

Modeling Hierarchical Spatial Interdependence for Limited Dependent Variables*

Ali Kagalwala[†] and Hankyeul Yang[‡]

Abstract

Multilevel modeling accounts for outcome dependence across lower-level units due to unobserved group effects, while spatial modeling accounts for outcome dependence across units in the same level of analysis due to diffusion. Outcome dependence can occur simultaneously due to both spatial diffusion in the lower-level units and spatial diffusion in the unobserved group effects. For example, counties are nested within states and diffusion processes might take place at both levels of analysis. Building on recent research from the spatial econometrics and multilevel modeling literature, we propose a class of spatial hierarchical models with binary outcomes. One method accounts for spatially independent, unobserved group effects and the other method accounts for spatially dependent unobserved group effects. We propose a Bayesian approach to estimate such effects while also accounting for lower-level diffusion in the outcome, and provide software to estimate these models. Our Monte Carlo results demonstrate that failing to correctly account for diffusion and/or the nested structure of data can lead to bias in both parameter estimates and substantive effects. We apply these models to analyze the causes of civil rights protests in the United States in the 1960s.

* Authors names are listed in alphabetical order.

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[†]Postdoctoral Researcher, Institute for Democracy, Journalism and Citizenship, Syracuse University, Email: akagalwa@syr.edu.

[‡]PhD candidate, Department of Political Science, Texas A&M University, Email: yang-hankyeul@tamu.edu.

1 Introduction

Theories about diffusion in political science are vast. These theories include diffusion of policy across states, conflict across countries, and defense policy across nations ([Simmons and Elkins, 2004](#); [Polo, 2020](#); [Sandler and Shimizu, 2014](#)). The introduction and advancement of spatial regression models in political science have empowered researchers to appropriately model such spatial processes. Failing to accurately model spatial processes results in inefficiency at best and bias, inconsistency, and inefficiency at worst. The introduction of spatial regression models also introduced a new dimension to substantive interpretation. Scholars could now analyze how a change in an independent variable affects the dependent variable in the same unit (direct effect), the dependent variable in other units (indirect effects), and the total effect in all units due to a change in that independent variable (total effects).

Analyzing multilevel data is also popular in political science. These data are structured in a manner such that lower-level or level-1 units are nested in higher-level or level-2 units.¹ Two examples of data that contain a nested structure include units (level-2) that are each observed across multiple time periods (level-1) (e.g., time-series-cross-section data), and units at different levels of analysis, (e.g., voters (level-1) in a county (level-2)). Due to the hierarchical nature of the data, estimating models that fail to account for the nested structure of these data lead to inaccurate inferences. This is because both level-1 and level-2 units can have their own effects. One such effect includes level-2 intercepts that account for unobserved effects that are invariant across level-1 units within each level-2 unit (fixed/random intercepts). At best, failing to account for the multilevel nature of data will

¹Data can have more than two levels. Without loss of generality, we focus our discussion to two hierarchical levels.

result in inefficiency, and at worst, there will be bias, inconsistency, and inefficiency.

A budding vein of research now combines advances in spatial econometrics and hierarchical modeling to model diffusion at multiple levels of analysis ([Dong and Harris, 2015](#)). Spatial models can account for outcome dependence across units in the same level of analysis due to diffusion and multilevel models account for outcome dependence across level-1 units due to unobserved group effects. However, there also exists the possibility that outcome dependence is a consequence of two processes occurring simultaneously. One such process is due to diffusion among level-1 units and the other process can be due to the presence of spatially dependent/independent level-2 unobserved group effects. Consider the civil rights protests in the US in the 1960s. A level-1 diffusion process is when a county was more likely to experience a civil rights protest if nearby counties experienced civil rights protests due to the spread of information about protests through local news media ([Andrews and Biggs, 2006](#)). As an example of spatially dependent level-2 effects, consider that the baseline propensities (unobserved state-level effects) for civil rights protests to occur in one state could have affected the baseline propensities for civil rights protests to occur in other states possibly due to interstate travel by civil rights activists ([Andrews and Biggs, 2006](#)). As an example of spatially independent level-2 effects, consider that, hypothetically, interstate travel during that time was prohibited for all individuals; each state could have had its own baseline propensities that were independent of other states' baselines propensities. [Dong and Harris \(2015\)](#) only account for level-1 and level-2 diffusion processes in their model and do not introduce a model that accounts for spatially independent level-2 unobserved group effects. Thus, estimating a model similar to theirs can lead to efficiency losses when there exists spatially independent level-2 unobserved group effects.

We introduce two hierarchical spatial strategies for binary outcomes. Our first proposed model, spatial autoregressive probit with random effects (SAR-RE), accounts for level-1 spatial diffusion in the outcome and only spatially independent, unobserved level-2 effects. Our second proposed model, hierarchical spatial autoregressive (HSAR) probit, accounts for diffusion at both level-1 and level-2 units and allows researchers to explicitly theorize about and test for the existence of multilevel diffusion processes.² We reach two main conclusions from a series of Monte Carlo experiments. First, as expected, estimating a spatial autoregressive (SAR) probit model or a SAR random effects probit model that only accounts for level-1 diffusion results in inaccurate inferences when there is also level-2 diffusion; the HSAR model performs well in this scenario. Second, we find that estimating a HSAR model results in accurate inferences when there is no spatial diffusion at either level and/or in the presence of non-spatial random effects. However, this comes with a cost of reduced precision because of the inefficiency from estimating extra parameters. We thus recommend that researchers at minimum consider testing for higher-level spatial diffusion and then, based on these results, decide between the HSAR or a model it encompasses, e.g., SAR-RE.

2 The importance of spatial probit models

Many processes in political science have outcomes that are binary and are also affected by outcomes in other units. These include studies of territorial claims ([Lee, 2024](#)), military regimes ([Caruso, Pontarollo and Ricciuti, 2020](#)), terrorism ([Polo, 2020](#)), conflict ([Lane, 2016](#)), and mediation ([Böhmelt, 2015](#)). Spatial probit models allow for researchers to test theories about such diffusion processes with binary outcomes. The spatial probit model can be

²We provide an R package that estimates these models and calculates substantial effects of interest.

written as

$$\mathbf{y}^* = \rho \mathbf{W} \mathbf{y}^* + \mathbf{X} \boldsymbol{\beta} + \boldsymbol{\epsilon},$$

where \mathbf{y}^* is an $N \times 1$ vector of latent outcomes in all N units ($y_i = 1$ if $y_i^* > 1$ and $y_i = 0$ otherwise), ρ is the spatial autoregressive coefficient, \mathbf{X} is a matrix of predictors with dimensions $N \times K$, $\boldsymbol{\beta}$ is a $K \times 1$ vector, and \mathbf{W} is an $N \times N$ spatial weights matrix. In general, an important motivation for modeling spatial processes is that level-1 observations are not independent from each other, and thus researchers need to appropriately calculate and interpret how a change in a predictor affects the outcome of interest. Failing to appropriately account for a spatial process can lead to an inefficient, biased, and inconsistent estimator. Similar to spatial autoregressive (SAR) models with continuous outcomes, researchers can calculate post-estimation substantive effects (direct, indirect, and total effects) of a change in a predictor using a spatial probit model. The direct effect measures the change in the propensity of an event occurring due to a change in a predictor in the same unit, the indirect effect measures the change in the propensity of an event occurring in other units due to a change in the propensity of an event occurring in the same unit as a result of a change in a predictor in that unit, and the total effect is the sum of the direct and indirect effects and measures the total change in the propensity of an event occurring due to a change in the predictor (e.g., [LeSage and Pace, 2009](#); [Franzese, Hays and Cook, 2016](#)).

3 The importance of modeling unobserved random effects in multilevel data with binary outcomes

The motivation behind multilevel models are also non-independent observations. However, in this case, the level-1 observations (e.g., students) within a level-2 group (e.g., school) are not independent and the observations across level-2 groups are traditionally assumed to be independent.³ One commonly used model is the random intercepts probit model in which level-2 (j) intercepts affect the level-1 (i) outcome:

$$\Pr(y_{ij} = 1|x_i) = \Phi(\alpha_j + \beta x_i)$$

where y_{ij} is a binary outcome, α_j is the level-2 intercept such that $\alpha_j \sim N(0, \sigma_\alpha^2)$, and x_i is a level-1 predictor. The random intercepts model relies on the assumption that the predictor(s) and random intercepts are uncorrelated. Failing to account for random intercepts in the model leads to observed attenuation in β_1 in models with binary outcomes due to incorrect normalization ([Neuhaus and Jewell, 1993](#); [Cramer, 2003](#)). As the residual variance increases, the normalized coefficient estimate decreases as the change in the residual variance unaccounted for affects the scaling of the coefficients ([Cramer, 2003](#), 81).

³Although traditionally assumed to be independent, level-2 units may not necessarily be independent. For example, a negative learning outcome in a school may result in learning from schools with positive learning outcomes.

4 Multilevel data and multiple diffusion processes

For the most part, hierarchical models and spatial econometrics have developed as two distinct fields.⁴ The focus of the spatial econometrics literature is to explicitly model theoretically interesting diffusion processes. For example, [Simmons and Elkins \(2004\)](#) examine how liberal economic policies have diffused across countries over time. The hierarchical modeling literature implicitly assumes that units in a common group share some similar characteristics. The typical example often used is that students in the same classroom may often have correlated errors because they have the same teacher and share similar experiences within the classroom. Hierarchical models explicitly deal with this by accounting for the multilevel data structure in which lower-level units are nested within high-level units.

There have been recent advances in the spatial econometrics literature that focus on combining multilevel and spatial modeling. For example, [Dong and Harris \(2015\)](#) proposed a hierarchical spatial autoregressive model with which they estimated the effects of various factors on the leasing price of land parcels in China, where land parcels are grouped into various districts. Advances in spatial and hierarchical modeling combined with data availability have created opportunities to improve our understanding of politics. For example, while [Mazumder \(2018\)](#) investigates the persistent effects of civil rights protests on political attitudes, researchers may also want to investigate the conditions under which protests are likely to occur in some counties, but not in others. In addition to the diffusion of civil rights protests across counties because of the spread of information ([Andrews and Biggs, 2006](#)), two other features are worth considering. First, counties are nested within states. Different

⁴Similar to [Gelman and Hill \(2006\)](#), we use the terms multilevel models and hierarchical models interchangeably.

states might have had different political, social and economic conditions that could have affected the common baseline propensity of civil rights protests to occur in counties within the same state. Second, the baseline propensity for civil rights protests could have diffused across states.

Our proposed model takes into account the nested structure of data and also tests for the existence of a diffusion process among higher-level units. If a scholar has a theory about how a certain predictor of interest affects the diffusion of civil rights protests and does not account for the multilevel structure of the data by modeling diffusion at both level-1 (across counties within states) and level-2 (across states) units, they may erroneously conclude that that predictor has a larger-than-true indirect effect in the diffusion of civil rights protests.⁵ In other words, they may overestimate the magnitude of how a change in the probability of civil rights protests due to a change in a predictor in one county affects the change in the probability of civil rights protests in other counties. We introduce a second model, SAR-RE, that can be derived from the HSAR model. This model accounts for the multilevel nature of the data by only accounting for spatially independent random effects. When there is no spatial dependence across unobserved group effects, estimating the HSAR model can lead to inefficiency due to the costs of estimating an additional parameter. We begin by discussing the HSAR model and its estimation in the next section followed by a discussion of the SAR-RE in Section 7.

⁵We provide a more detailed technical discussion of this in section 5.1.

5 Hierarchical Spatial Probit Autoregressive Model

We propose a binary outcome model for estimating spatial autoregressive coefficients that accounts for the hierarchical nature of data.⁶ Our proposed hierarchical spatial autoregressive (HSAR) probit model is an extension of the work of Dong and Harris (2015), who focused on the continuous outcomes case.⁷

We can first conceptualize the outcome as a continuous latent variable, \mathbf{y}^* , such that

$$\mathbf{y}^* = \rho \mathbf{W} \mathbf{y}^* + \mathbf{X} \boldsymbol{\beta} + \Delta \boldsymbol{\theta} + \boldsymbol{\epsilon},$$

where there are $i = 1, \dots, N$ lower-level units nested in $j = 1, \dots, J$ higher-level units. \mathbf{y}^* is an $N \times 1$ vector of the latent continuous outcome variable, \mathbf{W} is an $N \times N$ spatial weights matrix for lower-level units,⁸ \mathbf{X} is an $N \times K$ matrix of covariates, $\boldsymbol{\beta}$ is a $K \times 1$ vector of parameters, Δ is an $N \times J$ matrix mapping lower-level units to higher-level units, $\boldsymbol{\theta}$ is a $J \times 1$ vector of random effects, and $\boldsymbol{\epsilon}$ is an $N \times 1$ vector of white noise.⁹

⁶We provide a discussion and analysis of the ordinal outcomes case for the HSAR model in Appendix U.

⁷Without loss of generality, our model is a stylized version in which higher-level predictors have been omitted. As we show later, this is to make our model easily comparable to the original SAR probit model. Practitioners can easily extend the model presented here to include covariates for the higher-level units.

⁸Note that our model allows for level-1 units in one group to be connected to level-1 units in another group. This must be done in \mathbf{W} . Consider our civil rights protests example. It is likely that the spread of information through local media occurred between two contiguous counties in different states. One such example is Harrison County, TX and Caddo County (Parish), LA.

⁹As an example of a Δ matrix, consider a 6×3 Δ matrix consisting of 6 lower-level units with 2 lower-level units in each higher-level unit. This can be represented by the following Δ matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The model assumes that $\boldsymbol{\theta}$ follows its own autoregressive process and that there is a spatial weights matrix \mathbf{M} of dimensions $J \times J$ as shown below

$$\boldsymbol{\theta} = \lambda \mathbf{M} \boldsymbol{\theta} + \mathbf{u} \Rightarrow \boldsymbol{\theta} = (\mathbf{I}_J - \lambda \mathbf{M})^{-1} \mathbf{u}$$

$$\mathbf{u} \sim \mathcal{N}(\mathbf{0}, \sigma_u^2 \mathbf{I}_J)$$

$$\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_N)$$

$$\boldsymbol{\theta} \sim \mathcal{N}(0, \sigma_u^2 (\mathbf{B}' \mathbf{B})^{-1}) \text{ where } \mathbf{B} \equiv \mathbf{I}_J - \lambda \mathbf{M}$$

As is standard in the literature, we assume that we observe a value of 1 for the binary outcome, y_{ij} , if the latent variable y_{ij}^* is (weakly) positive and 0 otherwise:

$$y_{ij} = 1 \iff y_{ij}^* \geq 0$$

$$y_{ij} = 0 \quad \text{otherwise}$$

In terms of the civil rights protest example, y_{ij} would be coded as 1 if a civil rights protest occurred within county i in state j and 0 otherwise. \mathbf{W} and \mathbf{M} would be the spatial weights matrices at the county and state levels respectively. Δ is the matrix that maps counties to states and \mathbf{X} is the matrix of predictors, such as the percentage of urban population and the median age.

We need to use the truncated multivariate normal distribution to generate the entire vector of latent variables \mathbf{y}^* instead of (independent) truncated univariate normal distributions as is the case with the standard probit model (e.g., [Geweke, 1991](#)).¹⁰ We impose two addi-

¹⁰We use the **R** package **tmvtnorm** ([Wilhelm and Manjunath, 2010](#)) to draw the values.

tional assumptions. First, the variance of the error, σ_ϵ^2 , is 1. This is a standard identification assumption. Second, there is no covariance between $\boldsymbol{\epsilon}$ and $\boldsymbol{\theta}$ (the vector of random effects). These can be written as:

$$Var(\boldsymbol{\epsilon}) = \mathbf{I}_N$$

$$Cov(\boldsymbol{\epsilon}, \boldsymbol{\theta}) = 0$$

Based on these above assumptions, the variance-covariance matrix of \mathbf{y}^* in [Dong and Harris \(2015\)](#) is:

$$Var(\mathbf{y}^* | X) = \mathbf{A}^{-1} \left(\sigma_u^2 \Delta (\mathbf{B}' \mathbf{B})^{-1} \Delta' + \mathbf{I}_N \right) (\mathbf{A}^{-1})' \equiv \mathbf{V} \quad (1)$$

where $\mathbf{A} \equiv \mathbf{I}_N - \rho \mathbf{W}$ and $\mathbf{B} \equiv \mathbf{I}_J - \lambda \mathbf{M}$ as per equation 1. As will be seen later, \mathbf{V} will play an important role in using the truncated multivariate normal distributions for generating \mathbf{y}^* .

5.1 Direct, Indirect, and Total Effects

Researchers in spatial econometrics have warned that we cannot directly infer effects from estimates (e.g., [Franzese, Hays and Cook, 2016](#)). Since the dependent variable is discrete, we would be interested in finding out, for example, how the propensity for the i th observation to experience an event would change due to a change in the value of a variable, x_k , for that same unit, i , or due to a change in x_k in unit j . While the calculation of such effects are not simple, past scholars have shown the derivation for such quantities of interest ([Beron and Vijverberg, 2004; Franzese, Hays and Cook, 2016](#)). To calculate the *average* direct and

indirect effects, we use the formula from [Franzese, Hays and Cook \(2016, 155,160\)](#).¹¹

Let x_{ik} be the value of x_k for unit i . To calculate the change in propensity for the i th observation to experience an event due to a change in x_{ik} (direct effect), we can calculate

$$\frac{\partial p(y_i = 1 | \mathbf{X}, \mathbf{M}, \mathbf{W})}{\partial x_{ik}} = \phi \left\{ [(\mathbf{I} - \rho \mathbf{W})^{-1} \mathbf{X} \boldsymbol{\beta}]_i / \omega_i \right\} [(\mathbf{I} - \rho \mathbf{W})^{-1} \boldsymbol{\beta}_k]_{ii} / \omega_i,$$

where ϕ is the PDF of the standard normal distribution and ω_i is the i th element of the variance-covariance matrix $\Omega \equiv \mathbf{A}^{-1} (\mathbf{A}^{-1})'$ ([Franzese, Hays and Cook, 2016](#)).¹²

Similarly, the change in propensity for the i th observation to experience an event due to a change in x_k for some other unit j (indirect) would be

$$\frac{\partial p(y_i = 1 | \mathbf{X}, \mathbf{M}, \mathbf{W})}{\partial x_{ik}} = \phi \left\{ [(\mathbf{I} - \rho \mathbf{W})^{-1} \mathbf{X} \boldsymbol{\beta}]_i / \omega_i \right\} [(\mathbf{I} - \rho \mathbf{W})^{-1} \boldsymbol{\beta}_k]_{ij} / \omega_i$$

The total effect of a change in a predictor on both its own outcome and the outcome of other units can thus be calculated by summing the direct and indirect effects.

We previously noted that failing to account for the level-2 diffusion process will result in overestimated indirect effects. To see the intuition behind this argument, consider the conditions under which ρ is estimated to be a non-zero coefficient. From past research ([Franzese, Hays and Cook, 2016](#)), we know that the variance of the error in a spatial probit

¹¹We focus on the fixed effects components as is common when calculating marginal effects in mixed effects models ([Bürkner, 2017](#); [Wiley and Hedeker, 2025](#)). Conceptually, this is also similar to interpreting the marginal effects of a SEM probit model as that of a regular probit model ([Martinetti et al., 2022](#)).

¹²Note that while we use \mathbf{V} for the estimation process of the parameter coefficient estimates, we use Ω for calculating the quantities of interest, i.e. the average effects.

model $\mathbf{V}(\mathbf{e})_{SAR}$ is

$$\mathbf{V}(\mathbf{e})_{SAR} = [(I - \rho\mathbf{W})' (I - \rho\mathbf{W})]^{-1}$$

which is reduced to an identity matrix when ρ in the data generating process is zero. In other words, when errors are homoskedastic due to ρ being zero in the data generating process, researchers would (on average) estimate ρ to be zero as well. What scholars should note, however, is that heteroskedasticity can be caused with random effects in the intercept as well. In particular, even if ρ equals zero, the variance components from $\Delta\boldsymbol{\theta}$ can still cause heteroskedasticity.

$$\mathbf{V}(\mathbf{e})_{HSAR} = (I_N - \rho\mathbf{W})^{-1} Var(\Delta\boldsymbol{\theta} + \boldsymbol{\epsilon}) ((I_N - \rho\mathbf{W})^{-1})' \quad (2)$$

Equation 2 depicts the variance of the error of our proposed hierarchical spatial probit model ($\mathbf{V}(\mathbf{e})_{HSAR}$) that accounts for a level-2 diffusion process.¹³ As seen above, we need to estimate an additional variance component, $Var(\Delta\boldsymbol{\theta} + \boldsymbol{\epsilon})$, to obtain unbiased estimates of $\boldsymbol{\beta}$ and the marginal effects. Unlike linear models, incorrectly modeling the error structure by inadequately accounting for heteroskedastic errors leads to biased and inefficient parameter estimates (Yatchew and Griliches, 1985; Keele and Park, 2006; Greene, 2003). There are two interrelated implications from such a scenario. First, as previously discussed, the coefficient estimate of $\boldsymbol{\beta}$ is likely to suffer from attenuation bias (Neuhaus and Jewell, 1993; Cramer, 2003). Second, the estimate of ρ would be overestimated as the spatial probit model erro-

¹³ $\mathbf{V}(\mathbf{e})_{HSAR}$ directly follows from equation 1.

neously ascribes heteroskedasticity to a spatial diffusion process. Consequently, this would lead scholars to subsequently overestimate indirect effects when calculating quantities of interest.

6 Estimating the Model

We adopt a Bayesian Markov Chain Monte Carlo (MCMC) method for model estimation.¹⁴

We employ diffuse priors to let the data dominate the posterior. This approach is standard and has been adopted by spatial econometrics researchers in past works ([LeSage and Pace, 2009](#)).

The likelihood function in terms of the latent variable \mathbf{y}^* may be specified as follows:

$$\mathcal{L}(\mathbf{y}^* | \rho, \lambda, \boldsymbol{\beta}, \boldsymbol{\theta}, \sigma_u^2) = (2\pi)^{-N/2} |\mathbf{A}| \exp \left\{ -\frac{1}{2} (\mathbf{A}\mathbf{y}^* - \mathbf{X}\boldsymbol{\beta} - \Delta\boldsymbol{\theta})' (\mathbf{A}\mathbf{y}^* - \mathbf{X}\boldsymbol{\beta} - \Delta\boldsymbol{\theta}) \right\}$$

The basic strategy is to employ a combination of Gibbs sampling and the Metropolis-Hastings sampling algorithms.¹⁵ While most parameters can be estimated using the Gibbs sampling algorithm, the spatial coefficients estimates have to be estimated using the Metropolis-Hastings algorithm.

6.1 Generating \mathbf{y}^*

We generate samples of \mathbf{y}^* as follows ([Albert and Chib, 1993](#)):

¹⁴The derivations for the full conditional distributions for the case of continuous outcome are detailed in [Dong and Harris \(2015\)](#) and reproduced in Appendix T.

¹⁵We found that Stan ([Carpenter et al., 2017](#)) is too computationally inefficient for estimating spatial econometric models with a large number of observations. [Wolf, Anselin and Arribas-Bel \(2018\)](#) suggest that this is due to the computational burden of calculating the log-determinant term for each leapfrog step.

$$y_{ij}^* | \rho, \beta, \theta, \mathbf{y} \sim \begin{cases} \mathcal{MVN}(k_i, v_i) \mathbb{1}(y_{ij}^* \geq 0) & y_{ij} = 1 \\ \mathcal{MVN}(k_i, v_i) \mathbb{1}(y_{ij}^* < 0) & y_{ij} = 0 \end{cases} \quad (3)$$

where $\mathbb{1}$ is the indicator function, v_i denotes the \mathbf{V}_{ii} element in the variance-covariance matrix of \mathbf{y}^* , and k_i is the i^{th} element of the $N \times 1$ column vector $\mathbf{K} \equiv \mathbf{A}^{-1}(\mathbf{X}\beta)$. Thus, equation 3 allows us to generate the latent values, y_{ij}^* , while simultaneously accounting for spatially correlated errors.

6.2 Simulating β , θ , σ_u^2 , ρ and λ

As explained above, we use the Metropolis-within-Gibbs algorithm which is the standard Bayesian procedure for estimating limited dependent variable models in spatial econometrics. For each draw, we sequentially update the parameter values of β , θ , σ_u^2 , ρ and λ . We can draw the values from standard distributions for β , θ and σ_u^2 because of the conjugacy structure whereas we have to use the Metropolis algorithm to draw the values for ρ and λ since the full conditional distributions are not in recognizable forms.¹⁶

7 Model Implications and Expectations

We briefly discuss three important implications of our proposed HSAR probit model and their performance expectations in our Monte Carlo simulations:

1. **Multilevel random intercept probit model:** this is a special case of our model

¹⁶We provide the conditional posterior distributions for these parameters in Appendix T.

with the restrictions that $\rho = \lambda = 0$.

$$\mathbf{y}^* = \mathbf{X}\boldsymbol{\beta} + \Delta\mathbf{u} + \boldsymbol{\epsilon} \quad \text{where} \quad \mathbf{u} \sim \mathcal{N}(\mathbf{0}, \sigma_u^2 \mathbf{I}_J)$$

The main implication here is that the multilevel random intercept probit model is likely to perform similarly to our proposed HSAR probit model when both λ and ρ are relatively low. Similar to the simulation results in [Dong et al. \(2015\)](#), we expect the multilevel random intercept probit model to recover biased estimates of β_0 as ρ increases and biased estimates of σ_u^2 as λ increases because the multilevel random intercept probit model is not properly accounting for the additional variance due to the spatial diffusion processes among both lower- and higher-level units.

2. SAR probit model: our model simplifies into a SAR probit model when we restrict

$$\sigma_u^2 = 0.$$

$$\mathbf{y}^* = \rho \mathbf{W} \mathbf{y}^* + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{4}$$

We expect that SAR probit model to perform relatively well and similar to our proposed HSAR model when there are no higher-level random effects, $\sigma_u^2 = 0$. When $\sigma_u^2 > 0$, we expect $\boldsymbol{\beta}$ to be underestimated ([Neuhaus and Jewell, 1993](#); [Cramer, 2003](#)) and ρ to be overestimated because the SAR probit model mistakenly attributes the heteroscedasticity from omitted random effects to spatial effects, as per our discussion surrounding equation 2.

3. SAR probit model with random effects: when only $\lambda = 0$, our model simplifies into a SAR probit model with random intercepts. To the best of our knowledge, this model has not been discussed—at least, widely—in the spatial econometrics literature and we have never seen this model applied in the political science literature. We highlight this model to be an additional contribution that is derived from our more general HSAR probit model.

$$\mathbf{y}^* = \rho W \mathbf{y}^* + \mathbf{X}\boldsymbol{\beta} + \Delta \mathbf{u} + \boldsymbol{\epsilon} \quad \text{where} \quad \mathbf{u} \sim \mathcal{N}(\mathbf{0}, \sigma_u^2 \mathbf{I}_J)$$

As we previously mentioned in our discussion surrounding equation 2 in section 5.1, we expect the model to overestimate ρ and the spatial indirect effect when the DGP is a HSAR process. This is because this model does not account for the higher-level spatial diffusion process.¹⁷

If the researcher is uncertain about the existence of spatial processes, the cost of estimating our proposed HSAR solution when there is either none (e.g., multilevel probit DGP) or one spatial process across both levels (e.g., SAR or SAR-RE DGPs) is efficiency losses due to the estimation of additional parameters. The benefit of treating the HSAR model as a general model is unbiasedness (assuming all other model assumptions hold) regardless of the existence of no spatial processes at the both levels, a spatial process at one level, or spatial interdependence at both levels. Thus, the researcher should carefully weigh the benefits and costs and consider first testing for higher-level diffusion when deciding on the

¹⁷These expectations should hold regardless of whether the predictor is a function of independently generated data, a single-level diffusion process, or a hierarchical diffusion process because simply including the predictor in the model should suffice (Cook, An and Favero, 2019; Cook, Hays and Franzese, 2020).

type of multilevel model to estimate.

8 Monte Carlo Simulations

We conduct a series of Monte Carlo simulations to assess the validity of our proposed HSAR model. Our DGP is as follows:

$$\mathbf{y}^* = \rho \mathbf{W} \mathbf{y}^* + \mathbf{X} \boldsymbol{\beta} + \Delta \boldsymbol{\theta} + \boldsymbol{\epsilon}$$

$$\boldsymbol{\theta} = \lambda \mathbf{M} \boldsymbol{\theta} + \mathbf{u} \Rightarrow \boldsymbol{\theta} = (\mathbf{I} - \lambda \mathbf{M})^{-1} \mathbf{u}$$

$$\mathbf{u} \sim \mathcal{N}(\mathbf{0}, \sigma_u^2 \mathbf{I}_J)$$

$$\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_N)$$

$$\boldsymbol{\theta} \sim \mathcal{N}(0, \sigma_u^2 (\mathbf{B}' \mathbf{B})^{-1}) \quad \mathbf{B} \equiv \mathbf{I}_J - \lambda \mathbf{M}$$

$$y_{ij} = 1 \iff y_{ij}^* \geq 0$$

$$y_{ij} = 0 \quad \text{otherwise}$$

We use the same spatial weights matrices (\mathbf{M} and \mathbf{W}) as [Dong and Harris \(2015\)](#) for both levels, with $J = 111$ and $N = 1117$.¹⁸ Δ denotes the mapping matrix that maps lower-level units to higher-level units. \mathbf{X} denotes the matrix of predictors, which includes an intercept common to all units. The parameters that are estimated with the HSAR model are $\boldsymbol{\beta}$ (the coefficient estimates of \mathbf{X}), ρ , λ (the spatial autoregressive coefficients for lower- and higher-level units, respectively) and σ_u^2 (the variance of the group-level intercept). We test the performance of our proposed model across a wide range scenarios by varying the parameters of ρ , λ , and σ_u^2 , such that $\rho, \lambda \in \{0, 0.3, 0.5\}$, and $\sigma_u^2 \in \{0, 0.5, 1\}$. We set the

¹⁸These spatial weight matrices are available through the HSAR package in R ([Dong and Harris, 2015](#)).

values of the intercept (β_0) and the coefficient of X_1 (β_1) as -0.5 and 1.0 respectively, similar to [Wucherpfennig et al. \(2021\)](#). We generate values of X_1 from an independent standard normal distribution.¹⁹

We ran 100 trials for each combination of parameters. For reasons of computational demands, we set the number of draws to 1000 for each trial and discarded the first 200 draws as a burn-in. We compare the results of our proposed HSAR model to those of three other models: a multilevel probit, a SAR probit, and a SAR probit with random intercepts.²⁰ We present the results for the bias in the direct and indirect effects of X_1 and $\hat{\rho}$.²¹

9 Monte Carlo Results

The results from our simulations are consistent with our expectations. In Figure 1, although the SAR probit model performs well when $\sigma_u^2 = 0$, there is attenuation bias in the direct effect of the SAR probit model when $\sigma_u^2 > 0$. The severity of this bias increases as σ_u^2 increases and as λ increases when $\sigma_u^2 = 1$. The SAR probit model also underestimates the direct effect when there are independent group-level random intercepts, i.e., $\lambda = 0$ and $\sigma_u^2 > 0$. Overall, the HSAR, the SAR-RE, and the multilevel probit models perform similarly well.²²

¹⁹We have also tried different numbers of higher-level units with $J = \{16, 49\}$. We generate 20 random districts within each J . This results in a combined total of $N = \{320, 980\}$ lower-level units. The $N \times J$ matrix Δ maps each of the lower level units, i , to the higher-level units, j . The \mathbf{W} spatial weights matrix was generated by simulating fake counties on a map of U.S. states and using three nearest neighbors. The M spatial weights matrix was generated by using a rook contiguity matrix. We show the results in the appendix due to space constraints. In Appendices B, D, F, H, J, L, R we further conduct simulations in which we draw X_1 from a spatially correlated process. $X_1 = (\mathbf{I} - \rho_x \mathbf{W})^{-1} \boldsymbol{\epsilon}_x$ where we set ρ_x to 0.3 and $\boldsymbol{\epsilon}_x$ is distributed standard normal. The results from these simulations are similar to those shown in the manuscript when X_1 is drawn from a standard normal distribution, i.e., when $\rho_x = 0$.

²⁰The multilevel probit model is implemented using the **lme4** package and the SAR probit model is implemented using the **ProbitSpatial** package in R ([Bates et al., 2015](#); [Martinetti et al., 2022](#)).

