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THE STANDARD MODEL AND BEYOND

Einstein's discovery of general relativity in the last century enabled us for the first time in history to come up with a compelling, testable theory of the universe. The realization that the universe is expanding and was once much hotter and denser allows us to modernize the deep age-old questions "Why are we here?" and "How did we get here?" The updated versions are now "How did the elements form?", "Why is the universe so smooth?", and "How did galaxies form from this smooth origin?" Remarkably, these questions and many like them have quantitative answers, answers that can be found only by combining our knowledge of fundamental physics with our understanding of the conditions in the early universe. Even more remarkable, these answers can be tested against astronomical observations.

This chapter describes the idea of an expanding universe, without using the equations of general relativity. The success of the Big Bang rests on three observational pillars: the Hubble diagram exhibiting expansion; light element abundances which are in accord with Big Bang nucleosynthesis; and the blackbody radiation left over from the first few hundred thousand years, the cosmic microwave background. After introducing these pieces of evidence, I move beyond the *Standard Model* embodied by the three pillars. Developments in the last two decades of the 20^{th} century — both theoretical and observational — point to

- the existence of dark matter and perhaps even dark energy
- the need to understand the evolution of perturbations around the zero order, smooth universe
- inflation, the generator of these perturbations

The emergent picture of the early universe is summarized in the time line of Figure 1.15.

1.1 THE EXPANDING UNIVERSE

We have good evidence that the universe is expanding. This means that early in its history the distance between us and distant galaxies was smaller than it is today. It is convenient to describe this effect by introducing the scale factor a, whose present value is set to one. At earlier times a was smaller than it is today. We can picture space as a grid as in Figure 1.1 which expands uniformly as time evolves. Points on the grid maintain their coordinates, so the *comoving distance* between two points — which just measures the difference between coordinates — remains constant. However, the physical distance is proportional to the scale factor, and the physical distance does evolve with time.

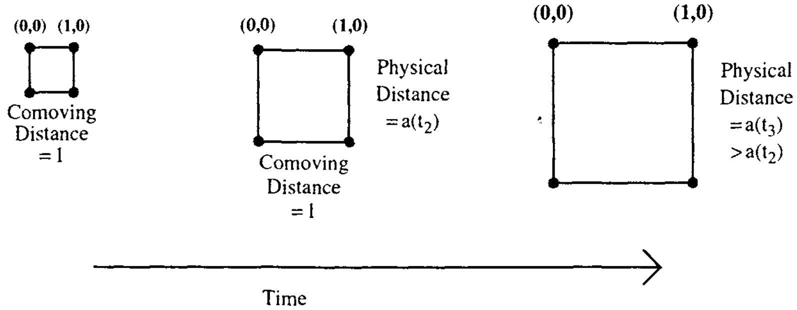


Figure 1.1. Expansion of the universe. The comoving distance between points on a hypothetical grid remains constant as the universe expands. The physical distance is proportional to the comoving distance times the scale factor, so it gets larger as time evolves.

In addition to the scale factor and its evolution, the smooth universe is characterized by one other parameter, its geometry. There are three possibilities: flat, open, or closed universes. These different possibilities are best understood by considering two freely traveling particles which start their journeys moving parallel to each other. A *flat* universe is Euclidean: the particles remain parallel as long as they travel freely. General relativity connects geometry to energy. Accordingly, a flat universe is one in which the energy density is equal to a critical value, which we will soon see is approximately 10^{-29} g cm^{-3}. If the density is higher than this value, then the universe is *closed*: gradually the initially parallel particles converge, just as all lines of constant longitude meet at the North and South Poles. The analogy of a closed universe to the surface of a sphere runs even deeper: both are said to have *positive curvature*, the former in three spatial dimensions and the latter in two. Finally, a low-density universe is *open*, so that the initially parallel paths diverge, as would two marbles rolling off a saddle.

To understand the history of the universe, we must determine the evolution of the scale factor a with cosmic time t. Again, general relativity provides the connection between this evolution and the energy in the universe. Figure 1.2 shows how the scale factor increases as the universe ages. Note that the dependence of a on t varies as the universe evolves. At early times, $a \propto t^{1/2}$ while at later times the dependence switches to $a \propto t^{2/3}$. How the scale factor varies with time is determined by the energy density in the universe. At early times, one form of energy, radiation, dominates, while at later times, nonrelativistic matter accounts for most of the energy density. In fact, one way to explore the energy content of the universe is to measure changes in the scale factor. We will see that, partly as a result of such exploration, we now believe that, very recently, a has stopped growing as $t^{2/3}$, a signal that a new form of energy has come to dominate the cosmological landscape.

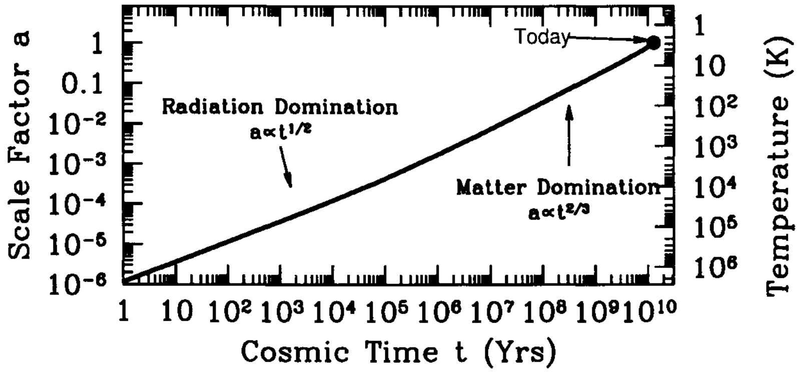


Figure 1.2. Evolution of the scale factor of the universe with cosmic time. When the universe was very young, radiation was the dominant component, and the scale factor increased as $t^{1/2}$. At later times, when matter came to dominate, this dependence switched to $t^{2/3}$. The right axis shows the corresponding temperature, today equal to 3K.

To quantify the change in the scale factor and its relation to the energy, it is first useful to define the Hubble rate

$$H(t) \equiv \frac{da/dt}{a} \tag{1.1}$$

which measures how rapidly the scale factor changes. For example, if the universe is flat and matter-dominated, so that $a \propto t^{2/3}$, then $H = (2/3)t^{-1}$. Thus a powerful test of this cosmology is to measure separately the Hubble rate today, H_0, and the age of the universe today. Here and throughout, subscript 0 denotes the value of a quantity today. In a flat, matter-dominated universe, the product $H_0 t_0$ should equal 2/3.

More generally, the evolution of the scale factor is determined by the Friedmann equation

$$H^2(t) = \frac{8\pi G}{3}\left[\rho(t) + \frac{\rho_{\rm cr} - \rho_0}{a^2(t)}\right] \tag{1.2}$$

where $\rho(t)$ is the energy density in the universe as a function of time with ρ_0 the present value. The *critical density*

$$\rho_{\rm cr} \equiv \frac{3H_0^2}{8\pi G} \tag{1.3}$$

where G is Newton's constant.

To use Einstein's equation, we must know how the energy density evolves with time. This turns out to be a complicated question because ρ in Eq. (1.2) is the sum of several different components, each of which scale differently with time. Consider first nonrelativistic matter. The energy of one such particle is equal to its rest mass energy, which remains constant with time. The energy density of many of these is therefore equal to the rest mass energy times the number density. When the scale factor was smaller, the densities were necessarily larger. Since number density is inversely proportional to volume, it should be proportional to a^{-3}. Therefore the energy density of matter scales as a^{-3}.

