**The Regression Part**

Firstly, let us look at linear regression. Since we have training data and testing data, the linear regression is a supervised machine learning problem. The idea of regression is to find a model that could fit the training data sets with the least loss.

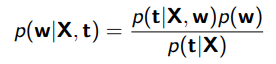
**Question 1**

**What are the models behind the two regression algorithms you chose and implemented?**

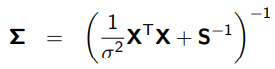
**Bayesian Linear Regression**

Bayesian linear regression is a regression approach that takes statistical analysis under the context of Bayesian inference. Its errors follow the normal distribution. We assume a particular form of the prior distribution and get the results from the posterior probability distributions.

According to the Bayes rule



Posterior density equals the likelihood of multiple prior density divide by marginal likelihood. But computing the posterior is hard since the marginal likelihood is hard to compute. Only in the case when the prior and likelihood are conjugate, we know the form of posterior, therefore we know the form of the normalizing constant. So, we do not need the marginal likelihood. By calculating the covariance and mean of the training dataset, we can simply find its covariance and mean.



By minimizing loss with regularization, we adapt multivariate normal with post mean and post covariance to get result w. Finally, multiple the test data set with w to get the test result.

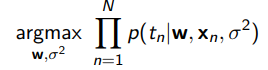
**Maximum Likelihood Regression**

Maximum likelihood Estimation of Linear Regression is a probabilistic framework for efficiently getting the probability distribution and finding the best model to describe a dataset. Specifically, the model choosing, and parameters define as a modeling hypothesis. We are trying to find the modeling hypothesis that maximizes the likelihood function

Here are functions for maximum likelihood estimator for w and sigma square



Likelihood evaluates the quantity obtained when evaluating the density. The higher the value, the more likely training data is fitted. For each input-response pair, we have a Gaussian likelihood to compute the log-joint likelihood at the maximum likelihood estimates. Assume that the training data is independent, we combine them to get the joint likelihood.



Then, we calculate the covariance matrix of w



Finally, we get sample w by multivariate normal between the calculated w and cov{w}. Multiple the test data set with sample w to get the test result.

**Question 2**

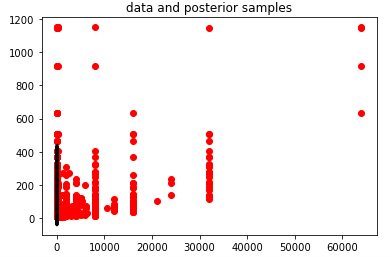
**What is your experiment setup for training these methods?**

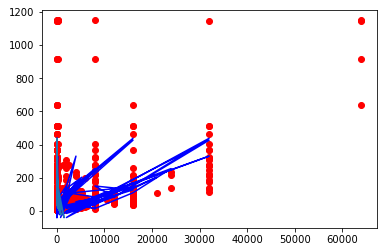
**Bayesian Linear Regression**

1. Split the dataset into six parts by its features ---- Cache memory size, the minimum and the maximum number of I/O channels, machine cycle time, and minimum and maximum main memory. Each of them is a (168,1) list.

2. Define a Gaussian prior over 𝐰, with mean 0, a (2,2) covariance matrix, and the fixed 𝜎2=2.

3. Using NumPy’s multivariate normal to generate samples from a multivariate Gaussian and sampling some 𝐰 vectors and plotting the models with all features.

4. Write functions to construct polynomial design matrix, and compute posterior mean and covariance where 𝐒 is the covariance matrix of the prior 𝑝(𝐰). Since there are 6 features, we add each polynomial dataset and compute the expected result sequentially.

5. Look at functions for posterior prediction by plotting the error bar for general checking.

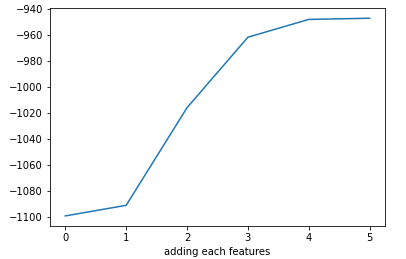
6. when the error bar looks fine, use w to generate test results from the test dataset.

