# The Potential of Potential Reconstruction

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## 1 Introduction

Inflation involves a period of rapid growth of the Universe. This is most easily illustrated by considering a homogeneous, isotropic Universe with a flat Friedmann–Robertson–Walker (FRW) metric described by a scale factor $a(t)$. Here, "rapid growth" means a positive value of $\ddot{a}/a = -(4\pi G_N/3)(\rho+3p)$ where $\rho$ is the energy density and $p$ the pressure. It is useful to identify the energy density driving inflation with some sort of scalar "potential" energy density $V > 0$ that is positive, and results in an effective equation of state $\rho \simeq -p \simeq V$, which satisfies $\ddot{a} > 0$. If one identifies the potential energy as arising from the potential of some scalar field $\phi$, then $\phi$ is known as the *inflaton* field.

The prime observational consequences of inflation derive from the stochastic spectra of density (scalar) perturbations and gravitational wave (tensor) modes generated during inflation. Each stretches from scales of order centimeters to scales well in excess of the size of the presently observable Universe. Once within the Hubble radius, gravitational waves redshift away and so their main influence is on the large-scale microwave background anisotropies, such as those probed by COBE [1]. The density perturbations are thought to lead to structure formation in the Universe. They produce microwave background anisotropies across a much wider range of angular scales than do the tensor modes, and constraints on the scalar spectrum are also available from the clustering of galaxies and galaxy clusters, peculiar velocity flows and a host of other measurable quantities [2].

Broadly speaking, inflation predicts a very nearly Gaussian spectrum of density perturbations that is *scale dependent*, i.e., the amplitude of the perturbation depends upon the length scale. Such a dependence typically arises because the Hubble expansion rate during the inflationary epoch changes, albeit slowly, as the field driving the expansion rolls towards the minimum of the scalar potential. This implies that the amplitude of the fluctuations as they cross the Hubble radius will be weakly time-dependent.

In the context of slow-roll inflation, any scale dependence for density perturbations is possible if one considers an arbitrary functional form for the inflaton potential, $V(\phi)$. In this sense, inflation makes no unique prediction concerning the form of the density perturbation spectrum and one is left with two options. Either one can aim to find a deeper physical principle that uniquely determines the potential, or observations that depend on $V(\phi)$ can be employed to limit the number of possibilities. One such observation is the amplitude of the tensor perturbations produced by inflation.

Recently, we provided a formalism which allows one to reconstruct the inflaton potential $V(\phi)$ directly from a knowledge of these spectra [3]. This developed an original but incomplete analysis by Hodges and Blumenthal [4]. An important result that follows from our formalism is that knowledge of the scalar spectrum alone is insufficient for a unique reconstruction. Reconstruction from only the scalar spectrum leaves an arbitrary integration constant, and since the reconstruction is nonlinear, different choices of this constant lead to different functional forms for the potential. A minimal knowledge of the tensor spectrum, say its amplitude at a single wavelength, is sufficient to lift this degeneracy.

The most ambitious aim of reconstruction is to employ observational data to deduce the inflaton potential over the range corresponding to microwave fluctuations and large-scale structure, although at present the observational situation is some way from providing the quality of data that this would require [3].

In this talk I will discuss the promise of potential reconstruction assuming one knows 1) the amplitude of the tensor spectrum at one point from microwave background fluctuations, presumably on quadrupole scales corresponding to $3000h^{-1}$Mpc, and 2) the scalar spectrum from microwave background fluctuations and the large-scale structure investigations from quadrupole scales down to scales of several Mpc.

## 2 Reconstruction Equations to Second Order

To some extent all inflationary calculations rely on the use of the slow-roll approximation. In the form we present here, the slow-roll approximation is an expansion in terms of quantities defined from derivatives of the Hubble parameter $H$. In general there are an infinite hierarchy of these which can in principle all enter at the same order in an expansion.