²¹We also vary the number of higher- and lower-level units with $J = \{16, 49\}$ and $N = \{320, 980\}$. We show the bias, standard deviation and the root mean squared error for $\hat{\rho}$, $\hat{\lambda}$, $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\sigma}_u^2$ for the above

Figure 1: Bias in the Direct Effect of X_1 for $J = 111$, $N = 1117$

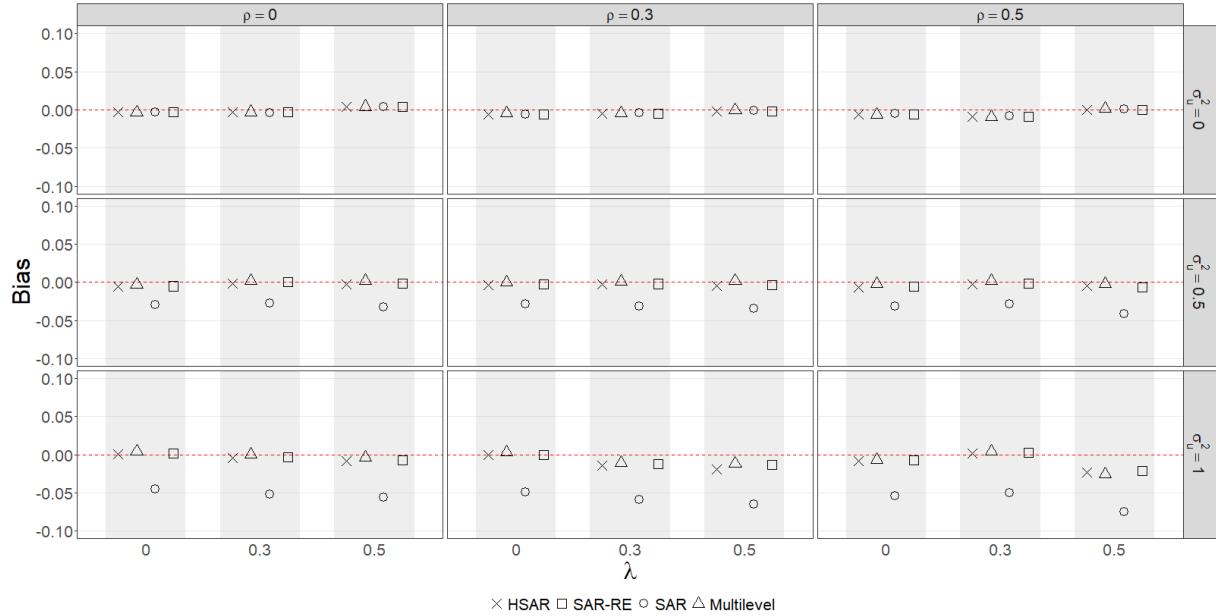
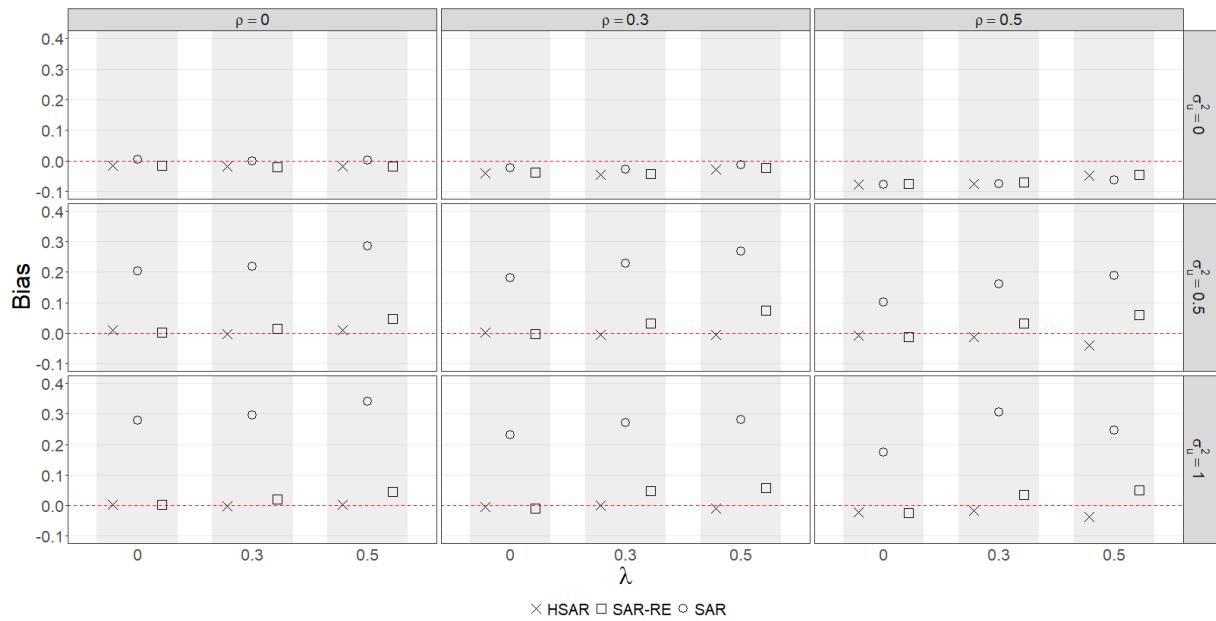
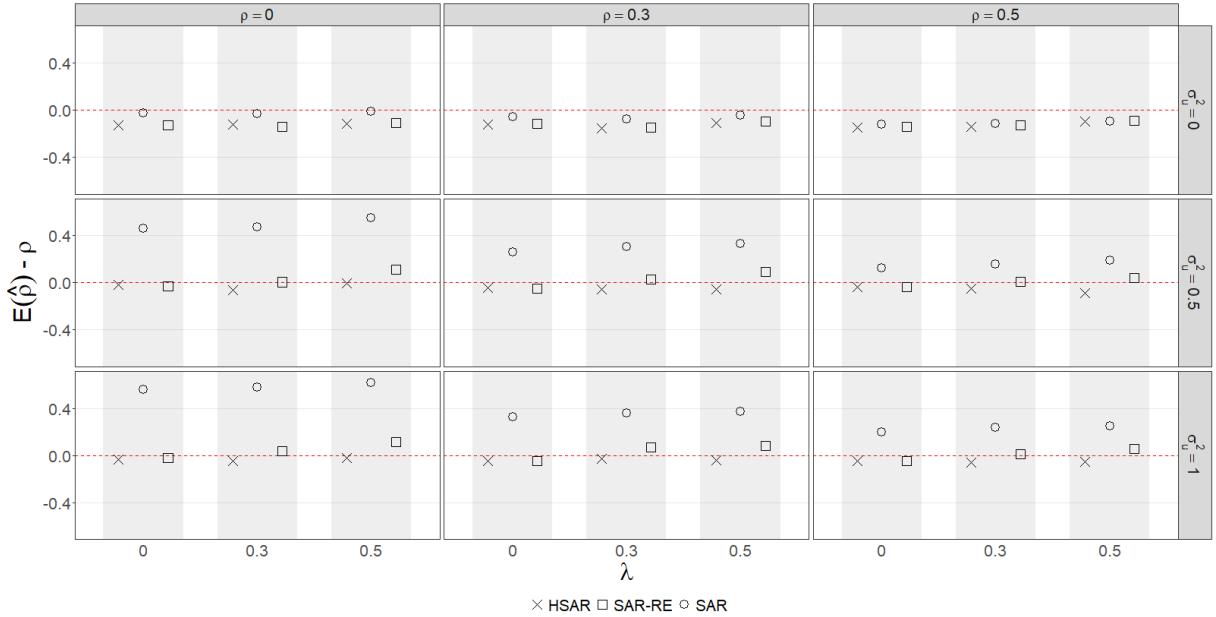


Figure 2: Bias in the Indirect Effect of X_1 for $J = 111$, $N = 1117$



We present the results for the bias in the indirect effect in Figure 2.²³ As expected, we find that the SAR probit model performs the worst and overestimates the indirect effect when the DGP contains unobserved effects, $\sigma_u^2 > 0$. This bias increases as σ_u^2 increases and/or as λ increases when $\sigma_u^2 > 0$. We also find that the performance of the SAR random effects model's performance worsens as λ increases when $\sigma_u^2 > 0$. This is consistent with our expectations as discussed in equation 2 in section 5.1. Our HSAR probit performs consistently well across the different combinations of ρ , λ , and σ_u^2 .

Figure 3: Bias in $\hat{\rho}$ for $J = 111$, $N = 1117$



As previously mentioned, since the SAR probit and SAR random effects models do not account for diffusion among the higher-level units, they mistakenly attribute this process to the spatial process among the lower-level units. This means that the SAR probit and SAR

combinations of J and N and also when $J = 111$ and $N = 1117$ in Appendices A-L, Q, and R.

²²As expected, the multilevel probit model recovers biased estimates of σ_u^2 and β_0 (Appendices Q and R) because it does not capture the additional variance caused by the higher-level spatial diffusion process (Dong et al., 2015). This can affect post-estimation calculations such as the intra-class correlation coefficient

²³The multilevel probit model does not allow for the calculation of indirect effects because it does not model spatial diffusion.

random effects models overestimate ρ , thus resulting in inflated indirect effects estimates. The results in Figure 3 corroborate these expectations. We find that the bias in $\hat{\rho}$ for the SAR probit and SAR random effects models increases as λ increases when $\sigma_u^2 \neq 0$.²⁴ For the SAR probit model, it is worth noting that we observe biased estimates of ρ even when $\lambda = 0$ —i.e., there is no spatial process among the higher-level units because omitting random intercepts ($\sigma_u^2 > 0$) is sufficient to induce bias (Neuhaus and Jewell, 1993; Cramer, 2003). Overall, we find that our HSAR model still performs the best in terms of $\hat{\rho}$. These findings highlight the importance of the indirect effects in particular, when comparing the HSAR model and the SAR-RE when $\lambda, \sigma_u^2 > 0$. The HSAR model performs well because it accounts for the multilevel nature of the data and the diffusion among unobserved group effects. The SAR and SAR-RE (despite its ability to recover overall unbiased direct effects) models recover biased $\hat{\rho}$ and consequently biased indirect effects because of their inability to account for the multilevel nature of the data and level-2 diffusion by restricting $\lambda = 0$, respectively. We also show that when $\lambda, \sigma_u^2 > 0$, the HSAR model, on average, outperforms the SAR-RE in terms of correctly classifying events and non-events (Appendix S, Table S109).

Across the different parameter combinations in the DGP and the estimates and effects calculations, we find that our proposed HSAR probit model is the best performing when $J = 111$ and $N = 1117$. In Appendices A-L, we vary the number of higher-level units, such that $J \in \{16, 49\}$. From the results in these appendices, when $J = 49$, $\rho = 0.5$, and $\sigma_u^2 \neq 0$, we notice that all models recover attenuated estimates of β_1 . Notably, the SAR probit model performs the worst and the multilevel probit, SAR random effects probit, and HSAR probit models perform similarly and recover only slightly attenuated estimates of β_1 . However,

²⁴The effect of β_1 is underestimated for the SAR probit model as per our expectations (Appendices A-R).

when $J = 111$, the multilevel probit, SAR random effects probit, and HSAR probit models, overall, recover unbiased estimates of β_1 .²⁵ We also note for $J \in \{16, 49\}$, the multilevel probit and the SAR-RE overestimate the variance of the group-level intercept, σ_u^2 , when $\lambda \neq 0$ and $\sigma_u^2 > 0$. $\hat{\sigma}_u^2$ for the SAR-RE is biased because it omits $\lambda \mathbf{M} \boldsymbol{\theta}$, implying that $\hat{\sigma}_u^2$ is also heteroskedastic since \hat{u} now includes the higher-level unmodeled spatial spillovers. Consequently, the standard errors will be incorrect, which will lead to incorrect inferences. Inaccurate estimates of σ_u^2 also causes problems estimating β_0 , β_1 , and θ , which in turn affect any inference using these parameters. The HSAR probit model overall recovers unbiased or close-to-unbiased estimates of σ_u^2 . Based on these observations, for the HSAR model, it is likely that there is small-sample bias in $\hat{\beta}_1$ when $J = 49$ which disappears when $J = 111$.

We argue that the HSAR model should be the first step when researchers believe that there are multilevel effects in the DGP. The HSAR model estimates λ , thus allowing researchers to explicitly theorize and test for the presence of a higher-level diffusion process. When there is no higher-level diffusion ($\lambda = 0$), researchers can still recover accurate estimates of the parameters and the substantive effects of interest. However, this comes at a cost to efficiency. As we demonstrate in Appendices Q and R, the SD of the HSAR model is higher than that of the SAR-RE. If the credible intervals of $\hat{\lambda}$ encompass 0, researchers may estimate the SAR-RE instead if they also find that the credible intervals of $\hat{\sigma}_u^2$ do not encompass 0.

²⁵The fact that the SAR random effects probit model performs relatively well could be an artifact of our Monte Carlo setup because the predictor, X_1 , is only a function of level-1 diffusion in its DGP. In Appendices S, we allow X_1 to be a function of both level-1 and level-2 diffusion processes. Our results are similar to those in the manuscript. This is because clustering on observables—i.e., the spatial diffusion processes in the DGP of X_1 —are unlikely to influence the estimates of predictors when modeling y . Simply including X_1 in the model specification of y is sufficient (Cook, An and Favero, 2019; Cook, Hays and Franzese, 2020). We provide these results in Appendix S.

10 Application

We now demonstrate the utility of our model by analyzing the diffusion process of civil rights protests in the United States in the 1960s. There are good theoretical reasons *not* to overlook the diffusion process of civil rights protests. Theoretically, scholars have debated whether and to what extent protests diffuse in various contexts (e.g., [Hale, 2019](#)). In the context of the United States' civil rights protests in the 1960s, sociologists have pointed out various mechanisms through which protests might have spread (e.g., local newspapers [Andrews and Biggs, 2006](#)). At the same time, we argue that the potential diffusion process across states has to be taken into account for at least two reasons. First, cities and counties in different states border each other. A cursory glance of the map showing where the civil rights protests in the 1960s occurred suggests that there might have been a potential diffusion process in the baseline propensities for civil rights protests for neighboring cities and counties across North Carolina and Virginia ([Mazumder, 2018](#), Figure 1). Second, historical accounts suggest that interstate diffusion process might be an important factor to take into account because, for example, activists traveled extensively with interstate buses as part of the civil rights movement ([Andrews and Biggs, 2006](#)).

We use the dataset provided by [Mazumder \(2018\)](#) and investigate the potential causes of civil rights protests across the 48 contiguous US states. The unit of analysis is county. The dependent variable is whether a civil rights protest took place at least once during the period, coded as 1 if any protest took place and 0 otherwise. For our covariates, we include the percentage of urban population, the percentage of black population, the median age and the median years of school education. We estimate four models: SAR probit, multilevel

probit, SAR-RE, and our proposed HSAR probit.²⁶ The multilevel probit, SAR-RE, and the HSAR probit models include state-level random intercepts. We used 50,000 iterations for the MCMC simulations and discarded the first 10,000 iterations as a burn-in.

Table 1: Comparison of Models (48 states)

	SAR Probit Model 1	Multilevel Probit Model 2	SAR Random Effects Model 3	HSAR Probit Model 4
Percentage of Urban Population	0.026 (0.022, 0.030)	0.031 (0.026, 0.036)	0.029 (0.024, 0.034)	0.028 (0.024, 0.033)
Percentage of Black Population	0.031 (0.025, 0.037)	0.038 (0.029, 0.046)	0.035 (0.026, 0.043)	0.034 (0.026, 0.043)
Median Age	0.003 (-0.016, 0.022)	0.009 (-0.019, 0.037)	0.011 (-0.015, 0.038)	0.010 (-0.014, 0.037)
Median School Years	0.105 (0.033, 0.178)	0.183 (0.075, 0.292)	0.148 (0.043, 0.256)	0.148 (0.041, 0.257)
Constant	-3.620 (-4.639, -2.600)	-5.569 (-7.069, -4.069)	-4.793 (-6.336, -3.294)	-4.730 (-6.337, -3.213)
$\hat{\rho}$	0.330 (0.198, 0.461)	—	0.143 (0.023, 0.251)	0.149 (0.022, 0.266)
$\hat{\lambda}$	—	—	—	0.452 (-0.065, 0.817)
$\hat{\sigma}_u^2$	—	0.377	0.190 (0.086, 0.357)	0.147 (0.057, 0.296)
N	3043	3043	3043	3043

Note: 95% confidence intervals for the multilevel probit model and credible intervals for the spatial models are shown in parentheses.

Table 1 presents the results. The credible intervals for $\hat{\rho}$ for the SAR probit, the SAR probit with random effects, and the HSAR probit models do not encompass 0, suggesting that there is a spatial diffusion process of civil rights protests among counties. However, we notice that the estimate of ρ from the SAR probit model is very large compared to the estimates from the SAR random effects and the hierarchical spatial probit models. This is consistent with our simulation results; the spatial probit model often overestimates ρ .²⁷ We

²⁶We implement the SAR probit using the **spatialprobit** package by [Wilhelm and de Matos \(2013\)](#) in R.

²⁷We note that although the estimates of ρ for the SAR probit model do not largely depend on the value of λ in the DGP in the simulations, we do find inflated estimates of ρ across all values of λ . In our application, while we can only assume what the true DGP is, we compare the results of our simulation to those of our

also notice that the SAR random effects and the HSAR models perform similarly, a result we also find in our simulations for smaller samples sizes of the higher-level units (Appendices A-L). The credible intervals for $\hat{\lambda}$ encompass 0 implying that we fail to find sufficient evidence in favor of the presence of diffusion in the baseline propensities for civil rights protests across states. The results from this analysis on this single sample thus suggest that only a lower-level spatial diffusion process across countries occurs. It is noteworthy that the estimates from both the SAR random effects and the HSAR probit are similar and researchers would reach similar conclusion regardless of the model estimates.²⁸ This is also evident in the direct and indirect effects shown in Figures 4 and 5. We would like to remind readers that the HSAR probit is advantageous in that it explicitly models the higher-level diffusion process, and thus allows researchers to test theoretical propositions about higher-level diffusion processes.

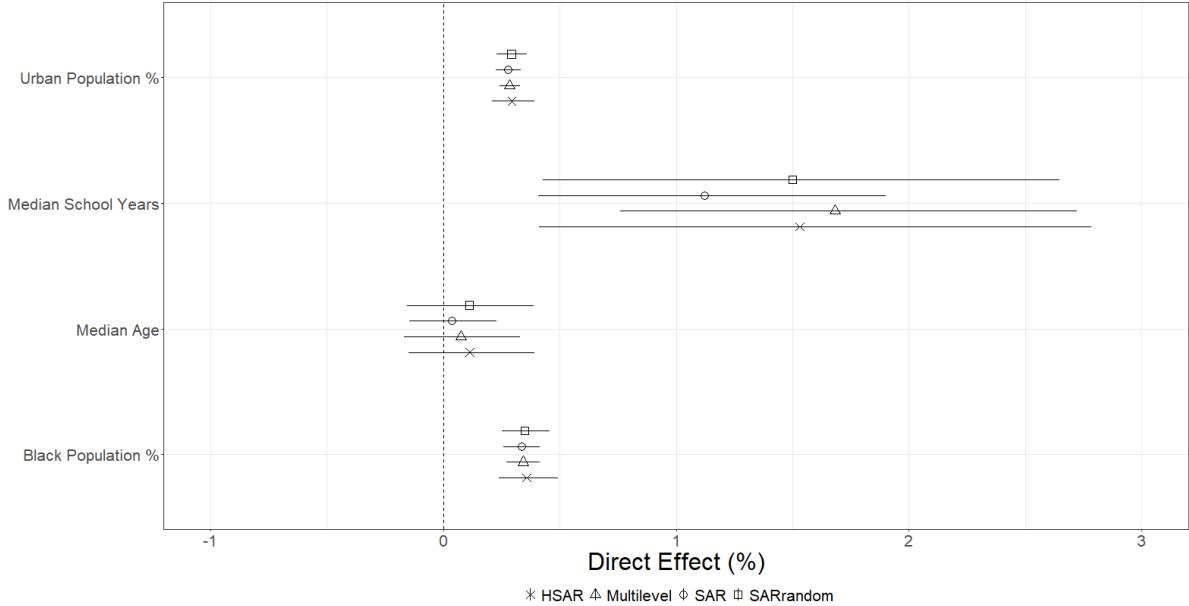
The coefficient estimates of the predictors by themselves are not very informative about their effects because a change in a predictor in one unit affects its own outcome and also those of other units. We thus present the estimates of the direct and indirect effects in Figures 4 and 5, respectively.²⁹ For the HSAR probit, in terms of the direct effect, a 1 percentage point increase in the urban population is, on average, positively associated with a 0.3% increase in the probability of a civil rights protest occurring in the same unit. For the indirect effect, a 1 percentage point increase in the urban population is, on average, positively associated

application for the SAR probit model with caution. In our application, we find an inflated estimate of ρ for the SAR probit model, but find a much lower value of $\hat{\rho}$ for the HSAR and SAR-RE. We also find that the estimates of $\sigma_u^2 > 0$. If we were to compare these results to those of what we found in our simulations, the inflation in $\hat{\rho}$ for the SAR probit is likely due to $\sigma_u^2 \neq 0$. As we also noted in the discussion surrounding equation 2, the SAR probit model mistakenly attributes the heteroscedasticity from omitted random effects to spatial effects, and thus overestimates ρ .

²⁸The estimate $\hat{\sigma}_u^2$ is larger for the SAR-RE because it does not explicitly model a potential higher-level spatial diffusion process as does the HSAR probit. The substantive effects from both models are similar (figures 4 and 5).

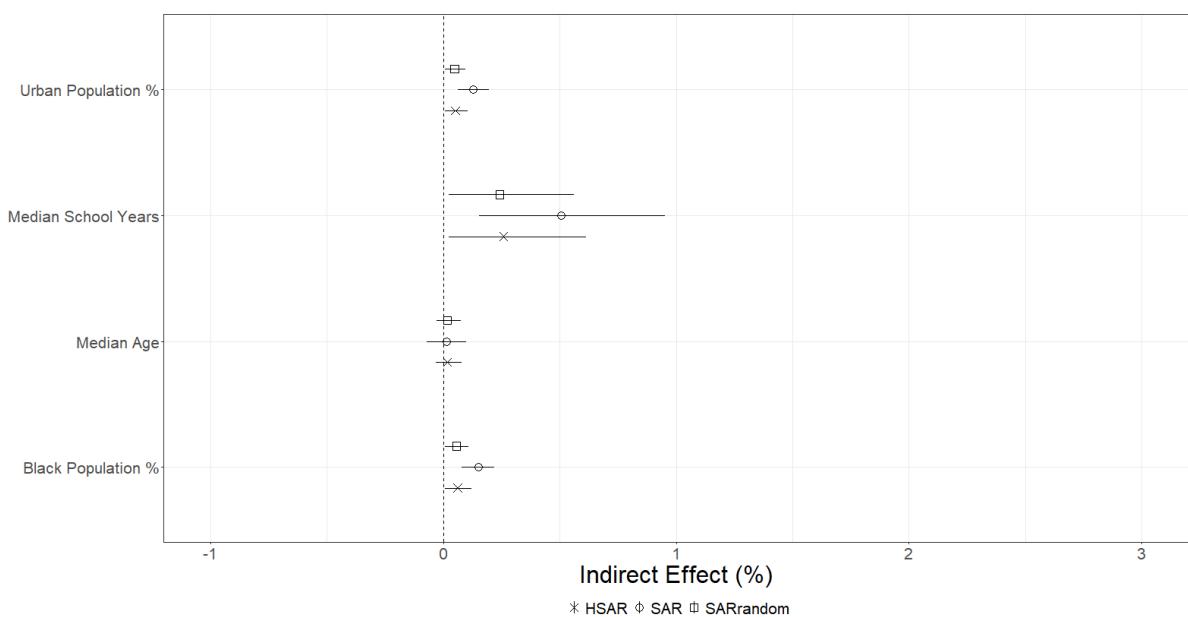
²⁹We used the simulated draws of β and ρ for the spatial models and a parametric bootstrap for the multilevel probit model to calculate the uncertainty of the effects' estimates.

Figure 4: Average Direct Effects of Predictors



with a 0.05% increase in the probability of a civil rights protest occurring in other units. The credible intervals of the HSAR model for both these effects do not encompass 0. In general, we see that the multilevel probit, the SAR random effects probit, and the HSAR probit models provide similar estimates for the direct effect. This is consistent with what we found in our simulations. In terms of the indirect effect, we see that the SAR probit model's estimates are in general much larger than those of the SAR random effects probit and the HSAR probit models. This is consistent with our argument around equation 2 and the findings from our simulations.

Figure 5: Average Indirect Effects of Predictors



11 Conclusion

We combine advances in spatial econometrics and multilevel modeling to introduce a class of binary outcome models, the SAR-RE and the HSAR models, that account for level-1 diffusion and spatially dependent or independent higher-level effects. We found that models, including the SAR-RE, that do not account for level-2 diffusion cannot accurately capture the spillover effects of a change in a predictor and lead to inaccurate inferences. Our proposed HSAR model, however, has widespread applicability because many datasets in political science are nested and can potentially have diffusion processes at multiple levels. Inefficiency is the only cost of estimating the HSAR model when: 1) there is no spatial diffusion, 2) there is spatial diffusion only at the lower-level and there are no random intercepts, 3) there is spatial diffusion only at the lower-level along with group-level random intercepts, 4) there is spatial diffusion only at the higher-level, or 5) there is spatial diffusion at both levels. This is because our proposed HSAR probit model nests the aforementioned scenarios. However, when there are only spatially independent unobserved group effects, the SAR-RE outperforms the HSAR model because of gains in efficiency by estimating one less parameter.

We conclude with some practical recommendations for applied researchers. If researchers have a theory about a spatial spillover, but suspect that there is likely to be dependence in outcomes due to unobserved group effects because of the nested structure of the data, they should opt for our proposed HSAR model. Estimating this model allows researchers to additionally test for a higher-level diffusion process even if theorized to be non-existent. That is, in addition to allowing researchers to explicitly theorize and test for higher-level diffusion, it provides an additional layer of robustness even when researchers do not theorize

about such diffusion. If researchers find that there is no higher-level diffusion and that the variance of the random effects is non-zero, they should consider the SAR-RE.

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Online Appendix for

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A Binary Probit $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho \in \{0, 0.3, 0.5\}$ and
 $\lambda \in \{0, 0.3, 0.5\}$

Table A1: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.005	-0.022	0.003	-0.002	0.066	-0.005	0.002	-0.002	0.111	0.371	0.271	-0.322	0.001	0.008	0.036
SD	0.079	0.26	0.178	0.074	0.316	0.075	0.174	0.074	0.317	0.052	0.079	0.081	0.161	0.074	0.276
RMSE	0.079	0.261	0.178	0.074	0.323	0.075	0.174	0.074	0.336	0.374	0.282	0.332	0.161	0.075	0.278

Δ

Table A2: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.007	-0.086	0.018	0.006	0.059	-0.003	0.022	0.007	0.134	0.377	0.283	-0.329	0.019	0.016	0.066
SD	0.073	0.232	0.216	0.082	0.3	0.074	0.219	0.081	0.313	0.062	0.099	0.075	0.205	0.084	0.282
RMSE	0.074	0.247	0.216	0.083	0.306	0.074	0.22	0.081	0.341	0.382	0.3	0.337	0.206	0.086	0.289

Table A3: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.013	-0.045	-0.031	0.012	0.055	-0.004	-0.048	0.013	0.344	0.411	0.271	-0.35	-0.054	0.026	0.271
SD	0.081	0.189	0.4	0.075	0.353	0.075	0.354	0.075	0.454	0.059	0.14	0.081	0.341	0.074	0.397
RMSE	0.082	0.195	0.401	0.076	0.357	0.075	0.358	0.076	0.569	0.416	0.305	0.359	0.345	0.079	0.48

Table A4: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.002	-0.013	0.012	-0.005	-0.057	0	0.029	-0.004	-0.025	0.25	0.271	-0.392	-0.196	-0.024	0.683
SD	0.058	0.228	0.146	0.079	0.257	0.062	0.13	0.081	0.265	0.037	0.065	0.084	0.181	0.08	0.416
RMSE	0.058	0.228	0.147	0.079	0.263	0.062	0.133	0.081	0.266	0.252	0.279	0.401	0.267	0.083	0.799

Table A5: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.006	-0.058	0.024	-0.016	-0.082	-0.001	0.019	-0.015	-0.011	0.259	0.274	-0.403	-0.203	-0.028	0.774
SD	0.055	0.219	0.234	0.078	0.254	0.057	0.215	0.077	0.269	0.038	0.11	0.079	0.309	0.082	0.469
RMSE	0.056	0.227	0.235	0.08	0.267	0.057	0.216	0.078	0.269	0.262	0.295	0.411	0.369	0.087	0.905

A3

Table A6: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.017	-0.052	0	-0.023	-0.084	-0.008	-0.001	-0.019	0.144	0.283	0.276	-0.444	-0.252	-0.016	1.155
SD	0.063	0.187	0.327	0.075	0.266	0.057	0.283	0.073	0.301	0.045	0.135	0.08	0.435	0.079	0.599
RMSE	0.065	0.194	0.327	0.078	0.279	0.057	0.283	0.076	0.334	0.286	0.308	0.451	0.503	0.08	1.301

Table A7: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.038	-0.048	0.054	-0.082	-0.222	-0.04	0.044	-0.083	-0.179	0.158	0.27	-0.464	-0.338	-0.138	1.14
SD	0.039	0.279	0.149	0.073	0.19	0.04	0.141	0.071	0.189	0.036	0.076	0.088	0.244	0.07	0.505
RMSE	0.054	0.283	0.158	0.11	0.292	0.057	0.147	0.11	0.26	0.162	0.28	0.473	0.417	0.154	1.247

Table A8: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.025	-0.113	-0.032	0.031	0.021	-0.024	-0.031	0.032	0.025	-0.026	-0.025	0.016	-0.379	-0.079	0.115
SD	0.036	0.245	0.059	0.067	0.009	0.037	0.059	0.067	0.012	0.032	0.056	0.068	0.085	0.062	0.055
RMSE	0.043	0.27	0.067	0.074	0.023	0.044	0.066	0.074	0.027	0.042	0.061	0.07	0.389	0.1	0.127

Table A9: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.052	-0.043	0.035	-0.093	-0.227	-0.044	0.05	-0.09	-0.047	0.193	0.291	-0.526	-0.407	-0.11	1.925
SD	0.056	0.167	0.319	0.09	0.199	0.05	0.261	0.09	0.225	0.043	0.126	0.097	0.503	0.082	0.912
RMSE	0.076	0.172	0.321	0.129	0.302	0.066	0.266	0.127	0.23	0.198	0.317	0.535	0.647	0.137	2.131

B Binary Probit $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho \in \{0, 0.3, 0.5\}$ and
 $\lambda \in \{0, 0.3, 0.5\}$

Table B10: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.007	-0.064	-0.021	0.017	0.066	-0.01	-0.024	0.018	0.115	0.346	0.253	-0.363	-0.022	0.027	0.033	
SD	0.066	0.228	0.164	0.082	0.298	0.07	0.161	0.083	0.338	0.059	0.088	0.08	0.159	0.082	0.264	
RMSE	0.066	0.237	0.165	0.084	0.305	0.071	0.163	0.085	0.357	0.351	0.268	0.372	0.161	0.086	0.266	

B5

Table B11: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.003	-0.068	-0.008	-0.003	-0.022	0.006	0.013	-0.005	0.045	0.357	0.266	-0.377	0.003	0.006	-0.002	
SD	0.069	0.24	0.264	0.077	0.284	0.065	0.218	0.078	0.293	0.058	0.108	0.076	0.22	0.078	0.275	
RMSE	0.069	0.249	0.264	0.078	0.285	0.066	0.219	0.078	0.296	0.361	0.287	0.384	0.22	0.078	0.275	

Table B12: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.015	-0.052	-0.039	-0.006	0.051	-0.004	-0.002	-0.008	0.288	0.394	0.284	-0.428	-0.009	0.004	0.222	
SD	0.066	0.179	0.35	0.064	0.301	0.064	0.307	0.066	0.367	0.058	0.131	0.078	0.301	0.063	0.322	
RMSE	0.068	0.186	0.352	0.064	0.305	0.064	0.307	0.066	0.466	0.398	0.313	0.436	0.301	0.064	0.391	

Table B13: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.017	-0.014	0.032	-0.03	-0.079	-0.019	0.022	-0.03	-0.033	0.239	0.274	-0.435	-0.178	0.015	0.628
SD	0.052	0.25	0.141	0.072	0.213	0.05	0.141	0.073	0.214	0.04	0.074	0.075	0.189	0.076	0.352
RMSE	0.055	0.25	0.145	0.078	0.228	0.054	0.143	0.079	0.216	0.243	0.284	0.442	0.259	0.078	0.719

Table B14: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.012	-0.065	0.032	-0.006	-0.087	-0.011	0.044	-0.007	-0.019	0.25	0.285	-0.437	-0.164	0.042	0.718
SD	0.051	0.212	0.226	0.089	0.231	0.048	0.22	0.086	0.227	0.04	0.105	0.08	0.314	0.095	0.447
RMSE	0.053	0.221	0.228	0.089	0.246	0.049	0.224	0.086	0.227	0.253	0.304	0.445	0.354	0.104	0.846

B6

Table B15: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.008	-0.04	0.06	-0.033	-0.088	-0.003	0.031	-0.032	0.149	0.283	0.291	-0.502	-0.214	0.033	1.215
SD	0.052	0.192	0.334	0.087	0.243	0.051	0.267	0.088	0.321	0.05	0.126	0.089	0.379	0.091	0.68
RMSE	0.052	0.196	0.339	0.093	0.259	0.052	0.269	0.093	0.354	0.287	0.317	0.51	0.435	0.097	1.392

Table B16: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.045	-0.027	0.057	-0.068	-0.205	-0.047	0.061	-0.068	-0.175	0.159	0.274	-0.469	-0.32	-0.019	1.169
SD	0.04	0.219	0.146	0.084	0.222	0.044	0.143	0.081	0.232	0.04	0.081	0.088	0.245	0.085	0.598
RMSE	0.06	0.221	0.157	0.107	0.302	0.064	0.155	0.106	0.29	0.164	0.286	0.477	0.403	0.087	1.313

Table B17: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.023	-0.174	-0.024	0.016	0.021	-0.021	-0.023	0.013	0.024	-0.02	-0.018	0.003	-0.359	0.004	0.121
SD	0.033	0.203	0.056	0.074	0.01	0.034	0.055	0.074	0.01	0.032	0.055	0.077	0.082	0.067	0.058
RMSE	0.04	0.267	0.062	0.076	0.024	0.04	0.06	0.075	0.025	0.038	0.058	0.077	0.368	0.067	0.135

B7

Table B18: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.052	-0.079	0.092	-0.103	-0.223	-0.043	0.109	-0.105	-0.072	0.194	0.312	-0.552	-0.3	-0.025	1.816
SD	0.047	0.196	0.293	0.086	0.206	0.043	0.265	0.084	0.238	0.041	0.127	0.093	0.488	0.088	0.798
RMSE	0.07	0.211	0.307	0.134	0.304	0.061	0.287	0.134	0.249	0.199	0.336	0.56	0.573	0.091	1.983

C Binary Probit $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho \in \{0, 0.3, 0.5\}$ and
 $\lambda \in \{0, 0.3, 0.5\}$