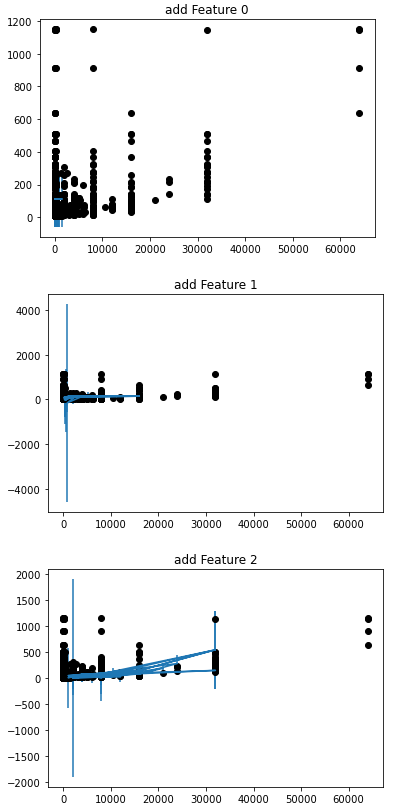
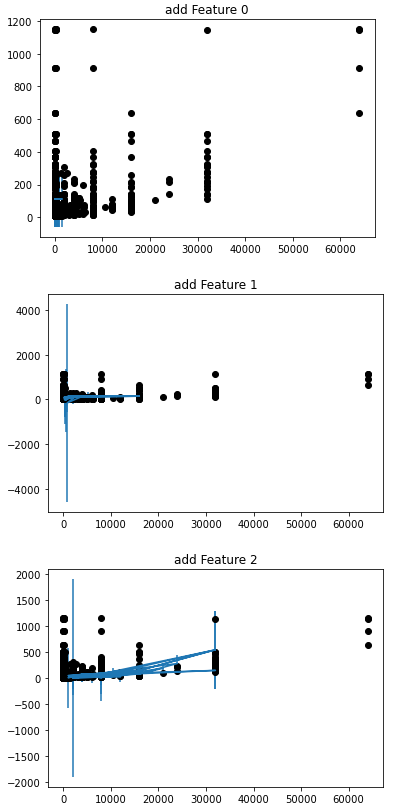
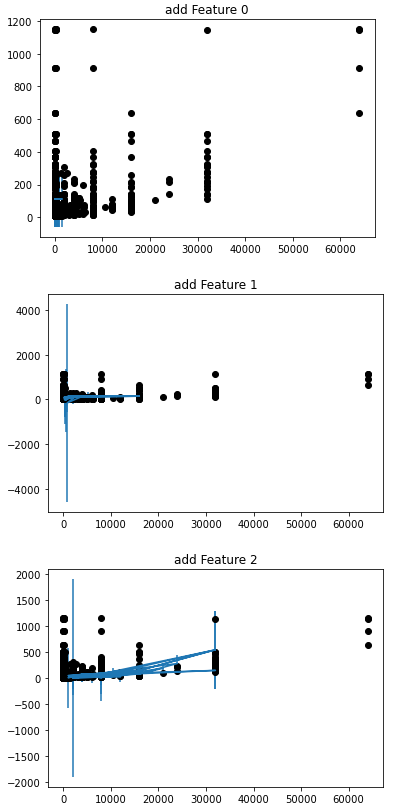
