CS 450-HW#3-Q3

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Eigenvalues of Special Matrices

1. Consider a projection matrix, P. What are the eigenvalues of P?

Let P be a $n \times n$ projection matrix, then P is idempotent, that is $P^2 = P$.

Let the eigenvalues of P be $\lambda_1, \lambda_2, ..., \lambda_n$

Then the eigenvalues of P^2 be $\lambda_1^2, \lambda_2^2, ..., \lambda_n^2$

For two identical matrices, they will share same eigenvalues.

Therefore, for $i \in [1, n]$

$$\lambda_i = \lambda_i^2$$
$$\lambda_i(\lambda_i - 1) = 0$$
$$\lambda_{i_1} = 1$$
$$\lambda_{i_2} = 0$$

Since we do not know exact form of P, we can only assert that the eigenvalues of a project matrix P is either 1 or 0.

2. What are the eigenvalues of the Householder transformation $H = I - 2\frac{vv^T}{v^Tv}$, where v is a nonzero vector?

Let H be a $n \times n$ Householder transformation matrix, then $H = H^T = H^{-1}$.

Let the eigenvalues of H be $\lambda_1, \lambda_2, ..., \lambda_n$

Then the eigenvalues of H^{-1} be $\lambda_1^{-1}, \lambda_2^{-1}, ..., \lambda_n^{-1}$

H^T will not affect the eigenvalues.

For two identical matrices, they will share same eigenvalues.

Therefore, for $i \in [1, n]$

$$\lambda_i = \lambda_i^{-1}$$
$$\lambda_i^2 = 1$$
$$\lambda_{i_1} = 1$$
$$\lambda_{i_2} = -1$$

Since we do not know exact form of v, we can only assert that the eigenvalues of a Householder transformation matrix P is either 1 or -1.