Unemployment Dynamics and Informality in Small Open Economies*

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Abstract

Despite the typically more pronounced aggregate fluctuations in emerging market economies (EMEs), this paper documents that EMEs exhibit a lower relative volatility and countercyclicality of unemployment rate than small open advanced economies. We link these differences to the larger informal economy in EMEs. We build a small-open-economy model that combines a formal sector featuring labor search frictions with a frictionless informal sector. A larger informal sector amplifies the impact of productivity shocks on formal output, consumption, and employment, while dampening their impact on unemployment. Varying the degree of informality explains a significant fraction of differences in unemployment dynamics across small open economies.

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1 Introduction

Compared to the small open advanced economies (AEs), emerging market economies (EMEs) exhibit distinct macroeconomic fluctuations. While most of the literature focuses on explaining the relatively larger aggregate fluctuations in EMEs, especially the excessively volatile consumption (e.g., Aguiar and Gopinath, 2007; Neumeyer and Perri, 2005; Uribe and Yue, 2006), only until recently do studies begin to examine the labor market dynamics.[1] The unemployment rate dynamics in small open economies, however, is largely overlooked, despite of its importance in household's welfare and policy making.

In this paper, we provide novel empirical evidence revealing that EMEs display a lower relative variability and countercyclicality of unemployment rate than AEs. We link these differences to the size of informal economy, which is substantially larger in EMEs. To rationalize the findings, we construct a small open economy model with a search-and-matching friction in the formal sector and a frictionless informal sector. Household members are employed by formal firms, informal firms, or unemployed, and can issue one-period non-contingent foreign debt. Firms face sector-specific productivity shocks. We calibrate the model to Mexico, a representative EME.

The primary message of this paper is that accounting for the degree of informality in a labor search framework captures important differences in business cycle features across small open economies.[2] Specifically, our main results are threefold. First, our model replicates the cross-country negative relationship between the size of informal sector and the relative volatility and countercyclicality of the unemployment rate. Second, in addition to the unemployment rate behavior, our baseline model simultaneously captures other salient features of EMEs that are usually difficult to replicate, including high output volatility and a more volatile consumption than output. Third, mismeasurement, a key feature of infor-

[1] See, e.g., Altug and Kabaca (2017); Boz, Durdu, and Li (2015); Colombo, Menna, and Tirelli (2019); Fernández and Meza (2015); Finkelstein Shapiro (2018); Leyva and Urrutia (2020).

[2] Informality in the model is defined as the fraction of total employment attributed to informal employment.

mality, further helps explain the differences in unemployment rate dynamics across small open economies.

In exploring the mechanism, our model reveals the central role of informality in propagating sector-specific productivity shocks and in reproducing the cross-country unemployment rate dynamics. When the economy is hit by a positive *formal* productivity shock, formal firms increase vacancy postings. The household reallocates members towards formal jobs and substitutes away from informal to formal consumption. As a result, unemployment decreases, while formal employment, output, and consumption increase. Importantly, we find that a larger informal sector leads to a smaller decrease in unemployment and a larger increase in formal output. This is because a larger informal sector provides a bigger pool of informal workers for reallocation, weakening the dependence on the unemployment margin. Thus, unemployment becomes less volatile and less countercyclical. Similar to Restrepo-Echavarria (2014), a larger informal sector also provides a stronger substitution between formal and informal consumption, generating a more volatile formal consumption relative to formal output.

In addition to their informal counterparts, a positive *informal* productivity shock also increases formal employment and output, while lowering unemployment. The reason is that the household substitutes from formal to informal consumption, raising the stochastic discount factor. Formal firms discount their future profits at a lower rate, which boosts vacancy postings and generates a small expansion of formal employment and output. Furthermore, the expansion is amplified by an increase of informality. A larger informal sector dampens the increases of informal employment and output due to diminishing returns to production. The changes in the responses of formal and informal employment cancel out, leaving the unemployment response largely unchanged. Consequently, the relative volatility and countercyclicality of the unemployment rate both decrease with informality.

Our paper resides in the small open economy literature understanding the differences in aggregate fluctuations between AEs and EMEs. Majority of the literature focuses on

explaining the higher volatility of consumption than of output and the more strongly countercyclical net exports in EMEs than in AEs. The studies offer explanations by relying on shocks to the level and volatility of interest rate at which countries borrow in international markets (Boz et al., 2015; Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe, 2011; Neumeyer and Perri, 2005; Rothert, 2020; Uribe and Yue, 2006), shocks to the trend component of productivity (Aguiar and Gopinath, 2007; Boz, Daude, and Durdu, 2011; Naoussi and Tripier, 2013), commodity prices (Bodenstein, Kamber, and Thoenissen, 2018; Charnavoki and Dolado, 2014; Fernández, González, and Rodriguez, 2018), the role of financial frictions (Chang and Fernández, 2013; Fernández and Gulan, 2015; Garcia-Cicco, Pancrazi, and Uribe, 2010), and government transfers (Michaud and Rothert, 2018). In contrast to the common notion of EMEs exhibiting a larger macroeconomic volatility, we document that the unemployment rate in EMEs tends to be less variable and less countercyclical than in AEs. Germane to our analysis, Choi and Shim (2018) report a smaller responsiveness of employment and hours worked to technology shocks in developing than in high-income countries. They attribute it to differential levels of subsistence consumption. We focus on explaining the moments related to the unemployment rate via its link to informal economy and center our attention on small open economies. By accounting for the large informal sector in EMEs we provide a complementary explanation for the distinct aggregate fluctuations in these countries.

Closely related to our framework, Colombo et al. (2019) analyze the impact of banking crisis and financial frictions on formal and informal labor markets in high-income and low-income countries, while Leyva and Urrutia (2020) emphasize the role of the out-of-labor force margin and interest rate shocks in driving the cyclicality of informality in Mexico. In comparison, we focus on the unemployment dynamics at the business cycle frequency and investigate the propagation of sector-specific productivity shocks. Our work also adds to the growing literature examining the role of informal economy in transmitting domestic and international shocks (Horvath, 2018; Fernández and Meza, 2015; Restrepo-Echavarria, 2014;

Yépez, 2019) and to studies analyzing the relationship between informality, regulation, and labor market frictions (Finkelstein Shapiro, 2018; Lama and Urrutia, 2011; Ulyssea, 2010). We contribute to this literature in two key respects. First, we highlight the importance of the size and mismeasurement of informal economy to macroeconomic volatility across small open economies, including the novel link between informality and unemployment dynamics. Second, our focus on the unemployment rate margin allows us to document the unique role of the informal sector in amplifying the responses of formal variables and dampening the responses of informal variables to sector-specific productivity shocks.

The paper proceeds as follows. Section 2 provides empirical evidence on the differences in informality and unemployment dynamics between AEs and EMEs. Section 3 lays out the theoretical framework. Section 4 discusses the calibration method. Section 5 presents the main results. Section 6 considers extensions and sensitivity analysis of the baseline model. Section 7 concludes.

2 Empirical evidence

In this section, we document significant differences in unemployment rate dynamics between AEs and EMEs. First, we find the unemployment rate in EMEs to be considerably less volatile and less countercyclical than in AEs. Second, we tie these two patterns to the difference in the size of informal sector, which tends to be much larger in EMEs.

2.1 Unemployment rate dynamics

We divide small open economies with sufficiently long quarterly unemployment and output data into advanced economies (AEs) and emerging market economies (EMEs), following the literature (e.g., Aguiar and Gopinath, 2007; Boz et al., 2015; Epstein, Shapiro, and Gómez, 2019).[3] This yields 14 AEs (Australia, Austria, Belgium, Canada, Denmark, Finland,

[3]Slight differences in country coverage are due to unemployment data availability since most of the related studies do not examine unemployment fluctuations.

Ireland, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, and Switzerland) and 14 EMEs (Argentina, Brazil, Chile, Czech Republic, Hungary, Israel, Malaysia, Mexico, Peru, Philippines, Slovakia, Slovenia, Thailand, and Turkey). The data are obtained from the International Financial Statistics (IFS) for the 1980:Q1-2018:Q2 sample period.[4] All data series are in real terms, seasonally adjusted using the US Census Bureau's X-12 ARIMA technique, and, after applying the natural logarithm, detrended using the HP filter with a smoothing parameter of 1600.

Table 1 provides an overview of unemployment rate patterns at the business cycle frequency. The numbers in brackets denote p-values for the Student's t-test and Mann-Whitney test for equality of means and medians between AEs and EMEs. Several differences between the two groups of countries stand out. First, output fluctuations in EMEs are more pronounced than in AEs, consistent with the related literature (e.g., Neumeyer and Perri, 2005). Second, EMEs exhibit a much lower absolute and relative volatility of unemployment rate compared to AEs. The mean (median) relative standard deviation of unemployment rate to output is 4.63 (4.61) in EMEs and 7.74 (7.79) in AEs. The difference in group means (medians) is statistically significant with a p-value of 0.000 (0.000). Third, the unemployment rate tends to be less countercyclical in EMEs than in AEs. The mean (median) correlation between the unemployment rate and output is -0.47 (-0.40) in EMEs and -0.62 (-0.69) in AEs. The correlations are significantly different with a p-value of 0.047 for means and 0.041 for medians.[5]

2.2 Unemployment rate dynamics and informality

We hypothesize that the distinct unemployment rate dynamics between AEs and EMEs may be attributed to the different size of informal economy, defined as the market-based

[4]The IFS unemployment rate data for Canada and Chile are complemented with data from the OECD and for Thailand from the International Labor Organization (ILO). See Appendix A for country-specific sample windows.

