

empirical observations:

n= 17, max= 12, avg= 7.31

n= 18, max= 13, avg= 8.02

n= 19, max= 14, avg= 8.31

n= 20, max= 16, avg= 8.58

n= 200, max= 105, avg= 86.00

n= 2000, max= 924, avg= 864.46

n=10000, max= 4464, avg= 4323.83

Based on the empirical observations, my algorithm has an upper bound of $0.43n$.

theoretical predictions:

First, call $\left\lfloor \frac{n}{4} \right\rfloor$ times QCOUNT to get results and corresponding indexes. There are about $\frac{3}{8}$ of the

results are 0, discard them. Thus, we get $\left\lfloor \frac{n}{4} \right\rfloor \times \frac{5}{8}$ useful results. And we put all 4s in front of the 2s.

Then we merge every 2 QCOUNT results into 1. We get 2 possible form of the pairs: (4,4) and (2,2).

To process case (4,4), we only need call QCOUNT once. After merge, the result will be either 0 or 8.

To process case (2,2), we need call QCOUNT 2.25 times in average. After merge, the result will be

either 0 or 4. In this step, we need to call QCOUNT $\left(\frac{4}{5} \times 2.25 + \frac{1}{5} \times 1 \right) \times \frac{1}{2} \times \left\lfloor \frac{n}{4} \right\rfloor \times \frac{5}{8} = \frac{5n}{32}$ times. After

this step and discard all 0s, only $\frac{1}{2}$ of the results remain. And those results are guaranteed ≥ 4 . So,

we can merge them using exactly $\frac{\text{number of results}}{2}$ QCOUNT calls. Continue merging until there is

only one result. Then return an arbitrary index from the result.

So, when $n=2000$, the theoretical AVG = $\frac{2000}{4} + \frac{5 \times 2000}{32} + \sum_{i=1}^4 \frac{5 \times 2000}{32 \times 4^i \times 2} = 812.5 + 51.9 = 864.4$

$$\text{theoretical WC} = \frac{2000}{4} + \frac{2000}{8} \times 3 + \sum_{i=1}^4 \frac{2000 \times 3}{8 \times 4^i \times 2} = 500 + 750 + 124.51 = 1374.51$$

$$\text{theoretical expected WC} = (1374.51 + 500) / 2 = 937.255$$

Observed WC and AVG:

$$n = 2000, \quad \text{max} = 924, \quad \text{avg} = 864.46$$

Since the samples are randomly generated, the worst sample may not be the real worst case. Thus, the observed WC is smaller than theoretical WC but close to theoretical expected WC. the observed AVG is close to theoretical AVG, this is because we have tested for 10000 times, the sample size is relatively large.