The slow-roll approximation arises in two separate places. The first is in simplifying the classical inflationary dynamics of expansion, with the lowest-order approximation ignoring the contribution of the inflaton's kinetic energy to the expansion rate. The second is in the calculation of the perturbation spectra; the standard expressions are true only to lowest-order in slow-roll. In the expressions in the previous section, we utilized the Hamilton-Jacobi approach [5] to treat the dynamical evolution exactly.

A very elegant calculation of the perturbation spectra to next order in slow-roll has now been provided by Stewart and Lyth [6]. The slow-roll approximation can be specified by parameters defined from derivatives of $H(\phi)$. There are in general an infinite number of these as each derivative is independent, but usually only the first few enter into any expressions. We shall require the first two, which

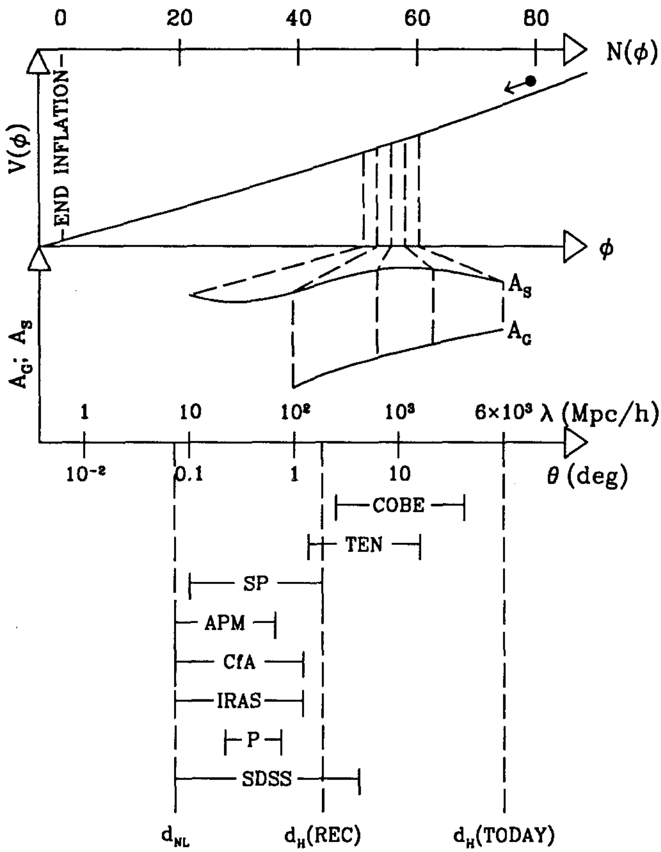


**Fig. 1:** The basic idea of inflation is that as the field evolves in the potential, quantum fluctuations in the inflaton field produce scalar density perturbations $A_S$, while fluctuations in the transverse, traceless metric components produce tensor gravitational wave perturbations, $A_G$. For reconstruction the two main steps involve converting the observations (lower half of figure) into the primordial scalar ($A_S$) and tensor ($A_G$) fluctuation spectra and then working in reverse to reconstruct the potential $V(\phi)$. The main observational information from the cosmic microwave background arises through the Cosmic Background Explorer (*COBE*) satellite [7], and the Tenerife (TEN) [8] and South Pole (SP) [9] collaborations. Galaxy surveys (APM [10], CfA [11], IRAS [12,13]) may provide useful information up to $100h^{-1}$ Mpc, while the Sloan Digital Sky Survey (SDSS) [14] should extend to the lowest scales measured by *COBE*. Peculiar velocity measurements using the POTENT (P) [15] methods are important on intermediate scales. The angle $\theta$ measures angular scales on the CMBR in degrees, and length scales $\lambda$ are in units of $h^{-1}$ Mpc. $d_H$ refers to the horizon size today and at recombination and $d_{\rm NL} \approx 8h^{-1}$ Mpc is the scale of non-linearity. Perfect observations will only reconstruct a small portion of the inflaton potential corresponding to between $53 \leq \Delta N \leq 60$ e-foldings before the end of inflation.

are all of the same order when defined by

$$\epsilon(\phi) = \frac{2}{\kappa^2}\left[\frac{H'(\phi)}{H(\phi)}\right]^2 ; \qquad \eta(\phi) = \frac{2}{\kappa^2}\frac{H''(\phi)}{H(\phi)} . \tag{1}$$

The slow-roll approximation applies when these slow-roll parameters are small in comparison to unity. The condition for inflation, $\ddot{a} > 0$, is precisely equivalent to $\epsilon < 1$.