The photons which make up the cosmic microwave background (CMB) today have a well-measured temperature $T_0 = 2.725 \pm 0.002K$ (Mather *et al.*, 1999). A photon with an energy $k_B T_0$ today has a wavelength $\hbar c/k_B T_0$. Early on, when the scale factor was smaller than it is today, this wavelength would have been correspondingly smaller. Since the energy of a photon is inversely proportional to its wavelength, the photon energy would have been larger than today by a factor of $1/a$. This argument applied to the thermal bath of photons implies that the temperature of the plasma as a function of time is

$$T(t) = T_0/a(t). \tag{1.4}$$

At early times, then, the temperature was higher than it is today, as indicated in Figure 1.2. The energy density of radiation, the product of number density times average energy per particle, therefore scales as a^{-4}.

Evidence from distant supernovae (Chapter 2; Riess *et al.*, 1998; Perlmutter *et al.*, 1999) suggests that there may well be energy, *dark energy*, besides ordinary matter and radiation. One possibility is that this new form of energy remains constant with time, i.e., acts as a *cosmological constant*, a possibility first introduced (and later abandoned) by Einstein. Cosmologists have explored other forms though, many of which behave very differently from the cosmological constant. We will see more of this in later chapters.

Equation (1.2) allows for the possibility that the universe is not flat: if it were flat, the sum of all the energy densities today would equal the critical density, and the last term in Eq. (1.2) would vanish. If the universe is not flat, the *curvature* energy scales as $1/a^2$. In most of this book we will work within the context of a flat universe. In such a universe, the evolution of perturbations is much easier to calculate than in open or closed universes. Further, there are several persuasive arguments, both theoretical and more recently observational, which strongly support the flatness of the universe. More on this in Chapters 2 and 8.

Figure 1.3 illustrates how the different terms in Eq. (1.2) vary with the scale factor. While today matter, and possibly a cosmological constant dominate the landscape, early on, because of the a^{-4} scaling, radiation was the dominant constituent of the universe.

Let's introduce some numbers. The expansion rate is a measure of how fast the universe is expanding, determined (Section 1.2) by measuring the velocities of distant galaxies and dividing by their distance from us. So the expansion is often

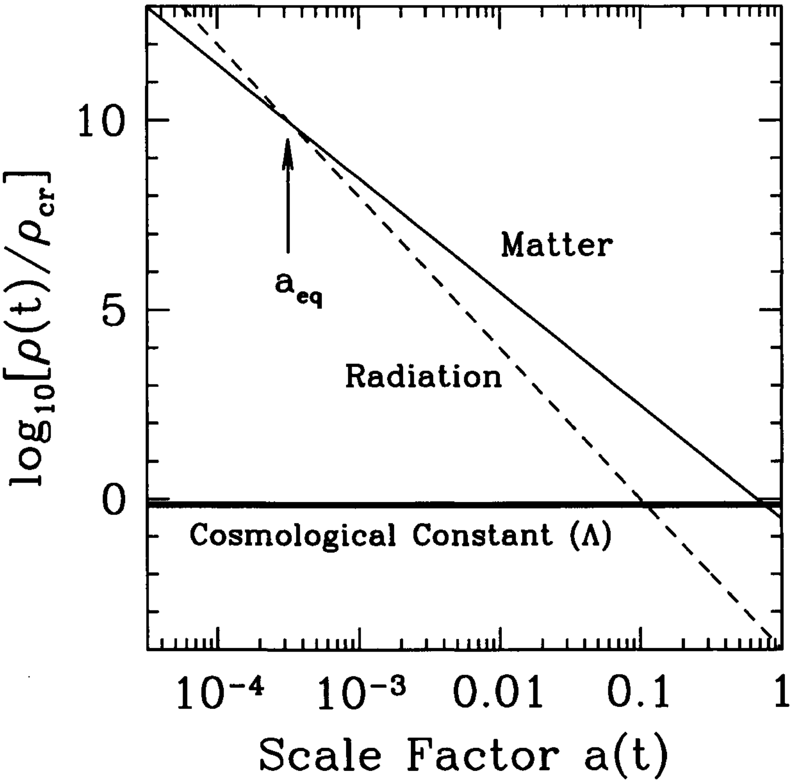


Figure 1.3. Energy density vs scale factor for different constituents of a flat universe. Shown are nonrelativistic matter, radiation, and a cosmological constant. All are in units of the critical density today. Even though matter and cosmological constant dominate today, at early times, the radiation density was largest. The epoch at which matter and radiation are equal is a_{eq}.

written in units of velocity per distance. Present measures of the Hubble rate are parameterized by h defined via

$$\begin{aligned} H_0 &= 100h \text{ km sec}^{-1}\,\text{Mpc}^{-1} \\ &= \frac{h}{0.98 \times 10^{10}\,\text{years}} = 2.133 \times 10^{-33} h \text{ eV}/\hbar \end{aligned} \tag{1.5}$$

where h has nothing to do with Planck's constant $\hbar$. The astronomical length scale of a megaparsec (Mpc) is equal to 3.0856×10^{24} cm. Current measurements set $h = 0.72 \pm 0.08$ (Freedman *et al.*, 2001).

The predicted age for a flat, matter-dominated universe, $(2/3)H_0^{-1}$, is then of order 8 to 10 Gyr. The current best estimate for the age of the universe is 12.6 Gyr, with a 95% confidence level lower limit of 10.4 Gyr (Krauss and Chaboyer, 2001), so this test suggests that a flat, matter-dominated universe is barely viable. You will show in Exercise 2 that the age of the universe with a cosmological constant is larger (for fixed h); in fact one of the original arguments in favor of this excess energy was to make the universe older.

Newton's constant in Eq. (1.3) is equal to $6.67 \times 10^{-8}\text{cm}^3\text{g}^{-1}\text{sec}^{-2}$. This, together with Eq. (1.5), enables us to get a numerical value for the critical density:

$$\rho_{\text{cr}} = 1.88h^2 \times 10^{-29}\text{g cm}^{-3}. \tag{1.6}$$

An important ramification of the higher densities in the past is that the rates for particles to interact with each other, which scale as the density, were also much higher early on. Figure 1.4 shows some important rates as a function of the scale

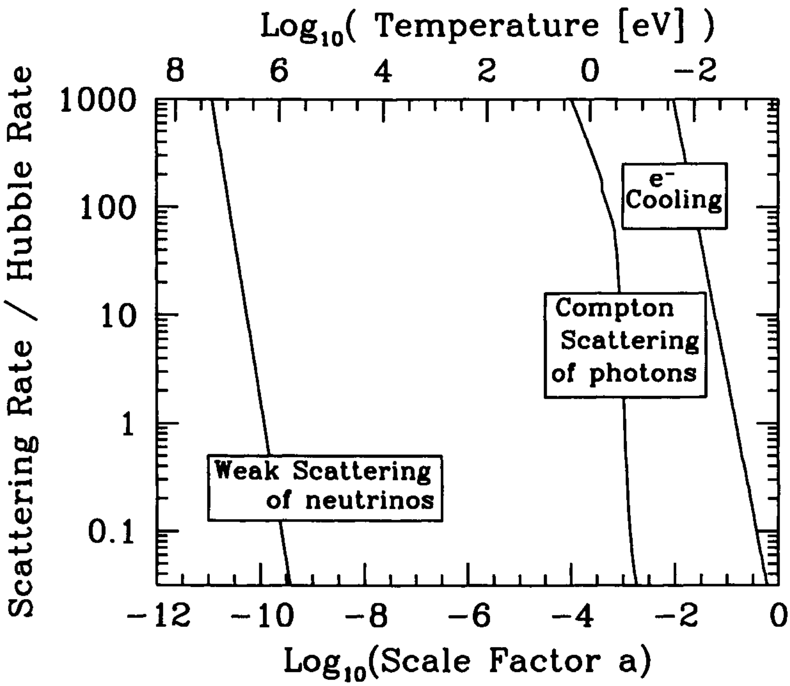


Figure 1.4. Rates as a function of the scale factor. When a given rate becomes smaller than the expansion rate H, that reaction falls out of equilibrium. Top scale gives (k_B times) the temperature of the universe, an indication of the typical kinetic energy per particle.

factor. For example, when the temperature of the universe was greater than several MeV/k_B, the rate for electrons and neutrinos to scatter was larger than the expansion rate. Thus, before the universe could double in size, a neutrino scattered many times off background electrons. All these scatterings brought the neutrinos into equilibrium with the rest of the cosmic plasma. This is but one example of a very general, profound fact: if a particle scatters with a rate greater than the expansion rate, that particle stays in equilibrium. Since rates were typically quite large, the early universe was a relatively simple environment: not only was it very smooth, but many of its constituents were in equilibrium. Chapter 2 explores some manifestations of the equilibrium conditions, while Chapter 3 touches on several cases where equilibrium could not be maintained because the reaction rates dropped beneath the expansion rate.

