Optimization Problem

$\min f_0(x)$

s.t. $f_i(x) \le b_i$ . $i = 1, \dots, m$

* $x = [x_1, \cdots x_n]$

$f_0 : R^n \rightarrow R$ Objective

$f_i : R^n \rightarrow R$ constraint

Optimal solution $x^*$ has smallest value among all.

$\left\{ \begin{array}{l} \text{Least-squares} \\ \text{Linear programming} \\ \text{Convex optimization} \end{array} \right.$ $\Rightarrow$ Solvable!

& Analyable!

- Least-squares

minimize $\|Ax - b\|_2^2$

- Analytical solution $x = (A^T A)^{-1} A^T b$
- Computation Complexity $n^2 k$ $(A \in R^{k \times n})$

- Linear Programming

minimize $c^T x$

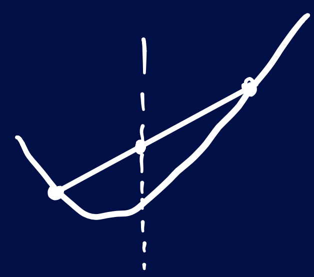
s.t. $a_i^T x \le b_i$ , $i = 1, \cdots m$

- No Analytical Solutions
- Computation Complexity $n^2 m$ if $m \ge n$

- Convex Optimization

minimize $f_0(x)$

s.t. $f_i(x) \le b_i$ , $i = 1 \dots m$.



$f_i(\alpha x + \beta y) \le \alpha f_i(x) + \beta f_i(y)$ if $\alpha + \beta = 1$

for $i = 0, 1, 2, \cdots, m$

- No Analytical Solutions
- Computation Complexity $\max\{n^3, n^2 m, F\}$

where $F$ is cost of $f_i$'s & 1st 2nd derivative

- Nonlinear Programming

$\alpha$. Local Optimization

- Find a near point that minimize $f_0$
- Fast & can handle large problems
- Needs initial guess!

$\beta$. Global Optimization

- Worst-case complexity grows exponentially!