ODEs and Stability

Consider the ODE $y\prime = -5y$ with initial condition $y(0) = 1$. We will solve this ODE numerically using a step size of $h = 0.5$.

Are solutions to this ODE stable? Explain.

$$\begin{aligned} y^{'} &= -5y \\ \lambda &= -5 \\ h &= 0.5 \\ G &= e^{-5\times 0.5} = e^{-2.5} \\ G &< 1 \end{aligned}$$

Solution to this ODE is stable

Is Euler's method stable for this ODE using this step size? Explain.

$$|G| = |1 + \lambda h| = |1 - 2.5| = 1.5 > 1$$

Euler's method at this situation is unstable

Compute the numerical value for the approximate solution at $t = 0.5$ given by Euler's method. Show your work.

$$\begin{aligned} y(0.5) &= y(0 + h) \\ &= Gy(0) \\ &= -1.5 \times 1 \\ &= -1.5 \end{aligned}$$

Is the backward Euler method stable for this ODE using this step size? Explain.

$$\begin{aligned} |G| &= |\frac{1}{1 - \lambda h}| \\ &= |\frac{1}{1 + 2.5}| < 1 \end{aligned}$$

Euler's backward method at this situation is stable

Compute the numerical value for the approximate solution at $t = 0.5$ given by the backward Euler method. Show your work.

$$\begin{aligned} y(0.5) &= y(0 + h) \\ &= Gy(0) \\ &= \frac{1}{3.5} \times 1 \\ &= \frac{2}{7} \end{aligned}$$