

CS 450-HW#3-Q5

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Comparing Fixed-Point Problems

For the equation

$$f(x) = x^2 - 3x + 2 = 0$$

Each of the following functions yields an equivalent fixed-point problem.

Analyze the convergence properties of each of the corresponding fixed-point iteration schemes for the root $x = 2$ by considering $|g'_i(2)|$.

1.

$$\begin{aligned} g_1(x) &= (x^2 + 2)/3 \\ g'_1(x) &= \frac{2}{3}x \\ |g'_1(2)| &= 4/3 > 1 \end{aligned}$$

\therefore Divergent.

2

$$\begin{aligned} g_2(x) &= \sqrt{3x - 2} \\ g'_2(x) &= \frac{3}{2\sqrt{3x - 2}} \\ |g'_2(2)| &= \frac{3}{4} < 1 \end{aligned}$$

\therefore Linear Convergent with rate $\frac{3}{4}$.

3.

$$\begin{aligned}g_3(x) &= 3 - \frac{2}{x} \\g'_3(x) &= \frac{2}{x^2} \\|g'_3(x)| &= \frac{1}{2} < 1\end{aligned}$$

\therefore Linear Convergent with rate $\frac{1}{2}$.

4.

$$\begin{aligned}g_4(x) &= \frac{x^2 - 2}{2x - 3} \\g'_4(x) &= \frac{2(x^2 - 3x + 2)}{(2x - 3)^2} \\g'_4(2) &= 0\end{aligned}$$

\therefore Rapid Convergent with rate at least quadratic.