The lowest-order expressions for the scalar ($A_S$) and tensor ($A_G$) amplitudes assume $\{\epsilon,\ \eta\}$ are negligible compared to unity. Improved expressions for the scalar and tensor amplitudes for finite but small $\{\epsilon,\ \eta\}$ were found by Stewart and Lyth [6]:

$$\begin{aligned} A_S &\simeq -\frac{\sqrt{2}\kappa^2}{8\pi^{3/2}}\frac{H^2}{H'}\left[1-(2C+1)\epsilon + C\eta\right] \\ A_G &\simeq \frac{\kappa}{4\pi^{3/2}} H \left[1-(C+1)\epsilon\right] , \end{aligned} \tag{2}$$

where $C = -2 + \ln 2 + \gamma \simeq -0.73$ is a numerical constant, $\gamma \approx 0.577$ being the Euler constant. The right hand sides of these expressions are evaluated when the scale in question crosses the Hubble radius during inflation, $2\pi/\lambda = aH$. The spectra can equally well be considered to be functions of wavelength or of the scalar field value.

The standard results to lowest-order are given by setting the square brackets to unity. Historically it has been common even for this result to be written as only an approximate equality (the ambiguity arising primarily because of a vagueness in defining the precise meaning of the density perturbation), though the precise normalization to lowest-order was established some time ago by Lyth [16] (see also the discussion in [2]).

The improved expressions for the spectra in Eqs. (2) are accurate in so far as $\epsilon$ and $\eta$ are sufficiently slowly varying functions that they can be treated adiabatically as constants while a given scale crosses outside the Hubble radius. Corrections to this would enter at next order. This differs from the usual situation in which $H$ is treated adiabatically. For the standard calculation to be strictly valid $H$ must be constant, but provided it varies sufficiently slowly (characterized by small $\epsilon$ and $|\eta|$), it can be evaluated separately at each epoch. This injects a scale dependence into the spectra. There is a special case corresponding to power-law inflation for which $\epsilon$ and $\eta$ are precisely constant and equal to each other. In this case there are exact expressions for the perturbation [6,17]. Furthermore, the corrections to each spectrum are the same and they cancel when the ratio is taken. In the general case $\epsilon$ and $\eta$ may be treated as different constants if it is assumed that the timescale for their evolution is much longer than the timescale for perturbations to be imprinted on a given scale. This assumption worsens as $\eta$ is removed from $\epsilon$, which would be characterized by the next order terms becoming large.

It is useful to define the dimensionless quantities $\widetilde{\phi}$ and $v$, and a dimensionless derivative denoted by a dot:

$$\widetilde{\phi} \equiv \frac{\kappa}{\sqrt{2}}\phi; \qquad v(\widetilde{\phi}) \equiv \frac{\kappa^4}{48\pi^3}V(\phi)\ ; \qquad \dot{X} \equiv \frac{dX}{d\widetilde{\phi}}\ . \tag{3}$$

In addition, we can use the identity $\eta = \epsilon + \dot{\epsilon}/2\sqrt{\epsilon}$ and adopt as the expansion variables $\sum_{n=0} \epsilon^{-n/2} d^n\epsilon/d\widetilde{\phi}^n$. In terms of these variables, the expressions for $A_S(\lambda)$, $A_G(\lambda)$, $v$, and the $\widetilde{\phi}$-$\lambda$ relation become

