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Quality criteria for an optical system

What will be study

- Spot size, resolution
- Encircled energy
- WFE PV and RMS
- Zernike polynomials (see special course later)
- Strehl ratio
- Rayleigh's criterion
- Maréchal's criterion
- MTF (see special course later)

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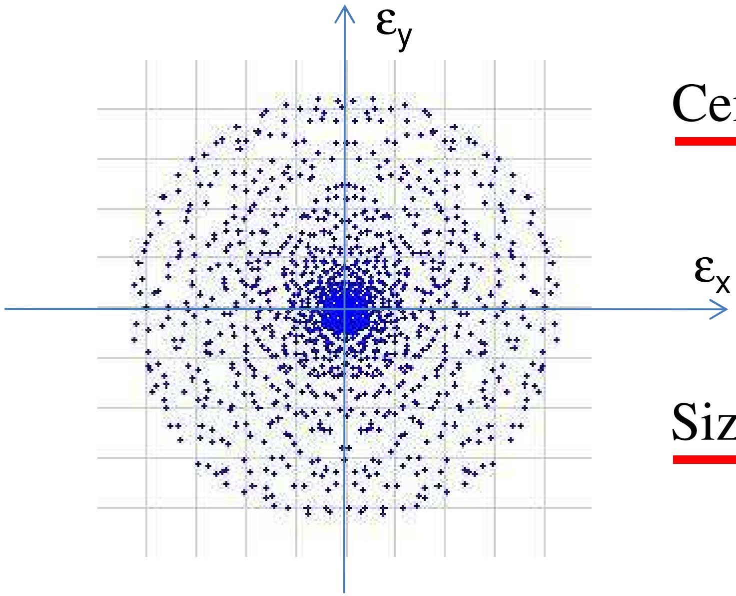
Sources of image degradation

- Diffraction
- Geometric and chromatic aberrations
- Diffusion from surfaces and volumes
- Fabrication and alignment (see course on tolerancing later)
- The detector (sampling)
- The environnement (temperature, turbulence…)
- …

Spot diagram

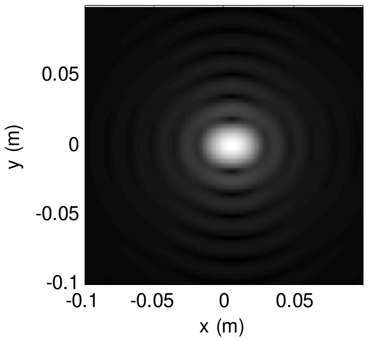


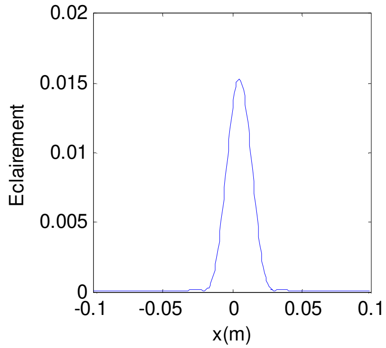
Centroïd : $\begin{cases} \overline{\varepsilon_{\mathrm{x}}} = \dfrac{1}{N}\displaystyle\sum_i (\varepsilon_{\mathrm{x}})_i \\ \overline{\varepsilon_{\mathrm{y}}} = \dfrac{1}{N}\displaystyle\sum_i (\varepsilon_{\mathrm{y}})_i \end{cases}$