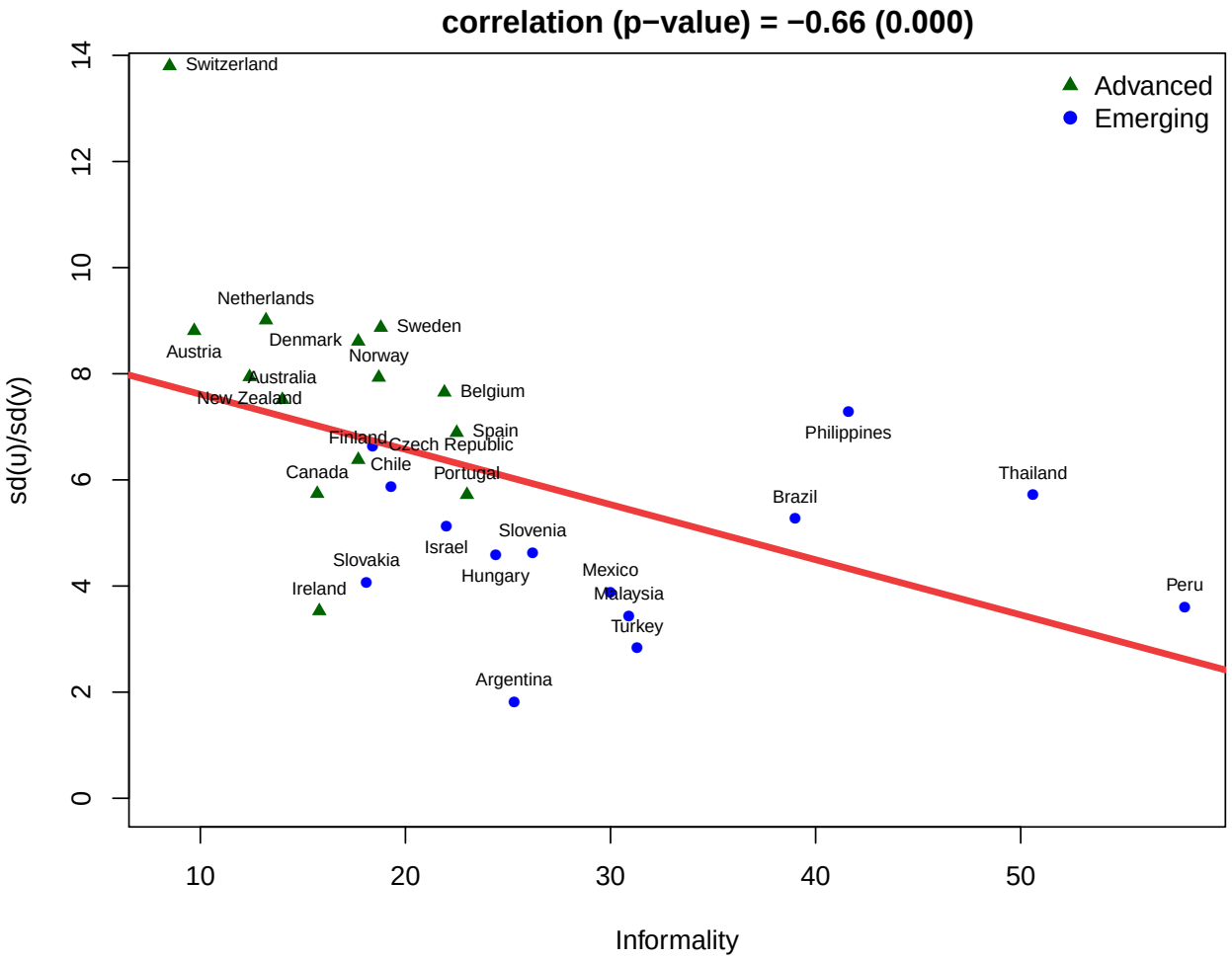
[5]The differences in unemployment rate dynamics between AEs and EMEs are robust to the consideration of a more balanced panel with a starting year of 1990.

Table 1: Unemployment rate dynamics in small open economies

Country	$\sigma(y)$	$\sigma(u)$	$\sigma(u)/\sigma(y)$	$\rho(u,y)$
Advanced				
Australia	1.20	9.04	7.51	-0.71
Austria	1.06	9.36	8.81	-0.33
Belgium	0.96	7.37	7.65	-0.59
Canada	1.44	8.26	5.74	-0.86
Denmark	1.30	11.21	8.61	-0.68
Finland	2.32	14.82	6.38	-0.72
Ireland	3.04	10.73	3.53	-0.52
Netherlands	1.21	10.92	9.01	-0.70
New Zealand	1.35	10.71	7.94	-0.42
Norway	1.80	14.29	7.93	-0.40
Portugal	1.46	8.35	5.72	-0.80
Spain	1.32	9.09	6.89	-0.72
Sweden	1.64	14.58	8.87	-0.48
Switzerland	1.14	15.74	13.80	-0.70
Mean	1.52	11.03	7.74	-0.62
Median	1.34	10.72	7.79	-0.69
Emerging				
Argentina	3.47	6.30	1.82	-0.62
Brazil	1.90	10.00	5.27	-0.38
Chile	1.80	10.56	5.88	-0.71
Czech Republic	1.88	12.49	6.64	-0.58
Hungary	1.40	6.44	4.59	-0.37
Israel	1.69	8.67	5.13	-0.33
Malaysia	2.14	7.36	3.44	-0.43
Mexico	3.24	12.62	3.89	-0.35
Peru	1.46	5.28	3.61	-0.34
Philippines	1.04	7.60	7.29	-0.05
Slovakia	2.29	9.34	4.07	-0.66
Slovenia	1.99	9.21	4.63	-0.69
Thailand	2.33	13.31	5.72	-0.29
Turkey	3.71	10.48	2.83	-0.78
Mean	2.17	9.26	4.63	-0.47
	[0.019]	[0.086]	[0.000]	[0.047]
Median	1.94	9.28	4.61	-0.40
	[0.008]	[0.104]	[0.000]	[0.041]

Notes: The table shows standard deviations and correlations of output and unemployment rate for small open advanced and emerging market economies. All series are HP-filtered with a smoothing parameter of 1600. The numbers in brackets denote p-values for the Student's t-test and Mann-Whitney test for equality of means and medians between advanced and emerging markets.

Figure 1: Relative volatility of unemployment rate to output versus informal economy



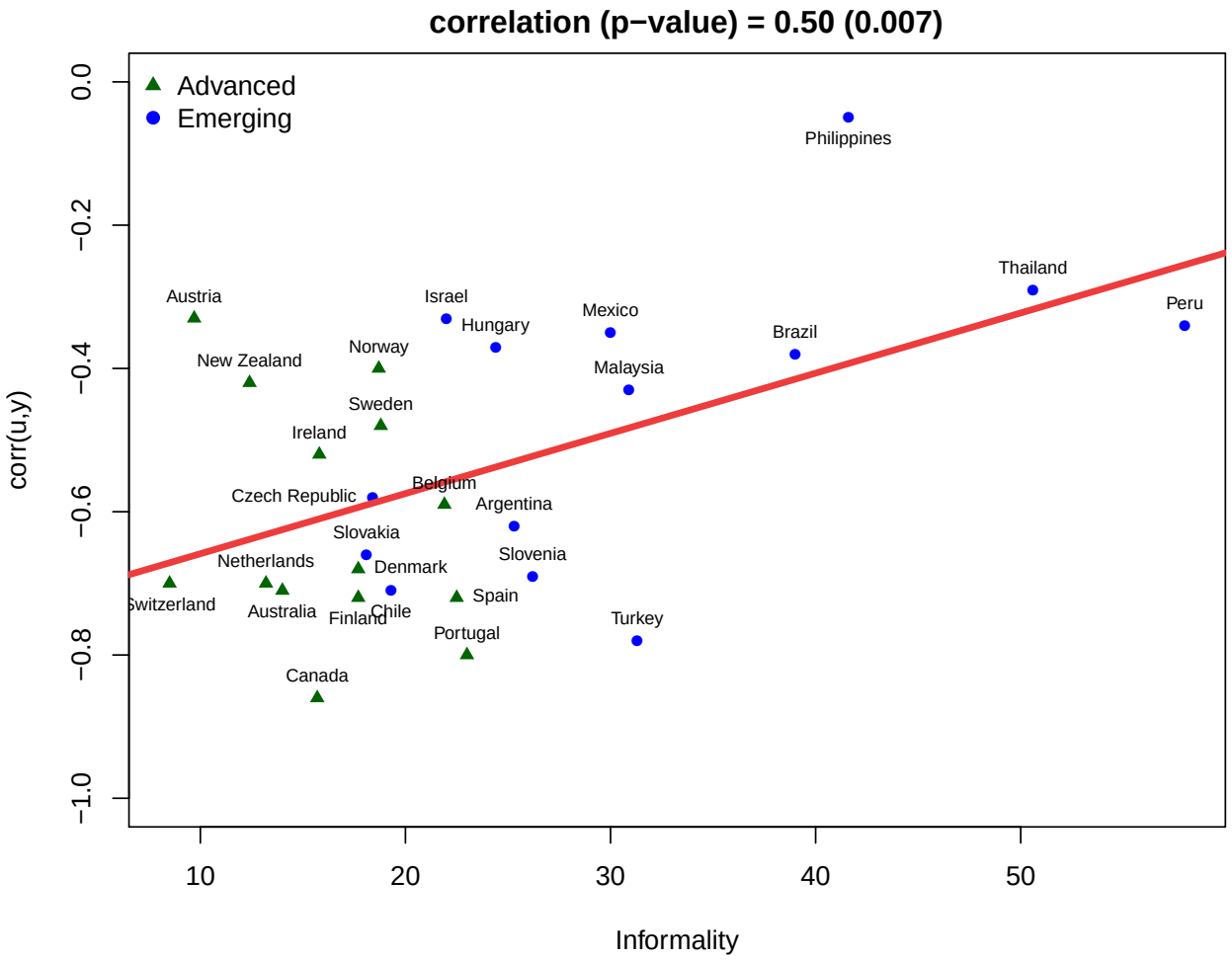
Notes: The figure plots the relative volatility of unemployment rate to output and informality – the size of informal economy as a percentage of GDP.

value added of productive legal economic activities that go unregistered by the government (Restrepo-Echavarria, 2014).[6] Using the data from Table 3.3.6 in Schneider, Buehn, and Montenegro (2010), we find that the average size of informality over the 1999-2007 period in our sample of AEs is 16.4% of GDP, while in EMEs it is almost twice as large at 30.8% of GDP.[7] Table B.1 in Appendix B highlights the importance of labor market flows between

[6]By definition, informal sector excludes home production. See Horvath (2018) for examples of various types of informal activities, and the difference between informal and illegal activities.

[7]There is a high correlation between the Schneider et al. (2010) informality measure and other measures, including constructing informality as a percentage of labor force (Leyva and Urrutia, 2020) or as self-employment (Loayza and Rigolini, 2011). We prefer Schneider et al.'s (2010) measure as it is available for all countries in our analysis.

Figure 2: Correlation between unemployment rate and output versus informal economy



Notes: The figure plots the correlation between unemployment rate and output, and informality – the size of informal economy as a percentage of GDP.

informal employment and unemployment for explaining the unemployment rate fluctuations in a representative emerging market economy, Mexico.

Figures 1 and 2 further demonstrate a close link between informality and unemployment rate fluctuations in small open economies. Figure 1 shows that the relative volatility of the unemployment rate decreases with the size of informal economy, while Figure 2 reveals that the unemployment rate becomes less countercyclical as the size of informal economy increases.[8] The red lines depict the fitted values from regressing the relative unemployment rate volatility, and in turn the correlation between unemployment rate and output, onto the

[8]We also find a negative relationship between informality and absolute volatility of unemployment rate.

size of informal economy. The regression coefficients (not reported) are found to be statistically significant with p-values of 0.000 (Figure 1) and 0.005 (Figure 2). The unconditional correlations, reported above each figure, are also found to be statistically significant.

In the next section, we use this evidence to construct a theoretical model that accounts for informal economy in order to examine the underlying mechanisms between the informal economy and the labor market dynamics in small open economies.

3 Economic model

We consider a small-open economy model with two sectors: a formal sector with a labor market search-and-matching friction and a frictionless informal sector. There is a representative household whose members pool their resources together to share consumption risk. Members can choose to work in the formal sector, informal sector, or not to work. The formal production and labor income, in contrast to the informal ones, are taxed. The sources of aggregate fluctuations in our framework are sector-specific technology shocks. To ease exposition we drop time subscripts and use a prime symbol ($'$) to denote a variable in the next period.

3.1 Search and matching

Each period, there is a fraction of household members working in the formal sector (n_f), in the informal sector (n_i), and a fraction of members not working (u). In the formal sector, firms post vacancies to attract workers for production. A matched formal worker-firm pair is dissolved with an exogenous separation rate s. Dissolved formal workers become unemployed or reallocate to the informal sector, and start searching for a formal sector job in the next period. Together, $n_f + n_i + u = 1$, implying that u also denotes the unemployment rate in the economy.