$$\begin{aligned}
A_G &= \frac{\kappa}{4\pi^{3/2}} H\left[1-(C+1)\epsilon\right] \\
A_S &= -\frac{\kappa}{4\pi^{3/2}} H \frac{1}{\sqrt{\epsilon}}\left[1-(C+1)\epsilon + \frac{C}{2}\frac{\dot{\epsilon}}{\sqrt{\epsilon}}\right] \\
v &= A_G^2\left[1+\left(2C+\frac{5}{3}\right)\epsilon\right] = A_S^2\epsilon\left[1+\left(2C+\frac{5}{3}\right)\epsilon - C\frac{\dot{\epsilon}}{\sqrt{\epsilon}}\right] \\
\frac{d\widetilde{\phi}}{d\lambda} &= \frac{\sqrt{\epsilon}}{\lambda}(1+\epsilon).
\end{aligned} \tag{4}$$

In the third expression, $v$ depends upon $A_G(\lambda)$ and $\epsilon$. Since we anticipate that we will only have information about $A_G(\lambda)$ at the largest scales, we have to use the "consistency" equation (also called the evolution equation) to relate $A_G(\lambda)$ to the more experimentally accessible $A_S(\lambda)$ at the expense of introducing the additional $\dot{\epsilon}/\sqrt{\epsilon}$ term.[4] This was done through the identity

$$\frac{A_G^2}{A_S^2} = \epsilon\left[1 - C\frac{\dot{\epsilon}}{\sqrt{\epsilon}}\right]. \tag{5}$$

which follows from the expressions for $A_S$ and $A_G$ in Eq. (4). Now we develop the evolution equation by taking the derivative of $A_S(\phi)$:

$$\begin{aligned}
\frac{\dot{A}_S}{A_S} &= \frac{1}{\epsilon^{1/2}}\left(\epsilon - \frac{1}{2}\frac{\dot{\epsilon}}{\sqrt{\epsilon}}\right) + \frac{1}{\epsilon^{1/2}}\left[\frac{C}{2}\frac{\ddot{\epsilon}}{\epsilon}\epsilon - (C+1)\frac{\dot{\epsilon}}{\sqrt{\epsilon}}\epsilon - \frac{C}{4}\left(\frac{\dot{\epsilon}}{\sqrt{\epsilon}}\right)^2\right] \\
&\quad \times \left[1+(C+1)\epsilon - \frac{C}{2}\frac{\dot{\epsilon}}{\sqrt{\epsilon}}\right].
\end{aligned} \tag{6}$$

Now in addition to the expansion in $\epsilon$ and its derivatives, a truncation is necessary. The truncation here is to assume $\ddot{\epsilon}/\epsilon \ll \dot{\epsilon}/\sqrt{\epsilon}$. With this truncation, to second order

$$\frac{\dot{A}_S}{A_S} = \frac{1}{\epsilon^{1/2}}\left[\epsilon - \frac{1}{2}\frac{\dot{\epsilon}}{\sqrt{\epsilon}}\right]. \tag{7}$$

Now we can express $\dot{A}_S/A_S$ in terms of the spectral index $1-n$ defined to be $d\ln A_S^2/d\ln\lambda$, and the evolution equation becomes

$$\frac{\dot{\epsilon}}{\sqrt{\epsilon}} = 2\epsilon - (1-n); \qquad \frac{d\epsilon}{d\lambda} = \frac{\epsilon}{\lambda}(1+\epsilon)\left[2\epsilon - (1-n)\right], \tag{8}$$

[4] Of course if the consistency equation is only used to evolve $\epsilon$, it can not be used as a check of inflation as discussed in [3].

where for the second equality we have used the $d\tilde{\phi}/d\lambda$ expression. This evolution equation serves two purposes. It removes $\dot{\epsilon}/\sqrt{\epsilon}$ from the equation for $v$, and it is a differential equation that can be evolved to give $\epsilon$ as a function of $\lambda$. To solve the equation it is necessary to know the spectral index as a function of $\lambda$, along with the initial condition $\epsilon(\lambda_0)$ as a function of $1 - n_0$, $A_G(\lambda_0)$ and $A_S(\lambda_0)$.

