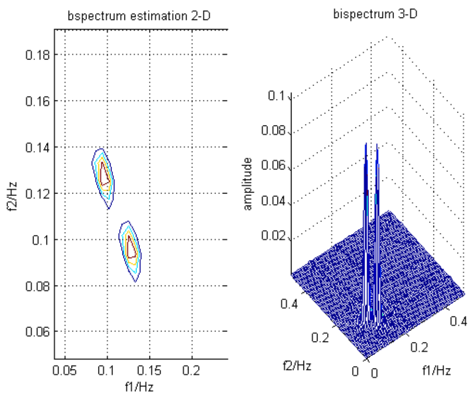
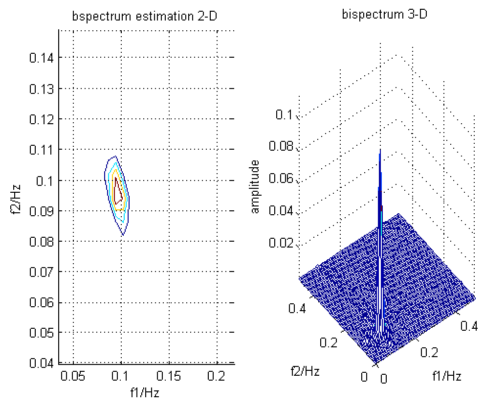
Bispectrum estimation

1. Sine signal Bispectrum estimation

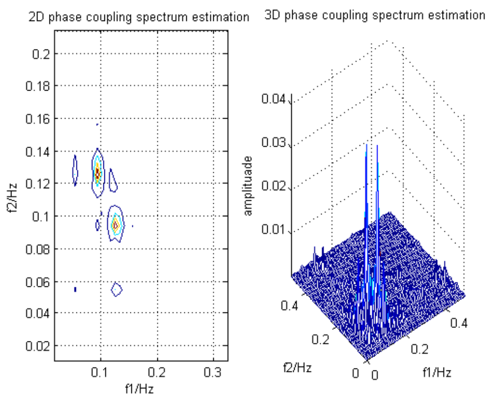
I set the f(x)=s1+s2+s3, s1, s2, s3 are the function of sine signal. I build two different f(x), one of f(x) has phase coupling and another one without phase coupling.

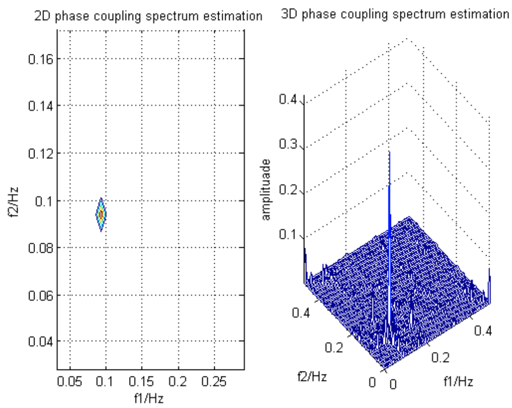
First of all, Let us see the bispectrum estimation. The figure on the left is coupling signal and the figure on the right does not have coupling signal.





Secondly, the difference between the both signals can be observed by using coupling bispectrum estimation.