The model abstracts from the out-of-labor force margin.[9] We are motivated by the evidence presented in Figure B.1 and Table B.1 in Appendix B, showing that the gross labor market flows between total (formal and informal) employment and unemployment account for a considerably larger fraction of the unemployment rate variance in Mexico over the 1987-2016 period than flows between out-of-labor force and unemployment.

We assume, in line with Cano-Urbina and Gibson (2018), that both the non-working household members and informal sector workers search every period for a formal job. We refer to these two groups as searchers. The assumption of on-the-job search by informal workers is based on the evidence presented in Table B.2 in Appendix B, where we document a comparable contribution of flows between formal and informal employment, and between formal employment and unemployment, to formal employment rate fluctuations in Mexico.[10]

The measure of vacancies and searchers for formal jobs is denoted by v and $u + n_i$. A standard constant-returns-to-scale matching technology, $M(u + n_i, v) = \omega(u + n_i)^{\alpha_m} v^{1-\alpha_m}$, determines the number of job matches each period as a function of vacancies and searchers, with ω denoting the matching efficiency and α_m the elasticity of matching. We define the probability of filling a vacancy as $q \equiv M(u + n_i, v)/v = \omega(v/(u + n_i))^{-\alpha_m} = \omega\theta^{-\alpha_m}$, and the probability that a searcher finds a job as $p \equiv M(u + n_i, v)/(u + n_i) = \omega(v/(u + n_i))^{1-\alpha_m} = \omega\theta^{1-\alpha_m}$. Accordingly, $\theta \equiv v/(u + n_i)$ is defined to be the labor market tightness.

3.2 Firms

Firms operate either in a formal sector f or in an informal sector i.[11] Formal labor market is subject to a search-and-matching friction, while informal labor market is frictionless, following Zenou (2008) and Colombo et al. (2019).

[9] See for example, Finkelstein Shapiro (2018) and Leyva and Urrutia (2020) for studies that examine the impact of labor force participation margin on labor market dynamics in emerging economies.

[10] Section 6.3 provides sensitivity analysis by lowering the degree of flows from informal to formal employment.

[11] The assumption of modeling informality as firms instead of self-employment is in line with La Porta and Shleifer (2008), who provide cross-country evidence that majority of informal firms consist of three or more employees (including the owner).

3.2.1 Formal firms

Every period, formal firms employ n_f workers with each worker earning a formal wage rate w_f, and pay a tax rate τ_n on their wage bill. Let $J(n_f, x)$ denote the current value of a formal firm successfully matched with a worker and $x \equiv (z_i, z_f)$ represent the exogenous aggregate state.[12] The formal firm chooses vacancies to maximize its present value of expected profits

$$J(n_f, x) = \max_{v} \{y_f - (1 + \tau_n) w_f n_f - \kappa v + E[\rho(x, x') J(n'_f, x')]\} \tag{1}$$

subject to the production technology $y_f = z_f (n_f)^{\alpha_f}$ and the evolution of formal employment

$$n'_f = (1 - s) n_f + q v. \tag{2}$$

The cost of posting a vacancy per period is denoted by κ. y_f denotes output produced by formal firms and α_f captures formal labor share of formal output. $\rho(x, x') \equiv \beta U_{c'_f} / U_{c_f}$ represents the stochastic discount factor with which firms discount their future profits, where β is the household's subjective discount factor and U_{c_f} is the marginal utility of formal consumption.

Optimality conditions yield the free-entry condition

$$\frac{\kappa}{q} = E[\rho(x, x') J_{n'_f}(n'_f, x')], \tag{3}$$

and the marginal value of an additional worker to the firm, $J_{n_f}(n_f, x)$,

$$J_{n_f}(n_f, x) = \alpha_f \frac{y_f}{n_f} - (1 + \tau_n) w_f + (1 - s) E[\rho(x, x') J_{n'_f}(n'_f, x')]. \tag{4}$$

The marginal value of a formal worker consists of firm's revenue minus the taxed wage bill plus the expected discounted value of the match continuing next period with a probability

[12] To ease notation, we abstract from prices and transfers being functions of state variables.

of $(1-s)$.

Combing the firm's two optimality conditions leads to the following job creation condition

$$\frac{\kappa}{q} = E\left[\rho(x,x')\left(\alpha_f \frac{y'_f}{n'_f} - (1+\tau_n)w'_f + (1-s)\frac{\kappa}{q'}\right)\right]. \tag{5}$$

The formal firms keep posting vacancies until the cost of an additional worker equals the expected discounted benefit of hiring an additional worker. The formal sector productivity (TFP), z_f, follows a standard AR(1) process $z'_f = \rho_{zf} z_f + \epsilon'_f$, where $\epsilon_f \overset{iid}{\sim} N(0, \sigma^2_{zf})$. ρ_{zf} captures the persistence and σ_{zf} captures the volatility of the productivity process.

3.2.2 Informal firms

We model the informal sector using a frictionless framework. A representative informal firm employs n_i workers and generates output, y_i, with production technology $y_i = z_i(n_i)^{\alpha_i}$. α_i denotes the informal labor share of informal output. The firm pays a competitive wage rate w_i and chooses n_i to maximize its profit given by:

$$\max_{n_i}[p_c y_i - w_i n_i]. \tag{6}$$

p_c denotes the relative price of informal to formal consumption goods. The efficiency condition yields the standard result that each period the informal wage rate equals to the marginal product of informal output, scaled by the relative consumption price:

$$w_i = p_c \alpha_i z_i (n_i)^{\alpha_i - 1}. \tag{7}$$

The informal sector productivity, z_i, follows a standard AR(1) process $z'_i = \rho_{zi} z_i + \epsilon'_i$, where $\epsilon_i \overset{iid}{\sim} N(0, \sigma^2_{zi})$. ρ_{zi} denotes the persistence and σ_{zi} the volatility of the informal productivity shocks.

3.3 Household

The infinitely lived representative household allocates a fraction of its members to formal employment n_f, informal employment n_i, and non-working u. The household members pool resources implying that each member's consumption is equal to household's total consumption (c). As in Fernández and Meza (2015) and Colombo, Onnis, and Tirelli (2016), the household aggregates formal and informal consumption according to the constant elasticity of substitution (CES) aggregator:

$$c = [\iota(c_f)^e + (1-\iota)(c_i)^e]^{1/e}, \tag{8}$$

where $1/(1-e)$ captures the elasticity of substitution between formal (c_f) and informal (c_i) consumption goods, and ι denotes the share of total consumption.

The household maximizes its expected lifetime utility represented as

$$V(n_f, d, x) = \max_{c_f, c_i, n_i, d'} \{U(c_f, c_i, n_f, n_i) + \beta E[V(n'_f, d', x')]\} \tag{9}$$

subject to the budget constraint

$$c_f + p_c c_i + d' = (1+r)d + (1-\tau_y)w_f n_f + w_i n_i + \Pi, \tag{10}$$

the law of motion of formal employment

$$n'_f = (1-s)n_f + p(u + n_i), \tag{11}$$

and the time endowment constraint

$$n_f + n_i + u = 1. \tag{12}$$

d' denotes the amount of non-contingent debt the household can issue in the international

financial markets at the interest rate $(1+r)$, τ_y denotes the tax rate on the formal labor income, and Π captures the received profit rebate from formal and informal firms. The right-hand side of equation (10) captures the total household's income, which can be spent on formal and informal consumption, and to pay-off last period's debt. Equation (11) states that the next period's formal employment equals the sum of the formal sector workers whose employment is not terminated in the current period and the newly matched workers.

Household's optimality conditions give rise to the following equations:

$$U_{c_i} = p_c U_{c_f}, \tag{13}$$

$$U_{c_f} = \beta E[U_{c'_f}(1+r')], \tag{14}$$

$$V_{n_f}(n_f, d, x) = [(1-\tau_y)w_f - w_i]U_{c_f} + U_{n_f} - U_{n_i} + \beta(1-s-p)E[V_{n'_f}(n'_f, d', x')]. \tag{15}$$

The first optimality condition in (13) pins down the relative consumption price, which equals the marginal rate of substitution between informal and formal consumption. Equation (14) captures the first order condition for debt holdings. Equation (15) represents the marginal value of a matched formal sector worker, $V_{n_f}(n_f, d, x)$, which comes from the sum of the wage differential expressed in the units of consumption goods, the difference in disutility of working in the formal versus informal sector, and the next period's expected continuation value.

The utility function takes the form

$$U(c_f, c_i, n_f, n_i) = \log(c) - \varphi(n)^\eta \tag{16}$$

with total consumption (c) given in equation (8) and total employment (n) given by $n = n_f + n_i$. $1/(\eta - 1)$ captures the Frisch elasticity of total labor supply and φ represents the disutility of work.

We set the real interest rate (r) on issued debt in world capital markets equal to a

combination of a constant world interest rate $\bar{r}$ and an interest rate premium

$$r = \bar{r} + \psi[exp(\tilde{d} - \bar{d}) - 1]. \tag{17}$$

$\psi > 0$ denotes the interest rate debt elasticity and $\bar{d}$ the steady state value of d. $\tilde{d}$ represents the aggregate debt level. This setup follows Schmitt-Grohé and Uribe (2003) and Garcia-Cicco et al. (2010) in order to achieve stationarity of our small open economy model.

3.4 Nash bargaining

Formal sector wage is set by Nash bargaining between formal sector firms and the household. Firms bargain to maximize their match surplus $J_{n_f}(n_f, x)U_{c_f}$, expressed in utils, while the household bargains to maximize its match surplus $V_{n_f}(n_f, d, x)$. Given the formal employment level n_f, exogenous aggregate state x, and formal sector workers' bargaining power $\alpha_b \in (0, 1)$, the formal wage rate solves

$$w_f = \arg\max_{w_f} [J_{n_f}(n_f, x)U_{c_f}]^{1-\alpha_b} [V_{n_f}(n_f, d, x)]^{\alpha_b}. \tag{18}$$

The optimal condition for the formal wage rate yields

$$(1 + \tau_n)(1 - \alpha_b)V_{n_f}(n_f, d, x) = (1 - \tau_y)\alpha_b J_{n_f}(n_f, x)U_{c_f}. \tag{19}$$

By combining equation (19) with the household's first order condition in (15), the free-entry condition in (5), and the marginal value of a formal worker in (4), we obtain the following formal wage equation

$$w_f = \frac{\alpha_b}{1 + \tau_n}(\alpha_f \frac{y_f}{n_f} + \kappa\theta) + \frac{1 - \alpha_b}{1 - \tau_y}(w_i - \frac{U_{n_f} - U_{n_i}}{U_{c_f}}). \tag{20}$$

The formal wage rate is a convex combination of two components. The first component

captures the marginal product of formal employment and the vacancy posting cost, whereas the second one consists of informal wage rate and marginal rate of substitution between (formal and informal) employment and formal consumption. The weights depend on the worker's bargaining power and tax rates on formal labor income and wage bill.

3.5 Government

The government observes formal activity, which allows it to levy a tax τ_y on the formal labor income and τ_n on the formal wage bill. Each period, the government balances its budget given by

$$g = \tau_n w_f n_f + \tau_y w_f n_f, \tag{21}$$

where the spending, g, is used for unproductive activities. The informal activity goes undetected by the government, and hence, it is not taxed.