Table C19: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.014	-0.085	-0.015	0.015	0.037	-0.015	-0.016	0.015	0.072	0.247	0.187	-0.194	-0.006	0.02	0.013
SD	0.068	0.271	0.112	0.07	0.158	0.067	0.109	0.07	0.162	0.061	0.074	0.063	0.099	0.07	0.133
RMSE	0.069	0.284	0.113	0.071	0.163	0.069	0.111	0.072	0.177	0.255	0.201	0.203	0.099	0.073	0.134

∞

Table C20: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	0.013	-0.057	0.002	0.013	0.012	0.014	-0.009	0.012	0.065	0.275	0.195	-0.2	-0.019	0.018	0.034
SD	0.069	0.26	0.21	0.071	0.15	0.07	0.151	0.071	0.156	0.057	0.098	0.073	0.154	0.071	0.135
RMSE	0.07	0.266	0.21	0.072	0.15	0.072	0.152	0.072	0.169	0.281	0.218	0.213	0.155	0.073	0.139

Table C21: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.001	-0.107	0.02	0.012	0.04	0.008	0.025	0.013	0.146	0.296	0.228	-0.219	0.016	0.017	0.106
SD	0.074	0.223	0.289	0.076	0.162	0.073	0.224	0.074	0.199	0.064	0.137	0.069	0.233	0.075	0.177
RMSE	0.074	0.247	0.29	0.077	0.167	0.074	0.226	0.075	0.247	0.303	0.266	0.23	0.233	0.077	0.206

Table C22: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.003	-0.08	0.018	0.001	-0.022	-0.006	0.023	0.001	0.008	0.174	0.209	-0.252	-0.166	-0.027	0.331
SD	0.055	0.267	0.136	0.076	0.149	0.057	0.12	0.077	0.147	0.035	0.072	0.082	0.155	0.073	0.204
RMSE	0.055	0.279	0.137	0.076	0.15	0.057	0.122	0.077	0.147	0.178	0.221	0.264	0.227	0.078	0.389

Table C23: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.009	-0.08	0.009	-0.002	-0.002	-0.002	0.008	-0.001	0.034	0.178	0.205	-0.274	-0.188	-0.026	0.396
SD	0.061	0.233	0.162	0.072	0.159	0.062	0.15	0.073	0.163	0.044	0.095	0.082	0.204	0.072	0.254
RMSE	0.062	0.246	0.162	0.072	0.159	0.062	0.15	0.073	0.167	0.183	0.226	0.286	0.277	0.076	0.47

C9

Table C24: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.004	-0.057	-0.005	0.004	-0.015	0.003	-0.012	0.007	0.119	0.2	0.209	-0.3	-0.224	-0.021	0.591
SD	0.056	0.204	0.237	0.07	0.149	0.055	0.216	0.071	0.201	0.05	0.132	0.083	0.3	0.074	0.401
RMSE	0.057	0.212	0.237	0.07	0.15	0.055	0.216	0.071	0.234	0.206	0.247	0.311	0.375	0.077	0.714

Table C25: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.022	-0.022	0.014	-0.035	-0.067	-0.021	0.006	-0.035	-0.054	0.103	0.187	-0.3	-0.376	-0.116	0.671
SD	0.048	0.218	0.138	0.075	0.117	0.048	0.129	0.076	0.11	0.038	0.083	0.076	0.214	0.072	0.267
RMSE	0.052	0.219	0.138	0.083	0.135	0.052	0.13	0.084	0.122	0.11	0.205	0.31	0.433	0.136	0.722

Table C26: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.025	-0.052	-0.004	-0.041	-0.047	-0.026	-0.011	-0.041	0.007	0.113	0.191	-0.349	-0.427	-0.109	0.872
SD	0.042	0.255	0.168	0.079	0.148	0.042	0.141	0.079	0.159	0.034	0.077	0.092	0.247	0.077	0.411
RMSE	0.049	0.26	0.168	0.09	0.156	0.049	0.142	0.089	0.16	0.118	0.206	0.361	0.493	0.134	0.964

C10

Table C27: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.04	-0.059	0.026	-0.049	-0.028	-0.036	0.02	-0.046	0.096	0.124	0.225	-0.389	-0.378	-0.105	1.123
SD	0.048	0.206	0.248	0.083	0.156	0.048	0.216	0.081	0.18	0.039	0.127	0.092	0.379	0.079	0.46
RMSE	0.062	0.214	0.249	0.096	0.159	0.06	0.217	0.093	0.204	0.13	0.259	0.4	0.535	0.132	1.214

D Binary Probit $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho \in \{0, 0.3, 0.5\}$ and
 $\lambda \in \{0, 0.3, 0.5\}$

Table D28: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.016	-0.037	-0.008	0.007	0.016	-0.019	-0.012	0.008	0.046	0.215	0.177	-0.23	-0.002	0.011	-0.015	
SD	0.067	0.241	0.123	0.066	0.162	0.068	0.124	0.065	0.162	0.066	0.085	0.066	0.114	0.067	0.131	
RMSE	0.069	0.244	0.124	0.066	0.163	0.07	0.124	0.066	0.168	0.225	0.196	0.239	0.114	0.067	0.132	

D11

Table D29: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.005	-0.05	-0.015	0.009	0.044	-0.001	0	0.007	0.088	0.25	0.196	-0.251	-0.002	0.014	0.043	
SD	0.055	0.207	0.18	0.067	0.176	0.054	0.141	0.066	0.182	0.057	0.098	0.067	0.149	0.068	0.162	
RMSE	0.055	0.213	0.181	0.068	0.182	0.054	0.141	0.067	0.202	0.256	0.22	0.259	0.149	0.07	0.167	

Table D30: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	0	-0.071	0.005	0.01	0.058	0.002	0.016	0.012	0.192	0.285	0.23	-0.285	0.018	0.018	0.139	
SD	0.068	0.204	0.252	0.07	0.198	0.066	0.228	0.07	0.218	0.057	0.132	0.066	0.228	0.07	0.178	
RMSE	0.068	0.216	0.252	0.071	0.207	0.066	0.229	0.071	0.29	0.291	0.265	0.292	0.228	0.072	0.226	

Table D31: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	0.002	-0.066	0.013	-0.005	-0.019	-0.002	0.017	-0.003	0.01	0.166	0.201	-0.299	-0.181	0.031	0.344
SD	0.047	0.259	0.128	0.074	0.144	0.048	0.128	0.07	0.134	0.042	0.079	0.068	0.165	0.076	0.231
RMSE	0.047	0.268	0.129	0.074	0.146	0.048	0.129	0.07	0.135	0.171	0.216	0.307	0.245	0.082	0.415

Table D32: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.01	-0.04	-0.007	-0.008	0.007	-0.007	-0.005	-0.008	0.054	0.178	0.2	-0.319	-0.201	0.027	0.433
SD	0.05	0.216	0.17	0.07	0.149	0.049	0.147	0.072	0.155	0.043	0.098	0.073	0.19	0.071	0.25
RMSE	0.051	0.219	0.17	0.07	0.149	0.05	0.147	0.073	0.164	0.183	0.222	0.327	0.276	0.076	0.5

D12

Table D33: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.003	-0.045	0.029	-0.01	0.009	0.005	0.034	-0.012	0.133	0.202	0.237	-0.363	-0.171	0.028	0.609
SD	0.056	0.194	0.274	0.08	0.165	0.056	0.21	0.079	0.184	0.041	0.123	0.076	0.302	0.079	0.295
RMSE	0.056	0.199	0.275	0.08	0.166	0.056	0.213	0.08	0.227	0.206	0.267	0.371	0.347	0.084	0.677

Table D34: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.028	-0.048	0.011	-0.044	-0.072	-0.029	0.004	-0.044	-0.051	0.094	0.18	-0.338	-0.375	-0.021	0.643
SD	0.044	0.239	0.128	0.071	0.129	0.046	0.128	0.072	0.131	0.036	0.082	0.083	0.204	0.075	0.303
RMSE	0.053	0.244	0.129	0.084	0.148	0.054	0.128	0.084	0.14	0.1	0.197	0.348	0.427	0.078	0.711

Table D35: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.039	-0.065	0.029	-0.039	-0.049	-0.032	0.033	-0.037	-0.003	0.106	0.214	-0.356	-0.331	-0.005	0.817
SD	0.044	0.261	0.162	0.077	0.123	0.045	0.163	0.075	0.136	0.041	0.099	0.083	0.269	0.079	0.357
RMSE	0.059	0.269	0.164	0.086	0.133	0.055	0.166	0.084	0.136	0.114	0.236	0.366	0.427	0.079	0.892

D13

Table D36: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.023	-0.074	0.063	-0.045	-0.07	-0.018	0.077	-0.044	0.033	0.13	0.247	-0.384	-0.29	-0.004	1.045
SD	0.043	0.18	0.194	0.077	0.129	0.043	0.2	0.077	0.164	0.039	0.131	0.095	0.351	0.079	0.449
RMSE	0.049	0.194	0.204	0.09	0.147	0.047	0.214	0.089	0.167	0.136	0.28	0.395	0.455	0.079	1.138

E Binary Probit $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho \in \{0, 0.3, 0.5\}$ and
 $\lambda \in \{0, 0.3, 0.5\}$

Table E37: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.024	-0.081	-0.037	0.037	0.021	-0.022	-0.034	0.036	0.025	-0.003	-0.017	0.025	-0.015	0.026	0.005	
SD	0.068	0.234	0.065	0.065	0.007	0.066	0.065	0.067	0.009	0.068	0.063	0.066	0.05	0.065	0.009	
RMSE	0.072	0.247	0.074	0.075	0.022	0.069	0.074	0.076	0.027	0.068	0.065	0.07	0.052	0.07	0.01	

E14

Table E38: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.024	-0.081	-0.037	0.037	0.021	-0.022	-0.034	0.036	0.025	-0.003	-0.017	0.025	-0.015	0.026	0.005	
SD	0.068	0.234	0.065	0.065	0.007	0.066	0.065	0.067	0.009	0.068	0.063	0.066	0.05	0.065	0.009	
RMSE	0.072	0.247	0.074	0.075	0.022	0.069	0.074	0.076	0.027	0.068	0.065	0.07	0.052	0.07	0.01	

Table E39: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.024	-0.081	-0.037	0.037	0.021	-0.022	-0.034	0.036	0.025	-0.003	-0.017	0.025	-0.015	0.026	0.005	
SD	0.068	0.234	0.065	0.065	0.007	0.066	0.065	0.067	0.009	0.068	0.063	0.066	0.05	0.065	0.009	
RMSE	0.072	0.247	0.074	0.075	0.022	0.069	0.074	0.076	0.027	0.068	0.065	0.07	0.052	0.07	0.01	

Table E40: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.022	-0.12	-0.026	0.027	0.02	-0.018	-0.023	0.025	0.022	0.004	-0.002	0.012	-0.199	-0.016	0.039
SD	0.054	0.266	0.057	0.067	0.008	0.053	0.056	0.065	0.009	0.048	0.052	0.065	0.06	0.062	0.031
RMSE	0.059	0.292	0.063	0.072	0.021	0.056	0.06	0.07	0.024	0.048	0.052	0.066	0.207	0.064	0.05

Table E41: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.022	-0.12	-0.026	0.027	0.02	-0.018	-0.023	0.025	0.022	0.004	-0.002	0.012	-0.199	-0.016	0.039
SD	0.054	0.266	0.057	0.067	0.008	0.053	0.056	0.065	0.009	0.048	0.052	0.065	0.06	0.062	0.031
RMSE	0.059	0.292	0.063	0.072	0.021	0.056	0.06	0.07	0.024	0.048	0.052	0.066	0.207	0.064	0.05

E15

Table E42: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.022	-0.12	-0.026	0.027	0.02	-0.018	-0.023	0.025	0.022	0.004	-0.002	0.012	-0.199	-0.016	0.039
SD	0.054	0.266	0.057	0.067	0.008	0.053	0.056	0.065	0.009	0.048	0.052	0.065	0.06	0.062	0.031
RMSE	0.059	0.292	0.063	0.072	0.021	0.056	0.06	0.07	0.024	0.048	0.052	0.066	0.207	0.064	0.05

Table E43: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.025	-0.113	-0.032	0.031	0.021	-0.024	-0.031	0.032	0.025	-0.026	-0.025	0.016	-0.379	-0.079	0.115
SD	0.036	0.245	0.059	0.067	0.009	0.037	0.059	0.067	0.012	0.032	0.056	0.068	0.085	0.062	0.055
RMSE	0.043	0.27	0.067	0.074	0.023	0.044	0.066	0.074	0.027	0.042	0.061	0.07	0.389	0.1	0.127

Table E44: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.025	-0.113	-0.032	0.031	0.021	-0.024	-0.031	0.032	0.025	-0.026	-0.025	0.016	-0.379	-0.079	0.115
SD	0.036	0.245	0.059	0.067	0.009	0.037	0.059	0.067	0.012	0.032	0.056	0.068	0.085	0.062	0.055
RMSE	0.043	0.27	0.067	0.074	0.023	0.044	0.066	0.074	0.027	0.042	0.061	0.07	0.389	0.1	0.127

E16

Table E45: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.025	-0.113	-0.032	0.031	0.021	-0.024	-0.031	0.032	0.025	-0.026	-0.025	0.016	-0.379	-0.079	0.115
SD	0.036	0.245	0.059	0.067	0.009	0.037	0.059	0.067	0.012	0.032	0.056	0.068	0.085	0.062	0.055
RMSE	0.043	0.27	0.067	0.074	0.023	0.044	0.066	0.074	0.027	0.042	0.061	0.07	0.389	0.1	0.127

F Binary Probit $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho \in \{0, 0.3, 0.5\}$ and $\lambda \in \{0, 0.3, 0.5\}$

Table F46: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.025	-0.122	-0.025	0.026	0.021	-0.021	-0.024	0.026	0.024	-0.005	-0.008	0.01	-0.006	0.011	0.004	
SD	0.061	0.227	0.061	0.059	0.007	0.059	0.059	0.06	0.009	0.062	0.057	0.058	0.051	0.057	0.009	
RMSE	0.066	0.258	0.066	0.064	0.022	0.063	0.064	0.065	0.026	0.062	0.058	0.059	0.051	0.058	0.01	

E17

Table F47: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.025	-0.122	-0.025	0.026	0.021	-0.021	-0.024	0.026	0.024	-0.005	-0.008	0.01	-0.006	0.011	0.004	
SD	0.061	0.227	0.061	0.059	0.007	0.059	0.059	0.06	0.009	0.062	0.057	0.058	0.051	0.057	0.009	
RMSE	0.066	0.258	0.066	0.064	0.022	0.063	0.064	0.065	0.026	0.062	0.058	0.059	0.051	0.058	0.01	

Table F48: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.025	-0.122	-0.025	0.026	0.021	-0.021	-0.024	0.026	0.024	-0.005	-0.008	0.01	-0.006	0.011	0.004	
SD	0.061	0.227	0.061	0.059	0.007	0.059	0.059	0.06	0.009	0.062	0.057	0.058	0.051	0.057	0.009	
RMSE	0.066	0.258	0.066	0.064	0.022	0.063	0.064	0.065	0.026	0.062	0.058	0.059	0.051	0.058	0.01	

Table F49: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.026	-0.09	-0.016	0.019	0.021	-0.024	-0.013	0.018	0.025	-0.002	0.008	-0.005	-0.179	0.038	0.043
SD	0.046	0.269	0.054	0.063	0.009	0.045	0.054	0.062	0.011	0.041	0.05	0.061	0.059	0.061	0.031
RMSE	0.053	0.283	0.057	0.066	0.023	0.051	0.055	0.064	0.027	0.042	0.051	0.062	0.189	0.072	0.053

Table F50: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.026	-0.09	-0.016	0.019	0.021	-0.024	-0.013	0.018	0.025	-0.002	0.008	-0.005	-0.179	0.038	0.043
SD	0.046	0.269	0.054	0.063	0.009	0.045	0.054	0.062	0.011	0.041	0.05	0.061	0.059	0.061	0.031
RMSE	0.053	0.283	0.057	0.066	0.023	0.051	0.055	0.064	0.027	0.042	0.051	0.062	0.189	0.072	0.053

F18

Table F51: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.026	-0.09	-0.016	0.019	0.021	-0.024	-0.013	0.018	0.025	-0.002	0.008	-0.005	-0.179	0.038	0.043
SD	0.046	0.269	0.054	0.063	0.009	0.045	0.054	0.062	0.011	0.041	0.05	0.061	0.059	0.061	0.031
RMSE	0.053	0.283	0.057	0.066	0.023	0.051	0.055	0.064	0.027	0.042	0.051	0.062	0.189	0.072	0.053

Table F52: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.023	-0.174	-0.024	0.016	0.021	-0.021	-0.023	0.013	0.024	-0.02	-0.018	0.003	-0.359	0.004	0.121
SD	0.033	0.203	0.056	0.074	0.01	0.034	0.055	0.074	0.01	0.032	0.055	0.077	0.082	0.067	0.058
RMSE	0.04	0.267	0.062	0.076	0.024	0.04	0.06	0.075	0.025	0.038	0.058	0.077	0.368	0.067	0.135

F19

Table F53: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.023	-0.174	-0.024	0.016	0.021	-0.021	-0.023	0.013	0.024	-0.02	-0.018	0.003	-0.359	0.004	0.121
SD	0.033	0.203	0.056	0.074	0.01	0.034	0.055	0.074	0.01	0.032	0.055	0.077	0.082	0.067	0.058
RMSE	0.04	0.267	0.062	0.076	0.024	0.04	0.06	0.075	0.025	0.038	0.058	0.077	0.368	0.067	0.135

Table F54: Binary Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.023	-0.174	-0.024	0.016	0.021	-0.021	-0.023	0.013	0.024	-0.02	-0.018	0.003	-0.359	0.004	0.121
SD	0.033	0.203	0.056	0.074	0.01	0.034	0.055	0.074	0.01	0.032	0.055	0.077	0.082	0.067	0.058
RMSE	0.04	0.267	0.062	0.076	0.024	0.04	0.06	0.075	0.025	0.038	0.058	0.077	0.368	0.067	0.135

G Binary Probit $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho \in \{0, 0.3, 0.5\}$ and
 $\lambda \in \{0, 0.3, 0.5\}$

Table G55: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.032	-0.048	-0.056	0.018	0.251	-0.038	-0.02	0.019	0.385	0.353	0.236	-0.308	-0.006	0.016	0.009
SD	0.122	0.316	0.352	0.123	0.575	0.118	0.321	0.13	0.624	0.085	0.156	0.101	0.284	0.126	0.408
RMSE	0.126	0.32	0.357	0.124	0.627	0.124	0.321	0.131	0.733	0.363	0.283	0.324	0.284	0.127	0.408

G20

Table G56: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.025	-0.111	0.005	0.025	0.239	-0.026	-0.005	0.03	0.399	0.362	0.247	-0.287	0.022	0.026	0.043
SD	0.102	0.302	0.53	0.135	0.633	0.109	0.428	0.133	0.694	0.112	0.194	0.123	0.391	0.131	0.505
RMSE	0.105	0.322	0.53	0.138	0.677	0.112	0.428	0.136	0.801	0.379	0.314	0.313	0.391	0.134	0.506

Table G57: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.014	-0.215	-0.114	0.039	0.273	-0.013	-0.025	0.035	0.539	0.38	0.233	-0.316	-0.001	0.04	0.225
SD	0.13	0.309	0.83	0.147	0.676	0.132	0.531	0.143	0.868	0.119	0.259	0.134	0.502	0.146	0.805
RMSE	0.131	0.376	0.838	0.152	0.729	0.133	0.532	0.147	1.022	0.399	0.348	0.343	0.502	0.151	0.836

Table G58: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.041	-0.053	-0.049	0.035	0.11	-0.043	-0.058	0.03	0.212	0.226	0.226	-0.346	-0.207	0.021	0.662
SD	0.104	0.325	0.345	0.13	0.522	0.104	0.326	0.128	0.507	0.069	0.152	0.14	0.392	0.129	0.737
RMSE	0.112	0.33	0.349	0.134	0.534	0.113	0.331	0.132	0.55	0.237	0.273	0.373	0.443	0.131	0.991

Table G59: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.033	-0.153	0.064	0.021	0.084	-0.035	0.03	0.023	0.245	0.242	0.255	-0.368	-0.12	0.014	0.745
SD	0.093	0.348	0.662	0.12	0.488	0.093	0.408	0.122	0.521	0.076	0.211	0.142	0.501	0.122	0.783
RMSE	0.098	0.38	0.665	0.122	0.495	0.099	0.409	0.124	0.576	0.254	0.331	0.394	0.516	0.123	1.081

G21

Table G60: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.053	-0.163	0.023	-0.006	0.184	-0.047	0.006	-0.002	0.481	0.266	0.255	-0.406	-0.161	0.01	1.229
SD	0.11	0.287	0.777	0.135	0.561	0.108	0.597	0.142	0.815	0.083	0.276	0.174	0.762	0.146	1.221
RMSE	0.123	0.331	0.777	0.135	0.59	0.118	0.597	0.142	0.946	0.279	0.376	0.442	0.779	0.146	1.732

Table G61: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.046	0.004	0.018	-0.026	-0.045	-0.044	0.046	-0.025	0.036	0.165	0.252	-0.397	-0.287	-0.073	1.294
SD	0.077	0.31	0.272	0.135	0.496	0.077	0.23	0.136	0.528	0.054	0.125	0.16	0.425	0.136	0.926
RMSE	0.09	0.31	0.273	0.138	0.498	0.089	0.235	0.138	0.529	0.174	0.281	0.428	0.513	0.155	1.591

Table G62: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.056	-0.044	-0.07	0.058	0.044	-0.054	-0.067	0.059	0.049	-0.036	-0.043	0.029	-0.314	-0.06	0.12
SD	0.078	0.18	0.115	0.125	0.028	0.077	0.112	0.125	0.029	0.066	0.103	0.125	0.141	0.108	0.104
RMSE	0.096	0.185	0.135	0.138	0.052	0.094	0.131	0.139	0.057	0.075	0.111	0.129	0.344	0.124	0.159

G22

Table G63: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.093	-0.16	0.02	-0.075	-0.02	-0.083	0.008	-0.074	0.233	0.17	0.258	-0.477	-0.358	-0.08	2.13
SD	0.097	0.338	0.818	0.143	0.49	0.084	0.45	0.142	0.558	0.079	0.221	0.165	0.865	0.144	2.066
RMSE	0.135	0.374	0.819	0.161	0.491	0.118	0.45	0.161	0.605	0.188	0.34	0.505	0.936	0.164	2.967

H Binary Probit $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho \in \{0, 0.3, 0.5\}$ and
 $\lambda \in \{0, 0.3, 0.5\}$

Table H64: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.021	-0.028	-0.025	0.029	0.284	-0.031	-0.057	0.033	0.418	0.324	0.219	-0.354	-0.029	0.026	0.062	
SD	0.13	0.271	0.355	0.133	0.688	0.129	0.331	0.136	0.763	0.113	0.151	0.134	0.298	0.126	0.608	
RMSE	0.131	0.272	0.356	0.136	0.744	0.132	0.336	0.14	0.871	0.343	0.266	0.378	0.3	0.128	0.611	

H23

Table H65: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.01	-0.159	0.016	0.022	0.197	-0.009	-0.003	0.023	0.327	0.345	0.24	-0.349	0	0.022	0.049	
SD	0.117	0.299	0.462	0.121	0.579	0.118	0.41	0.125	0.639	0.118	0.189	0.125	0.399	0.123	0.509	
RMSE	0.118	0.339	0.462	0.123	0.611	0.119	0.41	0.127	0.718	0.365	0.306	0.37	0.399	0.125	0.511	

Table H66: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.023	-0.143	0.055	0.028	0.194	-0.012	0.023	0.026	0.496	0.37	0.272	-0.385	0.028	0.027	0.199	
SD	0.113	0.261	0.655	0.125	0.502	0.113	0.532	0.132	0.726	0.111	0.259	0.121	0.52	0.133	0.654	
RMSE	0.115	0.298	0.658	0.128	0.538	0.113	0.533	0.135	0.879	0.387	0.375	0.403	0.521	0.135	0.683	

Table H67: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.035	-0.027	-0.017	0.031	0.145	-0.04	-0.027	0.03	0.241	0.218	0.234	-0.408	-0.156	0.072	0.631
SD	0.1	0.284	0.356	0.129	0.66	0.095	0.299	0.127	0.713	0.079	0.155	0.154	0.379	0.14	0.889
RMSE	0.106	0.285	0.356	0.132	0.676	0.103	0.3	0.13	0.753	0.232	0.281	0.436	0.41	0.157	1.091

Table H68: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.034	-0.115	0.02	0.011	0.135	-0.026	0.012	0.008	0.207	0.237	0.246	-0.419	-0.154	0.061	0.789
SD	0.096	0.27	0.514	0.148	0.552	0.09	0.395	0.144	0.561	0.081	0.202	0.157	0.572	0.159	0.923
RMSE	0.102	0.294	0.514	0.149	0.568	0.094	0.395	0.145	0.598	0.25	0.318	0.448	0.593	0.171	1.214

H24

Table H69: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.026	-0.154	-0.029	-0.033	0.054	-0.022	0.026	-0.029	0.352	0.271	0.269	-0.448	-0.141	0.024	1.212
SD	0.102	0.322	0.807	0.147	0.595	0.095	0.516	0.145	0.721	0.089	0.236	0.142	0.715	0.154	1.467
RMSE	0.105	0.357	0.807	0.151	0.597	0.098	0.517	0.148	0.802	0.285	0.358	0.47	0.729	0.156	1.903

Table H70: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.062	-0.068	-0.026	-0.051	-0.039	-0.064	-0.019	-0.046	0.059	0.154	0.231	-0.44	-0.328	-0.002	1.262
SD	0.091	0.321	0.389	0.148	0.417	0.091	0.315	0.144	0.415	0.066	0.155	0.146	0.491	0.17	0.868
RMSE	0.11	0.328	0.39	0.156	0.419	0.111	0.316	0.151	0.419	0.168	0.278	0.464	0.59	0.17	1.531

Table H71: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.031	-0.019	-0.056	0.056	0.041	-0.031	-0.041	0.055	0.046	-0.014	-0.025	0.039	-0.274	0.02	0.15
SD	0.058	0.205	0.152	0.133	0.02	0.058	0.082	0.132	0.023	0.054	0.082	0.139	0.125	0.121	0.109
RMSE	0.066	0.206	0.162	0.144	0.046	0.066	0.092	0.143	0.051	0.056	0.085	0.144	0.301	0.123	0.185

H25

Table H72: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.058	-0.229	-0.013	-0.068	-0.068	-0.063	0.017	-0.067	0.133	0.174	0.253	-0.471	-0.337	-0.004	2.054
SD	0.083	0.328	0.689	0.148	0.581	0.083	0.456	0.152	0.6	0.081	0.246	0.169	0.99	0.148	2.427
RMSE	0.101	0.4	0.689	0.163	0.585	0.104	0.456	0.166	0.614	0.192	0.352	0.501	1.046	0.148	3.179

I Binary Probit $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho \in \{0, 0.3, 0.5\}$ and $\lambda \in \{0, 0.3, 0.5\}$

Table I73: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.013	-0.047	0.001	0.019	0.035	-0.025	-0.054	0.025	0.124	0.217	0.134	-0.174	-0.028	0.017	-0.052	
SD	0.117	0.315	0.273	0.135	0.334	0.122	0.217	0.136	0.369	0.119	0.145	0.142	0.201	0.135	0.236	
RMSE	0.118	0.318	0.273	0.137	0.336	0.125	0.224	0.138	0.389	0.248	0.197	0.225	0.203	0.136	0.242	

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Table I74: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.024	-0.183	-0.078	0.031	0.153	-0.024	-0.058	0.033	0.241	0.246	0.15	-0.179	-0.025	0.027	0.045	
SD	0.115	0.306	0.35	0.112	0.368	0.114	0.286	0.112	0.398	0.135	0.191	0.117	0.25	0.11	0.287	
RMSE	0.118	0.356	0.358	0.116	0.399	0.116	0.292	0.116	0.466	0.281	0.243	0.214	0.251	0.114	0.291	

Table I75: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.046	-0.205	0.03	0.019	0.141	-0.047	0.009	0.013	0.287	0.242	0.189	-0.203	0.032	0.008	0.045	
SD	0.136	0.314	0.508	0.128	0.373	0.137	0.412	0.127	0.457	0.136	0.26	0.13	0.375	0.126	0.282	
RMSE	0.143	0.374	0.509	0.13	0.399	0.144	0.412	0.128	0.539	0.278	0.321	0.241	0.376	0.127	0.285	

Table I76: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.036	-0.094	-0.033	0.024	0.079	-0.033	-0.014	0.027	0.14	0.155	0.179	-0.235	-0.138	0.003	0.322
SD	0.098	0.297	0.323	0.115	0.306	0.103	0.236	0.115	0.331	0.061	0.135	0.132	0.273	0.11	0.367
RMSE	0.105	0.312	0.324	0.117	0.316	0.108	0.236	0.118	0.359	0.167	0.224	0.27	0.306	0.11	0.489

Table I77: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.027	-0.098	-0.083	0.023	0.058	-0.03	-0.052	0.021	0.161	0.167	0.16	-0.243	-0.212	0.003	0.414
SD	0.093	0.309	0.362	0.138	0.288	0.086	0.309	0.132	0.339	0.081	0.184	0.138	0.403	0.134	0.457
RMSE	0.097	0.324	0.371	0.14	0.294	0.091	0.314	0.133	0.376	0.186	0.244	0.28	0.456	0.134	0.617

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Table I78: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.044	-0.141	-0.116	0.017	0.156	-0.04	-0.058	0.017	0.37	0.197	0.192	-0.282	-0.192	0.002	0.696
SD	0.107	0.317	0.68	0.136	0.341	0.107	0.417	0.138	0.574	0.085	0.229	0.143	0.543	0.138	0.841
RMSE	0.116	0.347	0.69	0.137	0.375	0.114	0.421	0.139	0.683	0.215	0.299	0.316	0.576	0.138	1.092

Table I79: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.019	-0.034	0.012	-0.012	-0.014	-0.025	0.009	-0.01	0.04	0.097	0.18	-0.278	-0.266	-0.093	0.643
SD	0.069	0.294	0.24	0.141	0.285	0.074	0.211	0.143	0.308	0.061	0.128	0.149	0.334	0.121	0.58
RMSE	0.072	0.296	0.24	0.142	0.285	0.078	0.211	0.143	0.31	0.115	0.22	0.315	0.427	0.152	0.865

Table I80: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.042	-0.142	0.019	-0.009	0.051	-0.045	0.015	-0.007	0.169	0.106	0.202	-0.305	-0.251	-0.08	0.95
SD	0.087	0.34	0.287	0.129	0.294	0.09	0.25	0.13	0.376	0.069	0.157	0.148	0.407	0.125	0.814
RMSE	0.096	0.369	0.288	0.129	0.298	0.101	0.25	0.13	0.413	0.126	0.255	0.339	0.478	0.149	1.251

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Table I81: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.049	-0.191	0.029	-0.026	0.069	-0.046	0.027	-0.025	0.183	0.113	0.205	-0.335	-0.269	-0.078	1.067
SD	0.072	0.285	0.438	0.129	0.329	0.07	0.363	0.13	0.369	0.068	0.225	0.152	0.621	0.129	0.852
RMSE	0.087	0.343	0.439	0.131	0.336	0.083	0.364	0.132	0.412	0.131	0.304	0.368	0.677	0.151	1.366

J Binary Probit $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho \in \{0, 0.3, 0.5\}$ and $\lambda \in \{0, 0.3, 0.5\}$

Table J82: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.03	-0.072	-0.013	0.012	0.101	-0.028	-0.013	0.011	0.184	0.203	0.16	-0.224	0.019	0.003	-0.015
SD	0.107	0.313	0.306	0.129	0.353	0.113	0.254	0.127	0.455	0.114	0.163	0.116	0.218	0.12	0.256
RMSE	0.111	0.321	0.306	0.13	0.368	0.117	0.254	0.127	0.491	0.233	0.228	0.253	0.219	0.12	0.256

J29

Table J83: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.005	-0.143	-0.03	0.022	0.081	-0.007	-0.023	0.025	0.154	0.214	0.147	-0.216	-0.012	0.018	-0.01
SD	0.113	0.318	0.38	0.112	0.376	0.108	0.302	0.111	0.386	0.112	0.201	0.122	0.306	0.106	0.257
RMSE	0.113	0.348	0.381	0.114	0.385	0.108	0.302	0.114	0.416	0.242	0.25	0.248	0.306	0.107	0.257

Table J84: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.033	-0.229	-0.128	0.065	0.182	-0.019	-0.043	0.062	0.316	0.229	0.134	-0.226	-0.04	0.05	0.091
SD	0.128	0.325	0.611	0.126	0.468	0.124	0.477	0.125	0.562	0.143	0.303	0.138	0.431	0.113	0.348
RMSE	0.132	0.398	0.625	0.142	0.502	0.125	0.479	0.139	0.645	0.27	0.331	0.265	0.433	0.124	0.359

Table J85: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.023	-0.03	0.021	0.022	0.086	-0.031	-0.033	0.023	0.168	0.154	0.168	-0.266	-0.157	0.05	0.368
SD	0.098	0.345	0.364	0.135	0.372	0.097	0.213	0.129	0.362	0.074	0.129	0.118	0.253	0.134	0.412
RMSE	0.1	0.347	0.364	0.136	0.381	0.101	0.216	0.131	0.399	0.171	0.211	0.291	0.298	0.143	0.552

Table J86: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.029	-0.19	-0.041	0.04	0.156	-0.025	-0.04	0.046	0.231	0.164	0.179	-0.292	-0.171	0.073	0.482
SD	0.097	0.317	0.372	0.146	0.363	0.094	0.296	0.151	0.379	0.078	0.17	0.113	0.371	0.149	0.492
RMSE	0.101	0.369	0.374	0.151	0.395	0.097	0.298	0.158	0.444	0.181	0.247	0.314	0.409	0.166	0.688