So the system to be solved can be expressed as

$$\begin{aligned} v[\tilde{\phi}(\lambda)] &= A_S^2(\lambda)\epsilon(\lambda)\left\{1 + \frac{5}{3}\epsilon(\lambda) + C\left[1 - n(\lambda)\right]\right\} \\ \frac{d\tilde{\phi}}{d\lambda} &= \frac{\epsilon^{1/2}(\lambda)}{\lambda}\left[1 + \epsilon(\lambda)\right] \\ \frac{d\epsilon}{d\lambda} &= \frac{\epsilon}{\lambda}\left(1 + \epsilon\right)\left[2\epsilon - (1 - n)\right]. \end{aligned} \tag{9}$$

In the next section we discuss the simplification of the above expressions obtained by dropping the second-order terms and working to first order. In the section after that we solve an example first-order problem.

## 3 First-Order Approximation to Reconstruction

To first order in the slow-roll expansion variables the expressions simplify considerably. For example, to first order, $\epsilon = A_G^2/A_S^2$, $v = A_G^2$, and $\lambda d\tilde{\phi}/d\lambda = A_G/A_S$. The evolution–consistency equation is also quite simple. It can be written as

$$\frac{\lambda}{A_G(\lambda)}\frac{dA_G(\lambda)}{d\lambda} = \frac{A_G^2(\lambda)}{A_S^2(\lambda)}. \tag{10}$$

Again, the procedure will be the same as in the second-order case. The potential depends upon $A_G(\lambda)$, about which we will have information only on the largest scales (possibly only on one scale), so we specify the initial value of $A_G(\lambda)$, and use the consistency–evolution equation to evolve $A_G(\lambda)$ in terms of $A_S(\lambda)$. We can thus express the system to be solved in terms of two equations and a single first-order differential equation which can easily be solved in terms of the initial value $A_G(\lambda_0)$, yielding:

$$\begin{aligned} v[\tilde{\phi}(\lambda)] &= \left[A_G^{-2}(\lambda_0) - 2\int_{\lambda_0}^{\lambda}\frac{d\lambda'}{\lambda'}\frac{1}{A_S(\lambda')}\right]^{-1} \\ \pm\tilde{\phi} &= \int_{\lambda_0}^{\lambda}\frac{d\lambda'}{\lambda'}\frac{v^{1/2}[\tilde{\phi}(\lambda')]}{A_S(\lambda')} \end{aligned} \tag{11}$$

### 3.1 A Worked Example

Let's assume a simple power-law potential of the form $V(\phi) = \lambda_\phi \phi^4$ with $\lambda_\phi = 4 \times 10^{-14}$. This generates perturbation spectra of the form (evaluated at horizon crossing after inflation)

$$A_S(\lambda) = 4 \times 10^{-8}\left[50 + \ln(\lambda/\lambda_0)\right]^{3/2}; \quad A_G(\lambda) = 4 \times 10^{-8}\left[50 + \ln(\lambda/\lambda_0)\right]. \tag{12}$$

On any scale, the number of statistically independent sample measurements of the spectra that can be made is finite. Given that the underlying inflationary fluctuations are stochastic, one obtains only a limited set of realizations from the complete probability distribution function. Such a subset may insufficiently specify the underlying distribution, which is the quantity predicted by an inflationary model. The cosmic variance is an important matter of principle, being a source of uncertainty which remains even if perfectly accurate experiments could be carried out. At any stage in the history of the Universe, it is impossible to specify accurately the properties (most significantly the variance, which is what the spectrum specifies assuming gaussian statistics) of the probability distribution function pertaining to perturbations on scales close to that of the observable Universe.

Even assuming "perfect" observations, cosmic variance sets a lower limit on the uncertainty at any one scale. Assuming that the only errors come from cosmic variance, the determination of the spectra might look like in Fig. 2. In the realization generated by the random number generator, the value of $A_G(\lambda_0)$ is $1.87 \times 10^{-6}$, slightly below the ensemble mean of $2 \times 10^{-6}$.