1.2 THE HUBBLE DIAGRAM

If the universe is expanding as depicted in Figure 1.1, then galaxies should be moving away from each other. We should therefore see galaxies receding from us. Recall that the wavelength of light or sound emitted from a receding object is stretched out so that the observed wavelength is larger than the emitted one. It is convenient to define this stretching factor as the redshift z:

$$1 + z \equiv \frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} = \frac{1}{a}. \tag{1.7}$$

For low redshifts, the standard Doppler formula applies and $z \simeq \frac{v}{c}$. So a measurement of the amount by which absorption and/or emission lines are redshifted is a direct measure of how fast the structures in which they reside are receding from us.

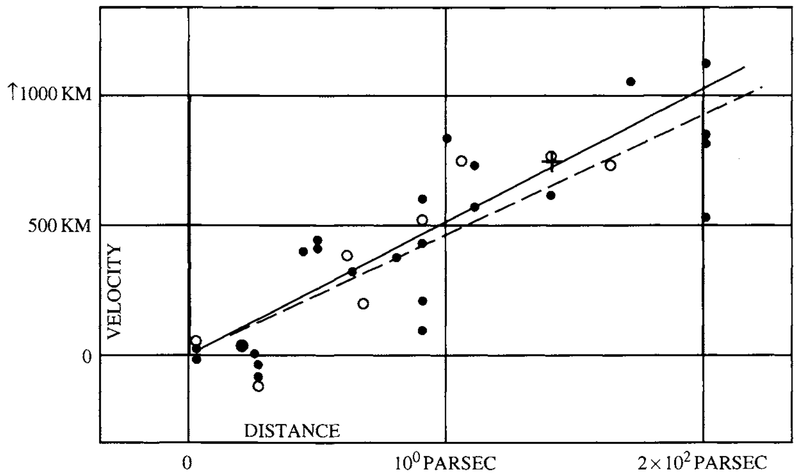


Figure 1.5. The original Hubble diagram (Hubble, 1929). Velocities of distant galaxies (units should be km sec^{-1}) are plotted vs distance (units should be Mpc). Solid (dashed) line is the best fit to the filled (open) points which are corrected (uncorrected) for the sun's motion.

Hubble (1929) first found that distant galaxies are in fact receding from us. He also noticed the trend that the velocity increases with distance. This is exactly what we expect in an expanding universe, for the physical distance between two galaxies is $d = ax$ where x is the comoving distance. In the absence of any comoving motion ($\dot{x} = 0$, no *peculiar* velocity) the relative velocity $v = \dot{d}$ is therefore equal to $\dot{a}x = Hd$. Therefore, velocity should increase linearly with distance (at least at low redshift) with a slope given by H, the Hubble constant. Hubble's Hubble constant can be easily extracted from Figure 1.5. It is simply $H = 1000/2$ km sec^{-1} Mpc^{-1}, almost a factor of 10 higher than current estimates. Also notice that Hubble's data went out to redshift $z = 1000$ km sec$^{-1}/c \simeq 0.003$.

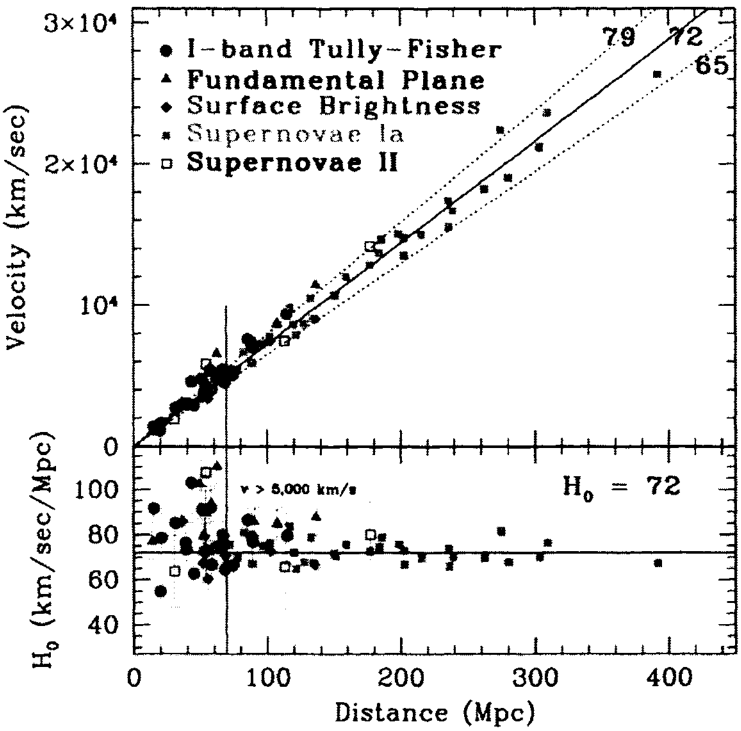


Figure 1.6. Hubble diagram from the Hubble Space Telescope Key Project (Freedman *et al.*, 2001) using five different measures of distance. Bottom panel shows H_0 vs distance with the horizontal line equal to the best fit value of 72 km sec^{-1} Mpc^{-1}.

The Hubble diagram is still the most direct evidence we have that the universe is expanding. Current incarnations use the same principle as the original: find the distance and the redshift of distant objects. Measuring redshifts is straightforward; the hard part is determining distances for objects of unknown intrinsic brightness. One of the most popular techniques is to try to find a *standard candle*, a class of objects which have the same intrinsic brightness. Any difference between the apparent brightness of two such objects then is a result of their different distances from us. This method is typically generalized to find a correlation between an observable and intrinsic brightness. For example, Cepheid variables are stars for which intrinsic brightness is tightly related to period. The Hubble Space Telescope measured the periods of thousands of Cepheid variables in galaxies as far away as 20 Mpc. With distances to these galaxies fixed, five different distance measures were used to go much further, as far away as 400 Mpc. Figure 1.6 shows that all of these five indicators agree with one another and have converged on $H_0 = 72$ km sec^{-1} Mpc^{-1} with 10% errors.

As shown in Figure 1.6 the standard candle that can be seen at largest distances is a Type Ia supernova. Since they are so bright, supernovae can be used to extend the Hubble diagram out to very large redshifts (the current record is of order $z \simeq 1.7$), a regime where the simple Doppler law ceases to work. Figure 1.7 shows a recent Hubble diagram using very these very distant objects. In the next chapter, we will derive the correct expression for the distance (in this case the *luminosity* distance) as a function of redshift. For now, I simply point out that this expression depends on the energy content of the universe. The three curves in Figure 1.7 depict three different possibilities: flat matter dominated; open; and flat with a cosmological constant (Λ). The high-redshift data are now good enough to distinguish among these possibilities, strongly disfavoring the previously favored flat, matter-dominated universe. The current best fit is a universe with about 70% of the energy in the form of a cosmological constant, or some other form of dark energy. More on this in Chapter 2.

1.3 BIG BANG NUCLEOSYNTHESIS

When the universe was much hotter and denser, when the temperature of order an MeV/k_B, there were no neutral atoms or even bound nuclei. The vast amounts of radiation in such a hot environment ensured that any atom or nucleus produced would be immediately destroyed by a high energy photon. As the universe cooled well below the binding energies of typical nuclei, light elements began to form. Knowing the conditions of the early universe and the relevant nuclear cross-sections, we can calculate the expected primordial abundances of all the elements (Chapter 3).