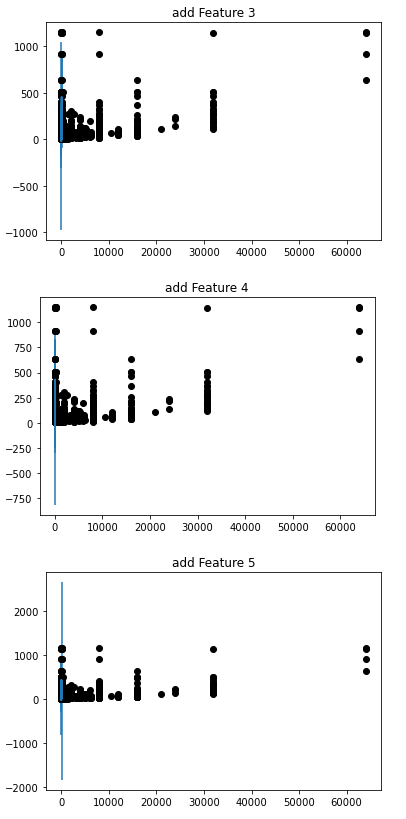
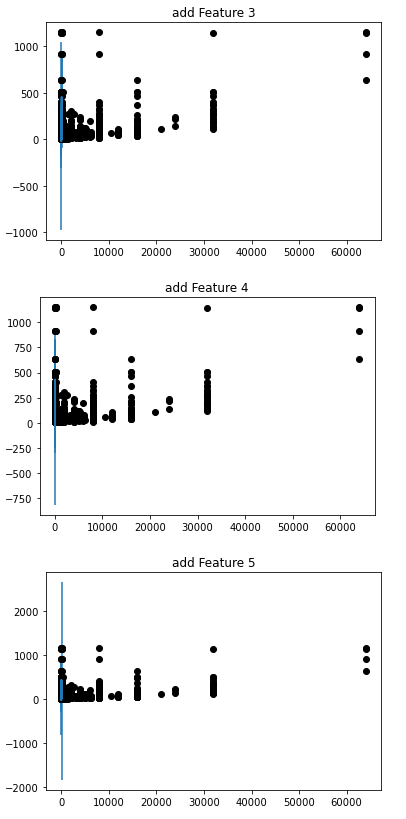
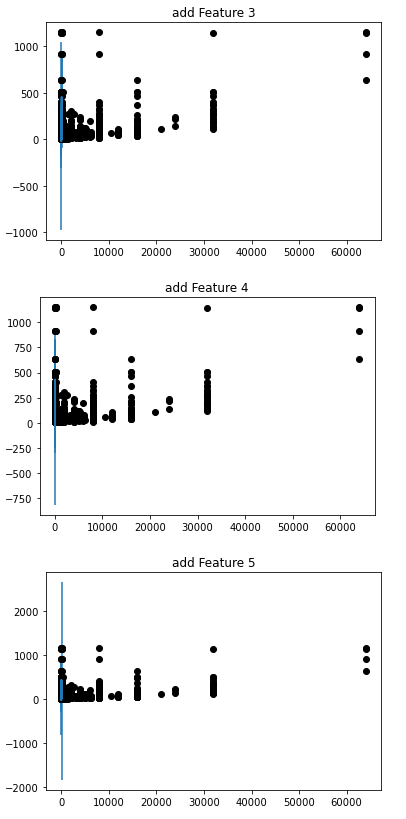
**Maximum Likelihood Regression**