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Table J87: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.032	-0.172	-0.063	0.021	0.055	-0.03	-0.133	0.029	0.246	0.169	0.116	-0.282	-0.305	0.062	0.543
SD	0.107	0.313	0.478	0.123	0.318	0.107	0.426	0.126	0.441	0.089	0.247	0.131	0.55	0.132	0.627
RMSE	0.111	0.358	0.483	0.125	0.322	0.111	0.447	0.129	0.505	0.19	0.273	0.311	0.629	0.146	0.83

Table J88: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.026	-0.012	0.007	-0.014	0.018	-0.025	0.037	-0.011	0.074	0.104	0.206	-0.304	-0.198	-0.007	0.747
SD	0.079	0.319	0.338	0.137	0.27	0.074	0.193	0.138	0.287	0.062	0.116	0.164	0.302	0.138	0.573
RMSE	0.083	0.319	0.338	0.137	0.271	0.078	0.197	0.138	0.296	0.121	0.236	0.345	0.361	0.138	0.942

Table J89: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.043	-0.164	0	-0.034	0.049	-0.041	-0.014	-0.027	0.13	0.096	0.177	-0.351	-0.289	-0.009	0.848
SD	0.074	0.332	0.429	0.12	0.286	0.072	0.257	0.129	0.339	0.061	0.171	0.159	0.43	0.139	0.722
RMSE	0.085	0.37	0.429	0.125	0.29	0.083	0.257	0.131	0.363	0.114	0.246	0.385	0.518	0.139	1.114

J31

Table J90: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.046	-0.217	-0.099	-0.044	0.028	-0.045	-0.093	-0.041	0.158	0.114	0.145	-0.361	-0.453	-0.004	1.065
SD	0.082	0.31	0.553	0.127	0.272	0.085	0.372	0.126	0.346	0.072	0.234	0.157	0.641	0.128	0.918
RMSE	0.094	0.379	0.562	0.134	0.273	0.096	0.383	0.132	0.38	0.135	0.275	0.394	0.785	0.128	1.405

K Binary Probit $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho \in \{0, 0.3, 0.5\}$ and
 $\lambda \in \{0, 0.3, 0.5\}$

Table K91: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.045	-0.009	-0.037	0.047	0.046	-0.048	-0.036	0.049	0.051	-0.011	-0.008	0.021	-0.004	0.023	0.01	
SD	0.103	0.157	0.105	0.108	0.033	0.103	0.096	0.105	0.031	0.104	0.091	0.1	0.081	0.102	0.025	
RMSE	0.113	0.157	0.111	0.118	0.056	0.114	0.103	0.116	0.059	0.105	0.091	0.102	0.081	0.105	0.027	

K32

Table K92: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.045	-0.009	-0.037	0.047	0.046	-0.048	-0.036	0.049	0.051	-0.011	-0.008	0.021	-0.004	0.023	0.01	
SD	0.103	0.157	0.105	0.108	0.033	0.103	0.096	0.105	0.031	0.104	0.091	0.1	0.081	0.102	0.025	
RMSE	0.113	0.157	0.111	0.118	0.056	0.114	0.103	0.116	0.059	0.105	0.091	0.102	0.081	0.105	0.027	

Table K93: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.045	-0.009	-0.037	0.047	0.046	-0.048	-0.036	0.049	0.051	-0.011	-0.008	0.021	-0.004	0.023	0.01	
SD	0.103	0.157	0.105	0.108	0.033	0.103	0.096	0.105	0.031	0.104	0.091	0.1	0.081	0.102	0.025	
RMSE	0.113	0.157	0.111	0.118	0.056	0.114	0.103	0.116	0.059	0.105	0.091	0.102	0.081	0.105	0.027	

Table K94: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.05	-0.016	-0.06	0.059	0.046	-0.048	-0.058	0.062	0.052	-0.009	-0.019	0.027	-0.17	0.013	0.046
SD	0.091	0.172	0.122	0.144	0.037	0.097	0.122	0.144	0.042	0.079	0.101	0.14	0.115	0.131	0.051
RMSE	0.104	0.173	0.136	0.156	0.059	0.108	0.135	0.156	0.067	0.079	0.103	0.142	0.205	0.132	0.068

Table K95: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.05	-0.016	-0.06	0.059	0.046	-0.048	-0.058	0.062	0.052	-0.009	-0.019	0.027	-0.17	0.013	0.046
SD	0.091	0.172	0.122	0.144	0.037	0.097	0.122	0.144	0.042	0.079	0.101	0.14	0.115	0.131	0.051
RMSE	0.104	0.173	0.136	0.156	0.059	0.108	0.135	0.156	0.067	0.079	0.103	0.142	0.205	0.132	0.068

K33

Table K96: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.05	-0.016	-0.06	0.059	0.046	-0.048	-0.058	0.062	0.052	-0.009	-0.019	0.027	-0.17	0.013	0.046
SD	0.091	0.172	0.122	0.144	0.037	0.097	0.122	0.144	0.042	0.079	0.101	0.14	0.115	0.131	0.051
RMSE	0.104	0.173	0.136	0.156	0.059	0.108	0.135	0.156	0.067	0.079	0.103	0.142	0.205	0.132	0.068

Table K97: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.056	-0.044	-0.07	0.058	0.044	-0.054	-0.067	0.059	0.049	-0.036	-0.043	0.029	-0.314	-0.06	0.12
SD	0.078	0.18	0.115	0.125	0.028	0.077	0.112	0.125	0.029	0.066	0.103	0.125	0.141	0.108	0.104
RMSE	0.096	0.185	0.135	0.138	0.052	0.094	0.131	0.139	0.057	0.075	0.111	0.129	0.344	0.124	0.159

Table K98: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.056	-0.044	-0.07	0.058	0.044	-0.054	-0.067	0.059	0.049	-0.036	-0.043	0.029	-0.314	-0.06	0.12
SD	0.078	0.18	0.115	0.125	0.028	0.077	0.112	0.125	0.029	0.066	0.103	0.125	0.141	0.108	0.104
RMSE	0.096	0.185	0.135	0.138	0.052	0.094	0.131	0.139	0.057	0.075	0.111	0.129	0.344	0.124	0.159

K34

Table K99: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.056	-0.044	-0.07	0.058	0.044	-0.054	-0.067	0.059	0.049	-0.036	-0.043	0.029	-0.314	-0.06	0.12
SD	0.078	0.18	0.115	0.125	0.028	0.077	0.112	0.125	0.029	0.066	0.103	0.125	0.141	0.108	0.104
RMSE	0.096	0.185	0.135	0.138	0.052	0.094	0.131	0.139	0.057	0.075	0.111	0.129	0.344	0.124	0.159

L Binary Probit $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho \in \{0, 0.3, 0.5\}$ and $\lambda \in \{0, 0.3, 0.5\}$

Table L100: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.043	-0.005	-0.043	0.03	0.048	-0.044	-0.046	0.029	0.056	-0.007	-0.014	-0.004	-0.013	-0.001	0.013	
SD	0.097	0.17	0.119	0.11	0.023	0.096	0.108	0.112	0.032	0.097	0.1	0.102	0.095	0.105	0.023	
RMSE	0.106	0.17	0.126	0.114	0.053	0.105	0.117	0.115	0.064	0.097	0.101	0.102	0.096	0.105	0.027	

L35

Table L101: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.043	-0.005	-0.043	0.03	0.048	-0.044	-0.046	0.029	0.056	-0.007	-0.014	-0.004	-0.013	-0.001	0.013	
SD	0.097	0.17	0.119	0.11	0.023	0.096	0.108	0.112	0.032	0.097	0.1	0.102	0.095	0.105	0.023	
RMSE	0.106	0.17	0.126	0.114	0.053	0.105	0.117	0.115	0.064	0.097	0.101	0.102	0.096	0.105	0.027	

Table L102: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR					SAR-RE					SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2	
Bias	-0.043	-0.005	-0.043	0.03	0.048	-0.044	-0.046	0.029	0.056	-0.007	-0.014	-0.004	-0.013	-0.001	0.013	
SD	0.097	0.17	0.119	0.11	0.023	0.096	0.108	0.112	0.032	0.097	0.1	0.102	0.095	0.105	0.023	
RMSE	0.106	0.17	0.126	0.114	0.053	0.105	0.117	0.115	0.064	0.097	0.101	0.102	0.096	0.105	0.027	

Table L103: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.038	-0.028	-0.025	0.056	0.044	-0.037	-0.042	0.053	0.052	-0.001	-0.01	0.019	-0.156	0.065	0.055
SD	0.073	0.194	0.222	0.142	0.03	0.079	0.099	0.136	0.038	0.067	0.087	0.127	0.115	0.139	0.061
RMSE	0.083	0.196	0.223	0.152	0.053	0.087	0.107	0.146	0.065	0.067	0.087	0.129	0.194	0.153	0.082

Table L104: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.038	-0.028	-0.025	0.056	0.044	-0.037	-0.042	0.053	0.052	-0.001	-0.01	0.019	-0.156	0.065	0.055
SD	0.073	0.194	0.222	0.142	0.03	0.079	0.099	0.136	0.038	0.067	0.087	0.127	0.115	0.139	0.061
RMSE	0.083	0.196	0.223	0.152	0.053	0.087	0.107	0.146	0.065	0.067	0.087	0.129	0.194	0.153	0.082

136

Table L105: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.038	-0.028	-0.025	0.056	0.044	-0.037	-0.042	0.053	0.052	-0.001	-0.01	0.019	-0.156	0.065	0.055
SD	0.073	0.194	0.222	0.142	0.03	0.079	0.099	0.136	0.038	0.067	0.087	0.127	0.115	0.139	0.061
RMSE	0.083	0.196	0.223	0.152	0.053	0.087	0.107	0.146	0.065	0.067	0.087	0.129	0.194	0.153	0.082

Table L106: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.031	-0.019	-0.056	0.056	0.041	-0.031	-0.041	0.055	0.046	-0.014	-0.025	0.039	-0.274	0.02	0.15
SD	0.058	0.205	0.152	0.133	0.02	0.058	0.082	0.132	0.023	0.054	0.082	0.139	0.125	0.121	0.109
RMSE	0.066	0.206	0.162	0.144	0.046	0.066	0.092	0.143	0.051	0.056	0.085	0.144	0.301	0.123	0.185

Table L107: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.031	-0.019	-0.056	0.056	0.041	-0.031	-0.041	0.055	0.046	-0.014	-0.025	0.039	-0.274	0.02	0.15
SD	0.058	0.205	0.152	0.133	0.02	0.058	0.082	0.132	0.023	0.054	0.082	0.139	0.125	0.121	0.109
RMSE	0.066	0.206	0.162	0.144	0.046	0.066	0.092	0.143	0.051	0.056	0.085	0.144	0.301	0.123	0.185

L37

Table L108: Binary Probit: $J = 16$, $N = 320$, $\sigma_u^2 = 0.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR					SAR-RE				SAR			Multilevel		
	ρ	λ	β_0	β_1	σ_u^2	ρ	λ	β_0	β_1	ρ	β_0	β_1	β_0	β_1	σ_u^2
Bias	-0.031	-0.019	-0.056	0.056	0.041	-0.031	-0.041	0.055	0.046	-0.014	-0.025	0.039	-0.274	0.02	0.15
SD	0.058	0.205	0.152	0.133	0.02	0.058	0.082	0.132	0.023	0.054	0.082	0.139	0.125	0.121	0.109
RMSE	0.066	0.206	0.162	0.144	0.046	0.066	0.092	0.143	0.051	0.056	0.085	0.144	0.301	0.123	0.185

M Binary Probit $J = 16, N = 320, \rho_x = 0.0, \rho \in \{0, 0.3, 0.5\}$,

$$\lambda \in \{0, 0.3, 0.5\}, \sigma_u^2 \in \{0, 0.5, 1.0\}$$

Figure M1: Bias in $\hat{\beta}_1$ for $J = 16, N = 320, \rho_x = 0.0$

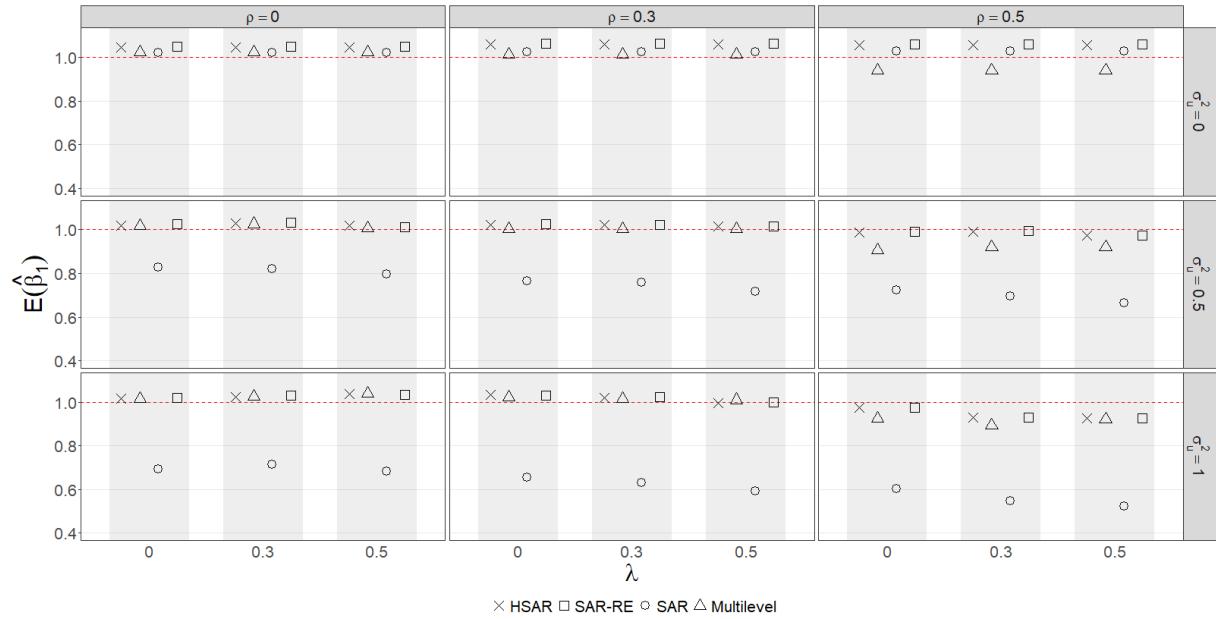


Figure M2: Bias in $\hat{\beta}_0$ for $J = 16$, $N = 320$, $\rho_x = 0.0$

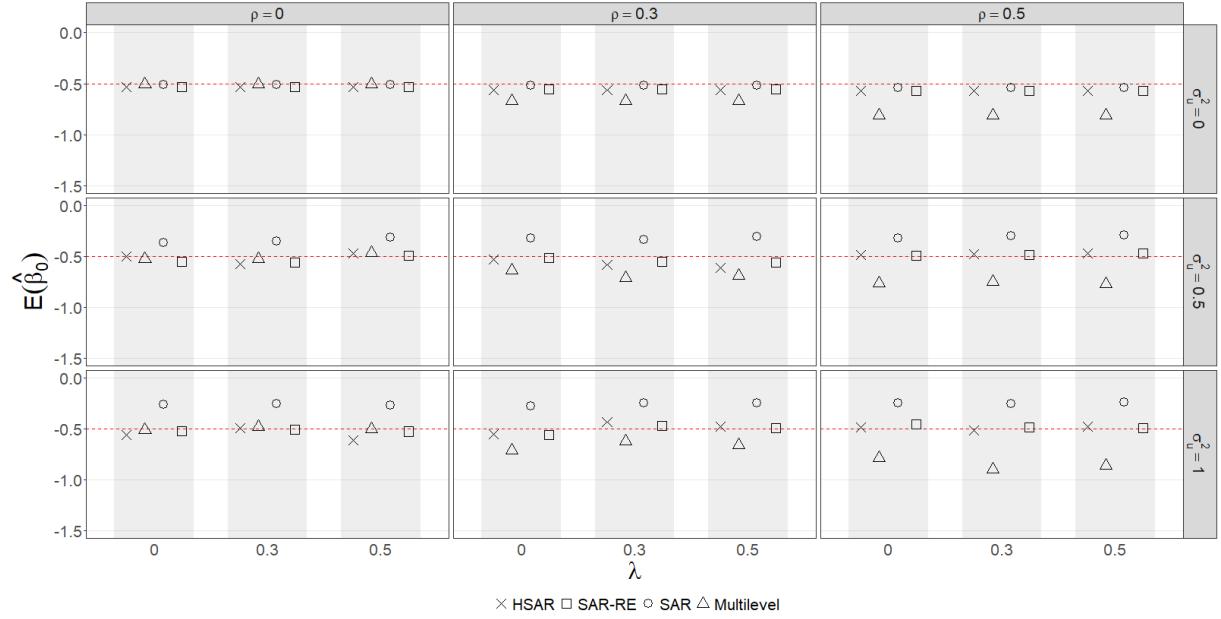


Figure M3: SD in $\hat{\beta}_0$ for $J = 16$, $N = 320$, $\rho_x = 0.0$

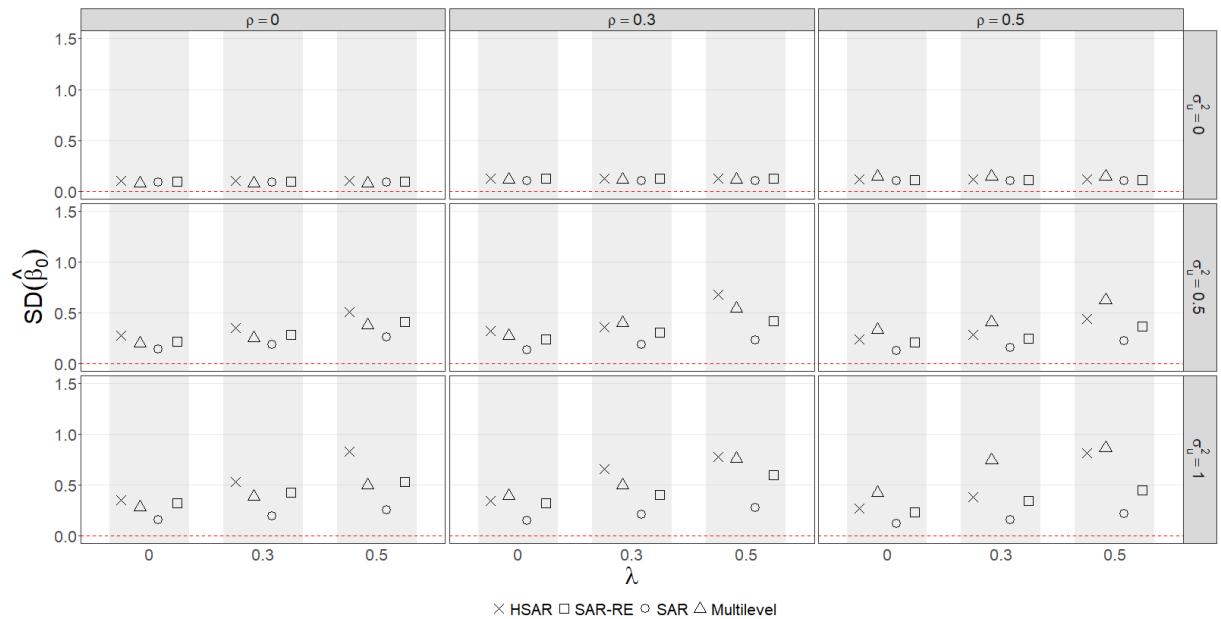


Figure M4: Bias in $\hat{\rho}$ for $J = 16$, $N = 320$, $\rho_x = 0.0$

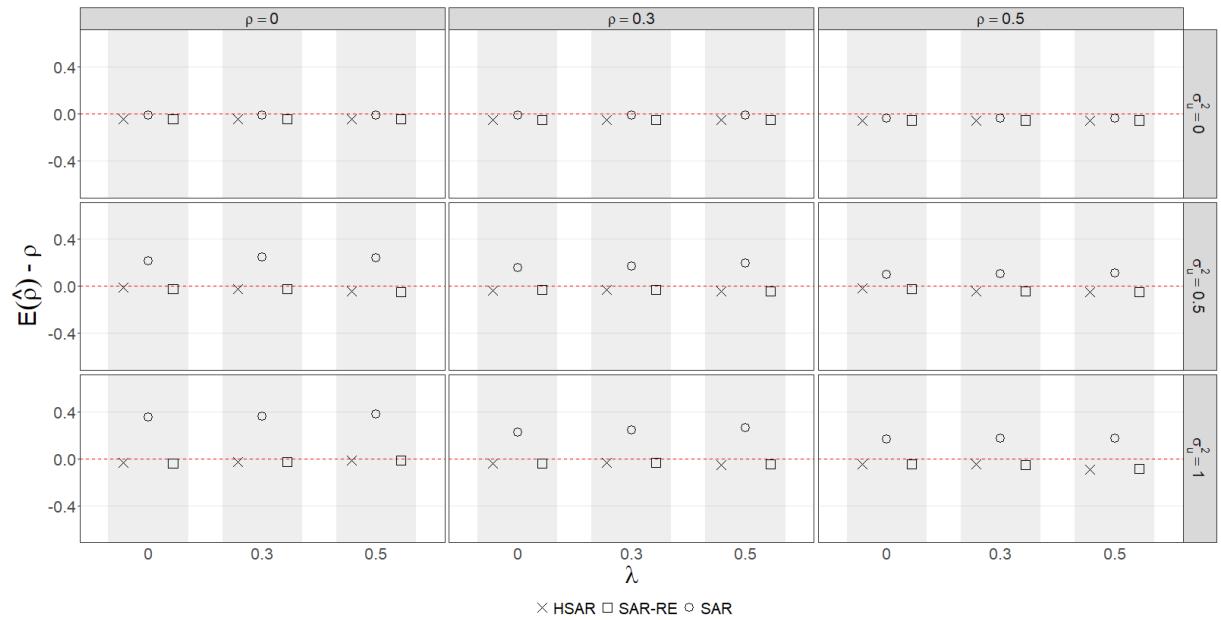


Figure M5: SD in $\hat{\rho}$ for $J = 16$, $N = 320$, $\rho_x = 0.0$

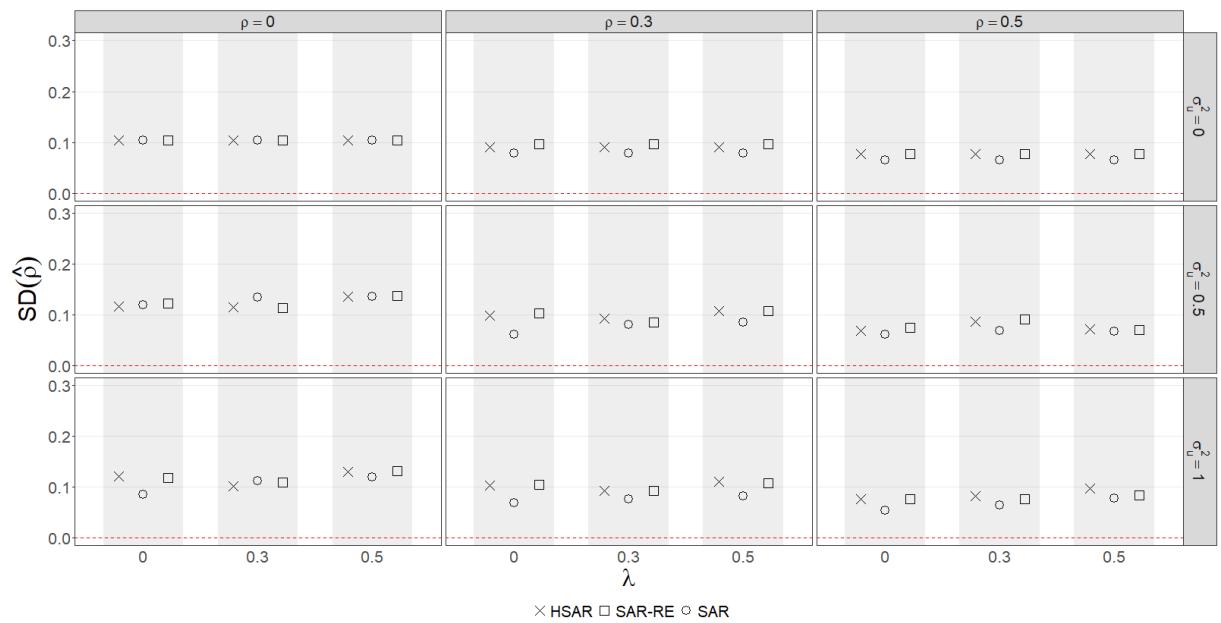


Figure M6: Bias in $\hat{\sigma}_u^2$ for $J = 16$, $N = 320$, $\rho_x = 0.0$

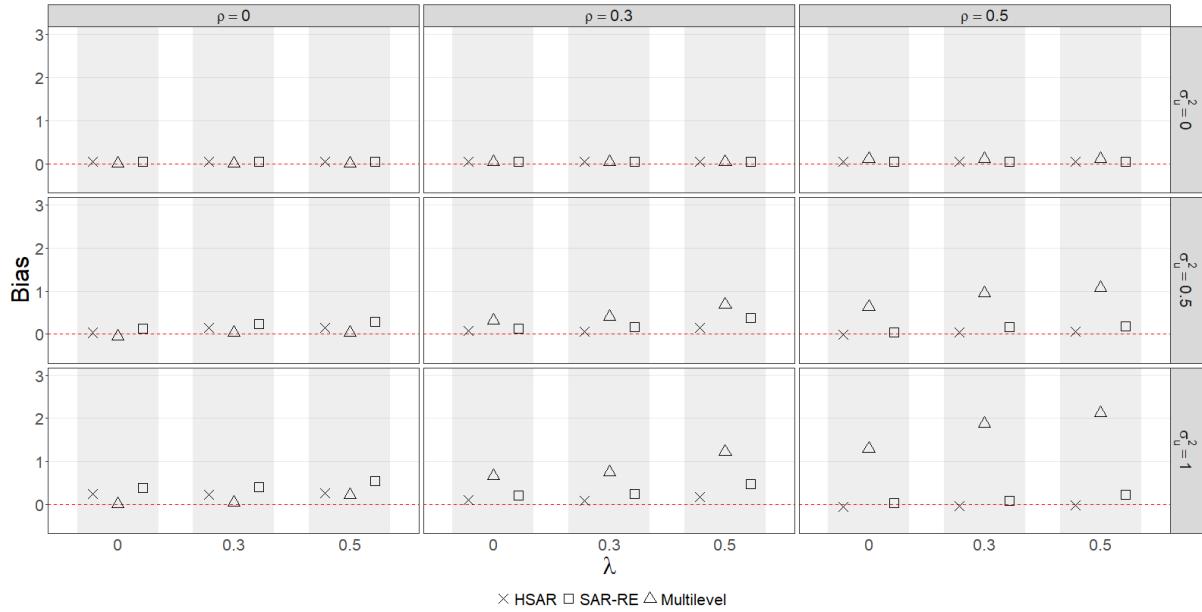
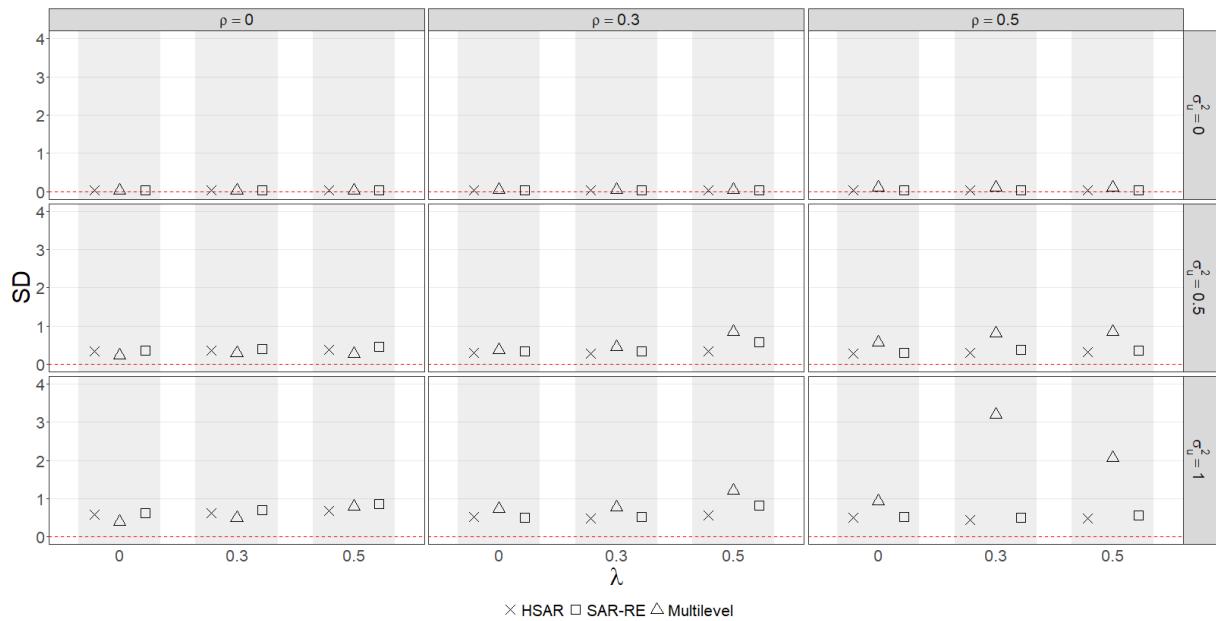