3.6 Market clearing

Formal and informal goods markets clear each period

$$y_i = c_i, \tag{22}$$

$$y_f = c_f + g + nx + \kappa v, \tag{23}$$

where net exports, nx, is equal to

$$nx = (1 + r)d - d'. \tag{24}$$

Formal output is used to cover formal consumption, government expenditures, net exports, and vacancy posting costs. As in Restrepo-Echavarria (2014) and Fernández and Meza (2015), we assume that informal output can only be used for informal consumption. This

assumption is also supported by the empirical evidence presented by Gasparini and Tornarolli (2009), who find that informal employment accounts for most of the employment in non-tradable sectors in Latin America and the Caribbean.

3.7 Equilibrium

Given formal employment n_f, the equilibrium is a set of quantity sequences $\{c_f, c_i, n_f, n_i, u, v\}$ such that the household maximizes its utility, formal and informal firms maximize their profits, formal employment follows the law of motion in equation (2), government balances its budget, and all markets clear. This yields the following 11 equations and 11 endogenous variables $\{q, w_f, w_i, \theta, v, u, n_f, n_i, c_f, c_i, d'\}$:

$$U_{c_f} = \beta E[U_{c'_f}(1 + r')] \tag{25}$$

$$n_f + n_i + u = 1 \tag{26}$$

$$n'_f = (1 - s)n_f + qv \tag{27}$$

$$q = \omega\theta^{-\alpha_m} \tag{28}$$

$$\theta = \frac{v}{u + n_i} \tag{29}$$

$$w_i = \frac{U_{c_i}}{U_{c_f}}\alpha_i z_i (n_i)^{\alpha_i - 1} \tag{30}$$

$$w_i = -\frac{U_{n_i}}{U_{c_f}} \tag{31}$$

$$w_f = \frac{\alpha_b}{1 + \tau_n}(\alpha_f (n_f)^{\alpha_f - 1} + \kappa\theta) + \frac{1 - \alpha_b}{1 - \tau_y}(w_i - \frac{U_{n_f} - U_{n_i}}{U_{c_f}}) \tag{32}$$

$$z_f (n_f)^{\alpha_f} = c_f + (1 + r)d - d' + \kappa v + \tau_y w_f n_f + \tau_n w_f n_f \tag{33}$$

$$z_i (n_i)^{\alpha_i} = c_i \tag{34}$$

$$\frac{\kappa}{q} = \beta \frac{U_{c'_f}}{U_{c_f}} [\alpha_f(z'_f)(n'_f)^{\alpha_f - 1} - (1 + \tau_n)w'_f + (1 - s)\frac{\kappa}{q'}]. \tag{35}$$

4 Calibration

We solve the model by log-linearizing it around the steady state following the approach of Schmitt-Grohé and Uribe (2004). The parameter values are disciplined using data for a representative emerging market economy, Mexico.

Table 2 presents the parameter calibration. The assigned value to the subjective discount factor, β, implies an average country interest rate of 2.2%, computed using the Emerging Market Bond Index (EMBI) spread data.[13] We fix η at 4, implying a Frisch elasticity of 0.33, which is an intermediate value of the range of estimates documented by Keane and Rogerson (2012). It also lies in the range of values used by other related studies for Mexico (e.g. Boz et al., 2015; Finkelstein Shapiro, 2018).[14] The elasticity of substitution between formal and informal consumption goods, $1/(1-e)$, is set to a standard value of 8, as adopted by Fernández and Meza (2015); Horvath (2018); Leyva and Urrutia (2020).

We calibrate the values for the disutility of employment, φ, and the share of formal consumption, ι, by using the equilibrium conditions for the formal wage, informal wage, and the job creation conditions. To do so, we use the steady state values of consumption, output, and employment for each sector in Mexico over 1987Q1-2018Q2.

We set the formal labor market separation rate, s, to 0.06, and the job finding rate, q, to 0.7, in line with the estimated values for Mexico documented by Bosch and Maloney (2008) and employed by Boz et al. (2015). The elasticity of matching, α_m, is set to 0.5, a common value used, for example, in Shimer (2005). To satisfy the Hosios condition, we set the workers' bargaining power, α_b, to 0.5. The tax rates on the formal wage bill τ_n and on the formal labor income τ_y are equal to 0.11 and 0.07, which are the corresponding estimates

[13] The data are sourced from Global Financial Data and are available over the 1998Q1-2016Q4 period.

[14] Section 6.3 presents sensitivity results for η, and several other parameters chosen externally to document the robustness of main results.

Table 2: Parameter calibration

parameter	description	value
β	subjective discount factor	0.98
$1/(\eta-1)$	Frisch elasticity of labor supply	0.33
φ	disutility of labor	0.29
$1/(1-e)$	elasticity of substitution between c_f and c_i	8
ι	share of total consumption	0.64
α_b	workers' bargaining power	0.5
α_m	elasticity of matching	0.5
τ_n	tax on formal wage bill	0.11
τ_y	tax on formal labor income	0.07
s	job separation rate	0.06
κ	unit cost of vacancy posting	0.173
ω	scale parameter of matching function	0.294
q	job finding rate	0.7
ψ	interest rate debt elasticity	0.7
α_f	formal labor share of formal output	0.65
α_i	informal labor share of informal output	0.7
ρ_{zf}	persistence of formal technology process	0.782
σ_{zf}	volatility of formal technology process	1.205
ρ_{zi}	persistence of informal technology process	0.782
σ_{zi}	volatility of informal technology process	1.205

for the Mexican economy in Fernández and Meza (2015).

The steady-state unemployment rate, $\bar{u}$, equals 3.84%, i.e., the average unemployment rate in Mexico from 1987Q1 to 2018Q2. The implied steady state fraction of employed workers, $(1-\bar{u})$, is allocated to formal and informal employment based on the size of informal sector.[15] In particular, using the Schneider et al. (2010) estimates for Mexico, we set the informal sector size (informality $\equiv \frac{\bar{n}_i}{\bar{n}}$) to 0.3, which leads to a steady state informal employment, $\bar{n}_i$, of $0.289(=\text{informality}\times(1-\bar{u}))$ and a steady state formal employment, $\bar{n}_f$, of $0.673(=1-\bar{u}-\bar{n}_i)$. This calibration approach of formal and informal employment being tied to the unemployment rate is similar to Ulyssea (2010). It is worth noting that informality for

[15]We do not study the causes of informality in this paper. Hence, the size of informality is calibrated, and varied in later sections to quantify its cross-country impact on unemployment fluctuations. See Aruoba (2018) and Quintin (2008) for potential causes of informality.

Mexico estimated by Schneider et al. (2010) is lower than the values reported by Fernández and Meza (2015) and Leyva and Urrutia (2020). We take a conservative stand and use the smaller value obtained from Schneider et al. (2010). The authors also provide comparable estimates for other countries, which we adopt in the cross-country analysis in Section 5.2.

We follow Andolfatto (1996) and Boz et al. (2015) by using a standard value of 0.01 for the total vacancy posting cost, $\bar{v}\kappa$. This together with the steady state formal employment value leads to steady state vacancies, $\bar{v}$, of 0.058, implying the unit cost of posting a vacancy, κ, of 0.173, and the matching function scaling parameter, ω, of 0.294. The interest rate debt elasticity, ψ, is fixed at 0.7, which is the estimated value (converted to quarterly frequency) for Mexico in Garcia-Cicco et al. (2010).

The remaining parameters pertain to the production functions. The labor share of output in the formal sector, α_f, is chosen to be 0.65, and in the informal sector, α_i, to be 0.7, as informal sector is usually more labor intensive, in line with Colombo et al. (2016). In line with Chen, Chu, and Lai (2018), we assume that the sector-specific technology processes are symmetric and uncorrelated. This makes our results more conservative and is also guided by the lack of data availability on the informal technology process.[16] We calibrate the persistence and the standard deviation of the technology processes such that the model replicates the observed auto-correlation and volatility of Mexican output during our sample period. This yields values of $\rho_{zf} = \rho_{zi} = 0.782$ and $\sigma_{zf} = \sigma_{zi} = 1.205$.[17]

5 Results

This section compares the model-generated moments with data, with the focus on unemployment rate fluctuations. We first examine the case of Mexico. Then, we vary the size of informal economy to contrast the model-generated relative volatility and countercyclical-

[16] In Section 6.2, we explore the importance of correlation and symmetry of the technology processes for our main results.

[17] Our calibrated values are in line with Boz et al. (2011) and Kemme and Koleyni (2017), among other studies focused on Mexico.

Table 3: Business cycle moments in Mexico: Data versus baseline model

Moment	Data	Baseline
Targeted moments		
$\sigma(y)$	3.24	3.24
Untargeted moments		
$\sigma(c)/\sigma(y)$	1.18	1.01
$\sigma(u)/\sigma(y)$	3.89	3.75
$\sigma(n)/\sigma(y)$	0.29	0.15
$\rho(c, y)$	0.94	1.00
$\rho(u, y)$	-0.35	-0.47
$\rho(n, y)$	0.37	0.47

Notes: The table reports the business cycle moments for Mexico. $\sigma(x)$ refers to a standard deviation of variable x. $\rho(x, z)$ refers to a correlation between x and z. Baseline model considers the case when none of the informal sector is captured in national statistics and unemployment is measured perfectly. In this case, y, c, nx, u denote y_f, c_f, nx, u in the model.

ity of unemployment rate with their cross-country data counterparts. Lastly, we examine the model mechanism through impulse responses of model variables to formal and informal productivity shocks.

5.1 Business cycle moments in Mexico

Table 3 shows that the baseline model successfully reproduces several business cycle moments of the Mexican economy, including features of EMEs that are typically difficult to generate. The first column presents the empirical moments.[18] The second column reports the model-generated moments for the baseline case, in which none of the informal sector is captured in national statistics.[19] Hence, y and c in the table denote y_f and c_f in the model. The model captures well the relative unemployment rate volatility (3.75 compared to 3.89 in the data), and the unemployment rate countercyclicality (correlation coefficient of -0.47 compared to -0.35 in the data). In addition, the model generates a more volatile consumption than output, a salient business cycle feature of EMEs that is typically difficult to reproduce.

[18] Due to the lack of data availability, we do not report vacancy-related moments.

[19] We consider the role of mismeasurement in Section 6.

Lastly, we find that total employment displays low volatility and cyclicality, in line with data. This is driven by a highly volatile and countercyclical informal employment, which offsets the procyclicality of formal employment, as documented for Mexico by Fernández and Meza (2015). Our baseline model also produces a strong countercyclicality of informality rate (n^i/n), consistent with the findings of Leyva and Urrutia (2020).