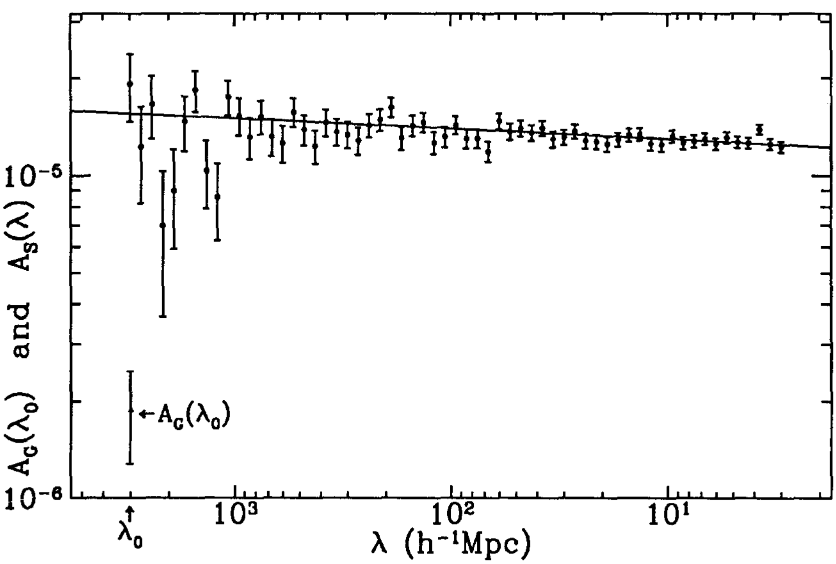


**Fig. 2:** An illustration of an anticipated data set limited by cosmic variance. The data was generated with a $\lambda_\phi \phi^4$ potential with $\lambda_\phi = 4 \times 10^{-14}$. The upper points are $A_S(\lambda)$, while the single lower point is $A_G(\lambda)$. The solid line is the mean $A_S(\lambda)$, while the mean $A_G(\lambda_0)$ is $2 \times 10^{-6}$.

As a first exercise, we simply perform a first-order reconstruction by doing a simple trapezoidal integration, and making the naïve assumption that the errors are uncorrelated. If we do that we obtain the reconstructed potential shown in Fig. 3. Also shown in Fig. 3 by the solid curve is the actual potential used to generate the synthetic data from which the potential was reconstructed.

There are several things we can notice in Fig. 3. First of all, reconstruction works: the true potential is within the error bars. The second obvious feature is that the slope of the reconstructed data is better than one might expect given the errors.

This feature can be explored by taking another approach to the uncertainty introduced in $A_G(\lambda_0)$ by cosmic variance. Let's ignore that error, and pick three realizations of $A_G(\lambda_0)$, one at the "measured" value, one $1\sigma$ above the measured value, and one $1\sigma$ below the measured value. (Here "$\sigma$" is the value determined by cosmic variance.) If we do that, we generate the three curves shown in Fig. 4. Although we can't tell which of the curves is the true potential, we know that the true potential is one of a family of curves bounded by the two extremes in the figure.

We can understand why this occurs, because if we look at the slope of $v^{-1}$, the initial value of $A_S(\lambda)$ drops out, and the contribution comes from adding together a large number of different $A_S(\lambda)$. Since we are combining a large number of data points, the central limit theorem tells us that the errors in the reconstructed potential will become small.

## 4 Conclusions

The quantum-mechanical fluctuations impressed upon the metric during inflation depend upon the inflaton potential. During inflation the Hubble expansion takes microscopic fluctuations of wavelength of order $10^{-28}$cm and stretches them to super-Hubble-radius size where they are frozen. Today they appear on scales as large as the observable Universe, $10^{+28}$cm. It is possible to read the fossil record of the fluctuations by observing cosmic microwave background fluctuations and the power spectrum of large-scale structure.

If the tensor perturbations are large enough to be identified, and if the scalar power spectrum is determined, the inflaton potential may be reconstructed.

Hence cosmology and astrophysics may provide the first concrete piece of the potential of energy scales of $10^{16}$GeV or so.