Figure M7: SD in $\hat{\sigma}_u^2$ for $J = 16$, $N = 320$, $\rho_x = 0.0$



N Binary Probit $J = 16, N = 320, \rho_x = 0.3, \rho \in \{0, 0.3, 0.5\},$

$\lambda \in \{0, 0.3, 0.5\}, \sigma_u^2 \in \{0, 0.5, 1.0\}$

Figure N8: Bias in $\hat{\beta}_1$ for $J = 16, N = 320, \rho_x = 0.3$

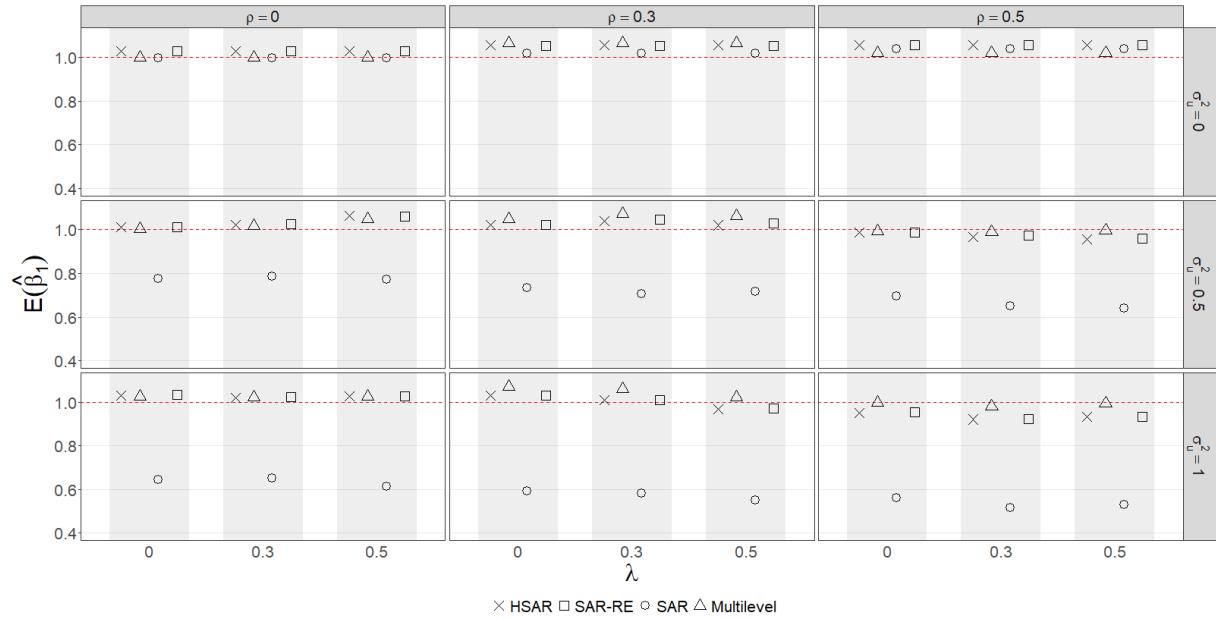


Figure N9: Bias in $\hat{\beta}_0$ for $J = 16$, $N = 320$, $\rho_x = 0.3$

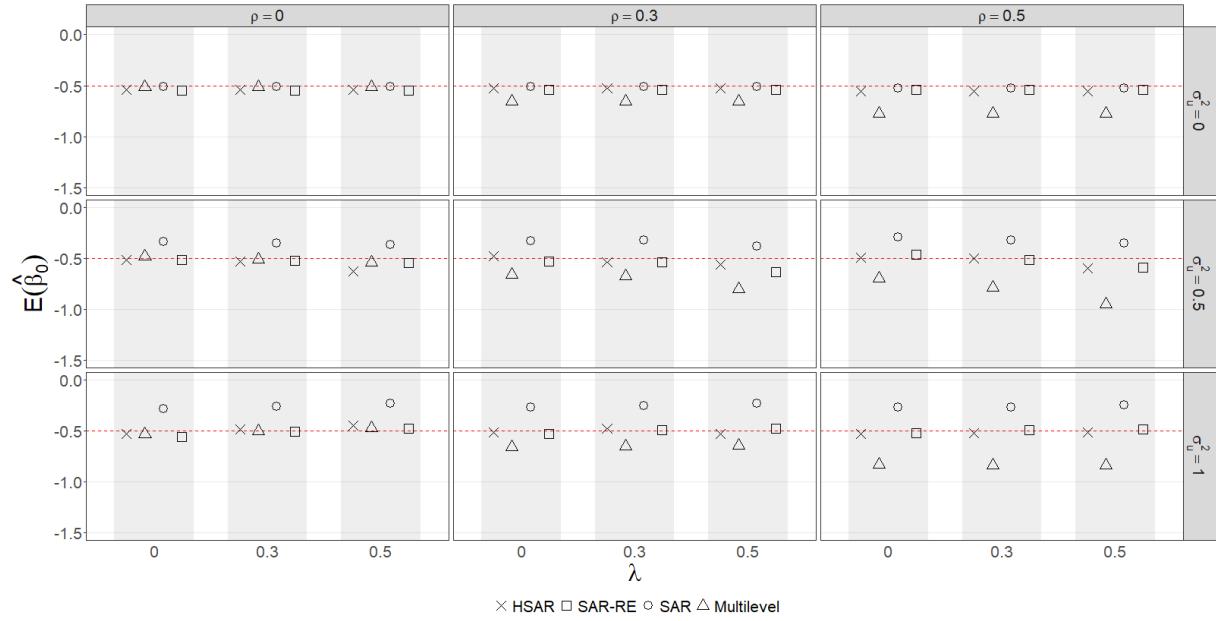


Figure N10: SD in $\hat{\beta}_0$ for $J = 16$, $N = 320$, $\rho_x = 0.3$

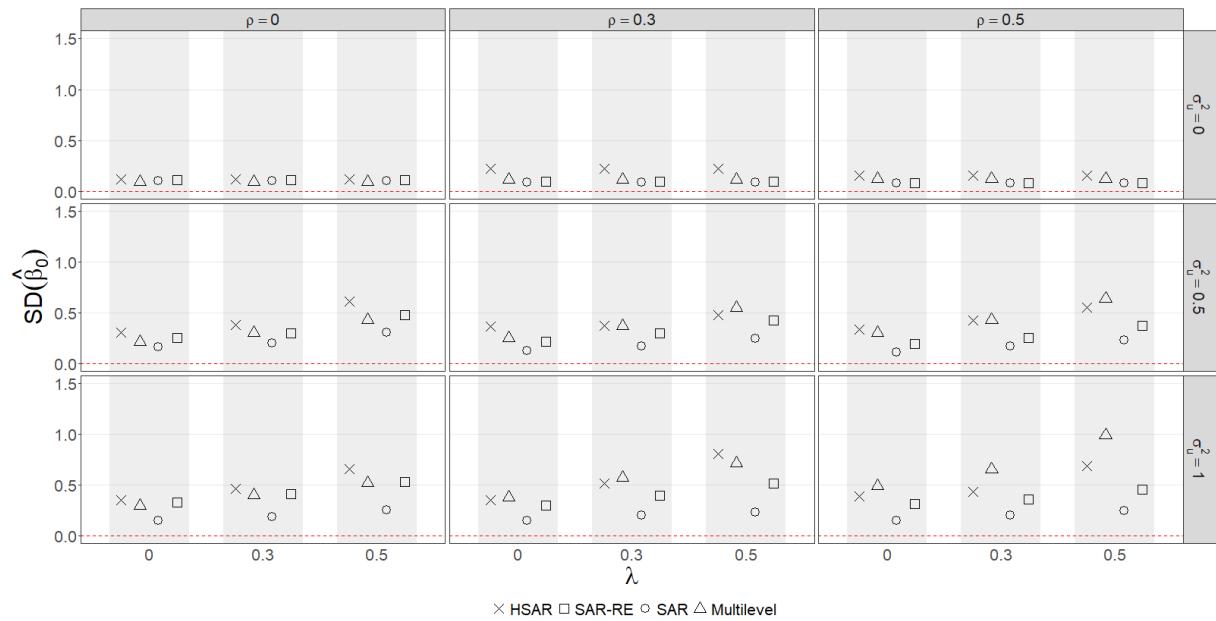


Figure N11: Bias in $\hat{\rho}$ for $J = 16$, $N = 320$, $\rho_x = 0.3$

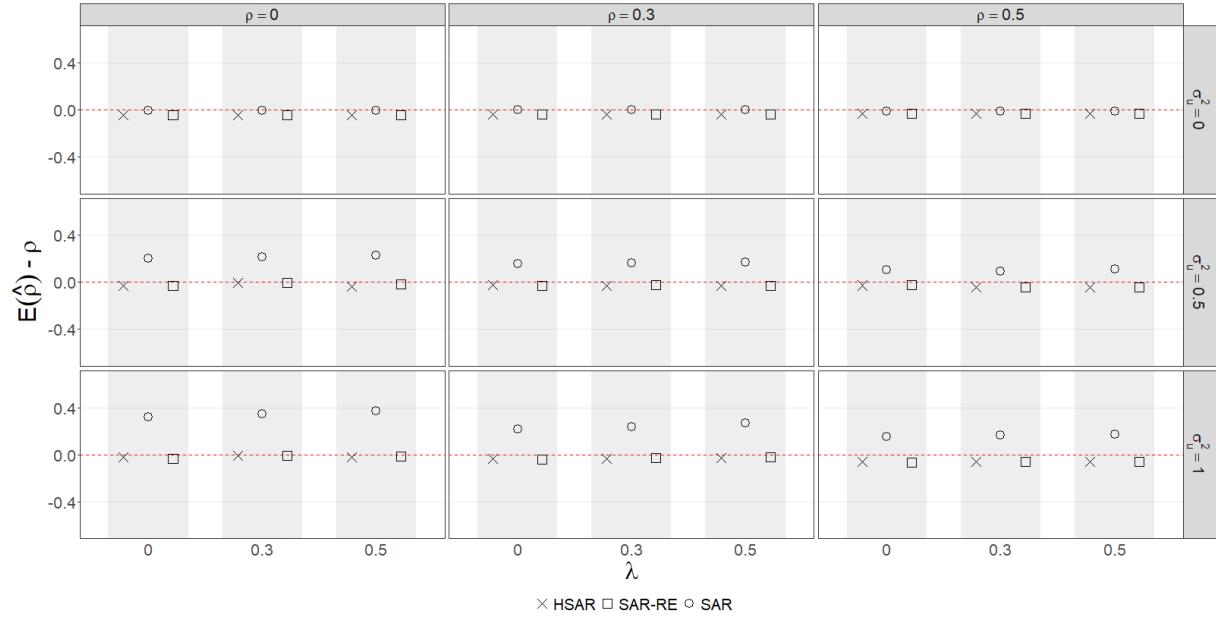


Figure N12: SD in $\hat{\rho}$ for $J = 16$, $N = 320$, $\rho_x = 0.3$

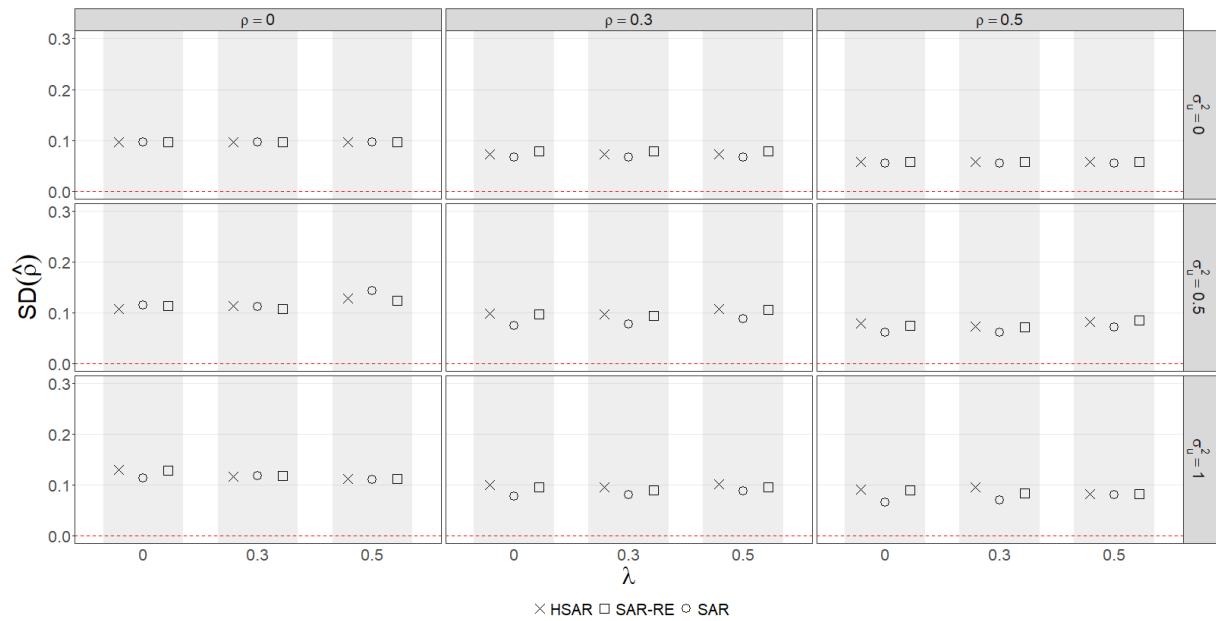


Figure N13: Bias in $\hat{\sigma}_u^2$ for $J = 16$, $N = 320$, $\rho_x = 0.3$

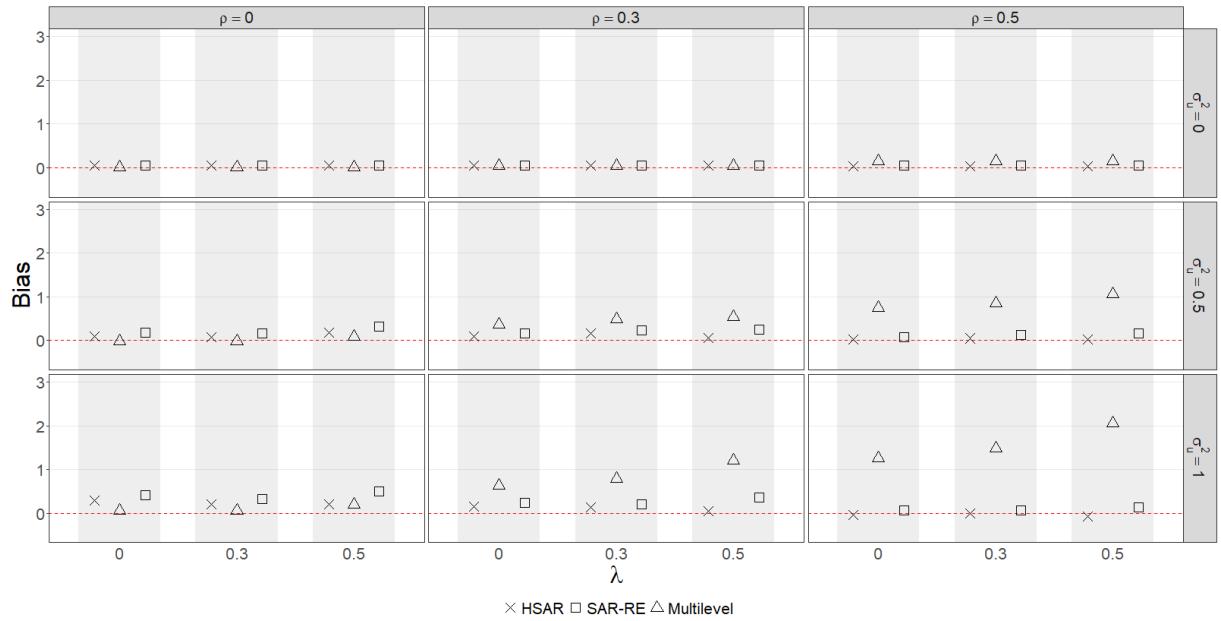
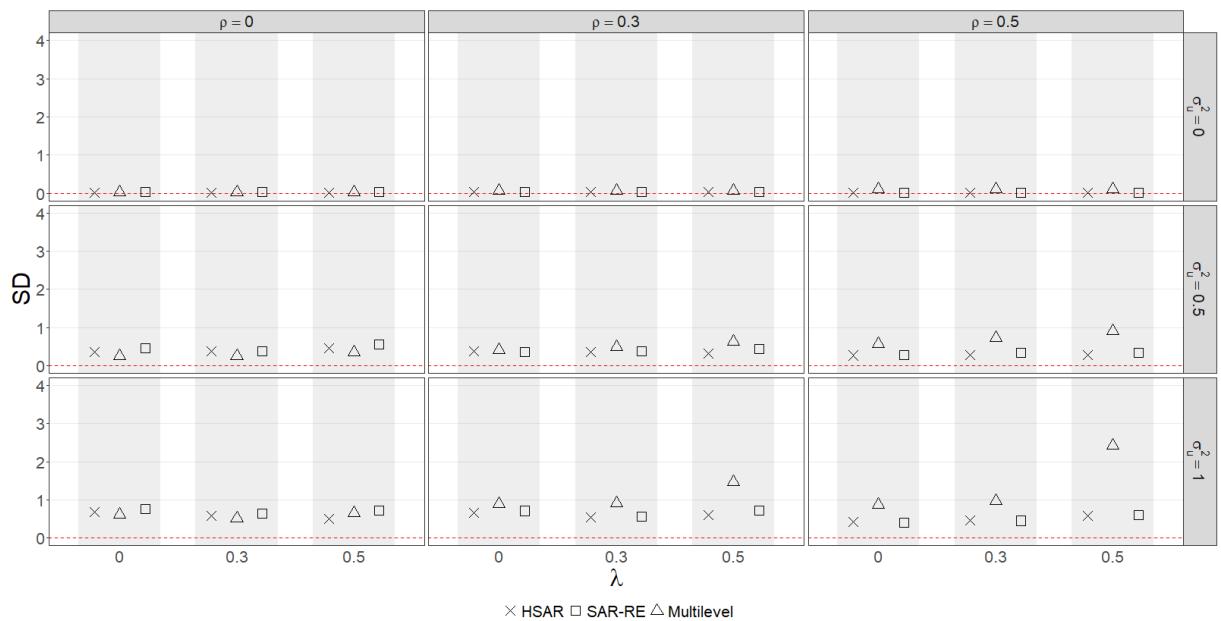


