Sunken Treasures in SUSTech Lake

Name:

Student ID:

1. Setup

It was reported in the news that some treasure chests full of gold and other precious artifacts had been recovered from Min River in Sichuan Province. The treasures sunk into the river bed during a war between the Ming Dynasty and the Daxi Dynasty of Zhang Xianzhong. Many geophysical methods, including underwater ERT, magnetic, geo-radar and electromagnetic (EM), were employed to help narrow down the prospective area for excavation. Inspired by the discovery in Sichuan, the president of SUSTech hopes ancient treasures can also be recovered on SUSTech campus. The president is particularly interested in the lakes near his residence. So, the Office of Campus Administration commissioned the geophysics department to carry out some geophysical surveys in the lake, in the hope of finding traces of ancient treasures.

As the first step, the Office of Campus Administration would like to know if there is any showing of suspected treasures. If possible, they would also like to obtain information about the material and shape, so the type of artifact can be preliminarily identified. Here is a list of their wishes:

1. Determine whether there is any possible ancient artifact in SUSTech lakes
2. Obtain the location of the suspected treasures
3. Determine the material (gold, silver, copper, iron, etc.)
4. Determine the shape (sphere, plate, rod, etc.)
5. Be quick and cheap but with sufficient information for the follow-up check-up

Questions:

Those requirements are mostly from the president. Do you think his dream of finding treasures is realistic? How confident are you of solving the problems above? Why? Provide your answers below. You may come back and revise your answers when you finish this worksheet.

Your answers:

1.
2.
3.
4.
5.

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2. Properties

Geophysical methods only work if there is a contrast in physical properties. In this underwater exploration project, the background is a layer of water overlying a sedimentary lakebed (mud and gravels); and the sought targets, possibly made of gold, silver, copper, iron or ceramics, are most likely sitting on the lakebed or buried at a relatively shallow depth.

Questions:

Find the approximate physical properties (or ranges) for the materials that you may encounter in this project. Also provide comments on whether these properties can be practically used to locate and identify underwater treasures.

Your answers:

Material	Density	Susceptibility (magnetic permeability)	Electrical conductivity
Lake water			
Lake bed			
Gold			
Silver			
Copper			
Iron			
Ceramics			

- Comments:

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3. Survey

Electrical and EM methods can both be used to detect anomalies in electrical conductivity. The electrical method works in the electrical steady state and relies on the anomalous charges built up on the conductivity interfaces. The EM methods rely on the interaction between the electrical and magnetic fields, as stated in the interfaces. The EM methods rely on the interaction between the electrical and magnetic fields, as stated in the Maxwell's equations.

Faraday's law: $\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$ in time domain or $\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}$ in frequency domain

Ampere's law: $\nabla \times \mathbf{H} = \sigma\mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}$ in time domain or $\nabla \times \mathbf{H} = \sigma\mathbf{E} + i\omega\varepsilon\mathbf{E}$ in frequency domain

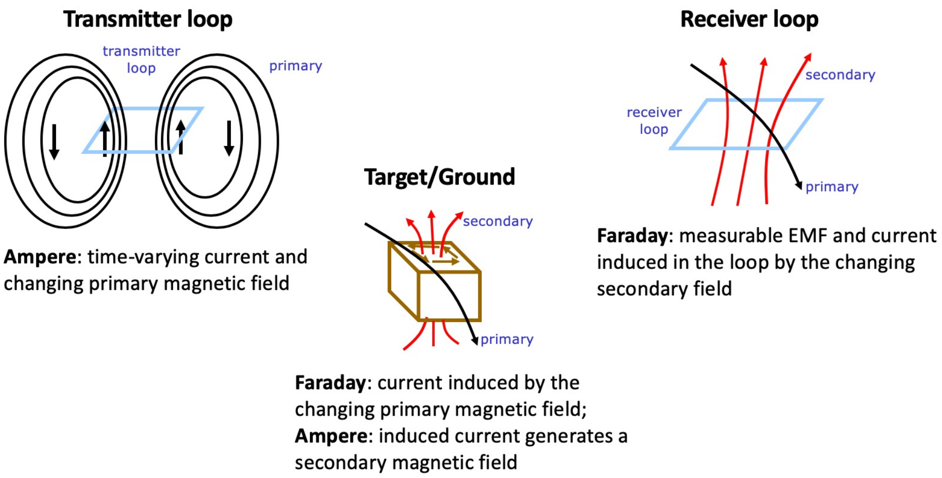
The two terms on the right hand side of Ampere's law are conduction and displacement currents respectively. When the condition $\sigma \gg \omega\varepsilon$ is satisfied, the displacement current can be safely ignored and the simplified equations are said to be in the **quasi-static** state.