5.2 Informal economy and unemployment dynamics

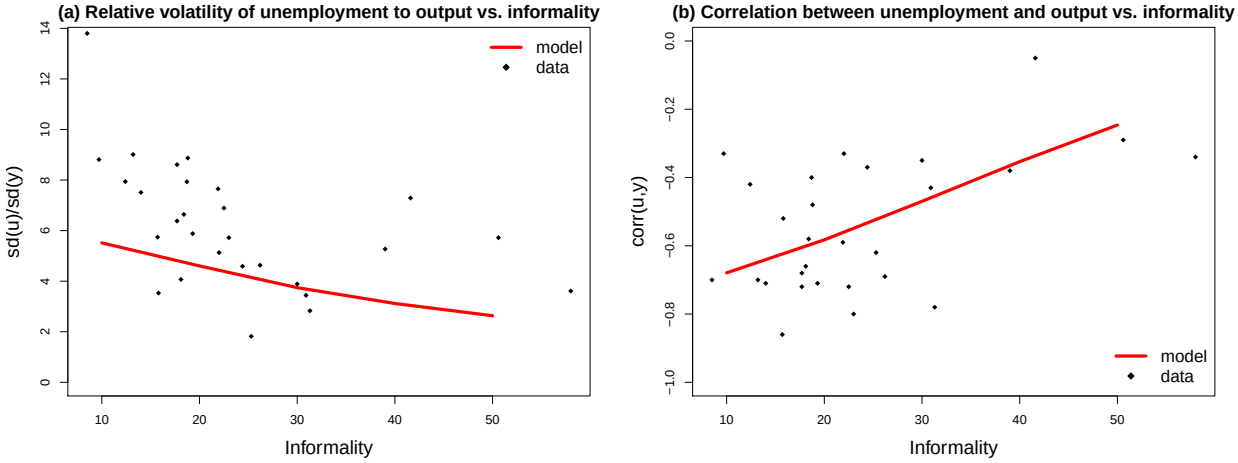
We vary the size of informality from 0.1 to 0.5 to study its impact on unemployment fluctuations across small open economies. The steady state values of informal and formal employment change according to $\bar{n}_i = \text{informality} \times (1 - \bar{u})$ and $\bar{n}_f = 1 - \bar{u} - \bar{n}_i$, while the steady state value of the unemployment rate ($\bar{u}$) remains unchanged.[20]

We report the results in Figure 3. The solid red line graphs the simulation results from the model. The black diamonds represent the data points from Figures 1 and 2. Overall, Figure 3 shows that the size of informal sector plays an important role in driving the cross-country differences in the unemployment rate behavior.

Panel (a) in Figure 3 shows that the baseline model generates a strong negative relationship between the size of informal sector and the relative volatility of unemployment rate to output. The relative volatility ranges from 5.52 (10% informality) to 2.63 (50% informality). Panel (b) in Figure 3 highlights that the model also reproduces the positive relationship between unemployment rate cyclicality and informality: the larger the informal sector, the less countercyclical the unemployment rate. The correlation between the unemployment rate and output goes from -0.68 (10% informality) to -0.25 (50% informality). These findings are consistent with our hypothesis that informal economy explains a significant portion of the cross-country variation in unemployment dynamics.

[20]The model parameters that are dependent on $\bar{n}_i$ and $\bar{n}_f$, are recalibrated accordingly.

Figure 3: Unemployment dynamics and informality: Data versus model



Notes: Panel (a) on the left plots the relative volatility of unemployment rate to output versus informality. Panel (b) on the right plots the correlation between unemployment rate and output versus informality. The black diamonds refer to data points. The solid red line denotes model-generated values

5.3 Mechanism

In this section, we show how sector-specific productivity shocks transmit through the economy. Because of the distinct frictions between formal and informal labor markets, the transmission of shocks depends on whether the shock originates in the formal or informal sector. By varying the size of informal sector, we illustrate the mechanism behind the impact of informality on unemployment fluctuations.

Figure 4 presents the impulse response functions, expressed in percentage deviations from the steady state, to a one percent increase in *formal* productivity. The blue solid line represents the baseline model responses when informality equals 0.3. Formal output, consumption, and employment increase in response to the shock, as formal firms increase vacancy postings and the household substitutes informal for formal consumption. However, the search friction prevents an immediate adjustment of formal employment. Together with the initial drop of informal employment, unemployment increases on impact. From the first period onward, formal employment gradually increases as informal sector workers and unemployed members reallocate to the formal sector in order to take advantage of the relatively higher formal

sector productivity. This, in turn, lowers the unemployment rate. The substitution to formal consumption and employment leads to a proportional decrease in informal consumption, employment, and output. Once the formal labor market fully absorbs the shock, all series gradually return to their steady state values.

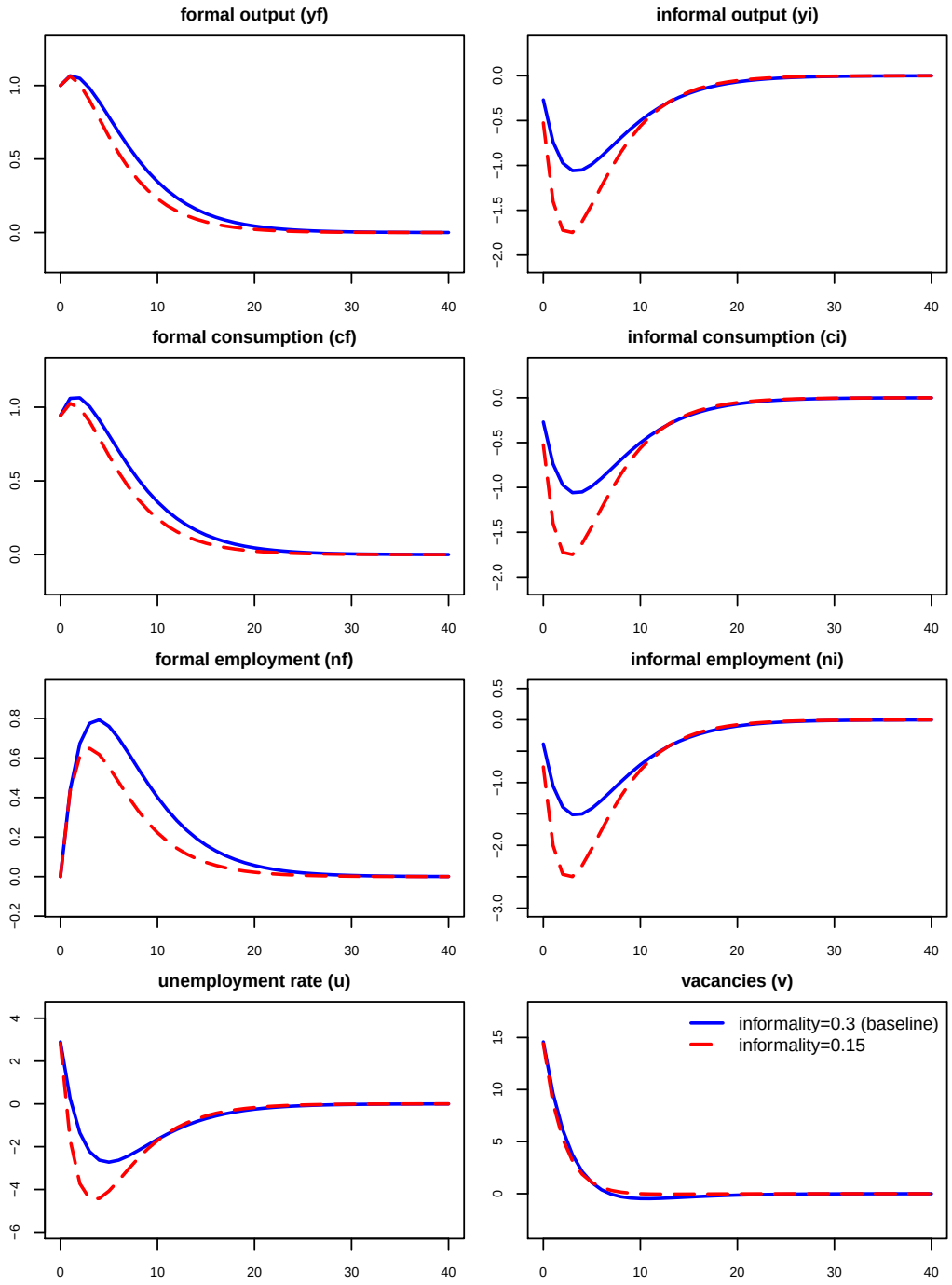
Our model allows for an additional source of flows to (and out of) formal employment from (to) the informal sector, compared to a standard one-sector search-and-match framework. The size of informal sector provides a key role in propagating the shocks throughout the economy. Figure 4 compares the baseline economy with 30% of total employment being informal (solid line) to an economy with informality of 15% (dashed line). A smaller informal sector leads to a larger decrease in unemployment and a smaller increase in formal output. A lower informality implies a smaller pool of informal workers for reallocation to the formal sector, forcing the household to depend more on unemployed members for adjustment when formal productivity increases. This is reflected in a larger drop of informal employment and simultaneously in a less pronounced increase of formal employment. As a result, more unemployed members are reallocated to the formal sector when the shock hits, leading to a more volatile and countercyclical unemployment rate.

Figure 5 shows the impulse responses to a one percent increase in *informal* productivity. The shock incentivizes the household to substitute from formal to informal consumption, and to reallocate unemployed members to the informal sector. This raises informal employment, consumption, and output, and decreases formal consumption. The response of formal and informal consumption raises the stochastic discount factor, which lowers the rate at which formal firms discount their future payoffs and, hence, makes vacancy postings more profitable—a link highlighted by the free-entry condition in equation (5). The resultant increase in vacancy postings, gradually stimulates formal employment and output.[21]

Similarly to Figure 4, Figure 5 shows that the size of informal sector dictates the strength of shock propagation. In particular, formal output, consumption, and employment become

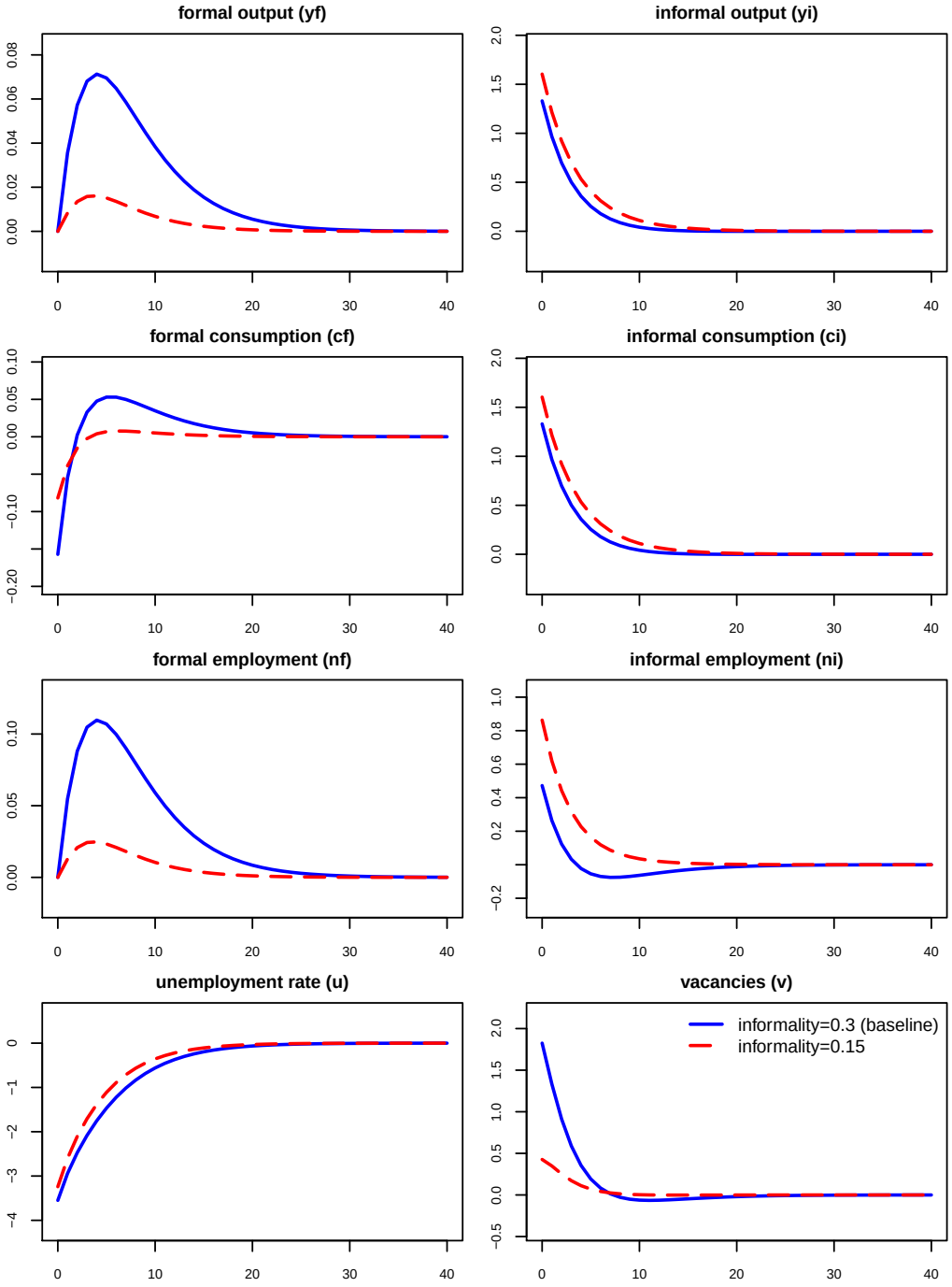
[21]Note that these increases are much smaller compared to the formal productivity shock. This is because an increase in informal productivity creates a relative decrease in formal productivity.

