Systems of ODEs

Write each of the following ODEs as an equivalent first-order system of ODEs:

Van der Pol equation:

$$y'' = y'(1-y^2)-y$$

$$\begin{cases} y_1 = y \\ y_2 = y' = y_1' \end{cases}$$

$$\mathbf{y}' = \frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y' \\ y'' \end{pmatrix} = \begin{pmatrix} y_2 \\ y_2(1-y_1)^2 - y_1 \end{pmatrix}$$

Blasius equation:

$$y''' = -yy''$$

$$\begin{cases} y_1 = y \\ y_2 = y' = y_1' \\ y_3 = y'' = y_2' \end{cases}$$

$$\mathbf{y}' = \frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} y' \\ y'' \\ y''' \end{pmatrix} = \begin{pmatrix} y_2 \\ y_3 \\ -y_1 y_3 \end{pmatrix}$$

Newton's Second Law of Motion for two-body problem:

$$y_1'' = -GMy_1/(y_1^2 + y_2^2)^{3/2},$$
$$y_2'' = -GMy_2/(y_1^2 + y_2^2)^{3/2}.$$

$$\begin{cases} y_1 = y_1 \\ y_2 = y_2 \\ y_3 = y_1' \\ y_4 = y_2' \end{cases}$$

$$\mathbf{y}' = \frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} y_1' \\ y_2' \\ y_1'' \\ y_2'' \end{pmatrix} = \begin{pmatrix} y_3 \\ y_4 \\ -GMy_1/(y_1^2 + y_2^2)^{3/2} \\ -GMy_2/(y_1^2 + y_2^2)^{3/2} \end{pmatrix}$$