The power of EM methods is that if we can manage to change the primary magnetic field, then according to the Faraday's law, an electromotive force (EMF) can be generated; such EMF can then drive a conduction current proportional to the conductivity σ. This induced current can again generate a secondary magnetic field. The information about the conductivity can be inferred from the measurements of the secondary magnetic field.

Loop-loop system

A variety of EM methods and systems are designed to exploit the mechanism of EM induction with different combinations and deployments of electrical and magnetic sensors. For the near surface exploration, an EM system consisting of two loops (coils), one as the transmitter and the other as the receiver, is often utilized. A qualitative explanation of how a loop-loop system works is illustrated below. Unlike the electrical method that relies on galvanic currents to probe the subsurface conductivity, loop-loop EM makes use of induced eddy currents and interacts with the subsurface through magnetic flux linkage. So, the most significant advantage of a loop-loop system is its mobility.

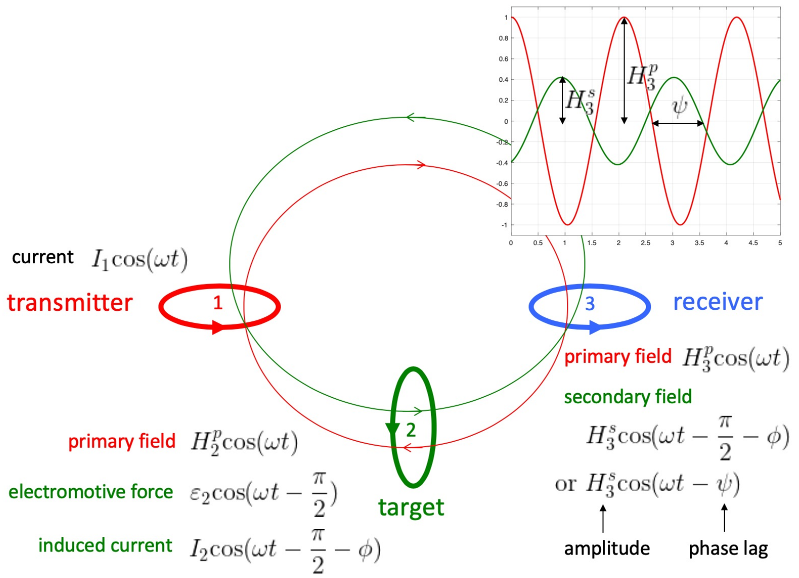
If the time-varying current in the transmitter loop is sinusoidal at a specific frequency and the measured signals at the receiver is also sinusoidal at the same frequency, the system is said to operate in the frequency domain. In contrast, if the transmitter current contains abrupt changes like a sudden turn-off, and the measured signals are the transient field in time, the system is considered to be in time domain.



Three-loop model

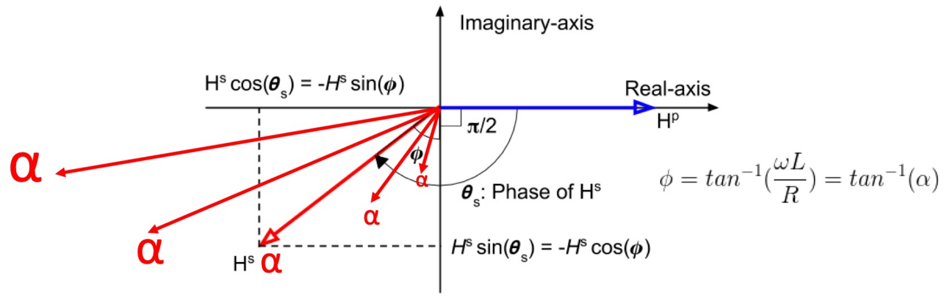
A quantitative understanding of the loop-loop EM system can be gained by analyzing a three-loop model - two loops as the transmitter and receiver and another loop as the subsurface object with its specific resistance (R) and self-inductance (L).