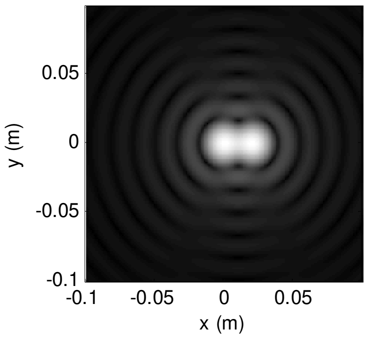
Size of the spot (RMS) : $\begin{cases} (\varepsilon_{\mathrm{x}})_{RMS} = \sqrt{\dfrac{1}{N}\displaystyle\sum_i \left((\varepsilon_{\mathrm{x}})_i - \overline{\varepsilon_{\mathrm{x}}}\right)^2} \\ (\varepsilon_{\mathrm{y}})_{RMS} = \sqrt{\dfrac{1}{N}\displaystyle\sum_i \left((\varepsilon_{\mathrm{y}})_i - \overline{\varepsilon_{\mathrm{y}}}\right)^2} \end{cases}$

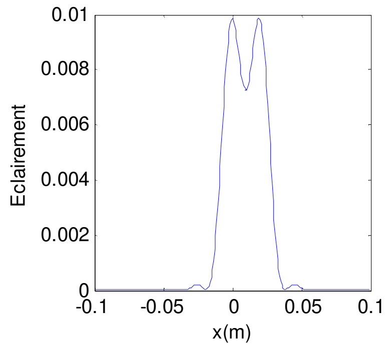
Geometric size of the spot : total width

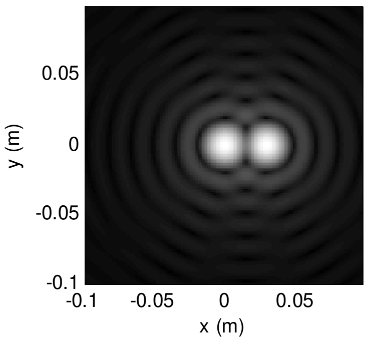
Resolution

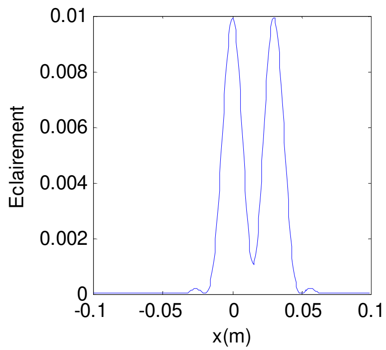

0.05
0
-0.05
-0.1
y (m)
-0.1
-0.05
0
0.05
x (m)


0.02
0.015
0.01
0.005
0
Eclairement
-0.1
-0.05
0
0.05
0.1
x(m)

