Accuracy of the Trapezoid Method

Assuming $\lambda \in \mathbb{R}$, what is the asymptotic value of the growth factor GG as $\lambda\Delta t \longrightarrow -\infty$ when using the trapezoid method for the model problem $y^{\bullet} = \lambda y$?

(Show a derivation rather than just writing the answer.)

What should the growth factor be for the analytical case as $\lambda\Delta t \longrightarrow -\infty$?

Using trapezoid rule for: $f(y,t) = y'(t) = \lambda y(t)$

$$
\begin{aligned}
y'(t) &= \frac{f(t_k, y_k) + f(t_{k+1}, y_{k+1})}{2} \\
y_{k+1} &= y_k + y'\Delta t \\
&= y_k + \frac{f(t_k, y_k)\Delta t}{2} + \frac{f(t_{k+1}, y_{k+1})\Delta t}{2} \\
y_{k+1} &= y_k + \frac{\lambda y_k}{2}\Delta t + \frac{\lambda y_{k+1}}{2}\Delta t \\
(1 - \frac{\lambda\Delta t}{2})y_{k+1} &= (1 + \frac{\lambda\Delta t}{2})y_k \\
y_{k+1} &= \frac{2+\lambda\Delta t}{2-\lambda\Delta t}y_k \\
G &= \frac{2+\lambda\Delta t}{2-\lambda\Delta t} \\
&\because \lambda\Delta t \longrightarrow -\infty \\
&\therefore G \longrightarrow -1
\end{aligned}
$$

Asymptotic value of the growth factor GG as $\lambda\Delta t \longrightarrow -\infty$when using the trapezoid method for the model problem $y^{\bullet} = \lambda y$ is -1 For analytical case:

$$
\begin{aligned}
&G = e^{\lambda\Delta t} \\
&\because \lambda\Delta t \longrightarrow -\infty \\
&\therefore G \longrightarrow 0
\end{aligned}
$$

Asymptotic value of the growth factor GG as $\lambda\Delta t \longrightarrow -\infty$when analytic case is 0