Piecewise Polynomial Interpolation

> In general, is it possible to interpolate nn data points by a piecewise quadratic polynomial, with knots at the given data points, such that the interpolant is
>
> Once continuously differentiable?
>
> Twice continuously differentiable?
>
> In each case, if the answer is "yes," explain why, and if the answer is "no," give the maximum value for nn for which it is possible.

Suppose we randomly take 3 consecutive abscissas, $(t_{i-1}, y_{i-1}), (t_i, y_i), (t_{i+1}, y_{i+1})$. Between each two abscissas, use a piecewise quadratic polynomial interpolation: $ax^2 + bx + c = 0$. At point (t_i, y_i), we have $a_1 x_i^2 + b_1 x_i + c_1 = a_2 x_i^2 + b_2 x_i + c_2$

Once continuously differentiable?

If the interpolation is once continuously differentiable, $2a_1 x_i + b_1 = 2a_2 x_i + b_2$

Overall there will be 5 equations:

$$\begin{cases} a_1 x_{i-1}^2 + b_1 x_{i-1} + c_1 = 0 \\ a_1 x_i^2 + b_1 x_i + c_1 = 0 \\ a_2 x_i^2 + b_2 x_i + c_2 = 0 \\ a_2 x_{i+1}^2 + b_2 x_{i+1} + c_2 = 0 \\ 2a_1 x_i + b_1 = 2a_2 x_i + b_2 \end{cases}$$

Overall there are 6 unknowns: $a_1, b_1, c_1, a_2, b_2, c_2$

Therefore we do can come up with two combination of coefficient, and once continuous differentiable interpolation is possible.

Twice continuously differentiable?

If the interpolation is twice continuously differentiable,

$$\begin{aligned} 2a_1 &= 2a_2 \\ a_1 &= a_2 \\ \Rightarrow b_1 &= b_2 \\ \Rightarrow c_1 &= c_2 \end{aligned}$$

Hence there can only have one interpolation equation, and thus twice continuously differentiable piecewise interpolation is not possible