Figure 1.8 shows the predictions of Big Bang Nucleosynthesis (BBN) for the light element abundances[1]. The boxes and arrows in Figure 1.8 show the current estimates for the light element abundances. These are consistent with the predictions, and this consistency test provides yet another ringing confirmation of the Big Bang. The measurements do even more though. The theoretical predictions, which we will explore in detail in Chapter 3, depend on the density of protons and neutrons at the time of nucleosynthesis. The combined proton plus neutron density

[1]Recall nuclear notation: The 4 in ^{4}He refers to the total number of nucleons (protons and neutrons). So ^{4}He has two neutrons and two protons, while ^{3}He has two protons and one neutron. See the box on page 63 for more details.

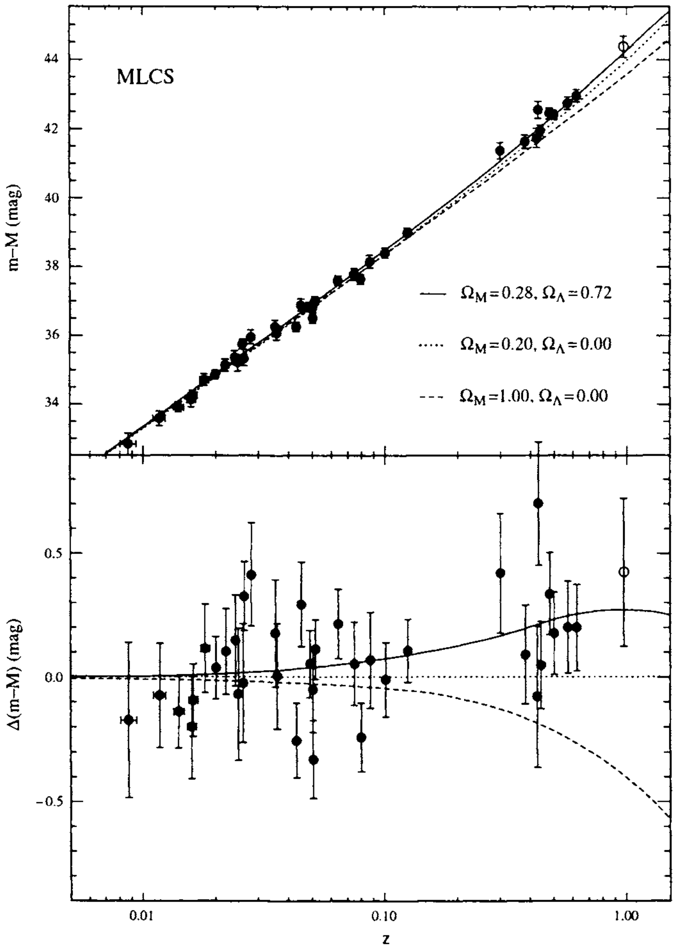


Figure 1.7. Hubble diagram from distant Type Ia supernovae. Top panel shows apparent magnitude (an indicator of the distance) vs redshift. Lines show the predictions for different energy contents in the universe, with Ω_M the ratio of energy density today in matter compared to the critical density and Ω_Λ the ratio of energy density in a cosmological constant to the critical density. Bottom panel plots the residuals, making it clear that the high-redshift supernovae favor a Λ-dominated universe over a matter-dominated one.

is called the *baryon* density since both protons and neutrons have baryon number one and these are the only baryons around at the time. Thus, BBN gives us a way of measuring the baryon density in the universe. Since we know how those densities scale as the universe evolves (they fall as a^{-3}), we can turn the measurements of light element abundances into measures of the baryon density today.

In particular, the measurement of primordial deuterium pins down the baryon density extremely accurately to only a few percent of the critical density. Ordi-

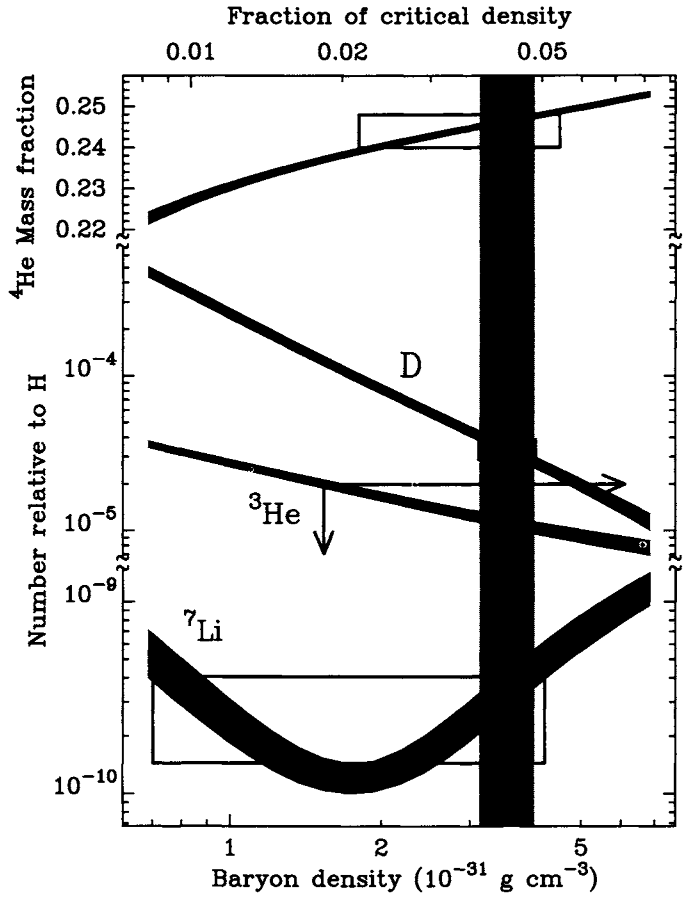


Figure 1.8. Constraint on the baryon density from Big Bang Nucleosynthesis (Burles, Nollett, and Turner, 1999). Predictions are shown for four light elements—^{4}He, deuterium, ^{3}He, and lithium—spanning a range of 10 orders of magnitude. The solid vertical band is fixed by measurements of primordial deuterium. The boxes are the observations; there is only an upper limit on the primordial abundance of ^{3}He.

nary matter (baryons) contributes at most 5% of the critical density. Since the total matter density today is almost certainly larger than this—direct estimates give values of order 20–30%— nucleosynthesis provides a compelling argument for nonbaryonic dark matter.

The deuterium measurements (Burles and Tytler, 1998) are the new developments in the field. These measurements are so exciting because they explore the deuterium abundance at redshifts of order 3–4, well before much processing could have altered the primordial abundances. Figure 1.9 shows one such detection. The basic idea is that light from distant QSOs is absorbed by intervening neutral hydrogen systems. The key absorption feature arises from transition from the ($n = 1$)