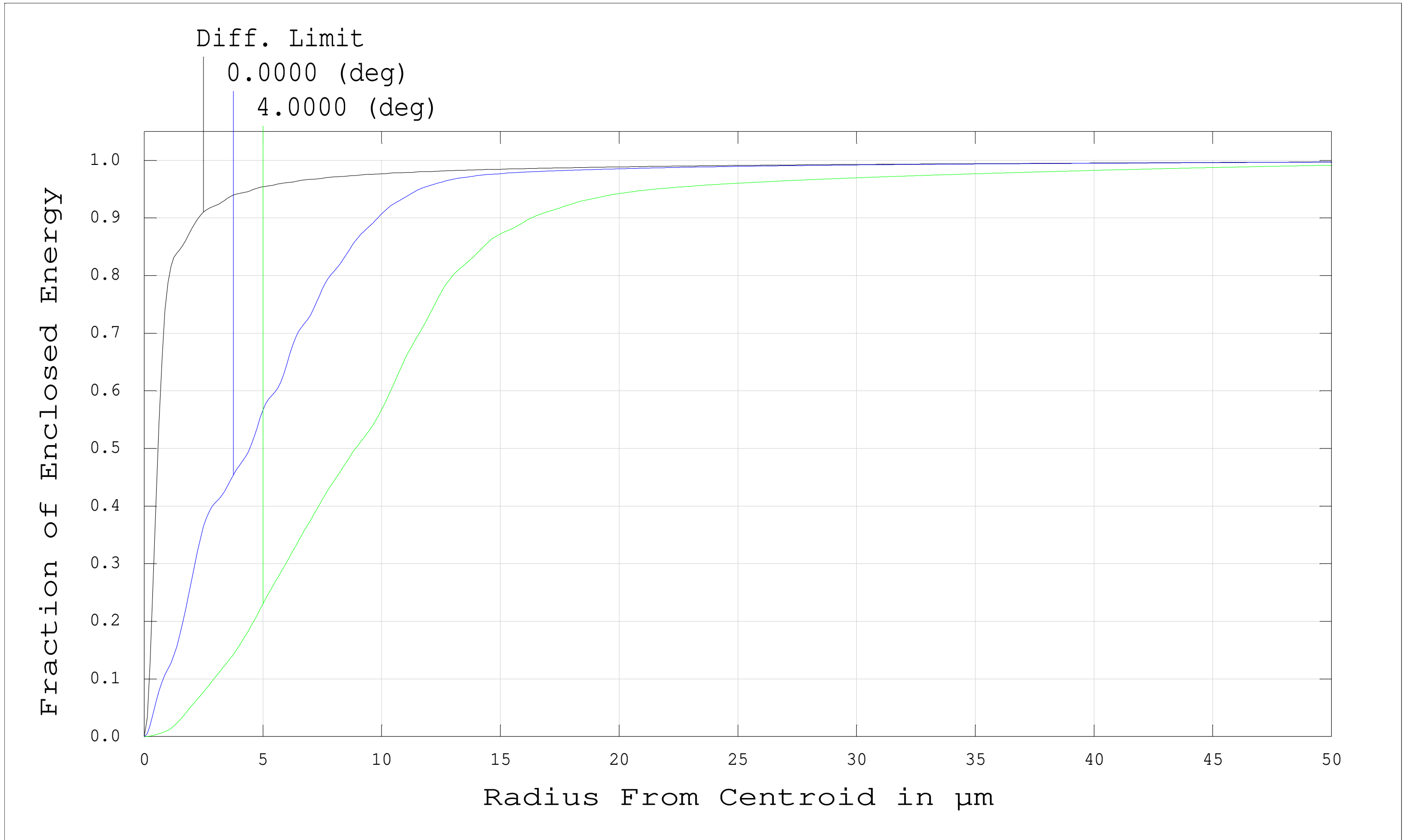

0.05
0
-0.05
-0.1
y (m)
-0.1
-0.05
0
0.05
x (m)


0.01
0.008
0.006
0.004
0.002
0
Eclairement
-0.1
-0.05
0
0.05
0.1
x(m)


0.05
0
-0.05
-0.1
y (m)
-0.1
-0.05
0
0.05
x (m)


0.01
0.008
0.006
0.004
0.002
0
Eclairement
-0.1
-0.05
0
0.05
0.1
x(m)

Encircled energy



FFT Diffraction Encircled Energy

PETZVAL LENS
01/09/2011
Wavelength: Polychromatic
Surface: Image

Petzval.zmx
Configuration 1 of 1

W_{PV} et W_{RMS}

- W_{PV} :

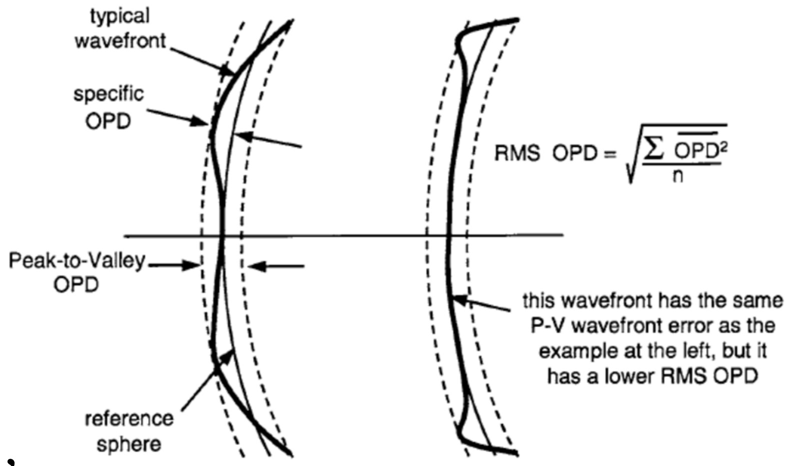
$$W_{PV} = \left|\max\left(W\left(x_P, y_P\right)\right) - \min\left(W\left(x_P, y_P\right)\right)\right|, \left(x_P, y_P\right) \in pupil$$