Not discussed here is the possibility of perturbative reconstruction, where the potential and its first few derivatives are reconstructed about a single point [3, 18, 19].

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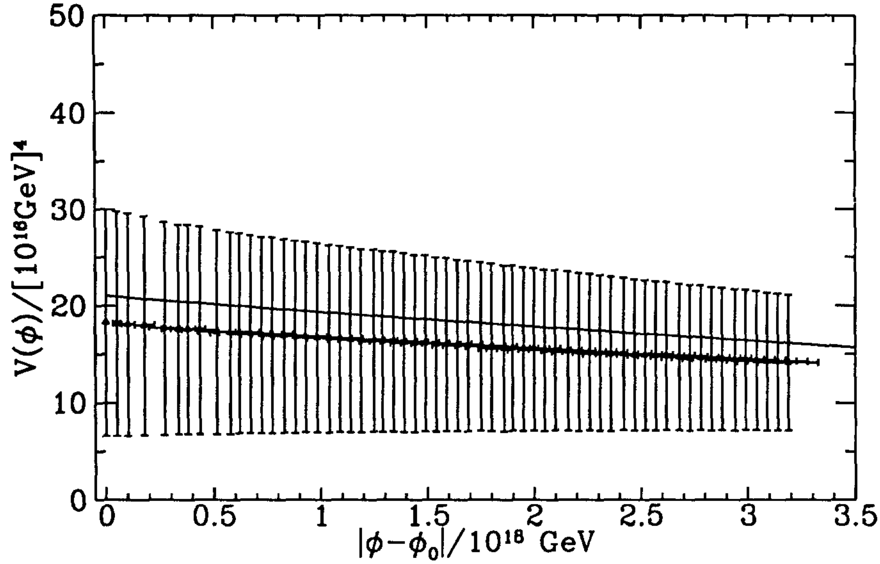


**Fig. 3:** First-order reconstruction of the example $\lambda_\phi \phi^4$ potential. The solid line is the actual potential, while the points and associated errors were generated from the data of Fig. 2.

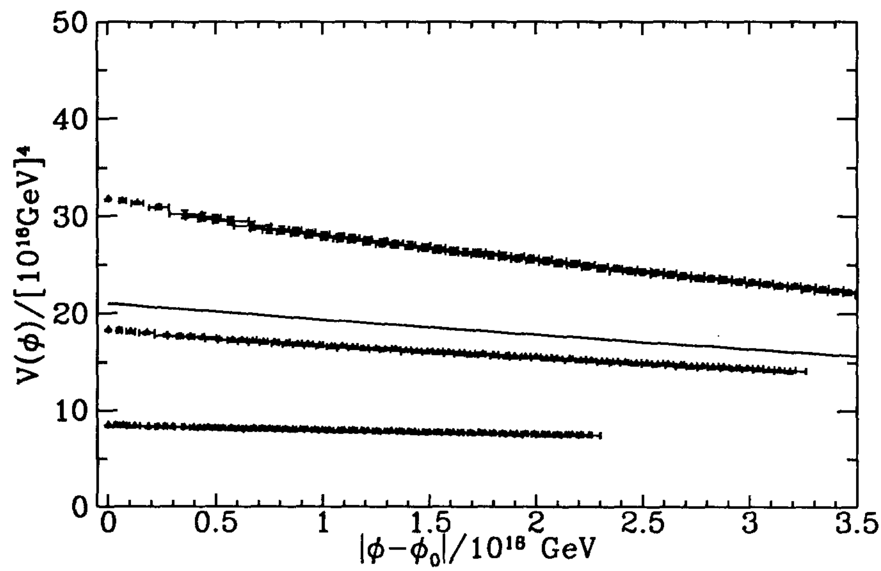


**Fig. 4:** The reconstructed potential ignoring uncertainty in $A_G(\lambda_0)$ for three choices of $A_G(\lambda_0)$ corresponding to the midpoint and $\pm 1\sigma$.

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