Figure N14: SD in $\hat{\sigma}_u^2$ for $J = 16$, $N = 320$, $\rho_x = 0.3$



O Binary Probit $J = 49$, $N = 980$, $\rho_x = 0.0$, $\rho \in \{0, 0.3, 0.5\}$,

$\lambda \in \{0, 0.3, 0.5\}$, $\sigma_u^2 \in \{0, 0.5, 1.0\}$

Figure O15: Bias in Direct Effect for $J = 49$, $N = 980$, $\rho_x = 0.0$

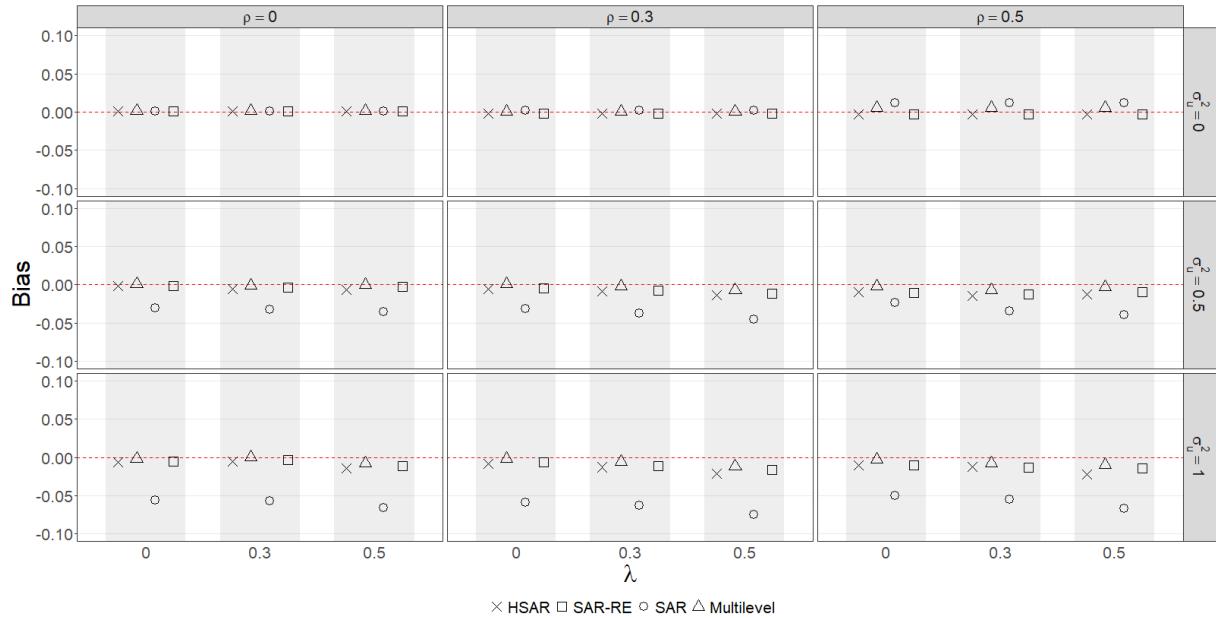


Figure O16: RMSE in Direct Effect for $J = 49$, $N = 980$, $\rho_x = 0.0$

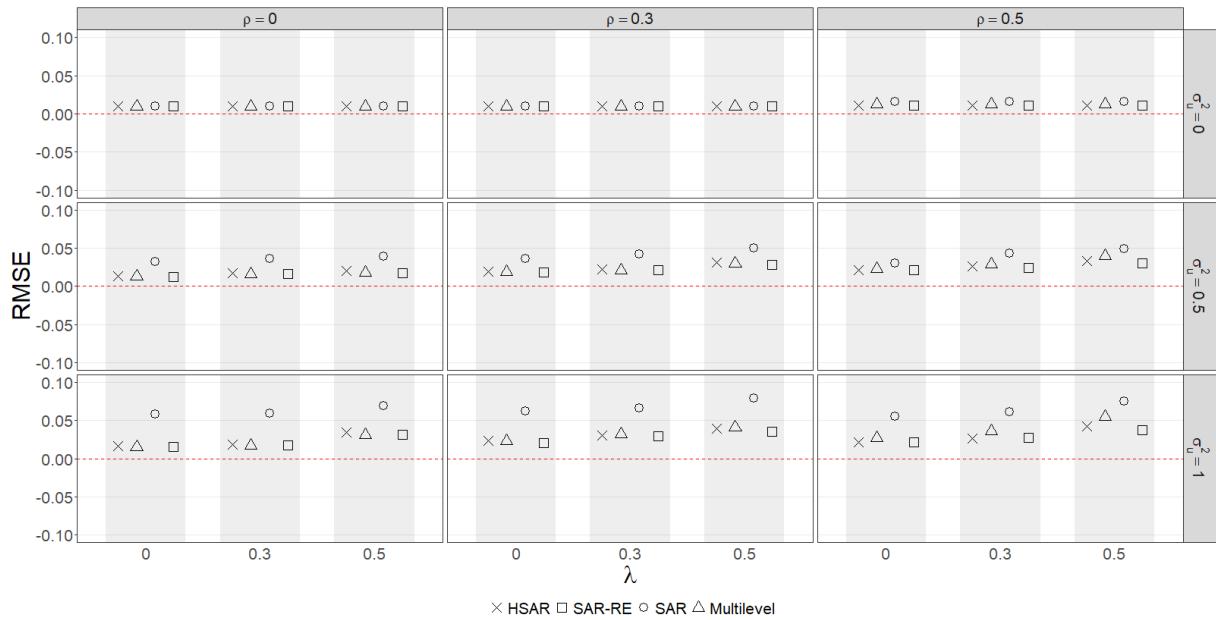


Figure O17: Bias in Indirect Effect for $J = 49$, $N = 980$, $\rho_x = 0.0$

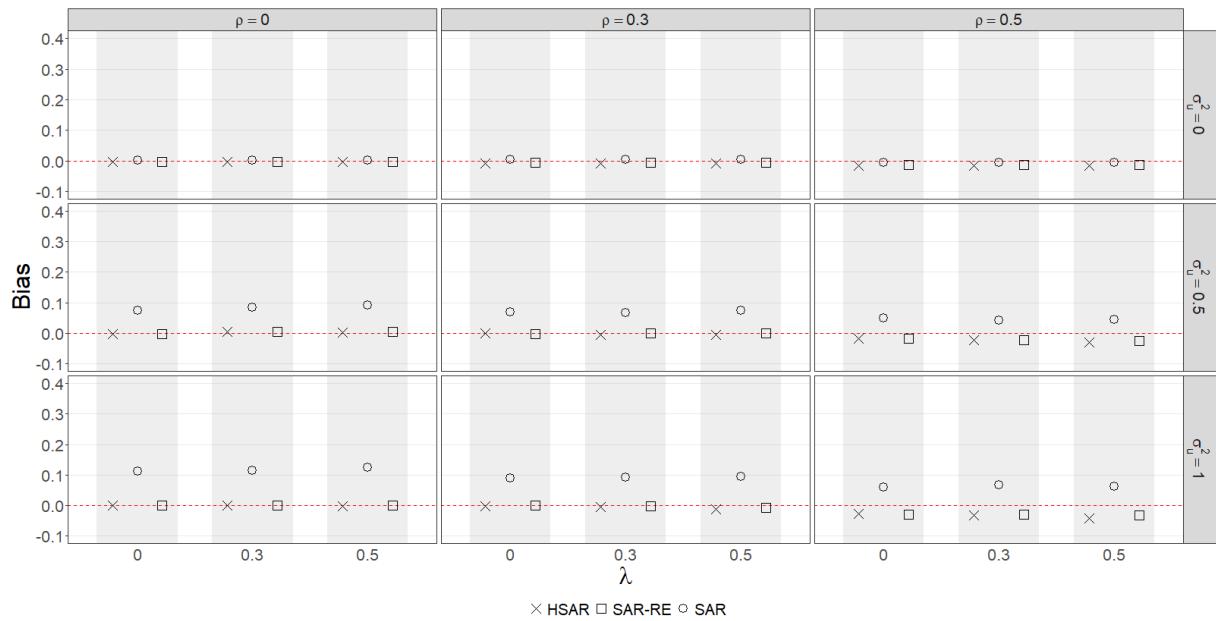


Figure O18: RMSE in Indirect Effect for $J = 49$, $N = 980$, $\rho_x = 0.0$

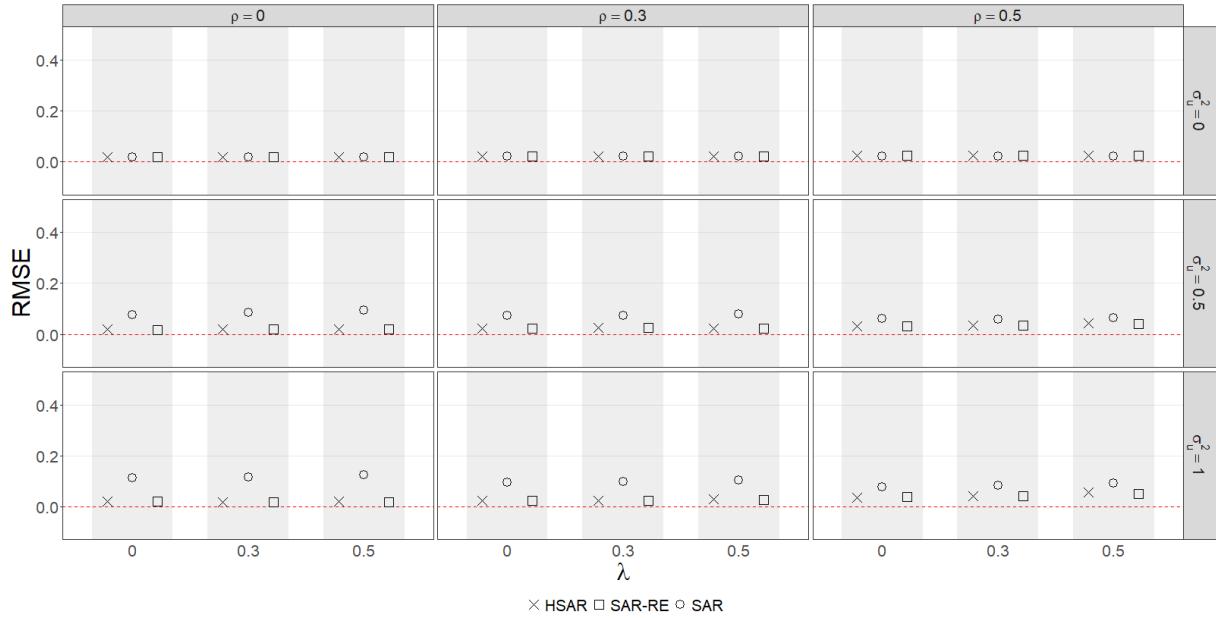


Figure O19: Bias in $\hat{\beta}_1$ for $J = 49$, $N = 980$, $\rho_x = 0.0$

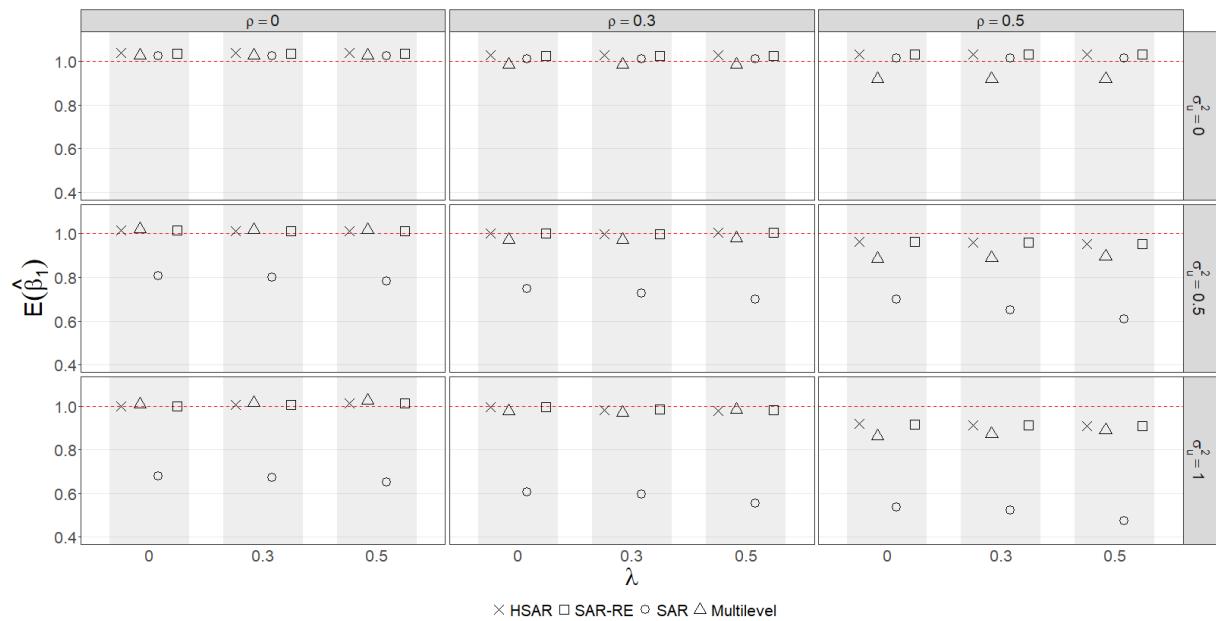


Figure O20: Bias in $\hat{\beta}_0$ for $J = 49$, $N = 980$, $\rho_x = 0.0$

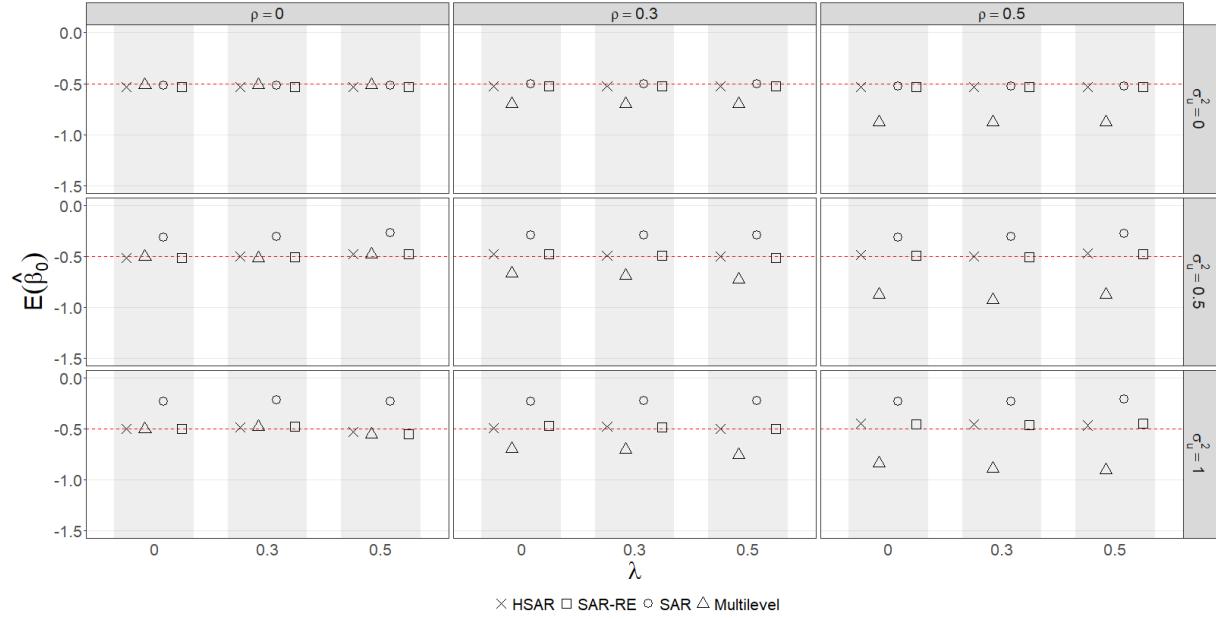


Figure O21: SD in $\hat{\beta}_0$ for $J = 49$, $N = 980$, $\rho_x = 0.0$

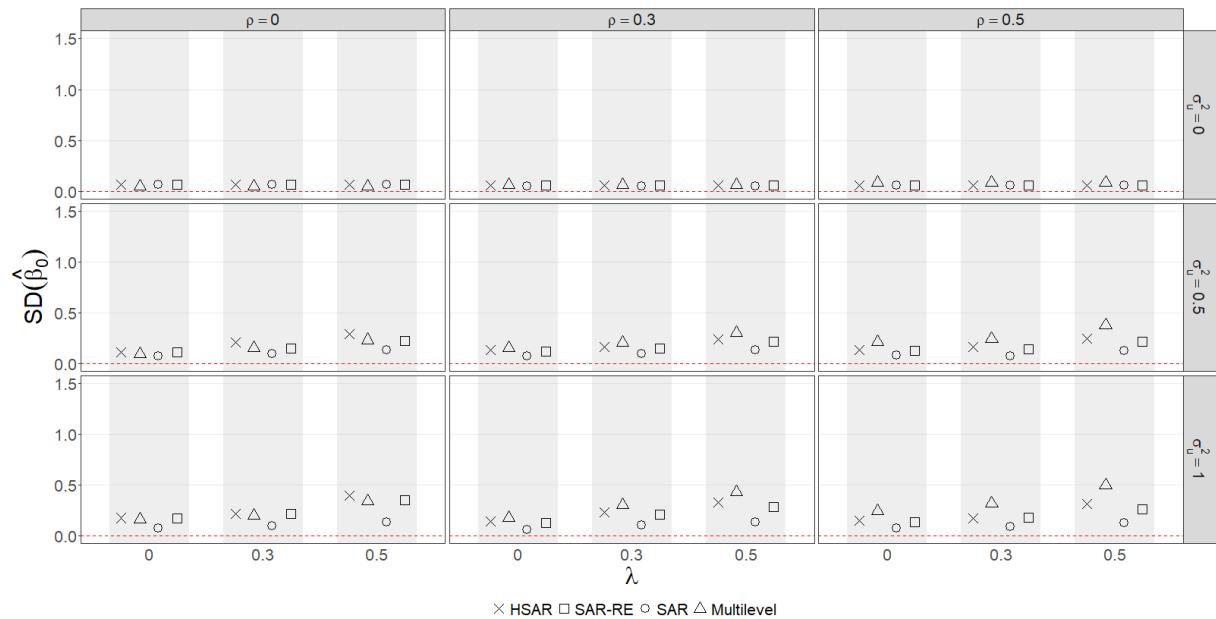


Figure O22: Bias in $\hat{\rho}$ for $J = 49$, $N = 980$, $\rho_x = 0.0$

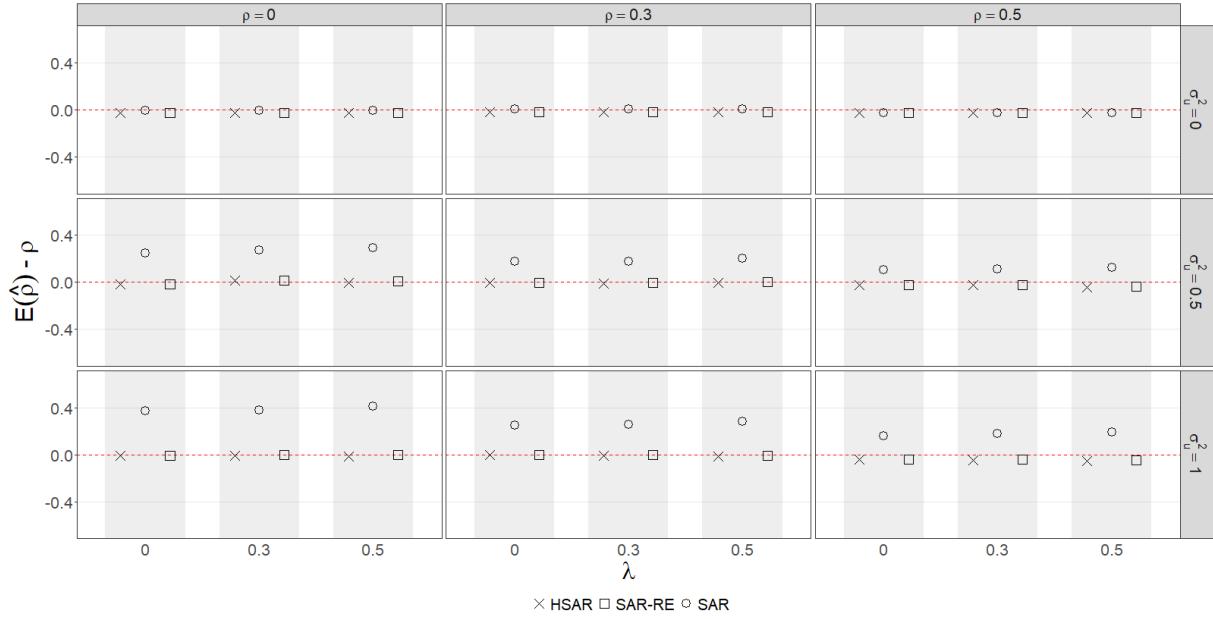


Figure O23: SD in $\hat{\rho}$ for $J = 49$, $N = 980$, $\rho_x = 0.0$

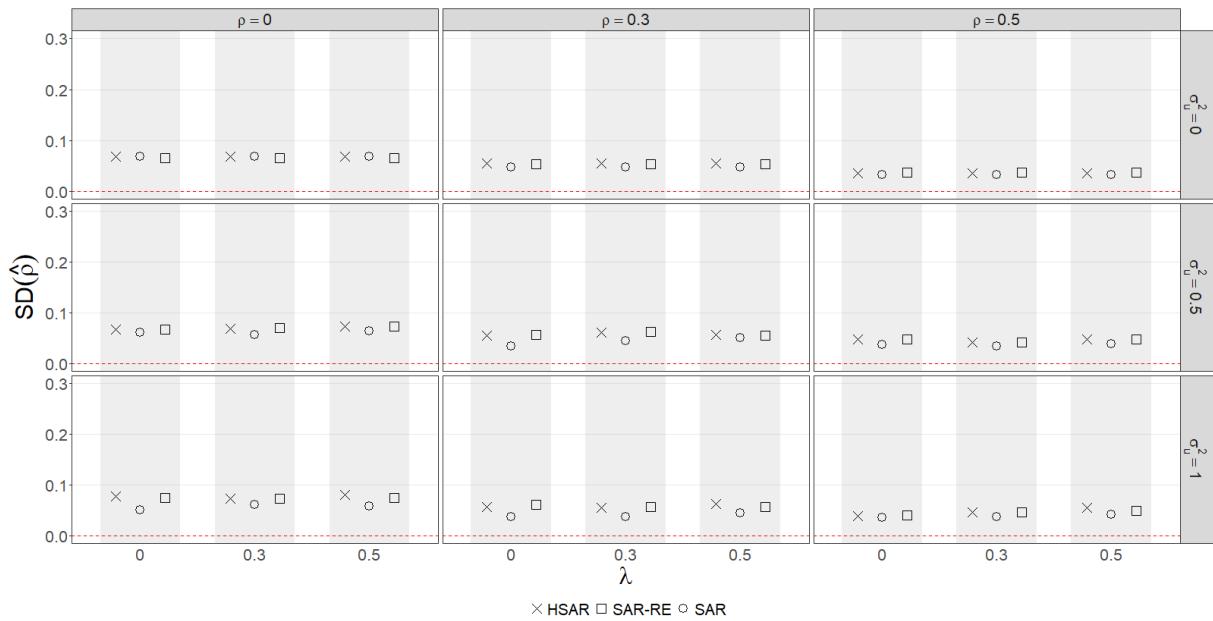


Figure O24: Bias in $\hat{\sigma}_u^2$ for $J = 49$, $N = 980$, $\rho_x = 0.0$

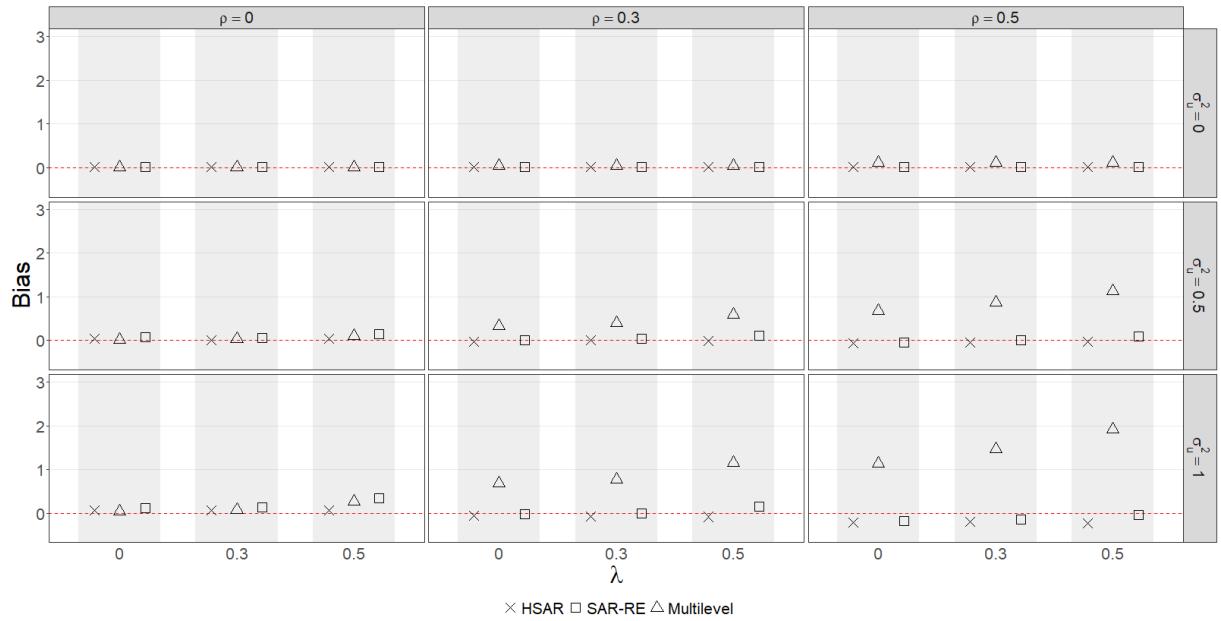
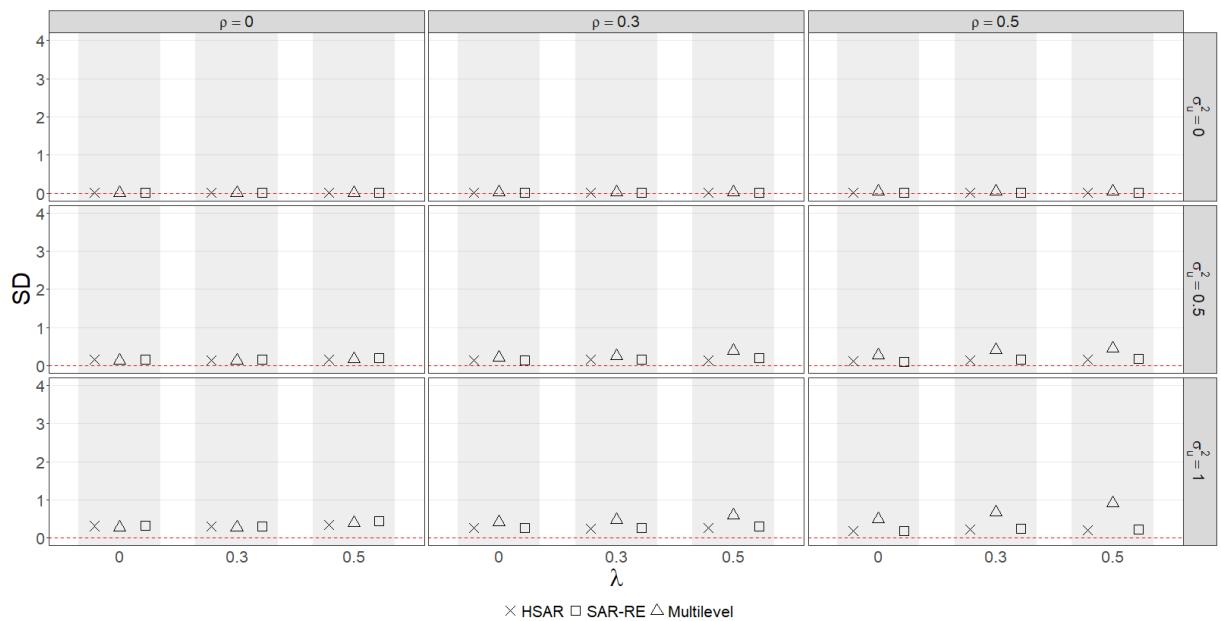


Figure O25: SD in $\hat{\sigma}_u^2$ for $J = 49$, $N = 980$, $\rho_x = 0.0$



P Binary Probit $J = 49$, $N = 980$, $\rho_x = 0.3$, $\rho \in \{0, 0.3, 0.5\}$,

$\lambda \in \{0, 0.3, 0.5\}$, $\sigma_u^2 \in \{0, 0.5, 1.0\}$

Figure P26: Bias in $\hat{\beta}_1$ for $J = 49$, $N = 980$, $\rho_x = 0.3$

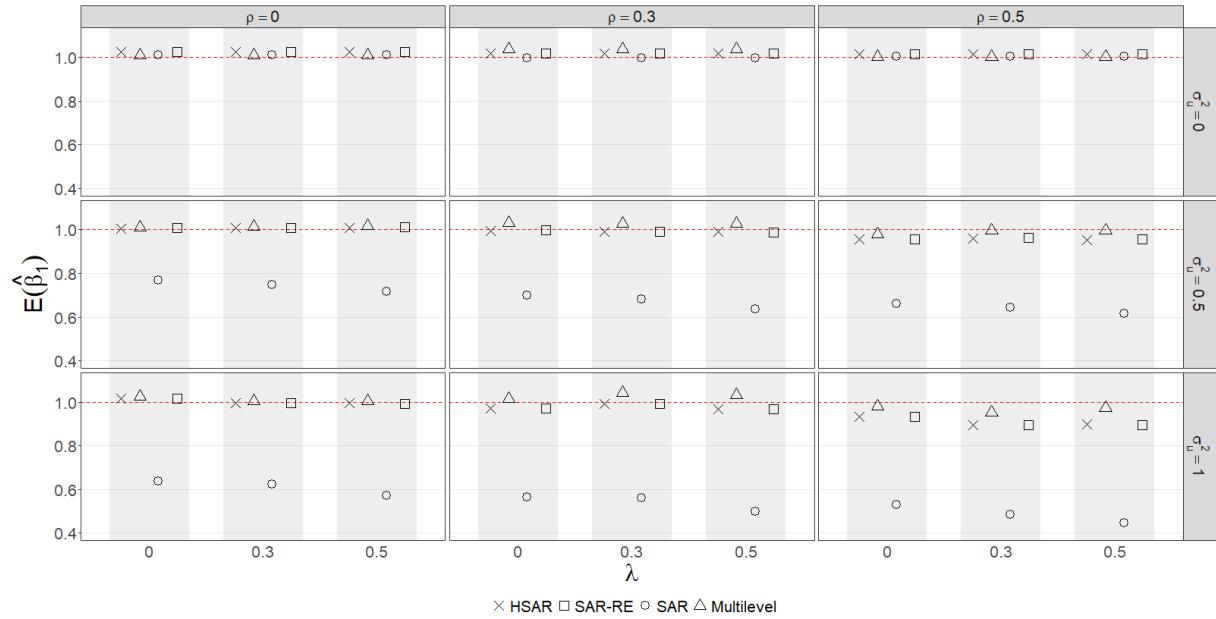


Figure P27: Bias in $\hat{\beta}_0$ for $J = 49$, $N = 980$, $\rho_x = 0.3$

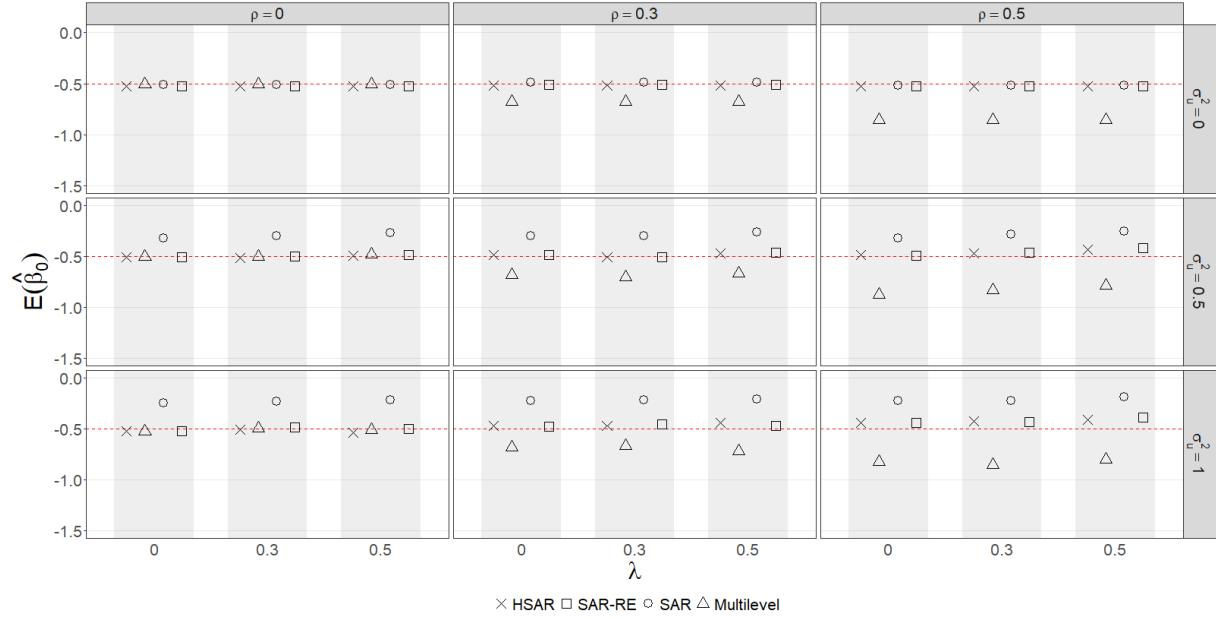


Figure P28: SD in $\hat{\beta}_0$ for $J = 49$, $N = 980$, $\rho_x = 0.3$

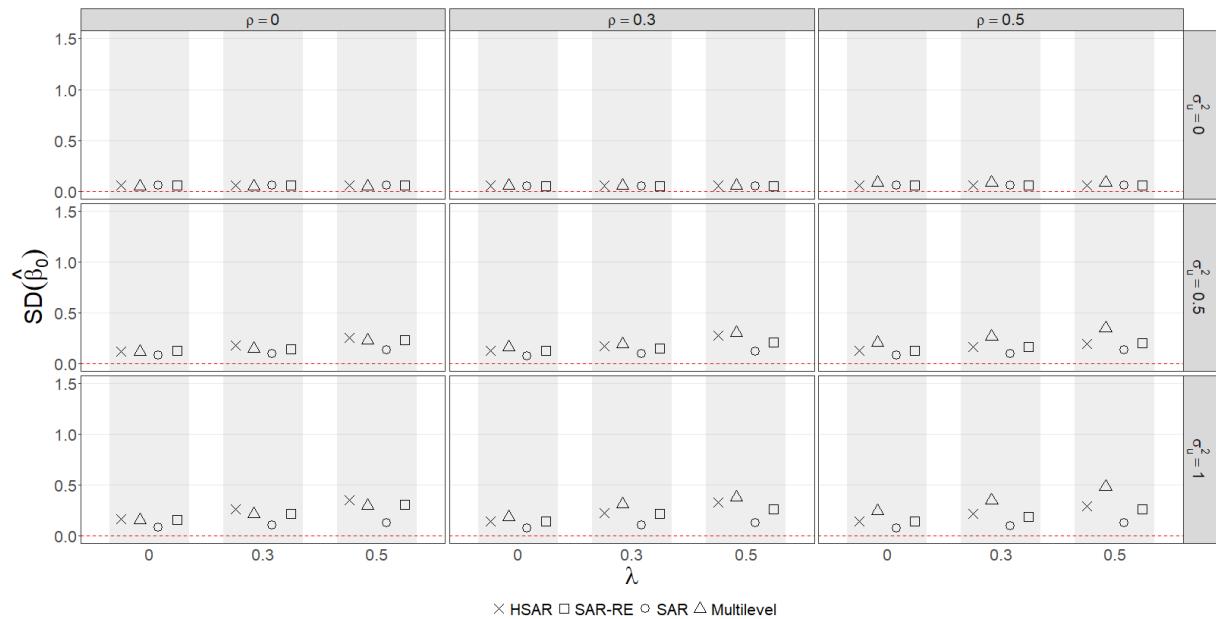


Figure P29: Bias in $\hat{\rho}$ for $J = 49$, $N = 980$, $\rho_x = 0.3$

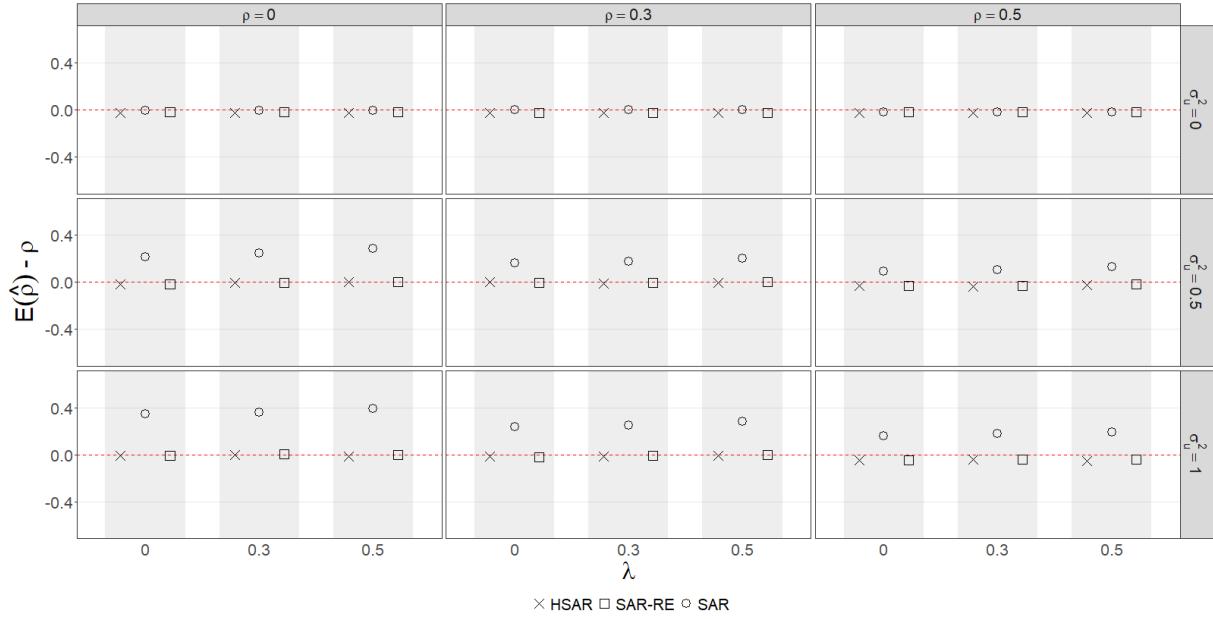


Figure P30: SD in $\hat{\rho}$ for $J = 49$, $N = 980$, $\rho_x = 0.3$

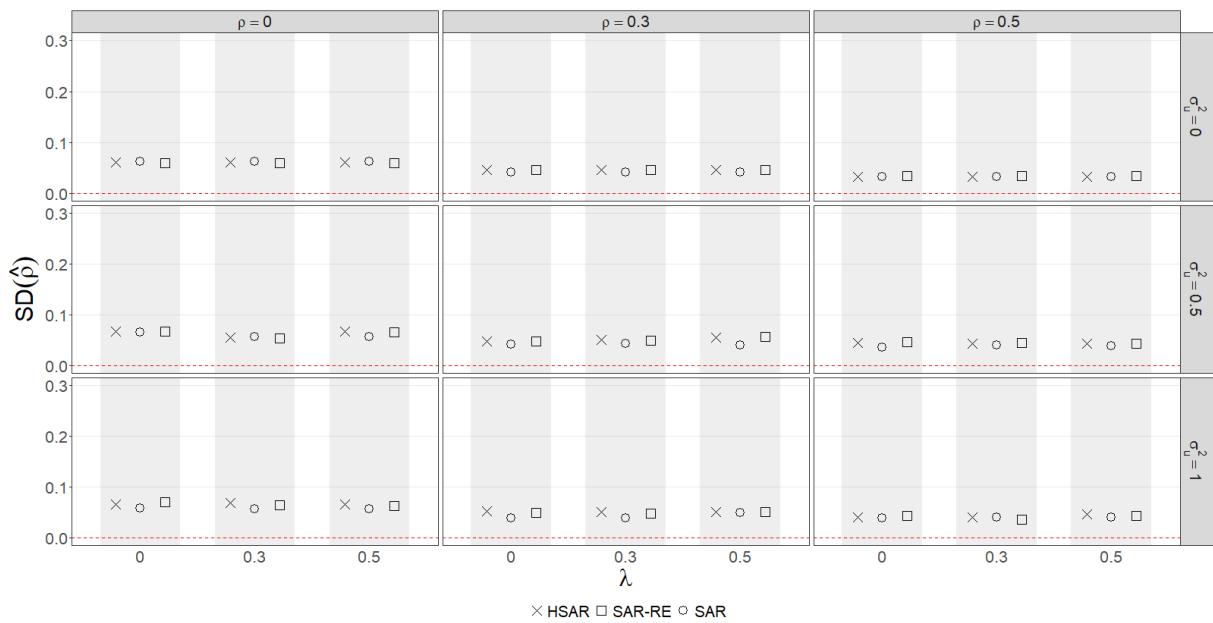


Figure P31: Bias in $\hat{\sigma}_u^2$ for $J = 49$, $N = 980$, $\rho_x = 0.3$

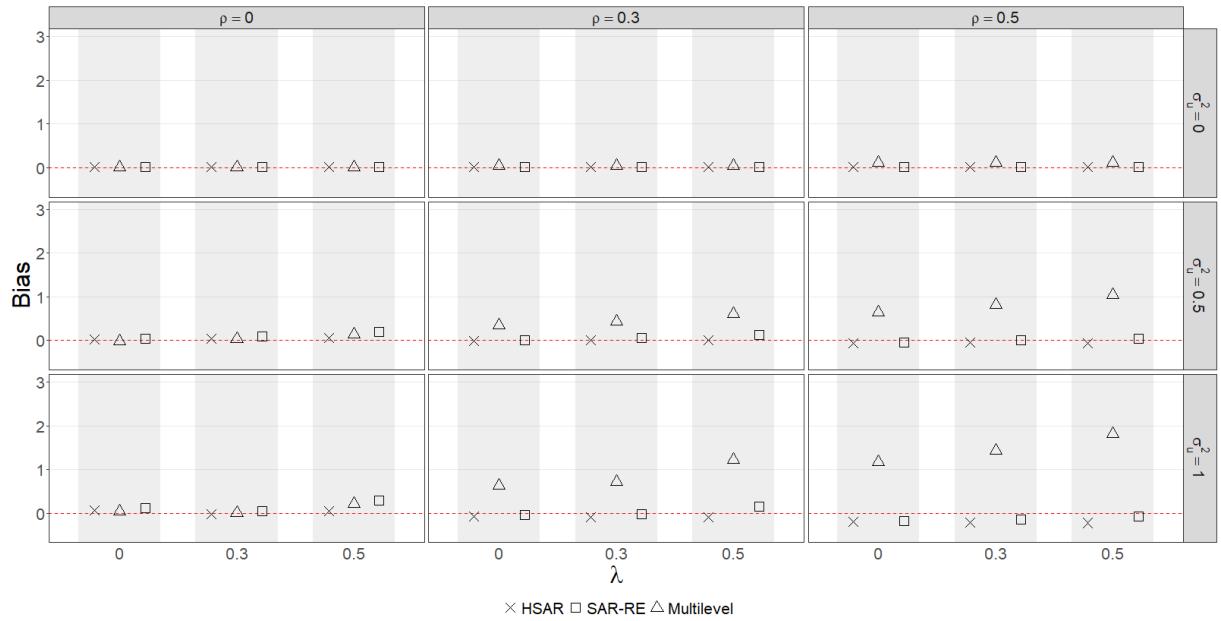
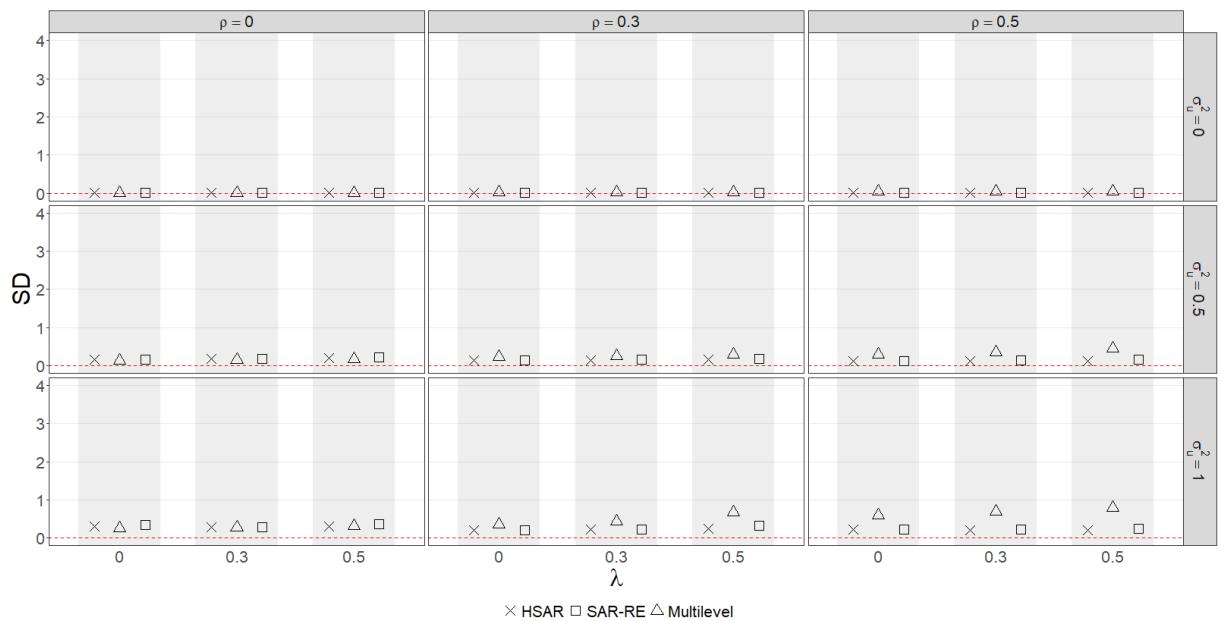


Figure P32: SD in $\hat{\sigma}_u^2$ for $J = 49$, $N = 980$, $\rho_x = 0.3$



Q Binary Probit $J = 111$, $N = 1117$, $\rho_x = 0.0$, $\rho \in \{0, 0.3, 0.5\}$,

$\lambda \in \{0, 0.3, 0.5\}$, $\sigma_u^2 \in \{0, 0.5, 1.0\}$

Figure Q33: Bias in $\hat{\beta}_1$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

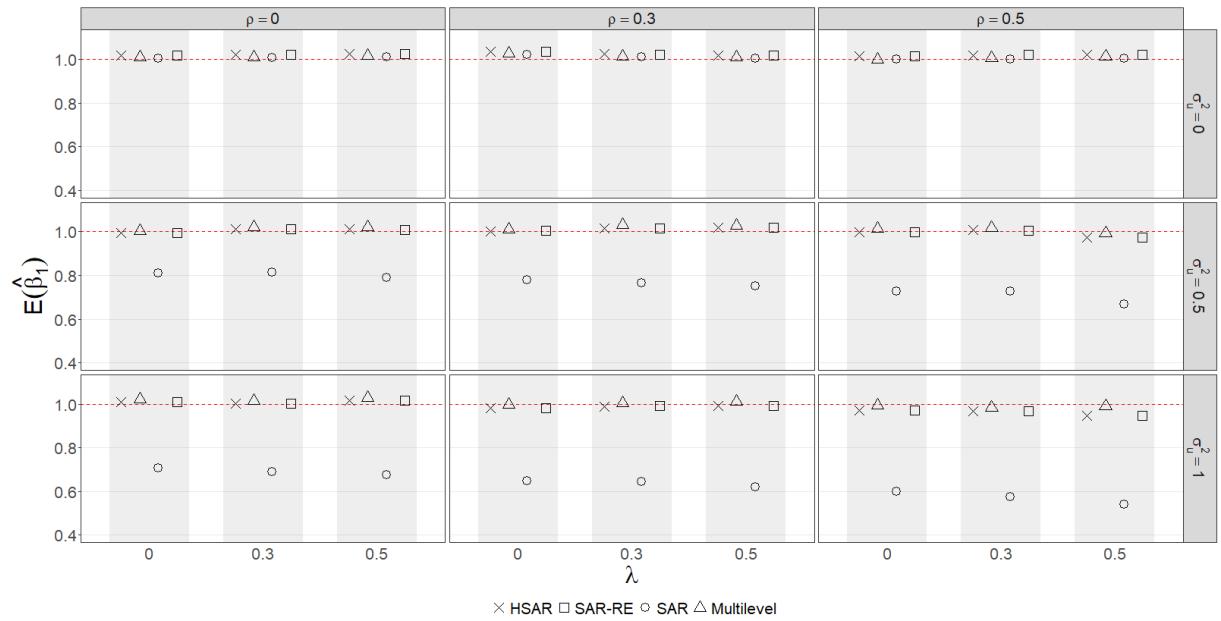


Figure Q34: SD in $\hat{\beta}_1$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

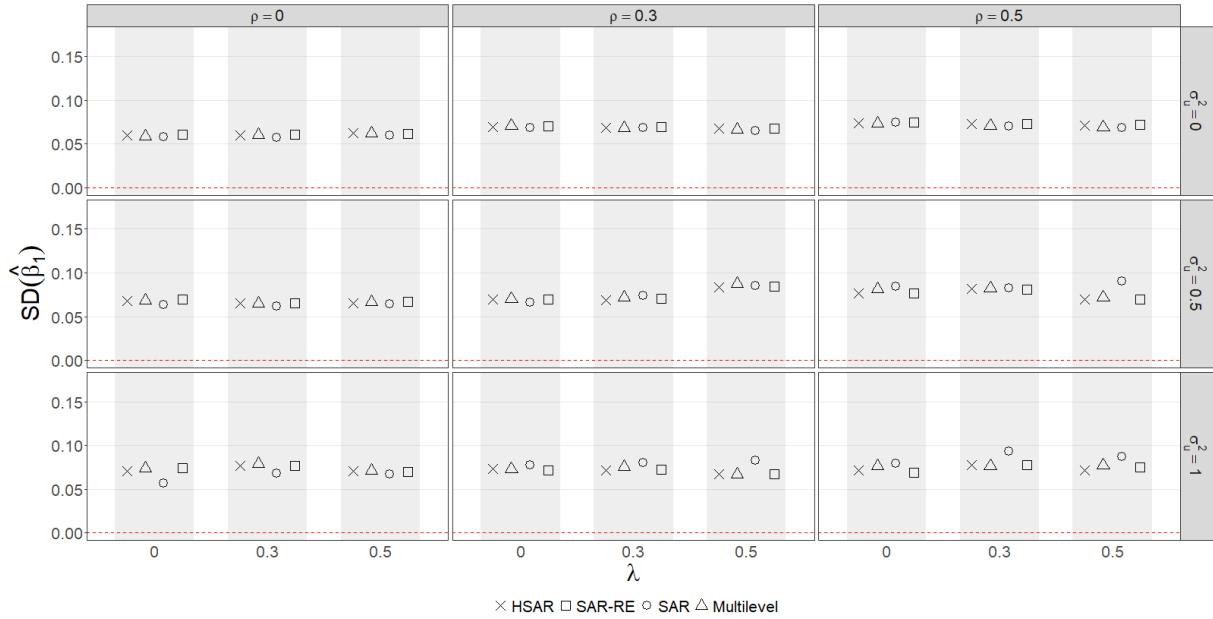


Figure Q35: RMSE in $\hat{\beta}_1$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