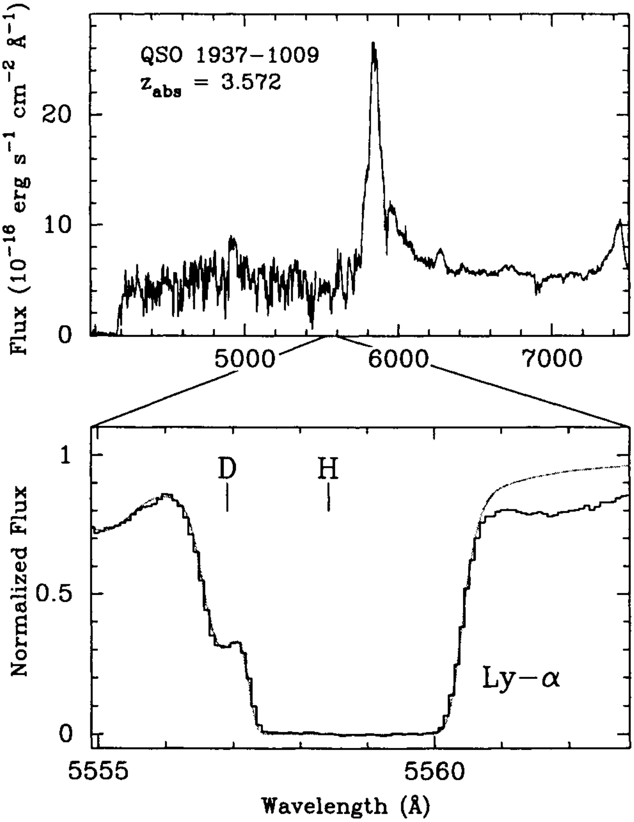


Figure 1.9. Spectrum from a distant QSO (Burles, Nollett, and Turner, 1999). Absorption of photons with rest wavelength 1216 Å corresponding to the $n = 1$ to $n = 2$ state of hydrogen is redshifted up to $1216\,(1 + 3.572)$ Å. Bottom panel provides details of the spectrum in this range, with the the presence of deuterium clearly evident.

ground state of hydrogen to the first excited state ($n = 2$), requiring a photon with wavelength $\lambda = 1215.7$ Å. Since photons are absorbed when exciting hydrogen in this fashion, there is a trough in the spectrum at $\lambda = 1215.7$ Å, redshifted by a factor of $1 + z$. The corresponding line from deuterium should be (i) shifted over by $0.33\,(1+z)$ Å(see Exercise 3) and (ii) much less damped since there is much less deuterium. Figure 1.9 shows just such a system; there are now half a dozen with detections precisely in the neighborhood shown in Figure 1.8. Note that the steep decline in deuterium as a function of baryon density helps here: even relatively large errors in D measurements translate into small errors on the baryon density.

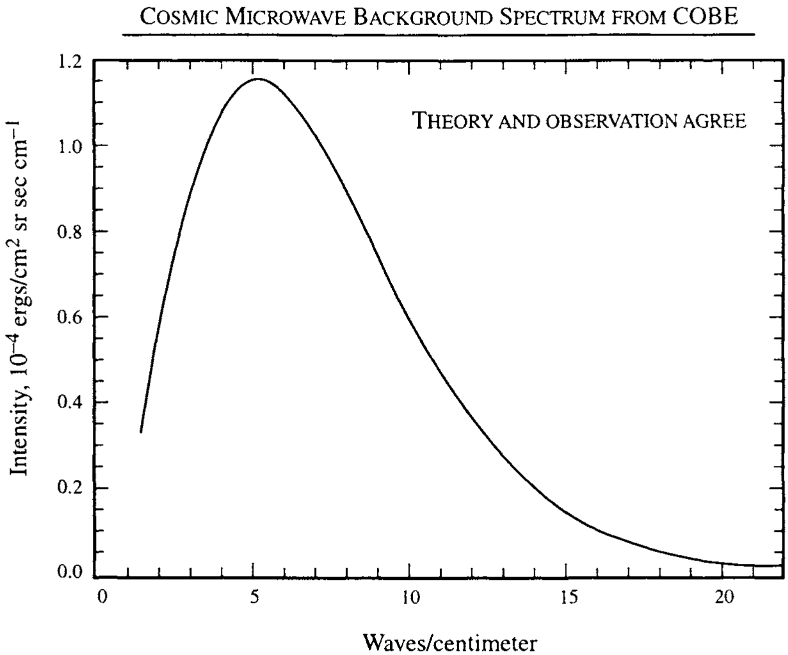


Figure 1.10. Intensity of cosmic microwave radiation as a function of wavenumber from Far InfraRed Absolute Spectrophotometer (FIRAS) (Mather *et al.*, 1994), an instrument on the COBE satellite. Hidden in the theoretical blackbody curve are dozens of measured points, all of which have uncertainties smaller than the thickness of the curve!

1.4 THE COSMIC MICROWAVE BACKGROUND

The CMB offers us a look at the universe when it was only 300,000 years old. The photons in the cosmic microwave background last scattered off electrons at redshift 1100; since then they have traveled freely through space. When we observe them today, they literally come from the earliest moments of time. They are therefore the most powerful probes of the early universe. We will spend an inordinate amount of time in this book working through the details of what happened before the epoch of last scattering and also developing the mathematics of the freestreaming process since then. A crucial fact about this history, though, is that the collisions with electrons before last scattering ensured that the photons were in equilibrium. That is, they should have a blackbody spectrum.

The specific intensity of a gas of photons with a blackbody spectrum is

$$I_\nu = \frac{4\pi\hbar\nu^3/c^2}{\exp\{2\pi\hbar\nu/k_BT\} - 1}. \tag{1.8}$$

Figure 1.10 shows the remarkable agreement between this prediction (see Exercise 4) of Big Bang cosmology and the observations by the FIRAS instrument aboard the

COBE spacecraft. We have been told[2] that detection of the 3K background by Penzias and Wilson in the mid-1960s was sufficient evidence to decide the controversy in favor of the Big Bang over the Steady State universe. Penzias and Wilson, though, measured the radiation at just one wavelength. If even their one-wavelength result was enough to tip the scales, the current data depicted in Figure 1.10 should send skeptics from the pages of physics journals to the far reaches of radical Internet chat groups.

The most important fact we learned from our first 25 years of surveying the CMB was that the early universe was very smooth. No anisotropies were detected in the CMB. This period, while undoubtedly frustrating for observers searching for anisotropies, solidified the view of a smooth Big Bang. We are now moving on. We have discovered anisotropies in the CMB, indicating that the early universe was not completely smooth. There were small perturbations in the cosmic plasma. To understand these, we must go beyond the Standard Model.

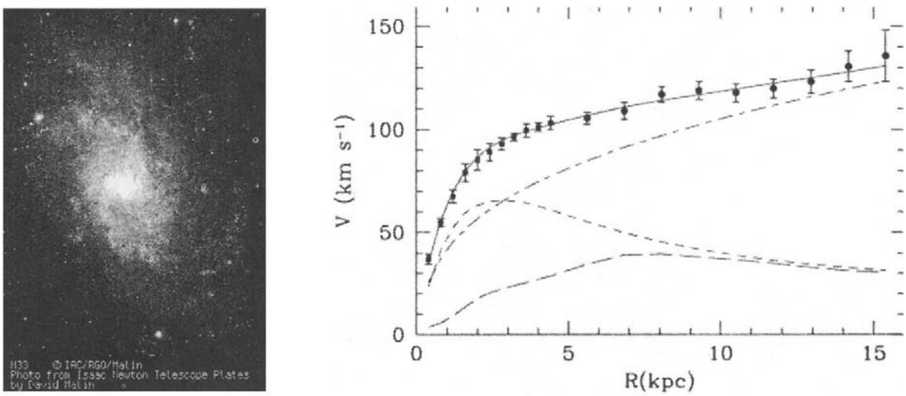
1.5 BEYOND THE STANDARD MODEL

While the three pillars put the Big Bang model on firm footing, other observations cry out for more details. I hinted above at one of these, the notion that there must be nonbaryonic matter in the universe. *Dark matter* is a familiar concept to astronomers; the first suggestion was put forth by Zwicky in 1933(!). Figure 1.11 illustrates the way dark matter can be found in galaxies, with the use of rotation curves probing the gravitational field. Indeed, a mismatch between the matter inferred from gravity and that we can see exists on almost all observable scales.

Because of the limits inferred from Big Bang nucleosynthesis, the dark matter, or at least an appreciable fraction of it, must be nonbaryonic. What is this new form of matter? And how did it form in the early universe? The most popular idea currently is that the dark matter consists of elementary particles produced in the earliest moments of the Big Bang. In Chapter 3, we will explore this possibility in detail, arguing that dark matter was likely produced when the temperature of the universe was of order hundreds of GeV/k_B. As we will see, the hypothesis that dark matter consists of fundamental relics from the early universe may soon be tested experimentally.

