Industrial Robots : Teaching / Playback Scheme

$\Rightarrow$ Intelligent Robots ~ what's the input

- Position Control : Desired position
- Trajectory Control : Desired geometric traj.
- Force Control : Desired force

1. Linear State-Variable System

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

* Robot : $H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$

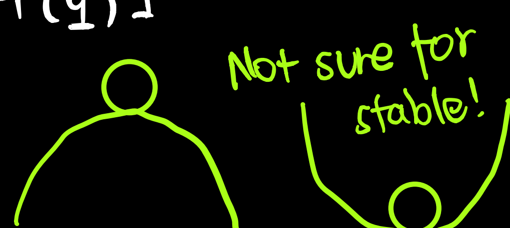
$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$. Then we can transform Robot Dynamics to

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ H^{-1}(q)\{-C(q,\dot{q}) - G(q)\} \end{bmatrix} + \begin{bmatrix} 0 \\ H^{-1}(q) \end{bmatrix}\tau$$

* For Non-linear System

$$\dot{x} = f(x,u)$$
$$y = g(x,u)$$

$\Rightarrow$ Equilibrium Point

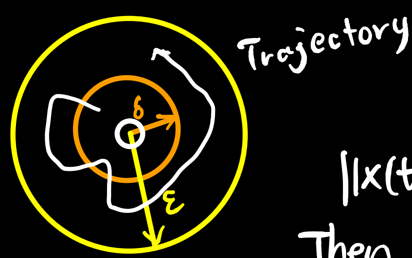


$\dot{x} = f(x,t)$. Then $x_e \sim 0 = f(x_e, t)$

⇒ Lyapunov Stability

x_e is stable at t_0. if $\forall \varepsilon > 0$. $\exists\, \delta(\varepsilon, t_0)$

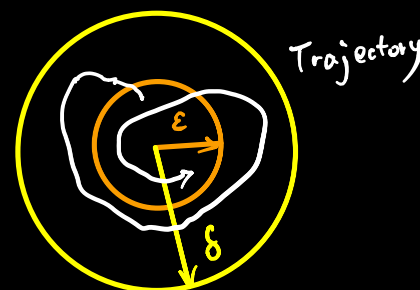
s.t. $\|x(t_0) - x_e\| < \delta(\varepsilon, t_0)$. then $\|x(t) - x_e\| < \varepsilon$



Stable

$\|x(t_0) - x_e\| < \delta \Rightarrow$

Then $\|x(t) - x_e\| < \varepsilon$



Convergent

⇒ Asymptotic Stability

$\exists\, \delta > 0$ s.t. $\|x(t_0) - x_e\| < \delta$. Then $\lim_{t\to\infty} x(t) = x_e$

⇒ Global Exponential Stability

$\exists\, a, \beta > 0$, s.t. $\|x(t) - x_e\| < a\|x(0) - x_e\| e^{-\beta t}$

* Lyapunov Theorem

$\dot{x} = f(x)$. $x(0) = x_0$ with $x_e = 0$

Stable $\Longleftrightarrow$ $\exists\, V(x)$ scalar func. $\begin{cases} V(x) \text{ Pos-def.} \\ \dot{V}(x) \text{ Neg-def. (or semi)} \end{cases}$

If not: Asymp.

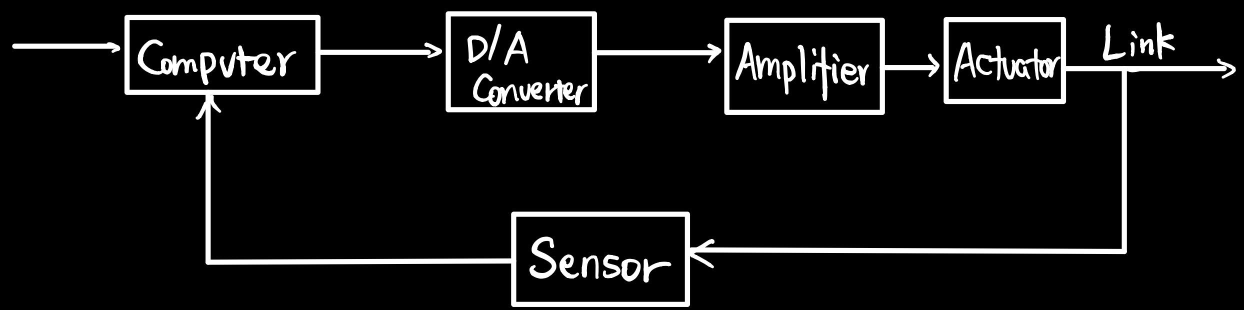
e.g. $\ddot{\theta} + \dot{\theta} + \sin\theta = 0$.