Matlab code

```
clc
clear all
close all
N = 128*64;
n = [0:N-1];
Nfft = 128;
Fs = 1;
t = n/Fs;
f1 = 0.6/2/pi;
f2 = 0.8/2/pi;
f3 = f1+f2;
```

```
fy1 = pi/6;
fy2 = pi/3;
fy3 = fy1+fy2;
s1 = sin(2*pi*f1*t + fy1);
s2 = sin(2*pi*f2*t + fy2);
s3 = sin(2*pi*f3*t + fy3);
s = s1+s2+s3;



%Fs = 256;
Nfft = 128;
Fs = 1;
t = n/Fs;

% s = awgn(s,0,'measured');
noi = zeros(1,N);
%Gaussian white noise
noi = randn(1,N);
s = s + noi;

nys = 0;
if (nys == 1)
    Ln = length(s);
    Nd = [1 -0.5 0.7 0.1];
    Nc = [1 0.5 0.2];
    nd = length(Nd)-1;
    nc = length(Nc)-1;
    xik = zeros(nc,1);
    ek = zeros(nd,1);
    xi = randn(Ln,1);
    for kn=1:Ln
        e(kn)=-Nd(2:nd+1)*ek+Nc*[xi(kn);xik];
        for nn=nd:-1:2
            ek(nn)=ek(nn-1);
        end
        ek(1)=e(kn);

        for nn=nc:-1:2
            xik(nn)=xik(nn-1);
        end
        xik(1)=xi(kn);
    end
    s = s+0.25*e;
```

```
    noi = 0.25*e;
end
%signal to noise ratio
S_N= 10*log10(sum(abs(s).^2)/sum(abs(noi).^2));

[bspec,waxis] = bispecd(s,Nfft,1,64,32);
close all

%bispectrum signal£¨including coupling signal magnitude and phase£©
SS = Nfft/2+1;
waxis1 = waxis(SS:end)*Fs;
figure(1)
subplot(121)
bspec1 = bspec(SS:end,SS:end);
contour(waxis1,waxis1,abs(bspec1),4);
grid on
title('bspectrum estimation 2-D')
xlabel('f1/Hz'), ylabel('f2/Hz')
[X1,Y1] = meshgrid(waxis1,waxis1);
subplot(122)
mesh(X1,Y1,abs(bspec1))
axis tight
xlabel('f1/Hz')
ylabel('f2/Hz')
zlabel('amplitude')
title('bispectrum 3-D');


figure(2)
plot(t,s);
axis([0,800,-500,500])
xlabel('t')
ylabel('amplitude')
title('time-domain signal ')
P = 10*log10(abs(fft(s-mean(s),Nfft).^2)/(Nfft+1));
f = (0:length(P)-1)*Fs/(length(P));

figure(3)
plot(f(1:Nfft/2),P(1:Nfft/2));
title('PSD estimation');
xlabel('f/Hz')
ylabel('P/dB')

%Ïà¸ÉË«Æ×ÐÅºÅ£¨ÆµÂÊf1,f2·ÇÏßÐÔÏàÎ»ñîºÏ²úÉúµÄÄÜÁ¿ÔÚf1+f2´¦×ÜÄÜÁ¿ÖÐËùÕ¼
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µÄ±ÈÀý£©
mask = hankel([1:Nfft],[Nfft,1:Nfft-1] );   % the hankel mask (faster)
P_d = abs(fft(s,Nfft).^2)/(Nfft+1);
P_d = P_d';
bspec_d = zeros(Nfft,Nfft);
Bspec_d = zeros(Nfft,Nfft);
bspec_d  = (P_d * P_d') .*reshape(P_d(mask), Nfft, Nfft);


Bspec_d = abs(bspec)./sqrt(bspec_d);
figure(4)
subplot(121)
contour(waxis1,waxis1,abs(Bspec_d(SS:end,SS:end)),4);
grid on
title('2D phase coupling spectrum estimation')
xlabel('f1/Hz'), ylabel('f2/Hz')
subplot(122)
mesh(X1,Y1,abs(Bspec_d(SS:end,SS:end)));
axis tight
xlabel('f1/Hz')
ylabel('f2/Hz')
zlabel('amplituade')
title('3D phase coupling spectrum estimation');

figure(5)
subplot(211)
diagBspec = diag(fliplr(bspec));
plot(waxis1,abs(diagBspec(SS:end)));
grid on;
xlabel('f/Hz')
ylabel('amplitude')
title('diabspec')

subplot(212)
diagBspec_d = diag(fliplr(Bspec_d));
plot(waxis1,abs(diagBspec_d(SS:end)));
grid on;
xlabel('f/Hz')
ylabel('amplitude')
title('phase coupling dia bspec')
```
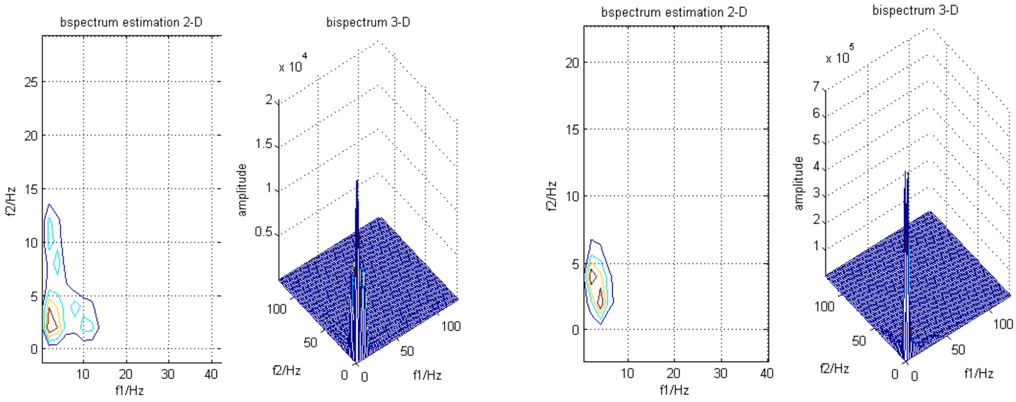
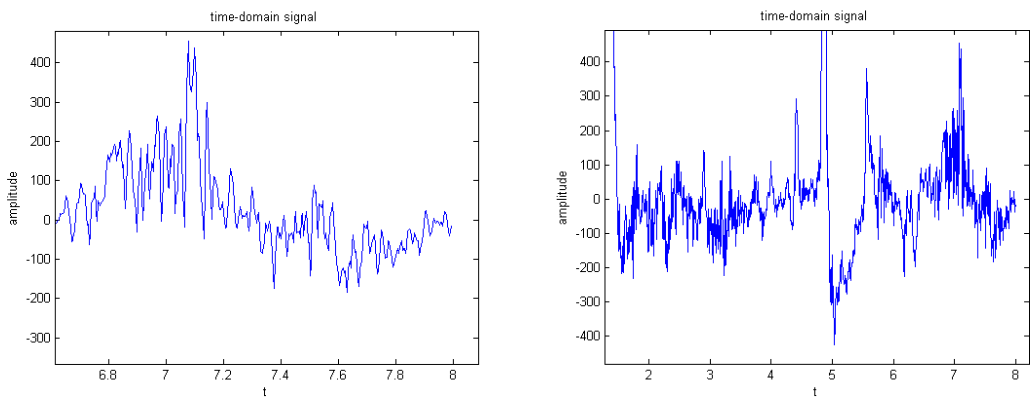