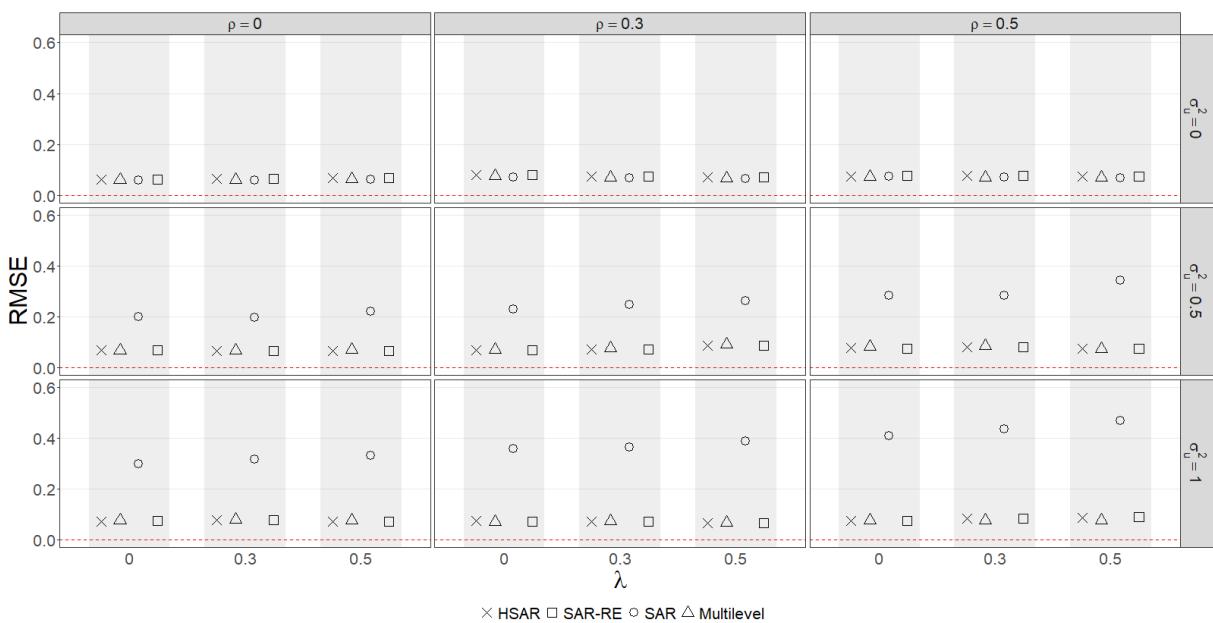


Figure Q36: RMSE in Direct Effect for $J = 111$, $N = 1117$, $\rho_x = 0.0$

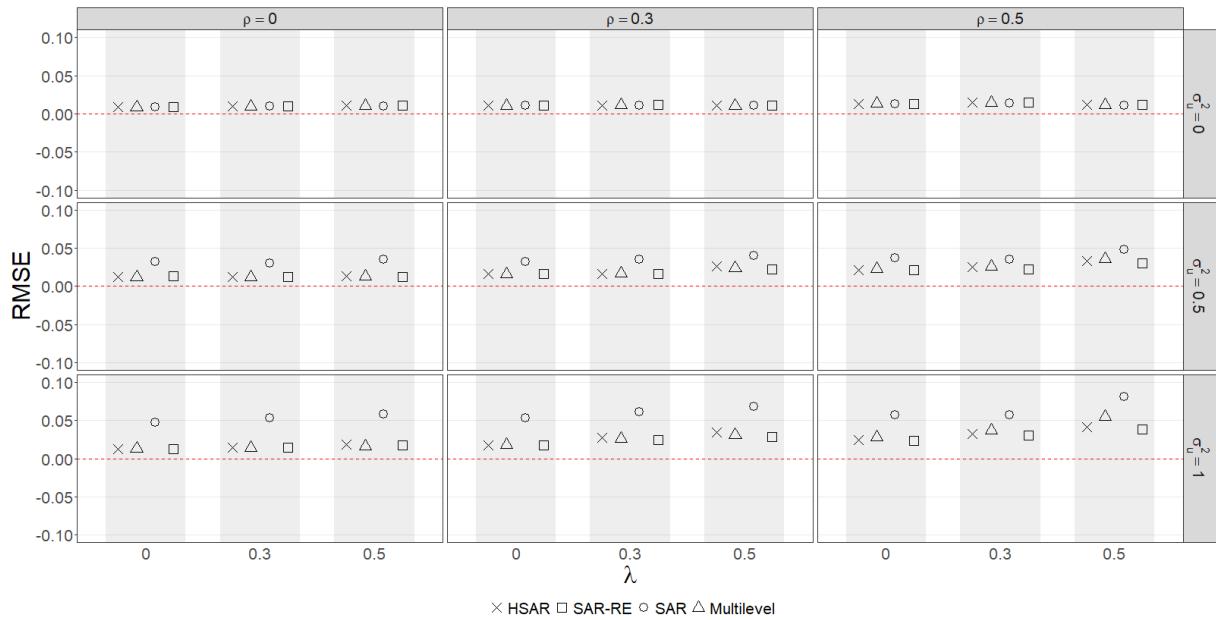


Figure Q37: RMSE in Indirect Effect for $J = 111$, $N = 1117$, $\rho_x = 0.0$

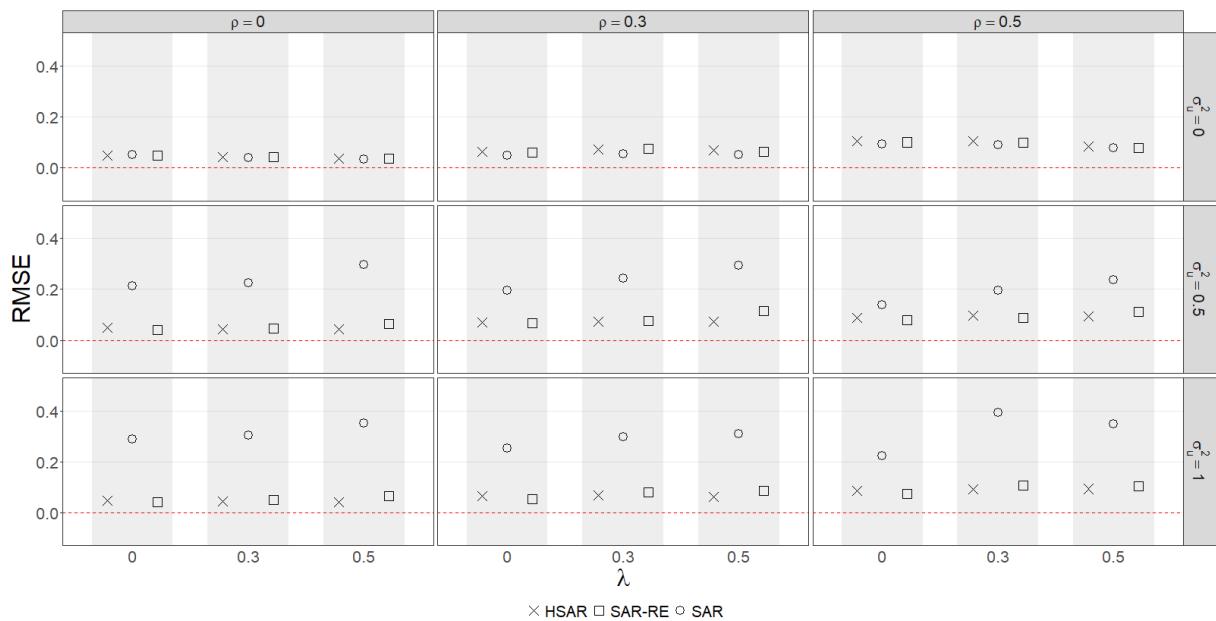


Figure Q38: Bias in $\hat{\sigma}_u^2$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

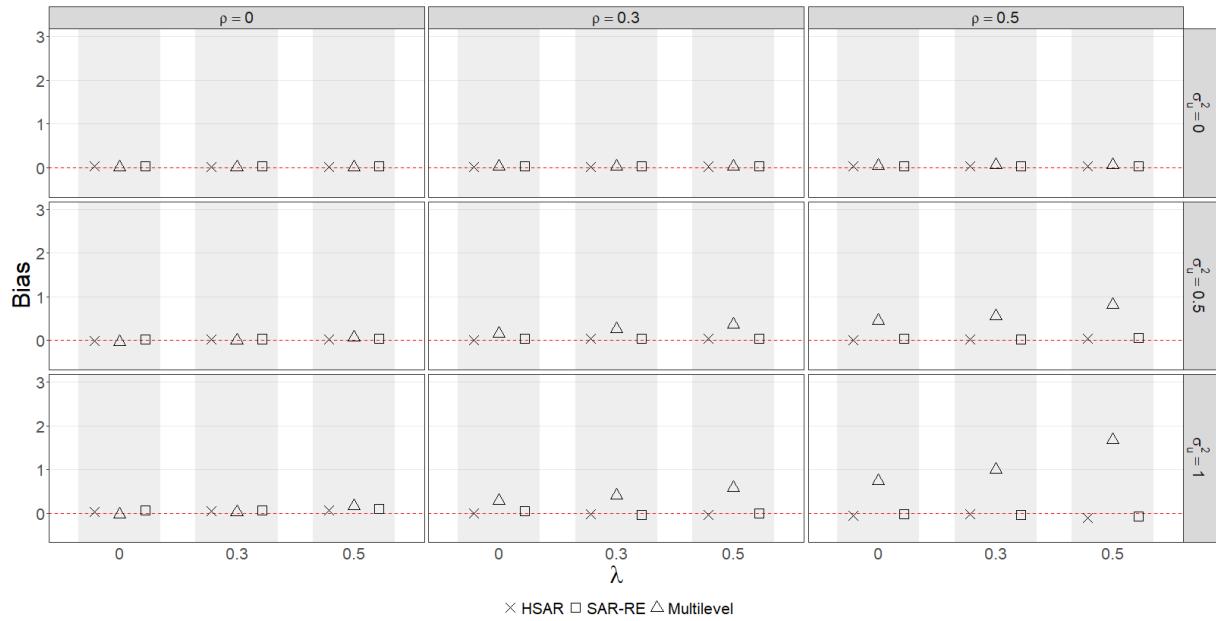


Figure Q39: SD in $\hat{\sigma}_u^2$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

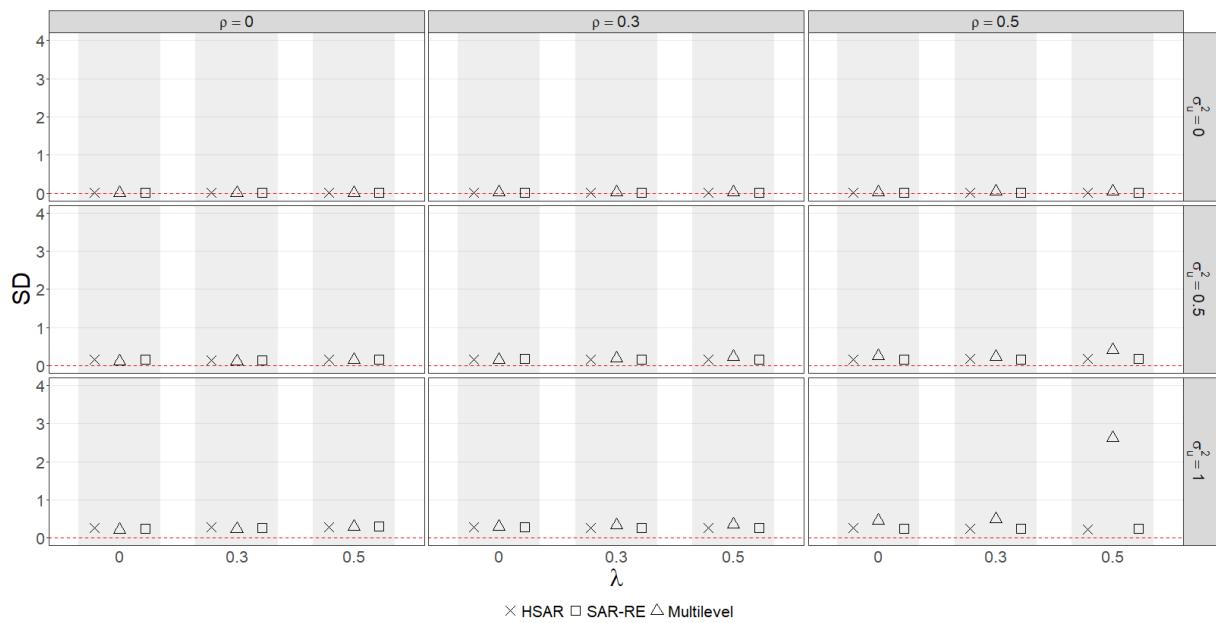


Figure Q40: RMSE in $\hat{\sigma}_u^2$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

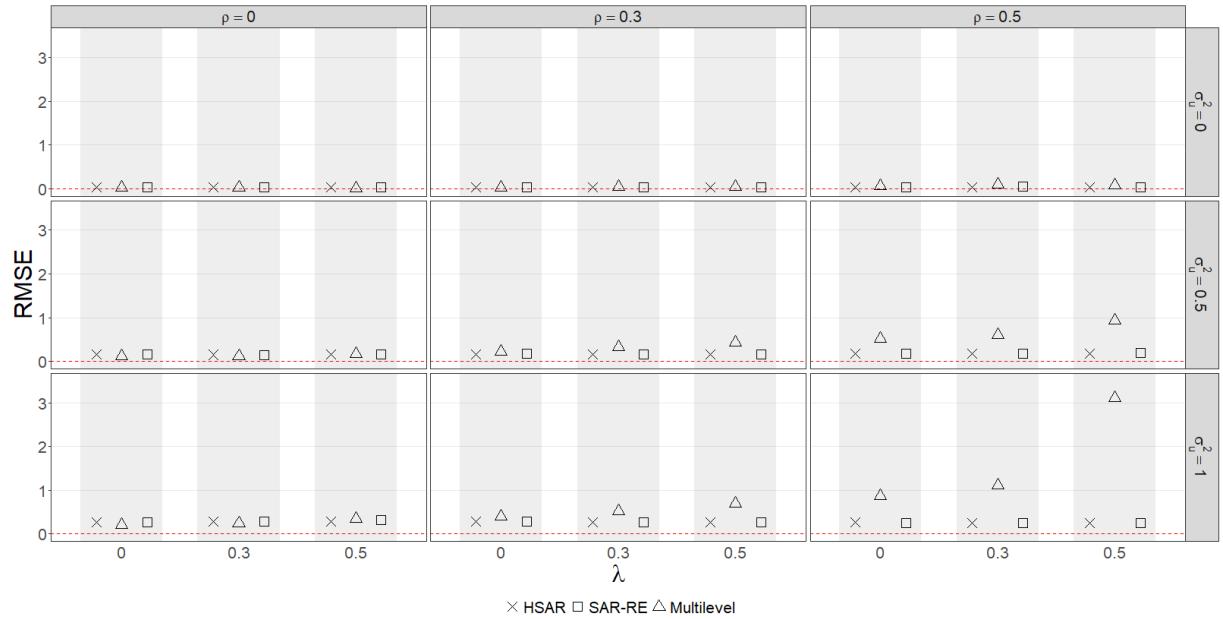


Figure Q41: Bias in $\hat{\beta}_0$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

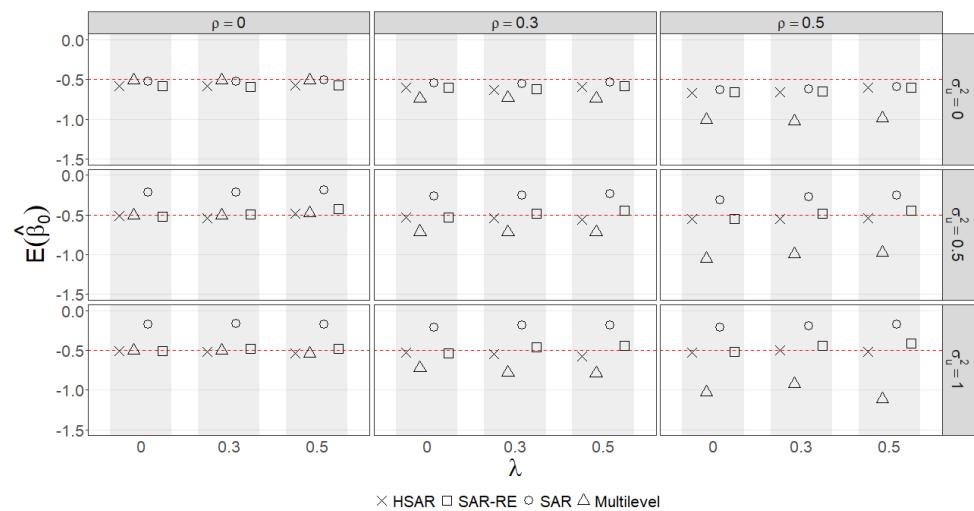


Figure Q42: SD in $\hat{\beta}_0$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

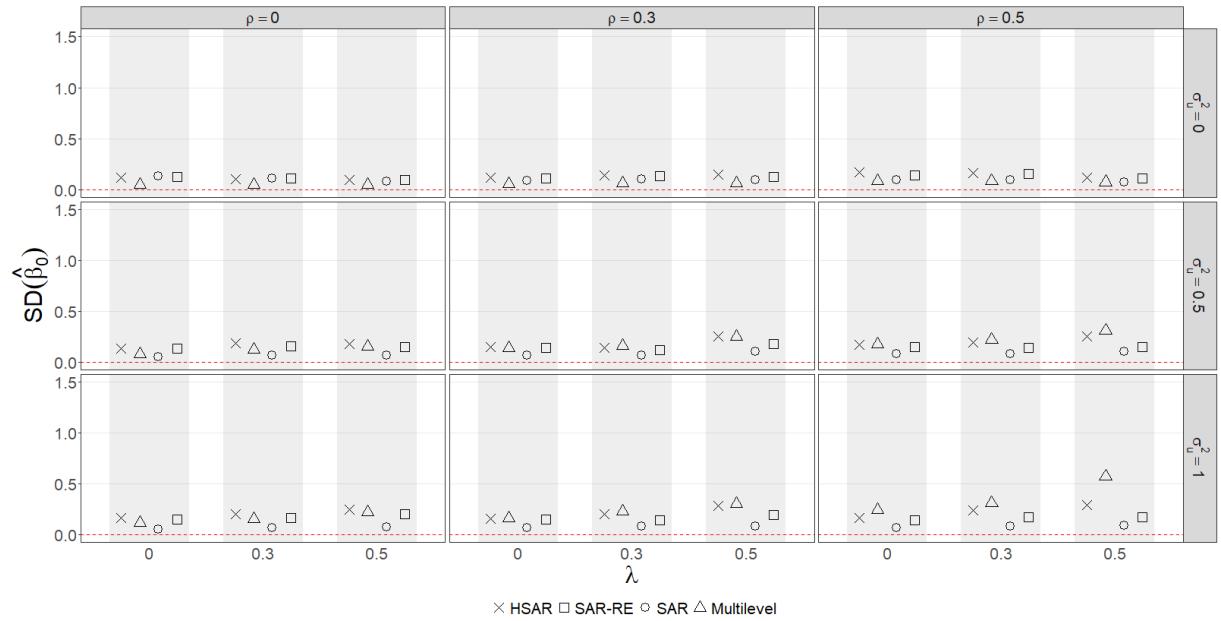


Figure Q43: RMSE in $\hat{\beta}_0$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

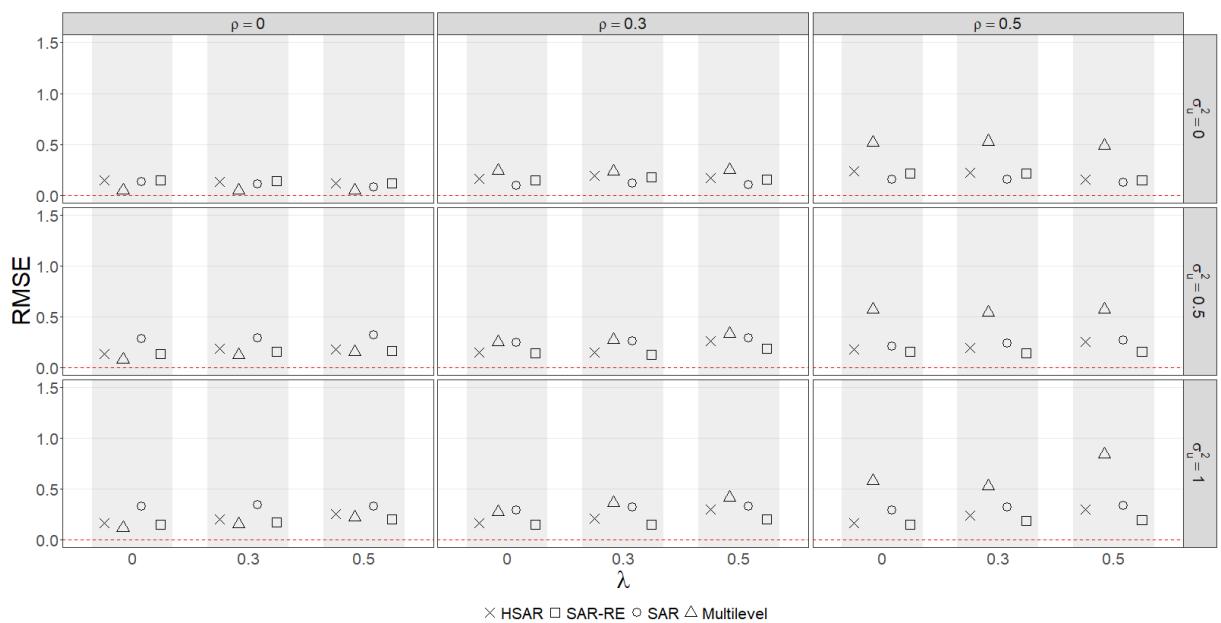


Figure Q44: SD in $\hat{\rho}$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

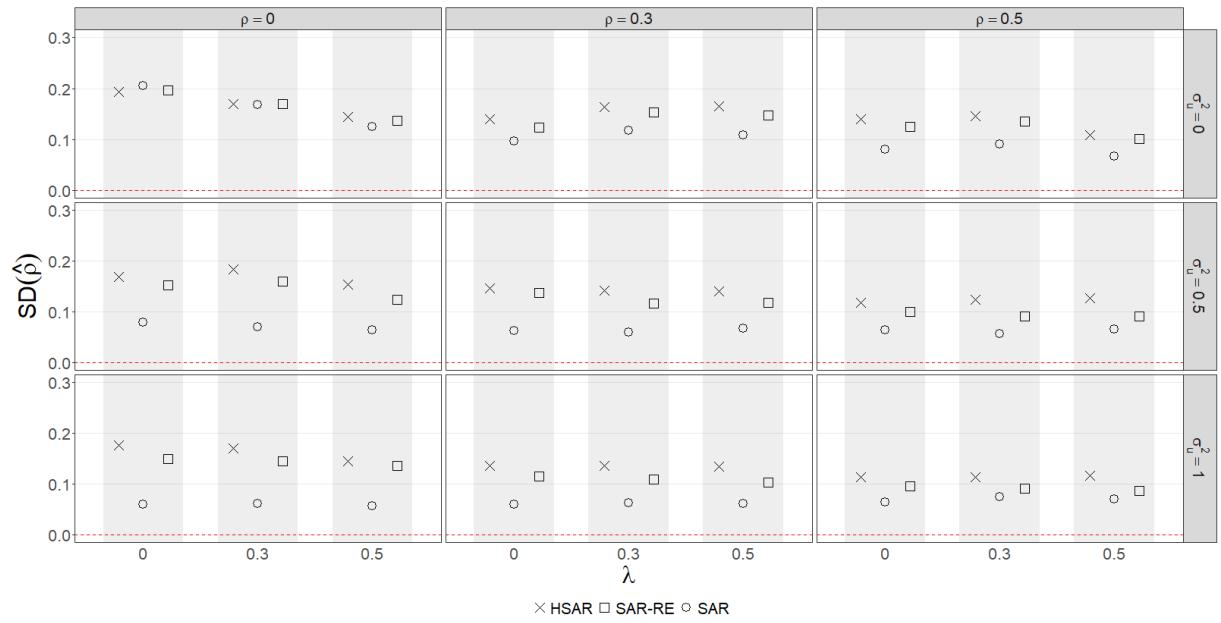


Figure Q45: RMSE in $\hat{\rho}$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

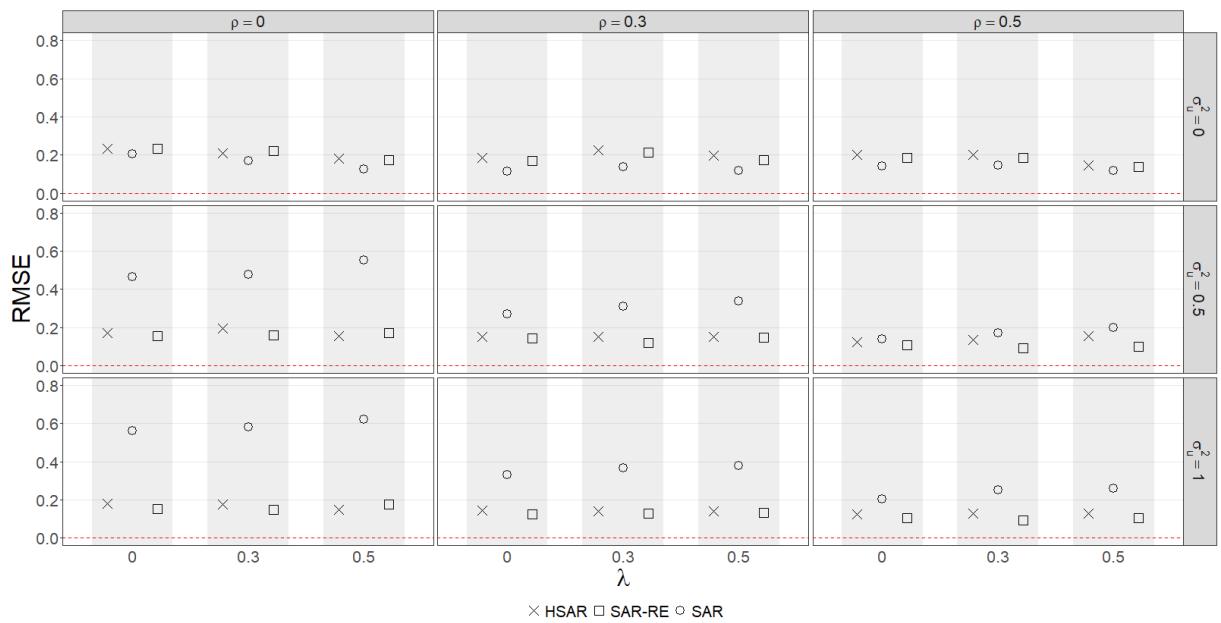


Figure Q46: Bias in $\hat{\lambda}$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

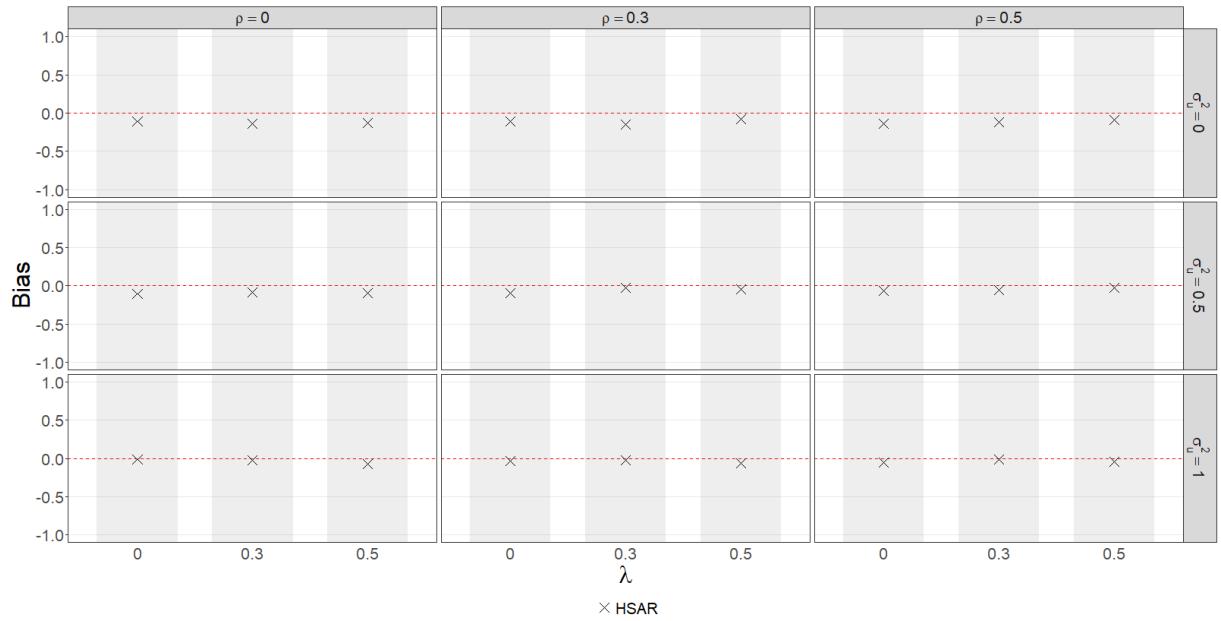


Figure Q47: SD in $\hat{\lambda}$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$

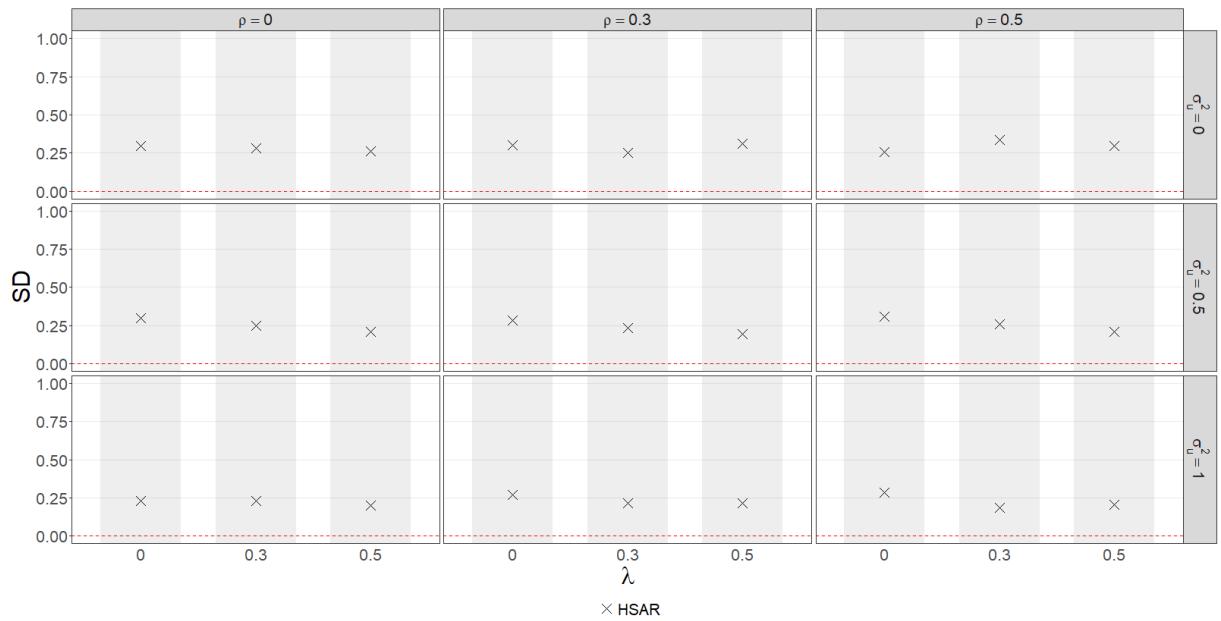
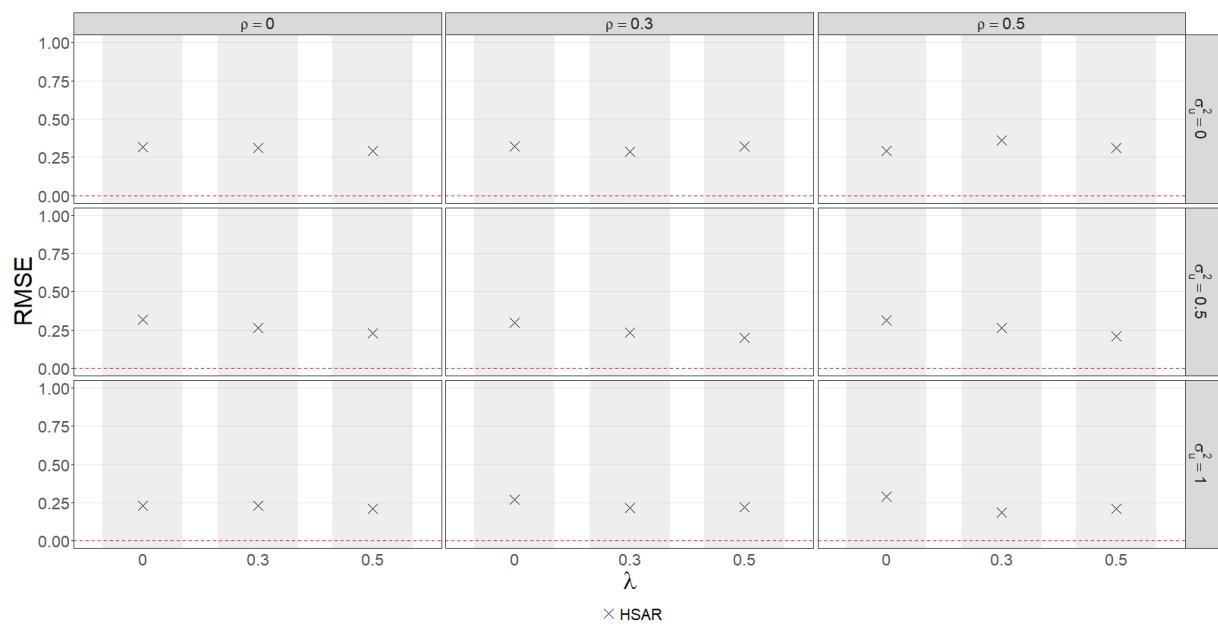


Figure Q48: RMSE in $\hat{\lambda}$ for $J = 111$, $N = 1117$, $\rho_x = 0.0$



R Binary Probit $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\rho \in \{0, 0.3, 0.5\}$,

$\lambda \in \{0, 0.3, 0.5\}$, $\sigma_u^2 \in \{0, 0.5, 1.0\}$

Figure R49: Bias in $\hat{\beta}_1$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

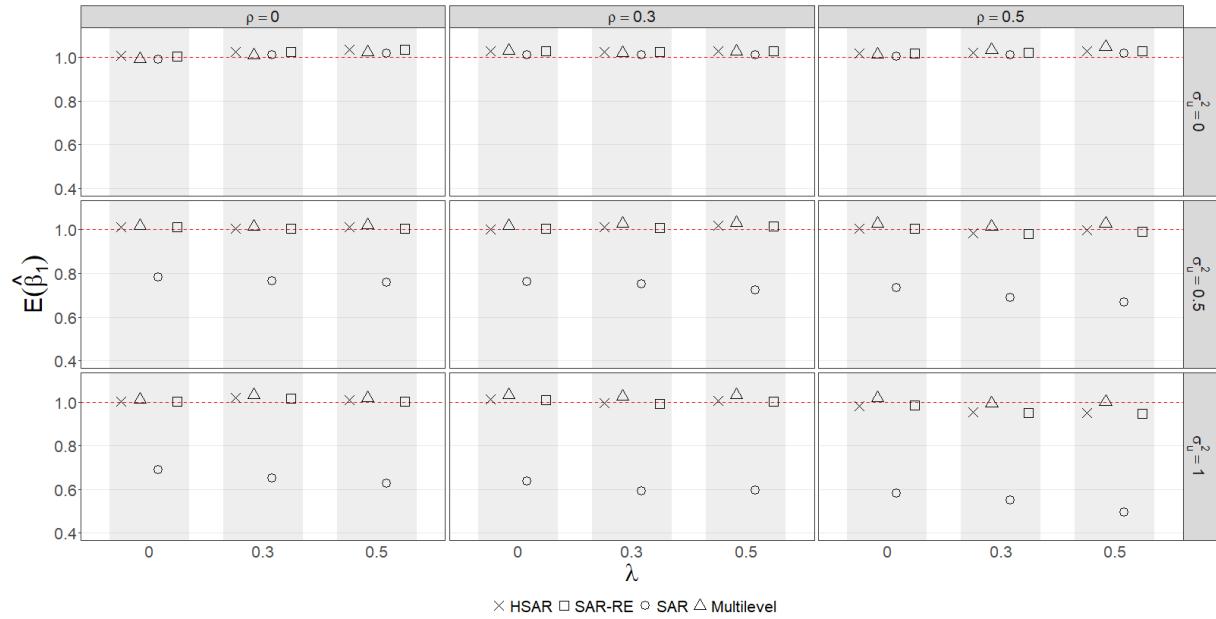


Figure R50: SD in $\hat{\beta}_1$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