- W_{RMS} :

$$W_{RMS} = \sigma_W = \sqrt{\overline{\left(W - \overline{W}\right)^2}}$$

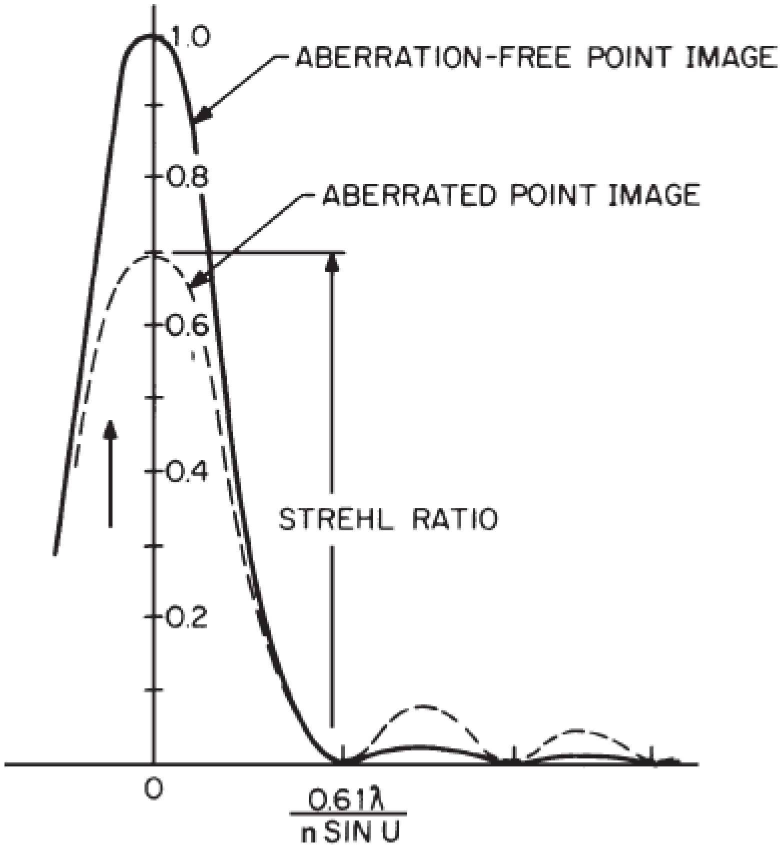
with :

$$\overline{W} = \frac{1}{area} \times \int_{pupil} W\left(x_P, y_P\right) dx_P dy_P$$



Modern optical engineering, Fisher

Strehl ratio



Modern optical engineering, W. Smith

$$Strehl \approx 1 - \frac{4\pi^2}{\lambda^2}\sigma_W^2$$

Proof

In the exit pupil : $E(x_P, y_P) = U(x_P, y_p) e^{-ikW(x_P, y_P)}$

In the focal plane (ie. far field) : $E(x_0, y_0) \propto \int_{-\infty}^{+\infty} U(x_P, y_P) e^{-ikW(x_P, y_P)} e^{-i\frac{k}{z}(x_0 x_P + y_0 y_P)} dx_P dy_P$

$$S = \frac{I_a(0,0)}{I(0,0)} = \left(\frac{\left| \int_{-\infty}^{+\infty} U(x_P, y_P) e^{-ikW(x_P, y_P)} e^{-i\frac{k}{z}(x_0 x_P + y_0 y_P)} dx_P dy_P \right|^2}{\left| \int_{-\infty}^{+\infty} U(x_P, y_P) e^{-i\frac{k}{z}(x_0 x_P + y_0 y_P)} dx_P dy_P \right|^2} \right)_{x_0=0, y_0=0} = \frac{\left| \int_{-\infty}^{+\infty} U(x_P, y_P) e^{-ikW(x_P, y_P)} dx_P dy_P \right|^2}{\left| \int_{-\infty}^{+\infty} U(x_P, y_P) dx_P dy_P \right|^2}$$