Figure 4: Impulse responses to a positive formal productivity shock



Notes: The figure plots the impulse response functions of formal and informal output, consumption, employment, unemployment rate, and vacancies to a one percent increase in the formal productivity. The solid line denotes the baseline model with an informal employment share of 0.3 of total employment. The dashed line denotes a model with an informal employment share of 0.15 of total employment.

Figure 5: Impulse responses to a positive informal productivity shock



Notes: The figure plots the impulse response functions of formal and informal output, consumption, employment, unemployment rate, and vacancies to a one percent increase in the informal productivity. The solid line denotes the baseline model with an informal employment share of 0.3 of total employment. The dashed line denotes a model with an informal employment share of 0.15 of total employment.

more pronounced when informality doubles from 15 to 30 percent. The responses of informal sector variables, however, become less pronounced due to diminishing returns to production as the pool of informal workers expands. The unemployment rate's response remains largely unchanged. The reason involves two offsetting forces. A bigger informal sector leads to a smaller increase in informal employment, but also to a larger increase in formal employment brought about by a more substantial increase in vacancies. The two effects largely cancel out, implying a very similar unemployment response. All in all, the relative unemployment rate volatility and countercyclicality both decrease with informality because of the relatively larger increase in output.

6 Extensions

In this section, we extend our baseline model to examine a key feature of informal economy: (mis)measurement of workers and output. We also explore the impact of sector-specific technology shocks on our main results. Lastly, we conduct sensitivity analysis of several model parameters.

6.1 Mismeasurement

Many countries employ various approaches to infer the extent of informal activities (see, e.g., Horvath, 2018; Restrepo-Echavarria, 2014). In Table 4, we compare our baseline results with three types of mismeasurement: mistreating informal workers to unemployed, mistreating informal workers to formal employment, and partially accounting for informal sector activities.

First, we consider the possibility that informal workers may under-report their employment. Column 2 (u_{IMi}) presents the results of attributing 5% of informal workers to unemployment.[22] The imperfectly measured unemployment is given by $u_{IMi} = 1 - n_f - (1 - \phi_u)n_i$,

[22] We chose 5% for illustration purposes due to lack of empirical evidence.

Table 4: Business cycle moments in Mexico: Mismeasurement

Moment	(1) Baseline	(2) u_{IMi}	(3) u_{IMf}	(4) y_{IM}
$\sigma(y)$	3.24	3.24	3.24	2.60
$\sigma(c)/\sigma(y)$	1.01	1.01	1.01	0.90
$\sigma(u)/\sigma(y)$	3.75	2.97	5.54	4.68
$\sigma(n)/\sigma(y)$	0.15	0.15	0.15	0.19
$\rho(c,y)$	1.00	1.00	1.00	1.00
$\rho(u,y)$	-0.47	-0.56	-0.36	-0.50
$\rho(n,y)$	0.47	0.56	0.36	0.50

Notes: The table reports the business cycle moments for Mexico. $\sigma(x)$ refers to a standard deviation of variable x. $\rho(x,z)$ refers to a correlation between variables x and z. 'Baseline' model considers the case when none of the informal sector is captured in national statistics and unemployment is measured perfectly. In this case, y, c, n, u denote y_f, c_f, n, u in the model. 'u_{IMi}' considers a scenario when 5% of informal workers (n_i) is not captured in total employment, i.e., $u_{IMi} = 1 - n_f - (1 - 0.05)n_i$. '$u_{IMf}$' considers a scenario when 5% of informal workers (n_i) are double-counted; they are included in both informal and formal employment, i.e., unemployment is measured as $u_{IMf} = 1 - n_f - (1 + 0.05)n_i$. '$y_{IM}$' considers a scenario when the contribution of the observed informal activities to total output is 12% and unemployment is measured perfectly, i.e., y, c, n, u denote y_{IM}, c_{IM}, n, u in the model.

where $\phi_u = 0.05$. Alternatively, informal workers can also simultaneously hold jobs in the formal sector, in which case a worker may be counted in both formal and informal employment, inflating the total labor force. Column 3 (u_{IMf}) considers a case of counting 5% of informal workers as both formal and informal. The unemployment is in this case calculated as $u_{IMf} = 1 - n_f - (1 + \phi_u)n_i$, where $\phi_u = 0.05$. Compared to the baseline model, the relative standard deviation of unemployment to output decreases and the countercyclicality of unemployment rate increases for the u_{IMi} case. The moments change in the opposite direction for the u_{IMf} case. These results are driven by the volatility of informal employment and its negative relationship with formal employment, which are scaled up or down based on the unemployment mismeasurement.

Fernández and Meza (2015) provide evidence that, based on the Mexican statistical agency INEGI, the contribution of the observed informal activities to total output was about 12% over the 1998-2003 period. To allow for this possibility, we define imperfectly measured

output, consumption, and net exports each period as follows:

$$y_{IM} = y_f + \phi p_c y_i, \tag{36}$$

$$c_{IM} = c_f + \phi p_c c_i, \tag{37}$$

$$nx_{IM} = y_{IM} - c_{IM} - g - \kappa v, \tag{38}$$

where ϕ captures the fraction of informal sector included in the national accounts. To be in line with the empirical evidence, we calibrate ϕ using the following equation

$$\Omega = \frac{\phi \bar{p}_c \bar{y}_i}{\bar{y}_f + \phi \bar{p}_c \bar{y}_i}, \tag{39}$$

where $\Omega = 0.12$, as in Fernández and Meza (2015). This implies that ϕ is calibrated in our setup to be 0.52, i.e., about a half of the informal sector is captured by the government.

Column 4 shows that the volatility of output decreases with improved measurement of informal sector. Informal output is negatively correlated with formal output, due to the substitutability between formal and informal consumption goods. Given that the measured output incorporates a fraction of informal activities in addition to formal ones, its volatility decreases. This points to an additional explanation of the difference in output volatility between AEs and EMEs, to the extent that AEs devote relatively more resources to the measurement of their informal economy and, hence, capture a larger share of it. Due to the lower output volatility, the relative volatility and countercyclicality of unemployment both increase. The relative volatility of consumption decreases as well with improvements in measuring the informal sector, as in Restrepo-Echavarria (2014). In our framework, it becomes less than one when roughly a half of the informal sector is accounted for, capturing another difference in the business cycle behavior between AEs and EMEs (see, e.g., Neumeyer and Perri, 2005; Aguiar and Gopinath, 2007).

Overall, these findings complement the results in Figure 3 and show that not only the size

Table 5: Business cycle moments in Mexico: The role of shocks

Moment	(1) Baseline	(2) $z_f = z_i$	(3) $corr(z_f, z_i) = 0.5$	(4) z_i only	(5) z_f only
$\sigma(y)$	3.24	3.44	3.66	0.23	3.23
$\sigma(c)/\sigma(y)$	1.01	0.96	0.97	1.14	1.01
$\sigma(u)/\sigma(y)$	3.75	4.09	4.00	33.85	2.89
$\sigma(n)/\sigma(y)$	0.15	0.16	0.16	1.35	0.12
$\rho(c, y)$	1.00	1.00	1.00	0.54	1.00
$\rho(u, y)$	-0.47	-0.93	-0.81	-0.73	-0.57
$\rho(n, y)$	0.47	0.93	0.81	0.73	0.57

Notes: The table reports the business cycle moments for Mexico. $\sigma(x)$ refers to a standard deviation of variable x. $\rho(x, z)$ refers to a correlation between variables x and z. y, c, n, u denote y_f, c_f, n, u in the model. 'Baseline' refers to the baseline model with uncorrelated formal and informal productivity shocks. '$z_f = z_i$' denotes a model with a common productivity shock in formal and informal sector. '$corr(z_f, z_i) = 0.5$' stands for a model with formal and informal productivity shocks, which have a correlation of 0.5. 'z_i only' denotes a model with only informal productivity shocks. 'z_f only' denotes a model with only formal productivity shocks.

of informal sector, but also its mismeasurement plays an important role for the unemployment rate dynamics in small open economies.

6.2 The role of shocks

In this subsection, we explore the role of the sector-specific uncorrelated productivity shocks in the baseline model for the model-generated second moments in Mexico. Table 5 reports the findings and shows that both formal and informal productivity shocks are important for explaining the unemployment rate fluctuations.

For comparison, column 1 of Table 5 reproduces the results of the baseline model. In column 2, we consider a common productivity process for both (formal and informal) sectors. The two sectors attract unemployed workers simultaneously in response to increases in productivity, leading to a larger volatility and countercyclicality of the unemployment rate. In relative terms, the unemployment rate volatility becomes slightly lower due to the larger increase in output volatility. By reducing inter-sectoral productivity differentials, the common shock process also decreases the relative volatility of consumption.

In column 3, we increase the correlation of the sector-specific technology shocks from 0 (baseline) to 0.5. Similar to the common productivity shock case, the relative volatility of consumption and unemployment slightly decrease, while the countercyclicality of unemployment increases compared to the baseline case.

In columns 4 and 5, we shut down one shock at a time. Eliminating formal productivity shocks (column 4) intuitively leads to a large reduction in output volatility, because none of the informal sector output is assumed to be included in measured output. However, through the substitution between formal and informal consumption goods, informal sector fluctuations still perturb formal consumption and unemployment dynamics, leading to a large increase in their relative volatility levels and the countercyclicality of unemployment rate. In column 5, we shut down the fluctuations in informal productivity, which dampens the fluctuations in the formal labor market, and results in a lower relative volatility and a larger countercyclicality of unemployment rate.

6.3 Sensitivity analysis

We document the robustness of our baseline results for Mexico by varying values of several parameters that we adopt from literature.

In column 2 we lower the elasticity of substitution between formal and informal consumption, $\frac{1}{1-e}$, to 5 from the benchmark value of 8. The lower value is usually considered for the elasticity between market and non-market consumption goods in the home production literature (e.g. Benhabib, Rogerson, and Wright, 1991; Chen et al., 2018). A household with a lower elasticity is less willing to substitute between formal and informal consumption goods when the relative price of consumption changes, which reduces the relative volatility of consumption. It also reduces the role of informal sector in absorbing shocks from the formal sector. Therefore, it results in a higher volatility and countercyclicality of unemployment.

