and K.C. Smith, *Microelectronic Circuits*, 6th ed., Oxford University Press publishing, New York, 2011.
2. A.S. Sedra and K.C. Smith, *Laboratory Manual for Microelectronic Circuits*, 3rd ed., Oxford University Press publishing, New York, 1997.