2. EEG signal Bispectrum estimation

In this part, we download one seizure signal and one non-seizure signal for further analysis. we extract the seizure part as the data (76800:78847) which length is 2048. First of all, we use function bspec() to build 2-D and 3-D spectrum estimation which is including the coupling signal magnitude and phase. The seizure signal and non-seizure signals' figures are

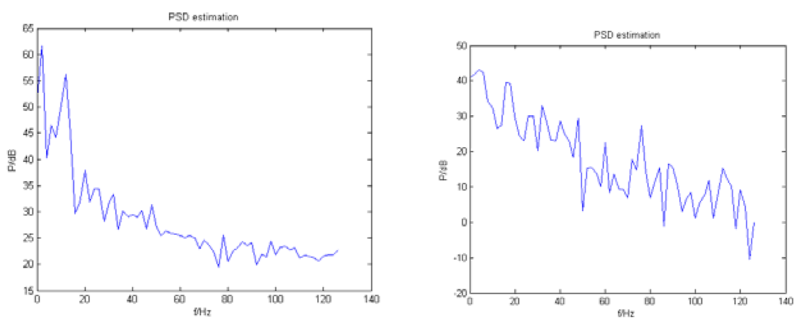


According to the figures above, we find that seizure signal got higher frequency and power by comparing to the non seizure signal.

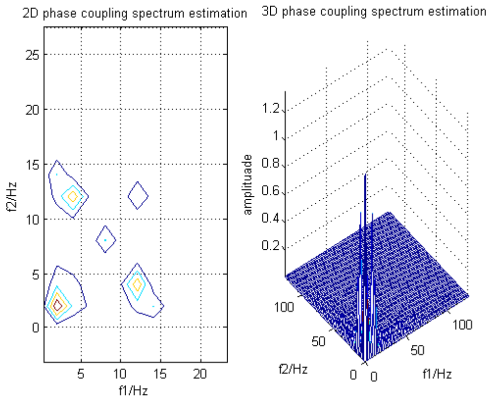
Secondly, we have a look in time-domain for both signals.

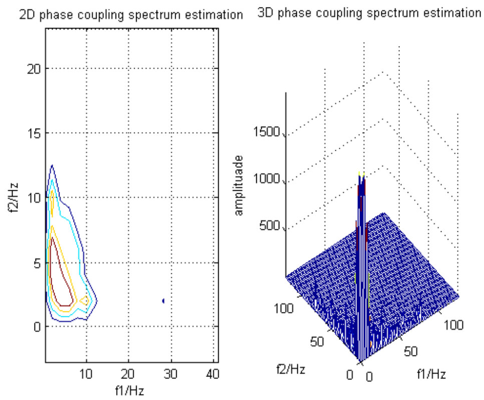


Thirdly, we compare the PSD of both signals. The peak of power of seizure signal is higher than non-seizure signal.

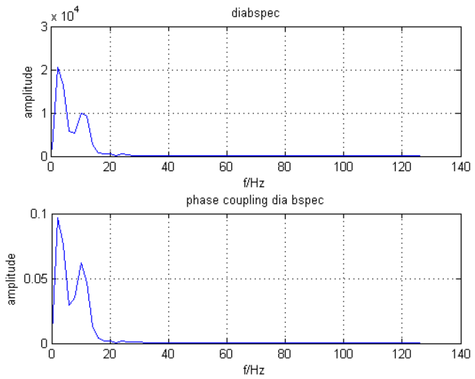


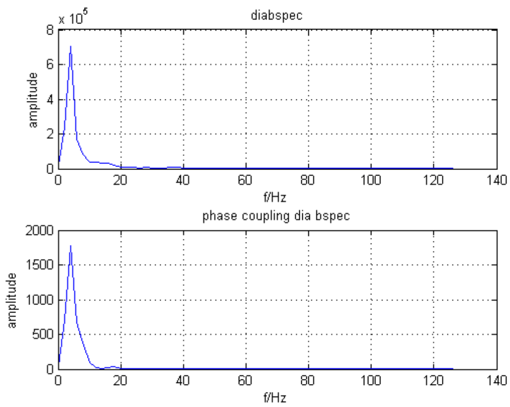
In addition, we can see phase coupling bispectrum analysis by using function bspec(). This part shows the proportion of each frequency in whole frequency. According to the spectrum below, we find that the coupling phase frequency is scattered in seizure signal and the non-seizures' is clustered.