Column 3 in Table 6 examines the impact of a higher labor supply elasticity. In particular, we increase the Frisch elasticity, $\frac{1}{\eta-1}$, from the baseline value of 0.33 to 0.5, which is

Table 6: Business cycle moments in Mexico: Sensitivity analysis

Moment	(1) Baseline	(2) $\frac{1}{1-e}=5$	(3) $\frac{1}{\eta-1}=0.5$	(4) $\kappa=0.1$	(5) $\alpha_i=0.8$	(6) $\xi=0.727$
$\sigma(y)$	3.24	3.31	3.00	3.64	3.27	3.22
$\sigma(c)/\sigma(y)$	1.01	0.99	1.09	1.01	1.02	1.01
$\sigma(u)/\sigma(y)$	3.75	4.05	4.31	4.01	3.37	3.73
$\sigma(n)/\sigma(y)$	0.15	0.16	0.17	0.16	0.13	0.15
$\rho(c,y)$	1.00	1.00	1.00	1.00	1.00	1.00
$\rho(u,y)$	-0.47	-0.61	-0.27	-0.63	-0.31	-0.32
$\rho(n,y)$	0.47	0.61	0.27	0.63	0.31	0.32

Notes: Baseline model considers the case when none of the informal sector is captured in national statistics and unemployment is measured perfectly. In this case, y, c, u, n denote y_f, c_f, u, n in the main text. ξ restricts the flow of informal workers to formal sector in the law of motion for formal employment as $n'_f = (1-s)n_f + p(u+\xi n_i)$.

the upper bound estimate in the preferred model of Cacciatore, Fiori, and Traum (2020). This amounts to setting $\eta = 3$. The higher Frisch elasticity raises the responsiveness of both formal and informal labor supply, which increases the fluctuations in unemployment and consumption, and decreases output variability. As a result, we observe a larger relative volatility of unemployment and consumption, and a smaller countercyclicality of unemployment.

In column 4, we lower the vacancy posting cost, κ, to 0.1, an alternative value also considered in Boz et al. (2015), from the baseline value of 0.173. We find that output and unemployment volatility increase as firms are more willing to adjust their vacancies in response to productivity shocks due to lower posting costs and a shorter employment duration. This increases the likelihood of finding a job in the formal sector and, hence, the flows between formal employment and unemployment, and in turn the countercyclicality of unemployment.

Column 5 considers a more labor-intensive informal production by setting α_i to 0.8, as in Fernández and Meza (2015), instead of the baseline value of 0.7. A larger share of labor in the informal production strengthens the link between informal employment and output fluctuations, implying a larger substitution between formal and informal labor, weakening the role of unemployment in absorbing the shocks. As a result, unemployment volatility and

countercyclicality both decrease for a bigger α_i.

Lastly, in column 6 we modify the number of job searchers to $u + \xi n_i$, where ξ is introduced to regulate the flow of informal sector workers searching for formal jobs.[23] Labor market tightness changes to $\theta = v/(u + \xi n_i)$, and the law of motion of formal employment becomes $n'_f = (1 - s)n_f + p(u + \xi n_i)$. Using the ratio of average flows from informal to formal employment and from unemployment to formal employment in Mexico over the 1987Q1-2016Q4 period, ξ is calibrated to be 0.727. Restricting the flows of workers dampens the impact of informal sector on unemployment. Consequently, unemployment volatility rises. As ξ restricts the number of informal workers searching for formal jobs, the remaining part of informal workers can only reallocate to unemployment, therefore, lowering the countercyclicality of unemployment.

Overall, the section shows that the baseline model-generated moments are robust to reasonable changes in several parameters pertinent to the labor search and informal economy literature.

7 Conclusion

We document significant differences in unemployment rate dynamics between small open advanced economies (AEs) and emerging market economies (EMEs). We link these differences to the size of informal economy. In particular, we show that the size of informal economy is negatively related to the relative volatility of unemployment rate to output, and the countercyclicality of unemployment rate. Our small-open economy two-sector model with asymmetric frictions between formal and informal labor markets replicates the documented cross-country unemployment rate dynamics, along with cross-country differences in consumption and output volatility. In addition to the flow between formal employment and unemployment, households have an alternative choice of working in the informal sector. A larger informal sector offers a stronger substitution between formal and informal employ-

[23] Note that in the baseline model $\xi = 1$.

ment over the business cycle, amplifying fluctuations in formal output, consumption, and employment, while dampening fluctuations in unemployment.

Declaration of Interest

All authors declare that they have no relevant information or potential conflicts of interest to disclose.

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Appendix A Data

In Table A.1, we provide country-specific sample windows for output (y) and unemployment rate (u) data used in our empirical analysis. The moments reported in Table 1 are computed on the overlapping sample of the output and unemployment rate series.

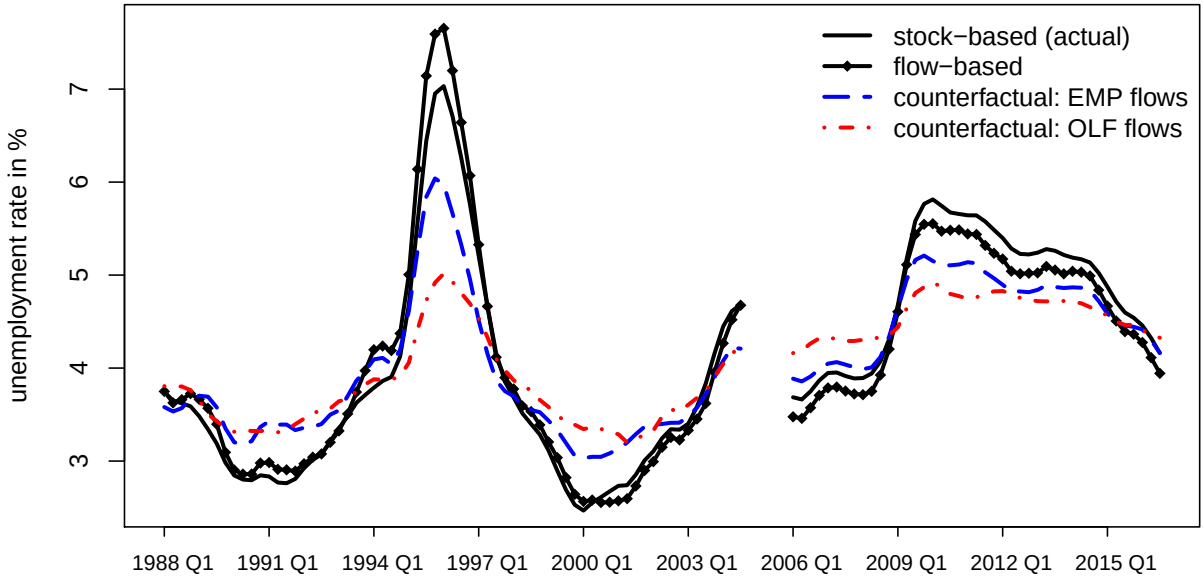
Table A.1: Sample windows for output and unemployment rate

Country	Sample(y)	Sample(u)	Country	Sample(y)	Sample(u)
Advanced			**Emerging**		
Australia	1980:1-2018:1	1980:1-2018:2	Argentina	1993:1-2017:1	2002:4-2017:2
Austria	1980:1-2018:2	1980:1-2018:2	Brazil	1995:1-2017:3	1981:1-2018:1
Belgium	1980:1-2018:2	1983:1-2017:4	Chile	1996:1-2017:3	1986:1-2018:2
Canada	1980:1-2017:4	1980:1-2018:2	Czech Republic	1995:1-2018:2	1993:1-2018:1
Denmark	1980:1-2018:2	1981:1-2017:4	Hungary	1991:1-2018:2	1992:1-2018:1
Finland	1980:1-2018:2	1988:1-2018:1	Israel	1980:1-2018:1	1990:1-2018:1
Ireland	1990:1-2018:2	1983:1-2017:4	Malaysia	1991:1-2017:1	1998:1-2017:4
Netherlands	1980:1-2018:2	1983:1-2018:2	Mexico	1981:1-2018:1	1987:1-2018:2
New Zealand	1980:1-2018:2	1986:1-2018:2	Peru	1980:1-2017:1	2001:2-2017:3
Norway	1980:1-2017:1	1980:1-2017:4	Philippines	1981:1-2017:3	1998:1-2018:1
Portugal	1980:1-2018:2	1992:1-2017:4	Slovakia	1995:1-2018:2	1994:1-2017:4
Spain	1980:1-2018:2	1986:2-2018:1	Slovenia	1995:1-2018:2	1996:1-2017:4
Sweden	1980:1-2017:2	1983:1-2018:2	Thailand	1993:1-2017:3	1998:1-2018:1
Switzerland	1980:1-2018:2	1993:1-2018:2	Turkey	1987:1-2018:2	2000:1-2018:1

Appendix B Informality, unemployment rate dynamics, and labor market flows

We examine the connection between unemployment rate fluctuations and other labor market statuses (out-of-labor force, formal employment, and informal employment) by analyzing the gross labor market flows in Mexico—a representative emerging market economy. We show that the flows of workers between total (formal and informal) employment and unemployment play a more important role for the unemployment rate dynamics than the flows between

Figure B.1: Labor market flows and unemployment rate dynamics in Mexico



Notes: The figure contrasts the contribution of employment-unemployment and out-of-labor force-unemployment flows to the unemployment rate fluctuations in Mexico over the 1987Q1-2004Q4 (ENEU survey) and 2005Q1-2016Q4 (ENOE survey) periods using data and methodology from Leyva and Urrutia (2020). 'stock based (actual)' plots the observed unemployment rate, while 'flow-based' plots the unemployment rate calculated using the composition of all gross flows among formal employed, informal wage-earner, informal self-employed, unemployed, and out-of-labor force. 'counterfactual EMP flows' denotes the unemployment rate that would have been observed if only the flows between employment and unemployment had been allowed to vary, while the rest of the flows would have equaled to their sample averages. The 'counterfactual: OLF flows' unemployment rate is calculated by allowing only the flows between out-of-labor force and unemployment to vary, while keeping the rest of the flow at their sample averages.

unemployment and inactivity (out-of-labor force).[24]

Using the data and approach in Leyva and Urrutia (2020), we obtain the gross flows among employment, unemployment, and out-of-labor force in Mexico over the 1987Q1-2004Q4 (ENEU survey) and 2005Q1-2016Q4 (ENOE survey) periods. Figure B.1 quantifies the contribution of gross flows between (from/to) employment and unemployment, and between out-of-labor force and unemployment to fluctuations in the unemployment rate. In addition to the actual unemployment rate (solid black line) we plot the implied 'flow-based' unemployment rate (solid black line with a diamond symbol) constructed by allowing all flows among formal employment, informal self-employment, informal wage-earners, unemployment, and out-of-labor force to vary over the sample period. The 'counterfactual: EMP

[24]This finding is in line with the evidence provided in Shimer (2012) for the U.S.