1. Split the dataset into six parts by its features ---- Cache memory size, the minimum and the maximum number of I/O channels, machine cycle time, and minimum and maximum main memory. Each of them is a (168,1) list.

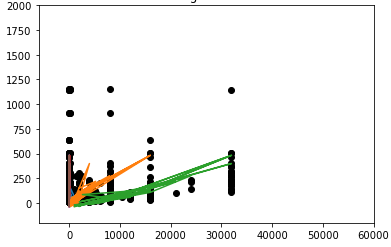
2. Write a function to compute the log of gaussian pdf, maximum likelihood estimate of w, maximum likelihood estimate of sigma square, and construct the polynomial design matrix

3. Test the joint likelihood against the adding of features to check the influence of each feature. Loop over all features, use the maximum likelihood estimate of w and maximum likelihood estimate of sigma square to compute the log-joint likelihood at the maximum likelihood estimates. The plot lost between adding features.



4. Predictive variance example, step up

5. Look at functions for posterior prediction by plotting the error bar for general checking.



6.When the error bar looks fine, use w to generate test results from the test dataset.

How are you going to measure the performance of your regression algorithms?



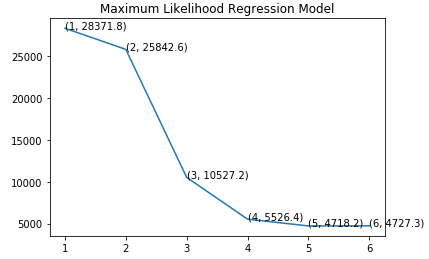
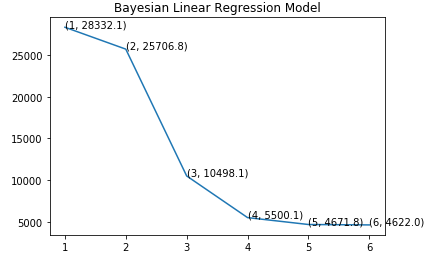
Plotting the predicted values against the real value is the best way to measure the goodness of a regression model. I use average squared loss, which measures the average error performed by the model in predicting the outcome for an observation. Mathematically, it is the average squared difference between the observed actual outcome values and the values predicted by the model. The lower the average squared loss reaches, the better the model accuracy will get.

Alternatively, we can test our test dataset on the Kaggle prediction. Somehow, it performs as a cross-validation way to check the general performance. However, it is not as explicit as the average squared loss.

**Question 3**

**Compare the performance of these two algorithms.**

In this section, I will adapt the average squared loss measurement onto the Bayesian Linear Regression Model and the Maximum Likelihood Regression Model.

In order to compare those two models in detail, I choose to construct polynomials on training data. For each constructed feature, I generate their model and use the original data in the generated model to calculate predict data. Finally, computing the loss between the actual train data and test data by average squared loss. The least result indicates the better performance this model may have.

As we can see in the plot, in general, adding features may decrease the model loss. For some reason, in the Maximum Likelihood Regression Model, the performance of 5 features is better than adding one more feature. The performance of the Bayesian Linear Regression Model is slightly better than the Maximum Likelihood Regression Model.

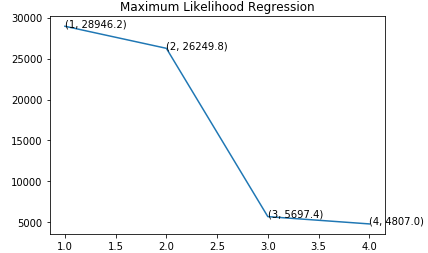
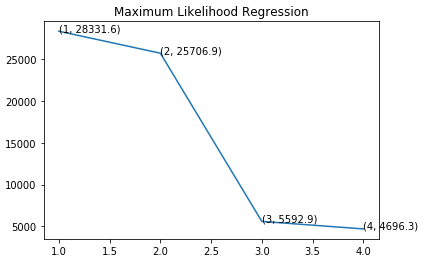
Bayesian Linear Regression Model fits the training data well and Maximum Likelihood Regression Model could perform a better prediction on a wide scope of the dataset. Thus, the Maximum Likelihood Regression Model has a likely equal performance on general practical datasets comparing with Bayesian Linear Regression Model.

**Question 4**

**Can you obtain better performance by using only a subset of the features?**

For both models, I tried to combine some “sharpen” features. I combined the minimum and the maximum main memory as the main memory feature and the minimum and the maximum number of I/O channels as the I/O channels feature. So, I subtracted the feature number from 6 to 4.

After that, I adapted the model based on my model setup. And I used the polynomial type of the original combined dataset to predict the test dataset. Finally, I calculated the average squared loss and compared the performance between the flatten feature prediction.



According to the average squared loss score, the combining feature penalizes the Maximum Likelihood Regression Model but slightly improves the Maximum Likelihood Regression Model. And in the Kaggle competition testing data score, there is not a better performance shown.

Thus, in our hypothesis to combine the minimum and maximum dataset features, we cannot improve model performance. However, there may exist other combination ways to eliminate dataset noise.