The last decades of the 20^{th} century saw a number of large surveys of galaxies designed to measure structure in the universe. These culminated in two large surveys, the Sloan Digital Sky Survey and the Two Degree Field Galaxy (Figure 1.12) Redshift Survey, which between them will compile the redshifts of, and hence the distances to, a million galaxies. Galaxies in Figure 1.12 are clearly not distributed randomly: the universe has structure on large scales. To understand this structure, we must go beyond the Standard Model not only by including dark matter, but also by allowing for deviations from smoothness. We must develop the tools to study

[2]For a fascinating first-hand account of the history of the discovery of the CMB, see Chapter 1 of Partridge (1995).



(a) (b)

Figure 1.11. (a) Image of spiral galaxy M33. The inner brightest region has a radius of several kpc. (b) Rotation curve for M33 (Corbelli and Salucci, 2000). Points with error bars come from the 21-cm line of neutral hydrogen. Solid line is a model fitting the data. Different contributions to the total rotation curve are: dark matter halo (dot-dashed line), stellar disk (short dashed line), and gas (long dashed line). At large radii, dark matter dominates.

perturbations around the smooth background of the Standard Model. We will see in Chapters 4 and 5 that this is straightforward in theory, as long as the perturbations remain small.

The best ways to learn about the evolution of structure and to compare theory with observations are to look at anisotropies in the CMB and at how matter is distributed on large scales. To compare theory with observations, we must at first try to avoid scales dominated by nonlinearities. As an extreme example, we can never hope to understand cosmology by carefully examining rock formations on Earth. The intermediate steps — collapse of matter into a galaxy; molecular cooling; star formation; planetary formation; etc. — are much too complicated to allow comparison between linear theory and observations. While perturbations to the matter on small scales (less than about 10 Mpc) have grown nonlinear, large-scale perturbations are still small. So they have been processed much less than the corresponding small-scale structure. Similarly, anisotropies in the CMB have remained small because the photons that make up the CMB do not clump.

Identifying large-scale structure and the CMB as the two most promising areas of study solves just one issue. Another very important challenge is to understand how to characterize these distributions so that theory can be compared to experiment. It is one thing to look at a map and quite another to quantitative tests of cosmological models. To make such tests, it is often useful to take the Fourier transform of the distribution in question; as we will see, working in Fourier space makes it easier to separate large from small scales. The most important statistic in the cases of

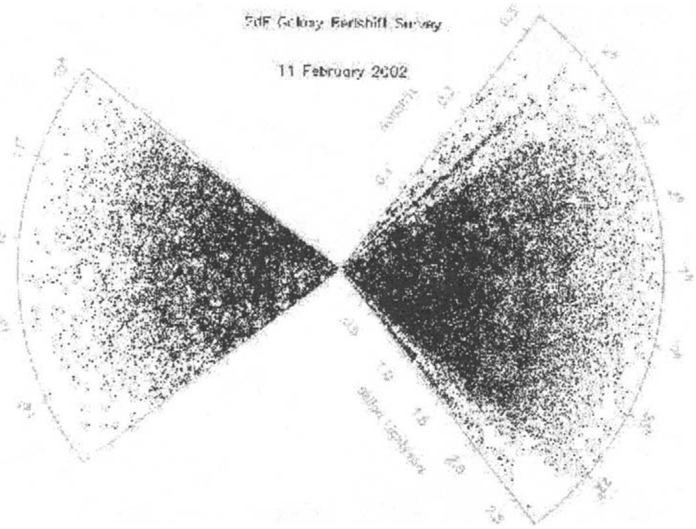


Figure 1.12. Distribution of galaxies in the Two Degree Field Galaxy Redshift Survey (2dF) (Colless *et al.*, 2001). By the end of the survey, redshifts for 250,000 galaxies will have been obtained. As shown here, they probe structure in the universe out to $z = 0.3$, corresponding to distances up to $1000\,h^{-1}$ Mpc away from us (we are located at the center). See color Plate 1.12.

both the CMB and large-scale structure is the *two-point function*, called the *power spectrum* in Fourier space. If the mean density of the galaxies is $\bar{n}$, then we can characterize the inhomogeneities with $\delta(\vec{x}) = (n(\vec{x}) - \bar{n})/\bar{n}$, or its Fourier transform $\tilde{\delta}(\vec{k})$. The power spectrum $P(k)$ is defined via

$$\langle \tilde{\delta}(\vec{k})\tilde{\delta}(\vec{k}')\rangle = (2\pi)^3 P(k)\delta^3(\vec{k} - \vec{k}'). \tag{1.9}$$

Here the angular brackets denote an average over the whole distribution, and $\delta^3()$ is the Dirac delta function which constrains $\vec{k} = \vec{k}'$. The details aside, Eq. (1.9) indicates that the power spectrum is the spread, or the variance, in the distribution. If there are lots of very under- and overdense regions, the power spectrum will be large, whereas it is small if the distribution is smooth. Figure 1.13 shows the power spectrum of the galaxy distribution. Since the power spectrum has dimensions of k^{-3} or (length)^3, Figure 1.13 shows the combination $k^3P(k)/2\pi^2$, a dimensionless number which is a good indication of the clumpiness on scale k.

The best measure of anisotropies in the CMB is also the two-point function of the temperature distribution. There is a subtle technical difference between the two power spectra which are used to measure the galaxy distribution and the CMB, though. The difference arises because the CMB temperature is a two-dimensional field, measured everywhere on the sky (i.e., with two angular coordinates). Instead of Fourier transforming the CMB temperature, then, one typically expands it in

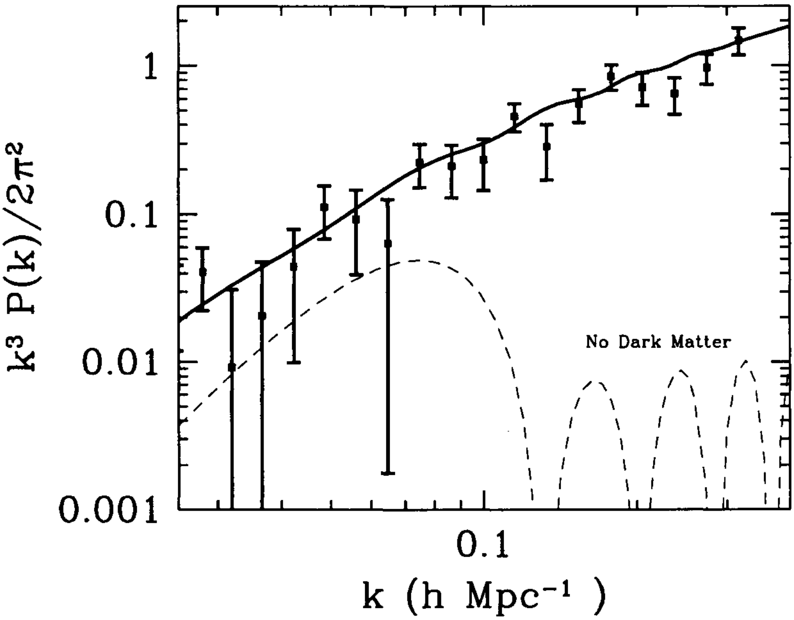


Figure 1.13. The variance $\Delta^2 \equiv k^3P(k)/2\pi^2$ of the Fourier transform of the galaxy distribution as a function of scale. On large scales, the variance is smaller than unity, so the distribution is smooth. The solid line is the theoretical prediction from a model in which the universe contains dark matter, a cosmological constant, with perturbations generated by inflation. The dashed line is a theory with only baryons and no dark matter. Data come from the PSCz survey (Saunders *et al.*, 2000) as analyzed by Hamilton and Tegmark (2001).

spherical harmonics, a basis more appropriate for a 2D field on the surface of a sphere. Therefore the two-point function of the CMB is a function of multipole moment l, not wave number k. Figure 1.14 shows the measurements of dozens of groups since 1992, when COBE first discovered large-angle (low l in the plot) anisotropies.