1. The transmitter Loop 1 carries a current $I_1 cos(\omega t)$ at an angular frequency ω.
2. The circular current in Loop 1 generates a primary magnetic field at the target Loop 2 and at the receiver Loop 3. The primary magnetic field is in-phase with the transmitter current. So we denote the primary field at Loop 2 as $H_2^p cos(\omega t)$ and at Loop 3 as $H_3^p cos(\omega t)$. H_2^p and H_3^p depends on the geometry of loops and can be calculated using Biot-Savart's law.
3. According to the Faraday's law, a time-varying magnetic field can create an EMF ε_2 but with a phase lag of $\pi/2$ (recall $i\omega$).
4. The EMF then drives an induced current I_2 in Loop 2. Because Loop 2 has a self-inductance, the induced current is not exactly in-phase with the EMF and is subject to an additional phase lag ϕ.
5. Again, I_2 in Loop 2 generates a secondary magnetic field H_3^s at the receiver Loop 3, which is in-phase with I_2. So, H_3^s is out-of-phase with H_3^p by a total phase lag of $\psi = \pi/2 + \phi$.



Suppose the primary field H^p has zero phase (on the real-axis), the total phase lag ψ makes the secondary field H_s swing in the third quadrant.

- How much the H^s swings away from the imaginary axis (ϕ) depends on a characteristic parameter of Loop 2 called **induction number** $\alpha = \frac{\omega L}{R}$
- The convention of data is to decompose the complex number H^s to two orthogonal components: one with a phase lag of π called **in-phase** or **real** ($H^s sin(\phi)$ on the negative real axis), and the other with a phase lag of $\pi/2$ called **out-of-phase** or **quadrature** or **imaginary** ($H^s cos(\phi)$ on the negative imaginary axis).
- The magnitude of H^s is usually recorded relative to H^p. So, the in-phase component of a loop-loop EM system data is $H^s sin(\phi)/H^p$ and the out-of-phase component is $H^s cos(\phi)/H^p$. The unit can be percentage or ppm.
- The convetion of sign is that data are positive if H^p and H^s are in the same direction at the receiver loop; data are negative if in the opposite direction.



Questions

How do the in-phase and out-of-phase components change as α increases (higher frequency, higher self-inductance, lower resistance)?

Your answers

- In-phase:
- Out-of-phase:

The quantitative relation between the induction number α and the responses in the in-phase and out-of-phase component can be described by the **response function**

$Q(\alpha) = \frac{\alpha^2 + i\alpha}{1+\alpha^2}$,

which is a property of Loop 2 and determines the relative magnitudes of the in-phase and out-of-phase.

Questions

Plot the responses of in-phase and out-of-phase components as a function of α from 10^{-3} to 10^3.

Think about a few situations when the in-phase part is much stronger than the out-of-phase part.

Your answers

-

In [ ]:

```
# Your code that plots the response function
```

The measured data at the receiver loop can be expressed as

$$\frac{H^s}{H^p} = -\frac{M_{12}M_{23}}{M_{13}L}Q(\alpha),$$

which includes two factors. The first factor is the coupling coefficient determined by the mutual inductances M_{12}, M_{23}, M_{13} and the self-inductance L of Loop 2. The second factor is the response function. Mutual inductance M_{ij} can be calculated as the magnetic flux linkage between Loop i and Loop j. Specifically, if both loops have an area of unity, it is the projection of magnetic flux (B field) from Loop i at Loop j to the normal direction of Loop j. You may find the equation of magnetic dipole field you have learned in the magnetic method useful in this calculation.

Questions

A small loop can be considered as a magnetic dipole, whose dipole direction can be specified by the inclination and declination following the same definition in the magnetic method. A commonly used horizontal co-plannar (HCP) loop-loop system has a transmitter loop (Tx) and a receiver loop (Rx) both with an inclination -90 degree and a declination 0 and separated by a constant distance. Suppose such a system moves from west to east and hovers across a subsurface object represented by another target loop with an inclination 0 and a declination 90 degree. The Tx-Rx separation is 1.66 m. The HCP loop-loop system is 1 m above the surface. The center of the target loop is 0.5 m below the surface.

Describe the instrument position relative to the target:

- When the transmitter loop has the worst geometric coupling (minimum magnetic flux linkage) with the target loop.
- When the transmitter loop has the best geometric coupling (maximum magnetic flux linkage) with the target loop.
- When the transmitter and receiver loops together have the best geometric coupling with the target loop.

Your answers

-
-
-

Questions

Now make a generic program to calculate the in-phase and out-of-phase components of H^s/H^p data of an HCP system when a conductive object is buried under surface.