Finally, we generate the figure by using function diagBspec() to compare both signals.





Matlag code

```
clc

close all
%load('chb01_01_edfm (1).mat')
load('chb01_03_edfm (5).mat')
x=val(76800:78847);
N = length(x);
n = [0:N-1];


%Fs = 256;
Nfft = 128;
Fs = 256;
t = n/Fs;
s = x
% s = awgn(s,0,'measured');
noi = zeros(1,N);
%Gaussian white noise
noi = randn(1,N);
s = s + noi;
```

```
nys = 0;
if (nys == 1)
   Ln = length(s);
   Nd = [1 -0.5 0.7 0.1];
   Nc = [1 0.5 0.2];
   nd = length(Nd)-1;
   nc = length(Nc)-1;
   xik = zeros(nc,1);
   ek = zeros(nd,1);
   xi = randn(Ln,1);
   for kn=1:Ln
      e(kn)=-Nd(2:nd+1)*ek+Nc*[xi(kn);xik];
      for nn=nd:-1:2
         ek(nn)=ek(nn-1);
      end
      ek(1)=e(kn);

      for nn=nc:-1:2
         xik(nn)=xik(nn-1);
      end
      xik(1)=xi(kn);
   end
   s = s+0.25*e;
   noi = 0.25*e;
end
%signal to noise ratio
S_N= 10*log10(sum(abs(s).^2)/sum(abs(noi).^2));

[bspec,waxis] = bispecd(s,Nfft,1,64,32);
close all

%bispectrum signal£¨including coupling signal magnitude and phase£©
SS = Nfft/2+1;
waxis1 = waxis(SS:end)*Fs;
figure(1)
subplot(121)
bspec1 = bspec(SS:end,SS:end);
contour(waxis1,waxis1,abs(bspec1),4);
grid on
title('bspectrum estimation 2-D')
xlabel('f1/Hz'), ylabel('f2/Hz')
[X1,Y1] = meshgrid(waxis1,waxis1);
subplot(122)
mesh(X1,Y1,abs(bspec1))
```

```
axis tight
xlabel('f1/Hz')
ylabel('f2/Hz')
zlabel('amplitude')
title('bispectrum 3-D');


figure(2)
plot(t,s);
axis([0,800,-500,500])
xlabel('t')
ylabel('amplitude')
title('time-domain signal ')
P = 10*log10(abs(fft(s-mean(s),Nfft).^2)/(Nfft+1));
f = (0:length(P)-1)*Fs/(length(P));

figure(3)
plot(f(1:Nfft/2),P(1:Nfft/2));
title('PSD estimation');
xlabel('f/Hz')
ylabel('P/dB')

%(phase coupling bispectrum estimation)
 mask = hankel([1:Nfft],[Nfft,1:Nfft-1] );   % the hankel mask (faster)
P_d = abs(fft(s,Nfft).^2)/(Nfft+1);
P_d = P_d';
bspec_d = zeros(Nfft,Nfft);
Bspec_d = zeros(Nfft,Nfft);
bspec_d  = (P_d * P_d') .*reshape(P_d(mask), Nfft, Nfft);


Bspec_d = abs(bspec)./sqrt(bspec_d);
figure(4)
subplot(121)
contour(waxis1,waxis1,abs(Bspec_d(SS:end,SS:end)),4);
grid on
title('2D phase coupling spectrum estimation')
xlabel('f1/Hz'), ylabel('f2/Hz')
subplot(122)
mesh(X1,Y1,abs(Bspec_d(SS:end,SS:end)));
axis tight
xlabel('f1/Hz')
ylabel('f2/Hz')
zlabel('amplituade')
```

```
title('3D phase coupling spectrum estimation');

figure(5)
subplot(211)
diagBspec = diag(fliplr(bspec));
plot(waxis1,abs(diagBspec(SS:end)));
grid on;
xlabel('f/Hz')
ylabel('amplitude')
title('diabspec')

subplot(212)
diagBspec_d = diag(fliplr(Bspec_d));
plot(waxis1,abs(diagBspec_d(SS:end)));
grid on;
xlabel('f/Hz')
ylabel('amplitude')
title('phase coupling dia bspec')
```