Figures 1.13 and 1.14 both have theoretical curves in them which appear to agree well with the data. The main goal of much of this book is to develop a first-principles understanding of these theoretical predictions. Indeed, understanding the development of structure in the universe has become a major goal of most cosmologists today. Note that this second aspect of cosmology beyond the Standard Model reinforces the first: i.e., observations of structure in the universe lead to the conclusion that there must be dark matter. In particular, the dashed curve in Figure 1.13 is the prediction of a model with baryons only, with no dark matter. The inhomogeneities expected in this model (when normalized to the CMB observations) are far too small. In Chapter 7, we will come to understand the reason why a baryon-only universe would be so smooth. For now, though, the message is clear: Dark matter is needed not only to explain rotation curves of galaxies but to explain

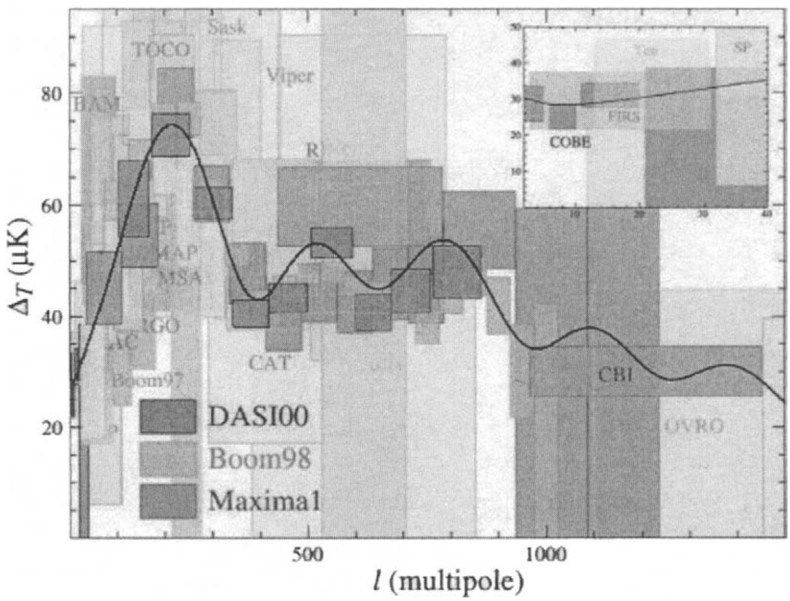


Figure 1.14. Anisotropies in the CMB predicted by the theory of inflation compared with observations. x-axis is multipole moment (e.g., $l = 1$ is the dipole, $l = 2$ the quadrupole) so that large angular scales correspond to low l; y-axis is the root mean square anisotropy (the square root of the two-point function) as a function of scale. The characteristic signature of inflation is the series of peaks and troughs, a signature which has been verified by experiment. See color Plate 1.14.

structure in the universe at large!

While trying to understand the evolution of structure in the universe, we will be forced to confront the question of what generated the initial conditions, the primordial perturbations that were the seeds for this structure. This will lead us to a third important aspect of cosmology beyond the Standard Model: the theory of inflation. Chapter 6 introduces this fascinating proposal, that the universe expanded exponentially fast when it was but 10^{-35} sec old. Until recently, there was little evidence for inflation. It survived as a viable theory mainly because of its aesthetic appeal. The discoveries of the past several years have changed this. They have by and large confirmed some of the basic predictions of inflation. Most notably, this theory makes concrete predictions for the initial conditions, predictions that have observable consequences today. For me, the most profound and exciting discovery in cosmology has been the observation of anisotropies in the CMB, with a characteristic pattern predicted by inflation.

The theory encompassing all these Beyond the Standard Model ingredients — dark matter plus evolution of structure plus inflation — is called *Cold Dark Matter*, or CDM. The "Cold" part of this moniker comes from requiring the dark matter particles to be able to clump efficiently in the early universe. If they are *hot* instead, i.e., have large pressure, structure will not form at the appropriate levels.

1.6 SUMMARY

By way of summarizing the features of an expanding universe that I have outlined above and that we will explore in great detail in the coming chapters, let's construct a time line. We can characterize any epoch in the universe by the time since the Big Bang; by the value of the scale factor at that time; or by the temperature of the cosmic plasma. For example, today, $a = 1$; $t \simeq 14$ billion years; and $T = 2.725\text{K} = 2.35 \times 10^{-4}$ eV$/k_B$. Figure 1.15 shows a time line of the universe using both time and temperature as markers. The milestones indicated on the time line range from those about which we are quite certain (e.g., nucleosynthesis and the CMB) to those that are more speculative (e.g., dark matter production, inflation, and dark energy today).

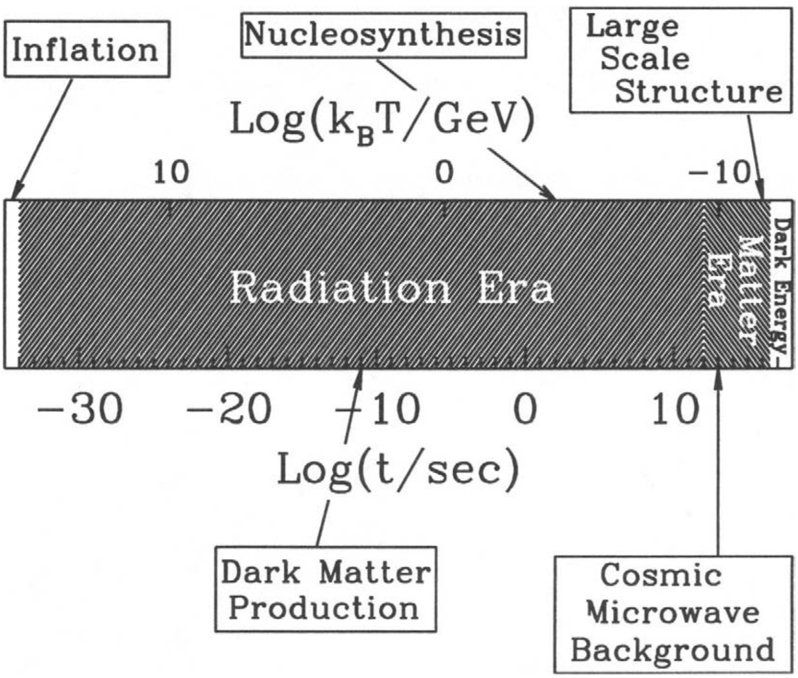


Figure 1.15. A history of the universe. Any epoch can be associated with either temperature (top scale) or time (bottom scale).

The time line in Figure 1.15 shows the dominant component of the universe at various times. Early on, most of the energy in the universe was in the form of radiation. Eventually, since the energy of a relativistic particle falls as $1/a$ while that of nonrelativistic matter remains constant at m, matter overtook radiation.

At relatively recent times, the universe appears to have become dominated not by matter, but by some dark energy, whose density remains relatively constant with time. The evidence for this unexplained form of energy is new and certainly not conclusive, but it is very suggestive.

The classical results in cosmology can be understood in the context of a smooth universe. Light elements formed when the universe was several minutes old, and the CMB decoupled from matter at a temperature of order $k_BT \sim 1/4$ eV. Heavy elementary particles may make up the dark matter in the universe; if they do, their abundance was fixed at very high temperatures of order $k_BT \sim 100$ GeV.

We will be mostly interested in this book in the perturbations around the smooth universe. The early end of the time line allows for a brief period of inflation, during which primordial perturbations were produced. These small perturbations began to grow when the universe became dominated by matter. The dark matter grew more and more clumpy, simply because of the attractive nature of gravity. An overdensity of dark matter of 1 part in 1000 when the temperature was 1 eV grew to 1 part in 100 by the time the temperature dropped to 0.1 eV. Eventually, at relatively recent times, perturbations in the matter ceased to be small; they became the nonlinear structure we see today. Anisotropies in the CMB today tell us what the universe looked like when it was several hundred thousand years old, so they are wonderful probes of the perturbations.

