Solving the Advection Equation (Part 2)

Answer the following questions regarding your solutions to the previous problem:

Consider your error plot from part 1 of the previous problem. Based on your plot, what is the order of convergence of this method with respect to Δx? Explain briefly how your plot demonstrates this.

In part 1: The order of convergence for $\Delta x < 10^{-2}$ is close to $O(x^2)$; The order of convergence for $\Delta x > 10^{-2}$ is close to $O(x)$. I will claim the convergence of this problem is $O(x^2)$.

What behavior do you observe in part 2 of the previous problem? Explain your observations in terms of the stability properties of the AB3 timestepper and the eigenvalues of the centered-difference matrix C.

the eigenvalues of the centered-difference matrix Chas the form

$$-\frac{c}{2\Delta x}\begin{bmatrix} 0 & 1 & & & \\ -1 & 0 & 1 & & \\ & -1 & \dots & \dots & \\ & & \dots & 0 & 1 \\ & & & -1 & 0 \end{bmatrix}.$$

For AB3, $|\lambda\Delta t| < 0.7236$. In this case, our CFL has to be less than 0.7236 to achieve stability. Therefore, when we choose Δt to make CFL=0.7, the solution shows clear stable pattern. However, when we set CFL=0.75, the solution is unstable.