- Input variables: Tx-Rx separation, midpoint position of the HCP system, operating frequency.
- The object is made of three orthogonal target loops (x-loop, y-loop, z-loop) buried at the same position, each of which has its own R and L; the three target loops interact with the EM system independently, so the principle of superposition holds. By adjusting R_x, R_y, R_z, L_x, L_y, L_z, the program can simulate objects with different aspect ratios and orientations.

Your answers

In [ ]:

```
# Your function definition
```

Questions

Simulate and plot the data profile when an HCP system operating at 1 kHz moves along the Tx-Rx line from west to east and over a gold plate buried 0.5 m deep. The Tx-Rx separation is 1.66 m; the HCP system is 0.5 m above the surface. The gold plate has R = 1.0 and L = 0.05. Plot the in-phase and out-of-phase data for the following four scenarios.

1. The gold plate has an inclination 0 and a declination 90 degree.
2. The gold plate has an inclination 45 degree and a declination 90 degree.
3. The gold plate has an inclination 90 degree and a declination 90 degree.
4. The object is an equiaxed gold sphere.

Your answers

In [ ]:

```
# Scenario 1: inclination 0, declination 90 (vertical plate)
```

In [ ]:

```
# Scenario 2: inclination 45, declination 90 (dipping plate)
```

In [ ]:

```
# Scenario 3: inclination 90, declination 90 (horizontal plate)
```

In [ ]:

```
# Scenario 4: gold sphere
```

4. Data

A HCP loop-loop system called GEM-2 is mounted on an inflatable boat to survey the SUSTech lake. The separation of Tx and Rx is 1.66 m. During operation, GEM-2 continously records the in-phase and out-of-phase data at some predefined frequencies and the GPS positions. The instrument is about 0.1 m above the water surface.

In [1]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline

# load GEM-2 data in SUSTech lake
df = pd.read_csv('37-xg-07ja-039_gem.csv', header=0)
df.describe() # show contents of data frame
```

Out[1]:

	Line	Sample	X	Y	Mark	Status	GF
count	14086.000000	14086.000000	14086.000000	1.408600e+04	14086.000000	14086.0	14086.0
mean	0.004260	7000.272114	807993.039224	2.502688e+06	0.009300	0.0	1.9
std	0.065128	4066.781473	12.816530	7.225577e+00	0.095991	0.0	0.2
min	0.000000	6.000000	807972.370000	2.502675e+06	0.000000	0.0	1.0
25%	0.000000	3479.250000	807979.260000	2.502682e+06	0.000000	0.0	2.0
50%	0.000000	7000.500000	807991.530000	2.502686e+06	0.000000	0.0	2.0
75%	0.000000	10521.750000	808005.620000	2.502694e+06	0.000000	0.0	2.0
max	1.000000	14043.000000	808013.280000	2.502705e+06	1.000000	0.0	2.0

8 rows × 32 columns

Questions

Plot a plan view of the GEM-2 data and locate the anomalies possibly associated with ancient artifacts on the lakebed. You can choose the component and frequency of your choice.

1. Describe the data pattern in general
2. Where are the anomalies that we are interested in?

Your answers

1.
2.

In [ ]:

```
# Plan view of the GEM-2 data
```

5. Processing

EM data are often contaminated by ambient noises. The GEM-2 data from the lake contain some strong noise that may distort useful signals. The GEM-2 data also contain the background EM responses from the lake water and lakebed. The background responses are not relevant if our goal is to find and characterize ancient artifacts.

Questions

Write a code to do the following

1. Extract the transects (profiles) that contain the interesting anomalies.
2. Suppress the high-frequency oscillation in the data from noise.
3. Remove the background responses.
4. Plot data profiles (use color and line style to indicate components and frequencies).

Your answers

In [ ]:

```
# Process GEM-2 data and make plots
```

6. Interpretation

The processed data contain no responses from the water and the lakebed as if the instrument and the sought targets are in a free space. This allows the simulation program you made above to be used for interpretation.

Questions

Adjust the parameters in your simulation program to match the data pattern on the processed data profiles. What information about the objects can you obtain?

Note:

1. The magnitude of data may be difficult to fit, so just focus on the data variation instead.
2. Matching all data using the three-loop model may be difficult, so just try your best.

Your answers

-

In [ ]:

```
# Run simulations to match the processed data patterns
# Plot and compare the observed field data and the simulated data
```

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7. Synthesis

Questions

1. Did you have any difficulties in matching the field data?
2. Are the anomalies in data from the same type of object? Why?
3. What conclusions about the existance and identification of ancient artifacts in the lake can you draw?

Your answers

1.
2.
3.

End of Worksheet

In [ ]:

```

```