Some of the elements in the time line in Figure 1.15 may well be incorrect. However, since most of these ideas are testable, the data which will be taken during the coming decade will tell us which parts of the time line are correct and which need to be discarded. This in itself seems a sufficient reason to study the CMB and large-scale structure.

SUGGESTED READING

There are many good textbooks covering the homogeneous Big Bang. I am most familiar with *The Early Universe* (Kolb and Turner), which has especially good discussions on nucleosynthesis and inflation. Peacock's *Cosmological Physics* is the most up-to-date and perhaps the broadest of the standard cosmology texts, with more of an emphasis on extragalactic astronomy than either *The Early Universe* or this book. A popular account which still captures the essentials of the homogeneous Big Bang (testifying to the success of the model: it hasn't changed that much in 25 years) is *The First Three Minutes* (Weinberg). More recently, three books of note are: *The Whole Shebang* (Ferris), *The Little Book of the Big Bang* (Hogan), and *A Short History of the Universe* (Silk).

A nice article summarizing the evidence for an expanding universe and some methods to quantify it is Freedman (1998). Two of the pioneers in the field of Big Bang nucleosynthesis, Schramm and Turner, wrote a very clear review article (1998) right before a tragic accident took the life of the first author. An excellent account of the evidence for dark matter in spiral galaxies is Vera Rubin's 1983 article in Scientific American.

I have not attempted to record the history of the discovery of the Big Bang. Three books I am familiar with which treat this history in detail are *Blind Watchers of the Sky* (Kolb), *3K: The Cosmic Microwave Background* (Partridge), and *Three Degrees Above Zero: Bell Labs in the Information Age* (Bernstein). An article which sheds light on this history is Alpher and Herman (1988).

EXERCISES

Exercise 1. Suppose (incorrectly) that H scales as temperature squared all the way back until the time when the temperature of the universe was $10^{19}\,\mathrm{GeV}/k_B$ (i.e., suppose the universe was radiation dominated all the way back to the *Planck time*). Also suppose that today the dark energy is in the form of a cosmological constant Λ, such that ρ_Λ today is equal to $0.7\rho_{\rm cr}$ and ρ_Λ remains constant throughout the history of the universe. What was $\rho_\Lambda/(3H^2/8\pi G)$ back then?

Exercise 2. Assume the universe today is flat with both matter and a cosmological constant, the latter with energy density which remains constant with time. Integrate Eq. (1.2) to find the present age of the universe. That is, rewrite Eq. (1.2) as

$$dt = H_0^{-1}\,\frac{da}{a}\left[\Omega_\Lambda + \frac{1-\Omega_\Lambda}{a^3}\right]^{-1/2} \tag{1.10}$$

where Ω_Λ is the ratio of energy density in the cosmological constant to the critical density. Integrate from $a = 0$ (when $t = 0$) until today at $a = 1$ to get the age of the universe today. In both cases below the integral can be done analytically.

(a) First do the integral in the case when $\Omega_\Lambda = 0$.

(b) Now do the integral in the case when $\Omega_\Lambda = 0.7$. For fixed H_0, which universe is older?

Exercise 3. Using the fact that the reduced mass of the electron–nucleus in the D atom is larger than in hydrogen, and the fact that the Lyman α ($n = 1 \rightarrow n = 2$) transition in H has a wavelength $1215.67\AA$, find the wavelength of the photon emitted in the corresponding transition in D. Astronomers often define

$$v \equiv c\frac{\Delta\lambda}{\lambda} \tag{1.11}$$

to characterize the splitting of two nearby lines. What is v for the H–D pair?

Exercise 4. Convert the specific intensity in Eq. (1.8) into an expression for what is plotted in Figure 1.10, the energy per square centimeter per steradian per second. Note that the x-axis is $1/\lambda$, the inverse wavelength of the photons. Show that the peak of a 2.73K blackbody spectrum does lie at $1/\lambda = 5\,\mathrm{cm}^{-1}$.

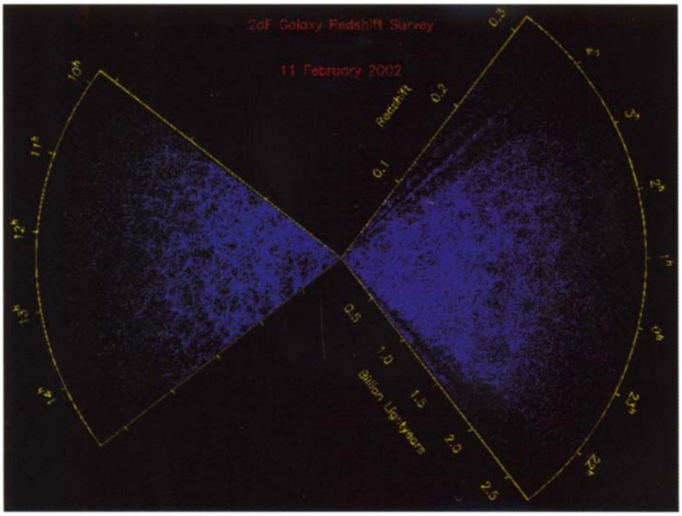


Plate 1.12. Distribution of galaxies in the Two Degree Field Galaxy Redshift Survey (2dF) (Colless *et al.*, 2001). By the end of the survey, redshifts for 250,000 galaxies will have been obtained. As shown here, they probe structure in the universe out to $z = 0.3$, corresponding to distances up to 1000 h^{-1} Mpc away from us (we are located at the center).

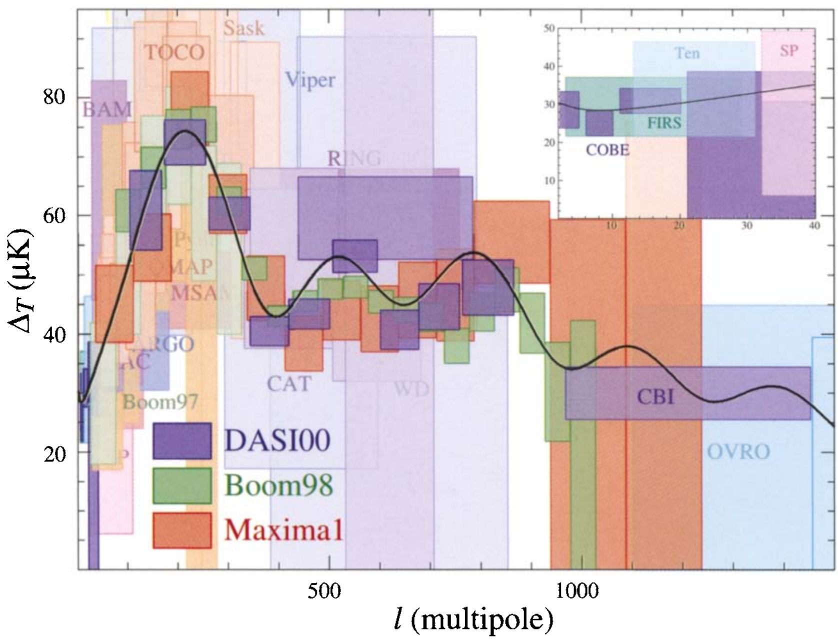


Plate 1.14. Anisotropies in the CMB predicted by the theory of inflation compared with observations. x-axis is multipole moment (e.g., l = 1 is the dipole, l = 2 the quadrupole) so that large angular scales correspond to low l; y-axis is the root mean square anisotropy (the square root of the two-point function) as a function of scale. The characteristic signature of inflation is the series of peaks and troughs, a signature which has been verified by experiment.