Hard to find

$\theta_e = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. $V(\theta, \dot{\theta}) = 1 - \cos\theta + \frac{1}{2}\dot{\theta}^2$

* LaSalle Theorem : What 's that crazy thing

2. Position Control



Objective : Move robot position $\vec{q}$ to desired $\vec{q}_d$

$u = -A\Delta q - B\dot{q}$. where $\Delta q = q - q_d$ * PD Control

$\Rightarrow H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = -A\Delta q - B\dot{q}$

Equlibrium Point $\Delta q_e = \underline{-A^{-1}G(q)}$ * Given by $\dot{x} = 0$

There is offset error!

$\begin{cases} A = 0 & \sim \text{Drawback a lot !} \\ G(q) = 0 & \sim \text{Sent to space !} \end{cases}$

Potential Energy

$V(t) = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \frac{1}{2}\Delta q^T A \Delta q + \boxed{U(q)}$

$$\dot{V}(t) = \dot{q}^T \left\{ H(q)\ddot{q} + A\Delta q + \underbrace{\boxed{\frac{\partial U(q)}{\partial q}}}_{G(q)} + \frac{1}{2}\dot{H}(q)\dot{q} \right\}$$

In Dynamics. $C(q,\dot{q})\dot{q} = \frac{1}{2}\dot{H}(q)\dot{q} + S(q,\dot{q})\dot{q}$

$$\Rightarrow \; = \dot{q}^T\left[-A\Delta q - B\dot{q} - S(q,\dot{q})\dot{q} - G(q) + A\Delta q + G(q)\right]$$

$$= \underbrace{\dot{q}^T\left[-S(q,\dot{q})\dot{q}\right.}_{=0 \,\sim\, \text{skew-symmetric } S} \left. - B\dot{q}\right]$$

$$= -\dot{q}^T B\dot{q} \leq 0$$

$\Rightarrow$ With Gravity Compensation

$$u = -A\Delta q - B\dot{q} + G(q)$$

3. Trajectory Tracking Control

Given $q_d(t)$. $\dot{q}_d(t)$ & $\ddot{q}_d(t)$ as desired state!

- Computed Torque

$$\tau = \left(\frac{1}{2}\dot{H}(q) + S(q,\dot{q})\right)\dot{q} + G(q) + \tau'$$

& $\tau' = H(q)\ddot{q} \triangleq H(q)\,u$ where $u = \ddot{q}$

$$u(t) = \ddot{q}_d(t) - K_1\Delta\dot{q}(t) - K_2\Delta q(t)$$

$$\Rightarrow \Delta\ddot{q}_d(t) + K_1\Delta\dot{q}(t) + K_2\Delta q(t) = 0 \,!$$

$*$ Controller $\tau = -K_1 \Delta \dot{q}(t) - K_2 \Delta q(t) + H(q)\ddot{q}_d(t)$

$+ \left(\frac{1}{2}\dot{H}(q) + S(q,\dot{q})\right)\dot{q}_d(t) + G(q)$

Consider $V(t) = \frac{1}{2}\Delta\dot{q}^T H(q)\Delta\dot{q} + \frac{1}{2}\Delta q^T K_2 \Delta q$

- Nominal Reference

$\dot{q}_r(t) = \dot{q}_d(t) - \lambda \Delta q(t)$, $\lambda > 0$ ~ Nominal Vector

def. $s = \dot{q}(t) - \dot{q}_r(t) = \underline{\Delta\dot{q}(t) + \lambda \Delta q(t)}$ ~ Error

Consider only 1 !