Table B.1: Contribution of labor market flows to unemployment rate variance in Mexico

Unemployment rate	[1] n to u	[1.1] n^f to u	[1.2] n^i to u	[2] u to n	[2.1] u to n^f	[2.2] u to n^i	[3] olf to u	[4] u to olf
2005Q1-2016Q4 (ENOE)	50.6	17.2	33.4	15.5	19.3	-3.7	25.8	2.1
	(1.8)	(1.9)	(1.1)	(1.1)	(1.3)	(0.9)	(1.9)	(1.0)
1987Q1-2004Q4 (ENEU)	46.2	14.2	31.7	7.8	9.8	-1.7	30.5	4.8
	(1.0)	(0.9)	(0.5)	(0.8)	(0.5)	(0.8)	(0.9)	(0.8)

Notes: The table shows the fraction of the flow-based unemployment rate (computed as a percentage of labor force) variance in percent explained by gross flows between unemployment and employment (n), formal employment (n^f), informal (self-employed and wage earners) employment (n^i), and out-of-labor force (olf). The numbers refer to the estimated coefficients of β from the following regression with a third-order polynomial time trend: counterfactual rate$_t = \alpha + \beta$flow-based rate$_t + e_t$. The numbers in parentheses denote the associated standard errors of β. The flow-based unemployment rate allows flows in all occupational categories to vary. The counterfactual unemployment rate allows only one type of flows, say from formal employment to unemployment (n^f to u), to vary and sets the remaining flows to their sample average.

flows' unemployment rate (blue long dash line) is computed by allowing only fluctuations in flows between total employment and unemployment and setting the remaining flows to their sample averages. The 'counterfactual: OLF flows' unemployment rate allows only the flows between inactivity and unemployment to change.[25]

Figure B.1 reveals that flows between employment and unemployment contribute significantly more to the unemployment rate fluctuations in Mexico than the participation margin. The counterfactual unemployment rate implied by the employment-unemployment flows tracks the observed unemployment rate much more closely, especially during the economic contractions of 1994-1996 and 2008-2009, than the one implied by the inactivity-unemployment flows.

In Table B.1 we use the variance decomposition and disaggragated labor market flows to provide supplementary evidence to Figure B.1, showing that labor market flows between informal employment (self-employed and wage-earners) and unemployment explain an important fraction of unemployment rate (computed as a percentage of labor force) fluctuations in Mexico.

Columns 1 and 2 show that the flows between total employment (n) and unemployment

[25]See Leyva and Urrutia (2020) and Shimer (2012) for more details on the labor market flow methodology.

Table B.2: Contribution of labor market flows to formal employment rate variance in Mexico

Formal employment rate	[1] n^i to n^f	[2] n^f to n^i	[3] u to n^f	[4] n^f to u
2005Q1-2016Q4 (ENOE)	18.6	29.7	15.7	14.4
	(3.2)	(7.6)	(2.3)	(2.4)
1987Q1-2004Q4 (ENEU)	39.5	3.5	11.2	16.4
	(5.9)	(4.6)	(1.4)	(2.2)

Notes: The table shows the fraction of the flow-based formal employment rate (computed as a percentage of working-age population) variance in percent explained by gross flows between unemployment and employment (n), formal employment (n^f), informal (self-employed and wage earners) employment (n^i), and out-of-labor force (olf). The numbers refer to the estimated coefficients of β from the following regression with a third-order polynomial time trend: counterfactual rate$_t = \alpha + \beta$flow-based rate$_t + e_t$. The numbers in parentheses denote the associated standard errors of β. The flow-based unemployment rate allows flows in all occupational categories to vary. The counterfactual formal employment rate allows only one type of flows, say from formal employment to unemployment (n^f to u), to vary and sets the remaining flows to their sample average.

(u) account for close to two thirds of unemployment rate variance during the 2005-2016 period and for over a half of unemployment rate variance during the 1987-2004 period. The flows between out-of-labor force (olf) and unemployment (columns 3 and 4) explain on average less than a third of fluctuations in the unemployment rate. Table B.1 also shows that in addition to the flows between formal employment (n^f) and unemployment (columns 1.1 and 2.1), the flows between informal employment (n^i) and unemployment (columns 1.2 and 2.2) are a significant driver of unemployment rate dynamics.

In table B.2 we provide evidence for the on-the-job search by informal workers. We report the relative importance of flows between formal employment and unemployment, and between formal employment and informal employment for fluctuations in the formal employment rate in Mexico. The comparison of the first two columns with the last two reveals that the flows between formal and informal employment account for a comparable fraction of the variance in the formal employment rate as the one explained by the flows between unemployment and formal employment. In both periods, the two types of flows account jointly for over 70% of the formal employment rate dynamics.

References

Aguiar, M. and G. Gopinath (2007). Emerging market business cycles: The cycle is the trend. *Journal of Political Economy 115*(1), 69–102.

Altug, S. and S. Kabaca (2017). Search frictions, financial frictions, and labor market fluctuations in emerging markets. *Emerging Markets Finance and Trade 53*(1), 128–149.

Andolfatto, D. (1996). Business cycles and labor-market search. *American Economic Review*, 112–132.

Aruoba, S. B. (2018). Institutions, tax evasion and optimal policy. *Working Paper*.

Benhabib, J., R. Rogerson, and R. Wright (1991). Homework in macroeconomics: Household production and aggregate fluctuations. *Journal of Political Economy 99*(6), 1166–1187.

Bodenstein, M., G. Kamber, and C. Thoenissen (2018). Commodity prices and labour market dynamics in small open economies. *Journal of International Economics 115*, 170–184.

Bosch, M. and W. Maloney (2008). *Cyclical movements in unemployment and informality in developing countries*. The World Bank.

Boz, E., C. Daude, and C. B. Durdu (2011). Emerging market business cycles: Learning about the trend. *Journal of Monetary Economics 58*(6-8), 616–631.

Boz, E., C. B. Durdu, and N. Li (2015). Emerging market business cycles: The role of labor market frictions. *Journal of Money, Credit and Banking 47*(1), 31–72.

Cacciatore, M., G. Fiori, and N. Traum (2020). Hours and employment over the business cycle: A structural analysis. *Review of Economic Dynamics 35*, 240–262.

Cano-Urbina, J. and J. Gibson (2018). Assessing the impact of informal sector employment on young less-educated workers. *Working Paper*.

Chang, R. and A. Fernández (2013). On the sources of aggregate fluctuations in emerging economies. *International Economic Review 54*(4), 1265–1293.

Charnavoki, V. and J. J. Dolado (2014). The effects of global shocks on small commodity-exporting economies: lessons from canada. *American Economic Journal: Macroeconomics 6*(2), 207–37.

Chen, K.-J., A. C. Chu, and C.-C. Lai (2018). Home production and small open economy business cycles. *Journal of Economic Dynamics and Control 95*, 110–135.

Choi, S. and M. Shim (2018). Labor market dynamics in developing economies: The role of subsistence consumption. *Working Paper*.

Colombo, E., L. Menna, and P. Tirelli (2019). Informality and the labor market effects of financial crises. *World Development 119*, 1–22.

Colombo, E., L. Onnis, and P. Tirelli (2016). Shadow economies at times of banking crises: Empirics and theory. *Journal of Banking & Finance 62*, 180–190.

Epstein, B., A. F. Shapiro, and A. G. Gómez (2019). Global financial risk, aggregate fluctuations, and unemployment dynamics. *Journal of International Economics 118*, 351–418.

Fernández, A., A. González, and D. Rodriguez (2018). Sharing a ride on the commodities roller coaster: Common factors in business cycles of emerging economies. *Journal of International Economics 111*, 99–121.

Fernández, A. and A. Gulan (2015). Interest rates, leverage, and business cycles in emerging economies: The role of financial frictions. *American Economic Journal: Macroeconomics 7*(3), 153–88.

Fernández, A. and F. Meza (2015). Informal employment and business cycles in emerging economies: The case of mexico. *Review of Economic Dynamics 18*(2), 381–405.

Fernández-Villaverde, J., P. Guerrón-Quintana, J. F. Rubio-Ramírez, and M. Uribe (2011). Risk matters: The real effects of volatility shocks. *American Economic Review 101*(6), 2530–61.

Finkelstein Shapiro, A. (2018). Labor force participation, interest rate shocks, and unemployment dynamics in emerging economies. *Journal of Development Economics 133*(C), 346–374.

Garcia-Cicco, J., R. Pancrazi, and M. Uribe (2010). Real business cycles in emerging countries? *American Economic Review 100*(5), 2510–31.

Gasparini, L. and L. Tornarolli (2009). Labor informality in Latin America and the Caribbean: Patterns and trends from household survey microdata. *Desarrollo y Sociedad* (63), 13–80.

Horvath, J. (2018). Business cycles, informal economy, and interest rates in emerging countries. *Journal of Macroeconomics 55*, 96–116.

Keane, M. and R. Rogerson (2012). Micro and macro labor supply elasticities: A reassessment of conventional wisdom. *Journal of Economic Literature 50*(2), 464–76.

Kemme, D. M. and K. Koleyni (2017). Exchange rate regimes and welfare losses from foreign crises: The impact of the US financial crisis on Mexico. *Review of International Economics 25*(1), 132–147.

La Porta, R. and A. Shleifer (2008). The unofficial economy and economic development. *Brookings Papers on Economic Activity*.

Lama, R. and C. Urrutia (2011). Employment protection and business cycles in emerging economies. *IMF Working Papers*, 1–39.

Leyva, G. and C. Urrutia (2020). Informality, labor regulation, and the business cycle. *Journal of International Economics*, https://doi.org/10.1016/j.jinteco.2020.103340.

Loayza, N. V. and J. Rigolini (2011). Informal employment: Safety net or growth engine? *World Development 39*(9), 1503–1515.

Michaud, A. and J. Rothert (2018). Redistributive fiscal policies and business cycles in emerging economies. *Journal of International Economics 112*, 123–133.

Naoussi, C. F. and F. Tripier (2013). Trend shocks and economic development. *Journal of Development Economics 103*, 29–42.

Neumeyer, P. A. and F. Perri (2005). Business cycles in emerging economies: The role of interest rates. *Journal of Monetary Economics 52*(2), 345–380.

Quintin, E. (2008). Contract enforcement and the size of the informal economy. *Economic Theory 37*(3), 395–416.

Restrepo-Echavarria, P. (2014). Macroeconomic volatility: The role of the informal economy. *European Economic Review 70*, 454–469.

Rothert, J. (2020). International business cycles in emerging markets. *International Economic Review 61*(2), 753–781.

Schmitt-Grohé, S. and M. Uribe (2003). Closing small open economy models. *Journal of International Economics 61*(1), 163–185.

Schmitt-Grohé, S. and M. Uribe (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control 28*(4), 755–775.

Schneider, F., A. Buehn, and C. E. Montenegro (2010). Shadow economies all over the world: New estimates for 162 countries from 1999 to 2007. *World Bank policy research working paper* (5356).

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review 95*(1), 25–49.

Shimer, R. (2012). Reassessing the ins and outs of unemployment. *Review of Economic Dynamics 15*(2), 127–148.

Ulyssea, G. (2010). Regulation of entry, labor market institutions and the informal sector. *Journal of Development Economics 91*(1), 87–99.

Uribe, M. and V. Z. Yue (2006). Country spreads and emerging countries: Who drives whom? *Journal of International Economics 69*(1), 6–36.

Yépez, C. A. (2019). Informality and international business cycles. *Journal of International Financial Markets, Institutions and Money 62*, 252–263.

Zenou, Y. (2008). Job search and mobility in developing countries. theory and policy implications. *Journal of Development Economics 86*(2), 336–355.