Assuming that the amplitude $\mathrm{U}(x_P, y_P)$ is constant other the pupil, we get :

$$\mathrm{S} \approx \frac{\left| \int_{-\infty}^{+\infty} e^{-ikW(x_P, y_P)} dx_P dy_P \right|^2}{\left| \int_{-\infty}^{+\infty} dx_P dy_P \right|^2} = \frac{1}{A^2} \left| \int_{-\infty}^{+\infty} e^{-ikW(x_P, y_P)} dx_P dy_P \right|^2$$

Proof

If $W << \dfrac{\lambda}{2\pi}$, we can write:

$$S \approx \frac{1}{A^2}\left|\int_{-\infty}^{+\infty} e^{-ikW(x_P,y_P)}dx_P dy_P\right|^2 = \left|\frac{1}{A}\int_{-\infty}^{+\infty}\left(1-ikW(x_P,y_P)-\frac{1}{2}k^2W^2\right)dx_P dy_P\right|^2$$

Hence:

$$S \approx \left|\frac{1}{A}\int_{-\infty}^{+\infty} dx_P dy_P - \frac{1}{A}\int_{-\infty}^{+\infty} ikW(x_P,y_P)dx_P dy_P - \frac{1}{2}\frac{1}{A}\int_{-\infty}^{+\infty} k^2W^2(x_P,y_P)dx_P dy_P\right|^2$$

$$S \approx \left|1 - i\frac{1}{A}\int_{-\infty}^{+\infty} kW(x_P,y_P)dx_P dy_P - \frac{1}{2}\frac{1}{A}\int_{-\infty}^{+\infty} k^2W^2(x_P,y_P)dx_P dy_P\right|^2$$

$$S \approx \left|1 - i\overline{kW} - \frac{1}{2}\overline{k^2W^2}\right|^2$$

$$S \approx 1 - \left(\overline{k^2W^2} - \overline{kW}^2\right)$$

Hence: $S \approx 1 - \sigma_{kW}^2 = 1 - k^2\sigma_W^2$

Rayleigh's criterion **(1879)**

Studying the spherical aberration, Rayleigh noticed that, if $\boxed{|W| \leq \frac{\lambda}{4}}$ on the edge of the pupil, then the decrease of the irradiance in the image is less than 20 % and the optical system is still visually diffraction limited

Maréchal's criterion (1943)

Extending Rayleigh's criterion to the other aberrations, and considering that the system is diffraction limited if the decrease of the irradiance of the image is less than 20 %, we get :

$$S \approx 1 - \frac{4\pi^2}{\lambda^2}\sigma_W^2 > 0{,}8 \Leftrightarrow \boxed{\sigma_W \underset{\approx}{<} \frac{\lambda}{14}}$$

P-V OPD	RMS OPD	Strehl ratio	% energy in Airy disk	% energy in Rings
0.0	0.0	1.00	84	16
0.25RL = λ/16	0.018λ	0.99	83	17
0.5RL = λ/8	0.036λ	0.95	80	20
1.0RL = λ/4	0.07λ	0.80	68	32
2.0RL = λ/2	0.14λ	0.4*	40	60
3.0RL = 0.75λ	0.21λ	0.1*	20	80
4.0RL=λ	0.29λ	0.0*	10	90

*The smaller values of the Strehl ratio do not correlate well with image quality.

Modern optical engineering, W. Smith