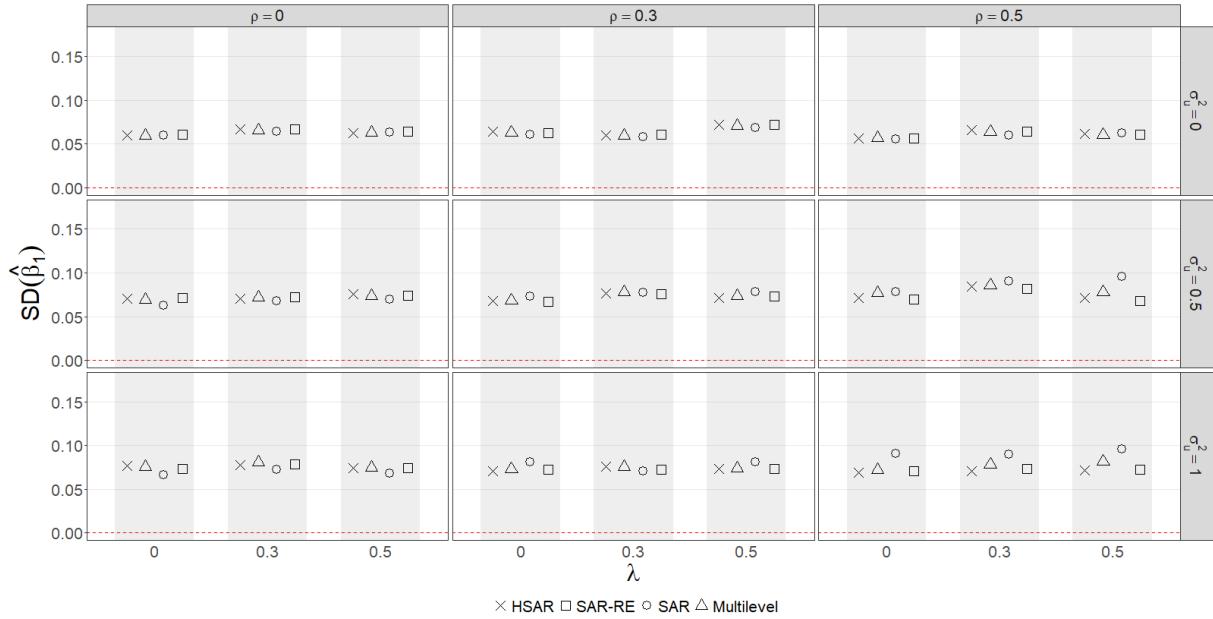


Figure R51: RMSE in $\hat{\beta}_1$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

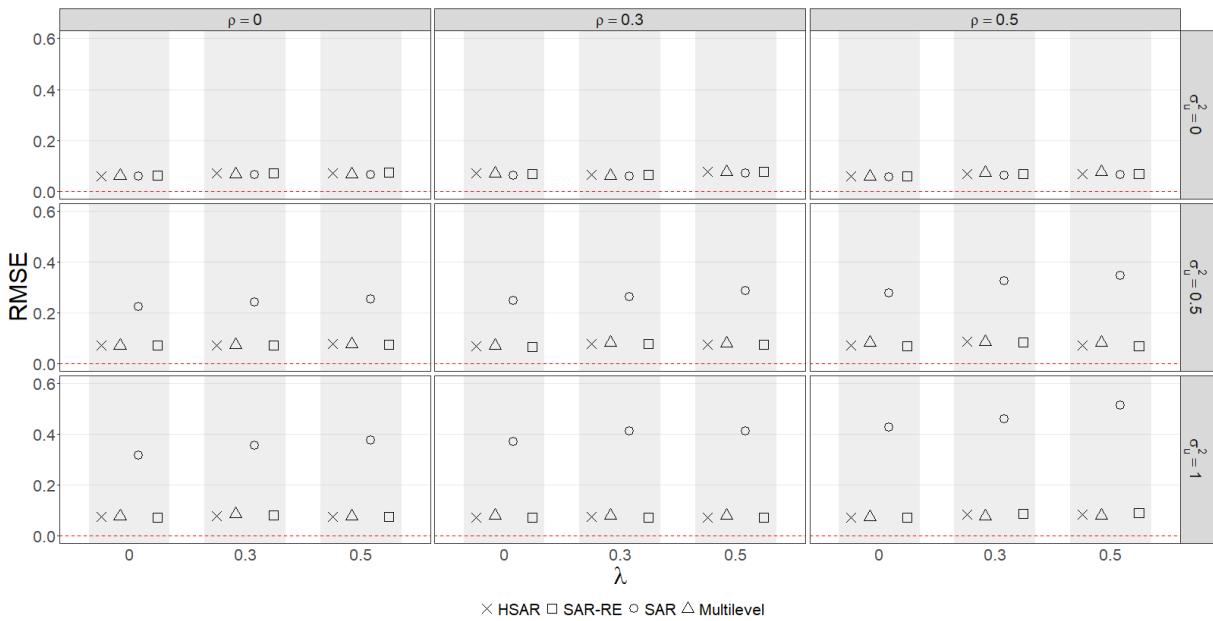


Figure R52: Bias in Direct Effect for $J = 111$, $N = 1117$, $\rho_x = 0.3$

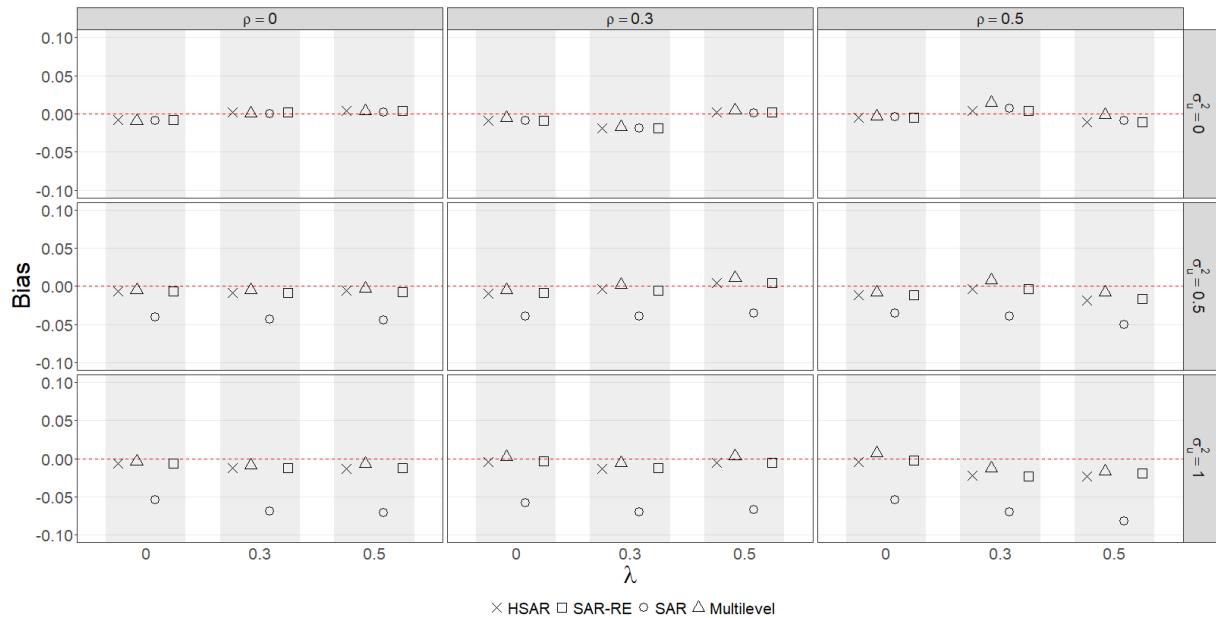


Figure R53: RMSE in Direct Effect for $J = 111$, $N = 1117$, $\rho_x = 0.3$

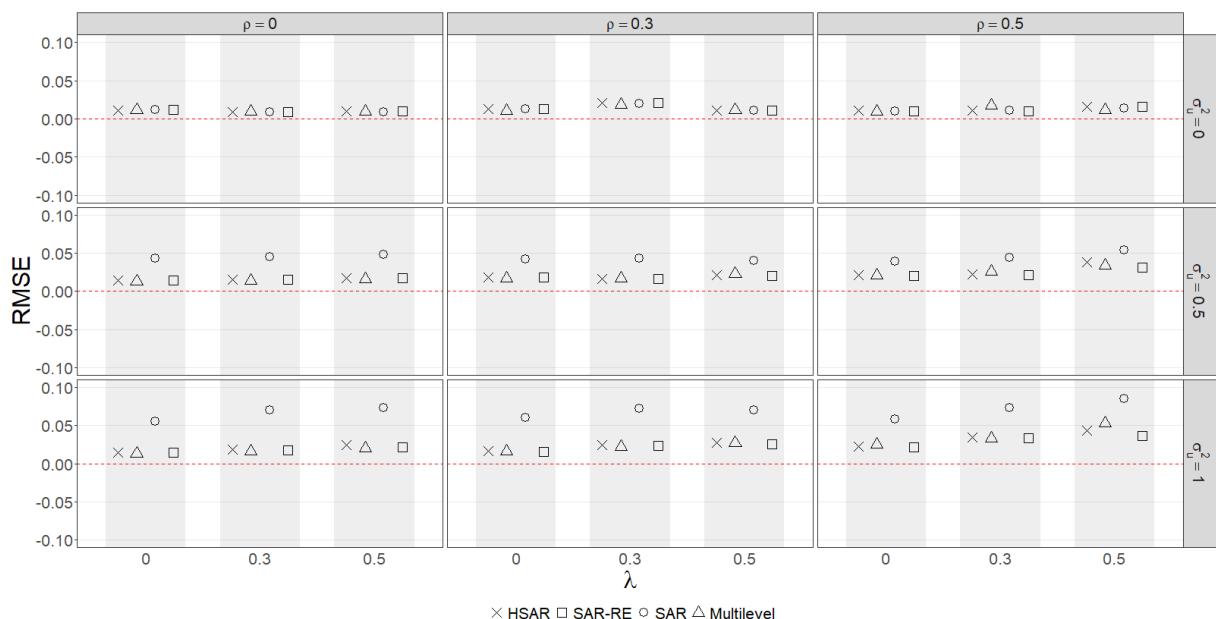


Figure R54: Bias in Indirect Effect for $J = 111$, $N = 1117$, $\rho_x = 0.3$

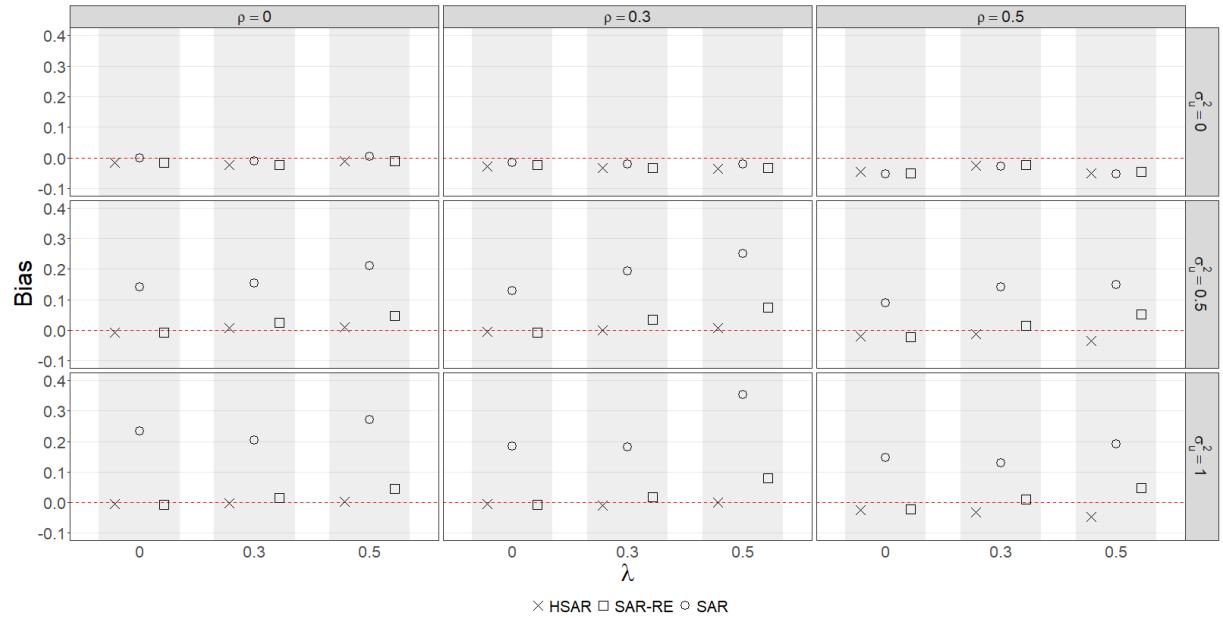


Figure R55: RMSE in Indirect Effect for $J = 111$, $N = 1117$, $\rho_x = 0.3$

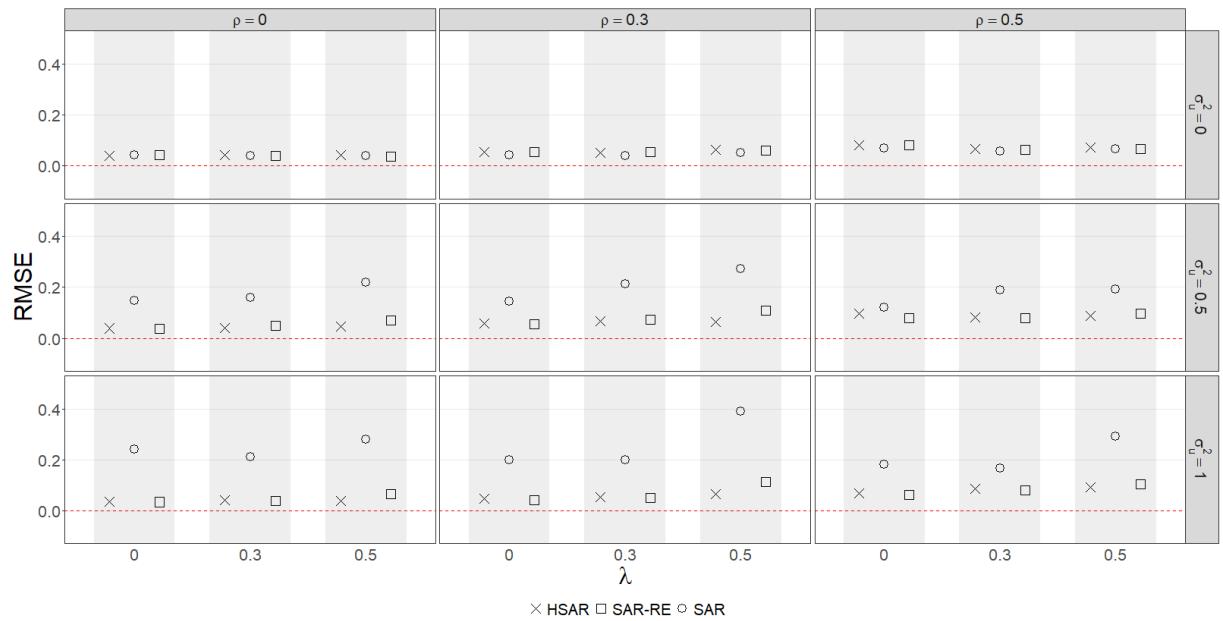


Figure R56: Bias in $\hat{\sigma}_u^2$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

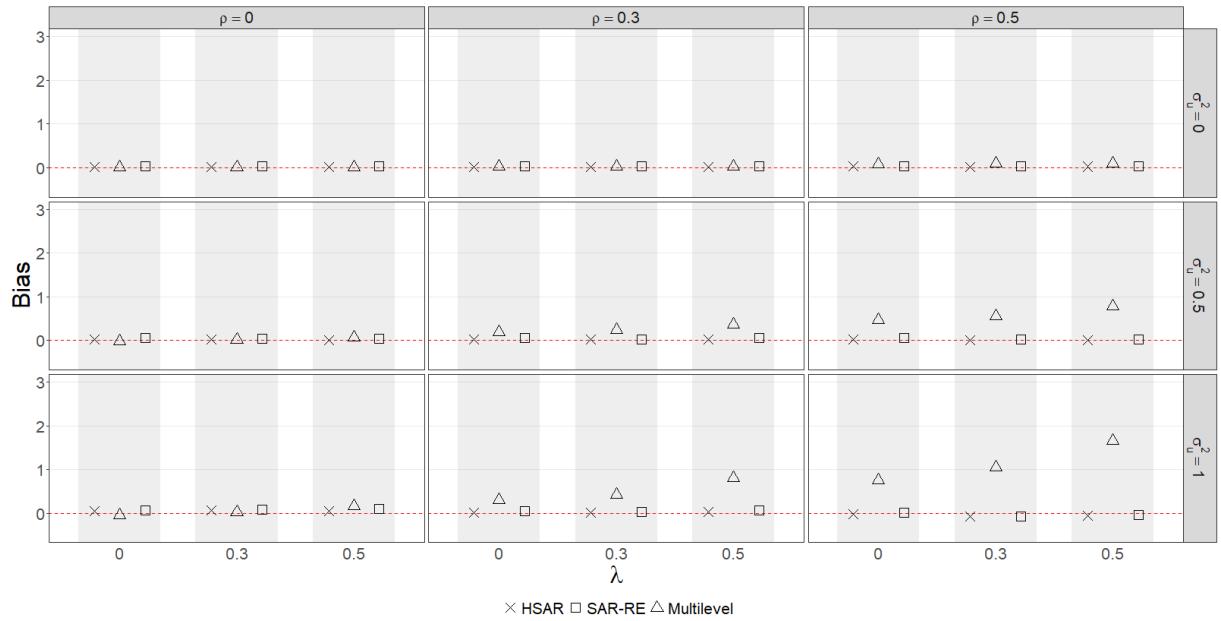


Figure R57: SD in $\hat{\sigma}_u^2$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

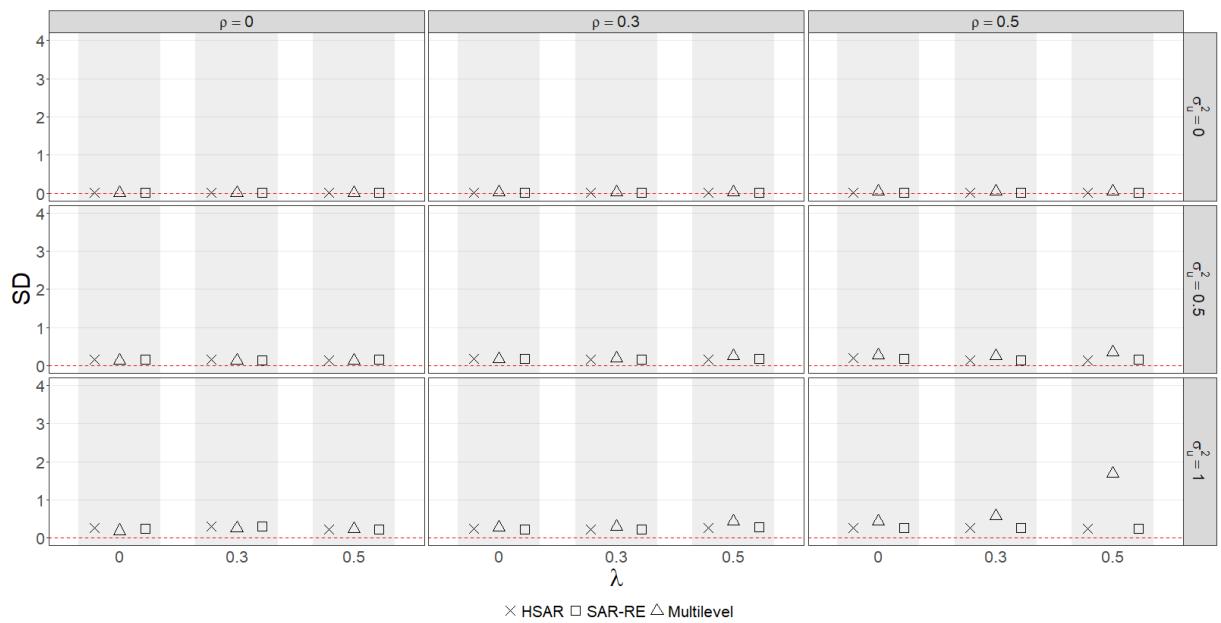


Figure R58: RMSE in $\hat{\sigma}_u^2$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

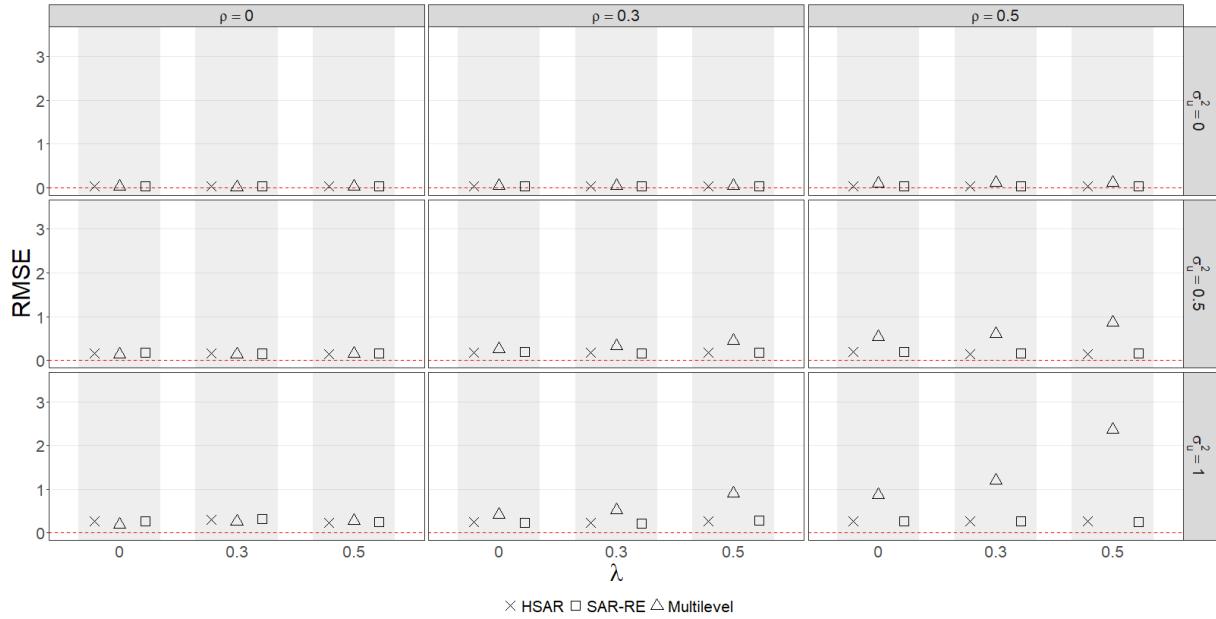


Figure R59: Bias in $\hat{\beta}_0$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

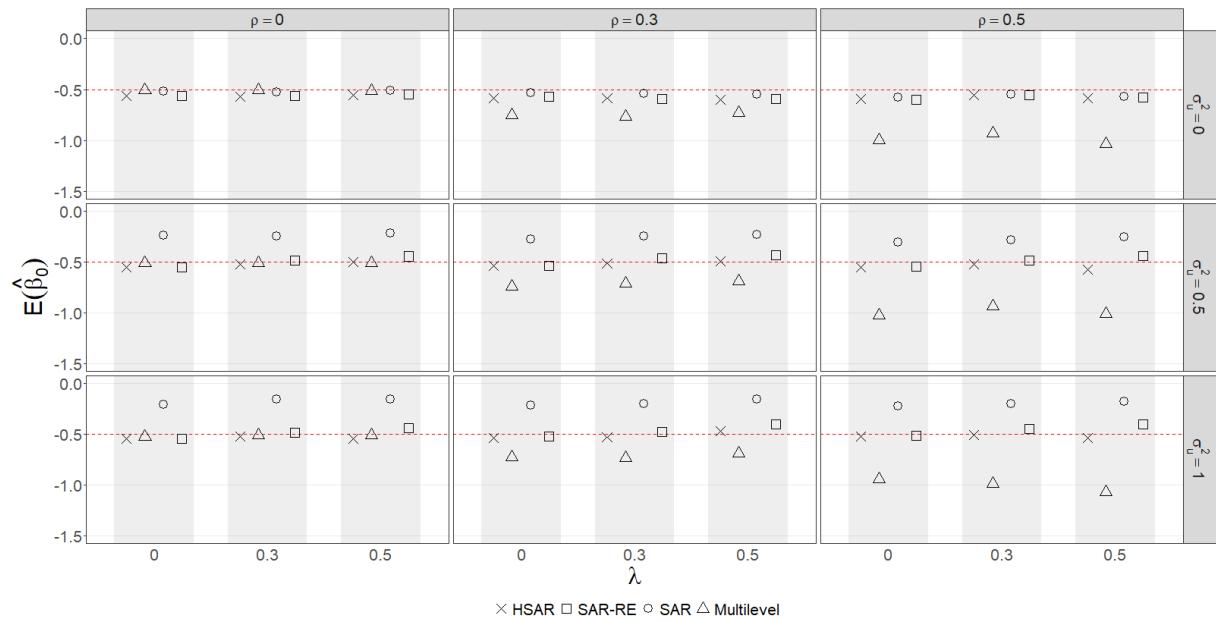


Figure R60: SD in $\hat{\beta}_0$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

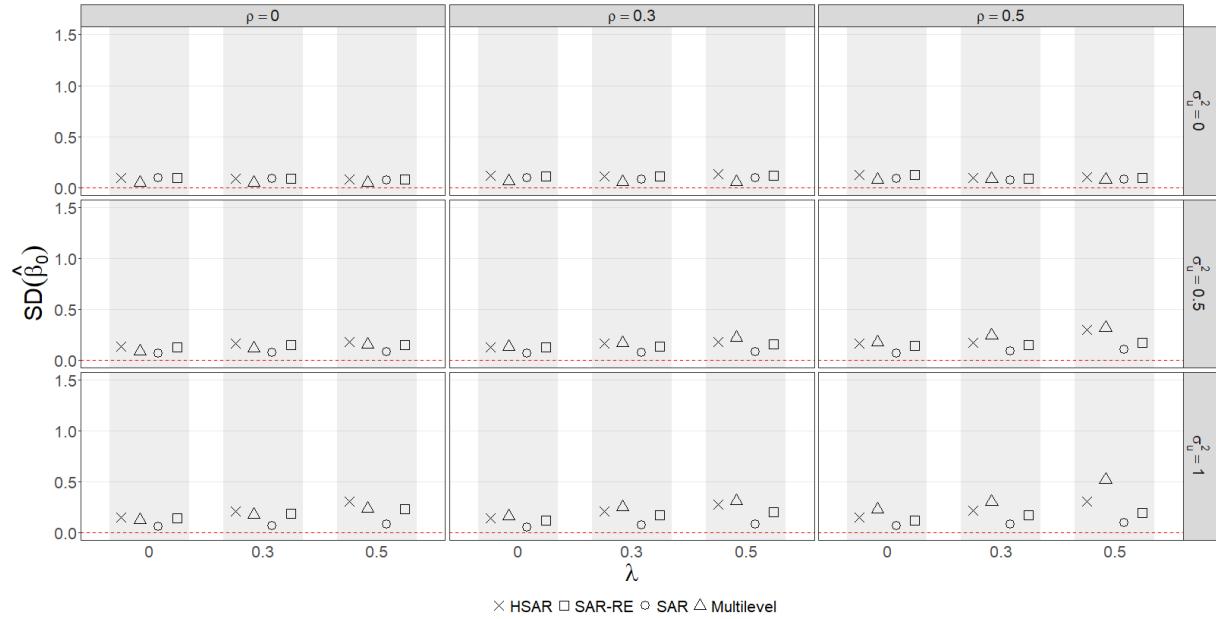


Figure R61: RMSE in $\hat{\beta}_0$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

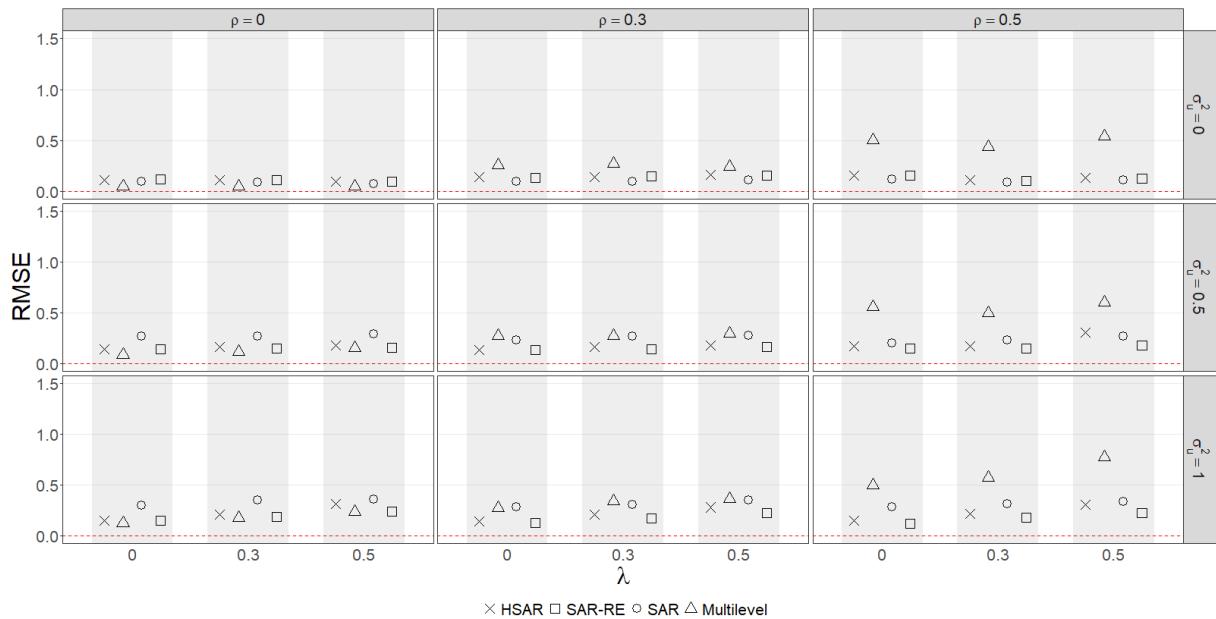


Figure R62: Bias in $\hat{\rho}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

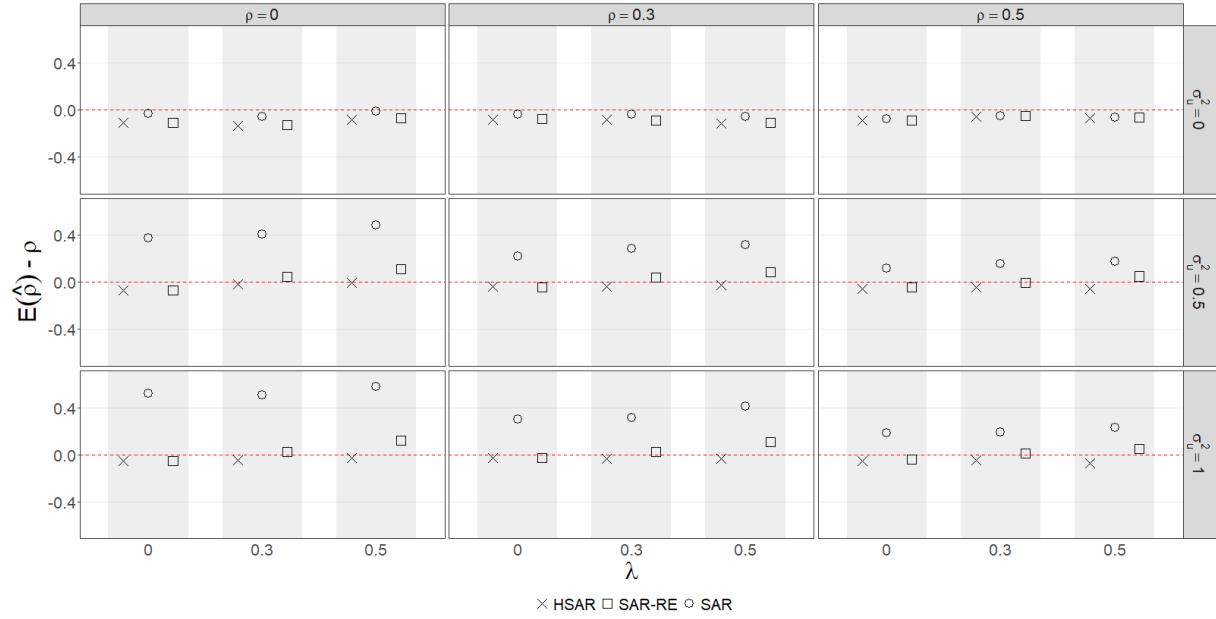


Figure R63: SD in $\hat{\rho}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

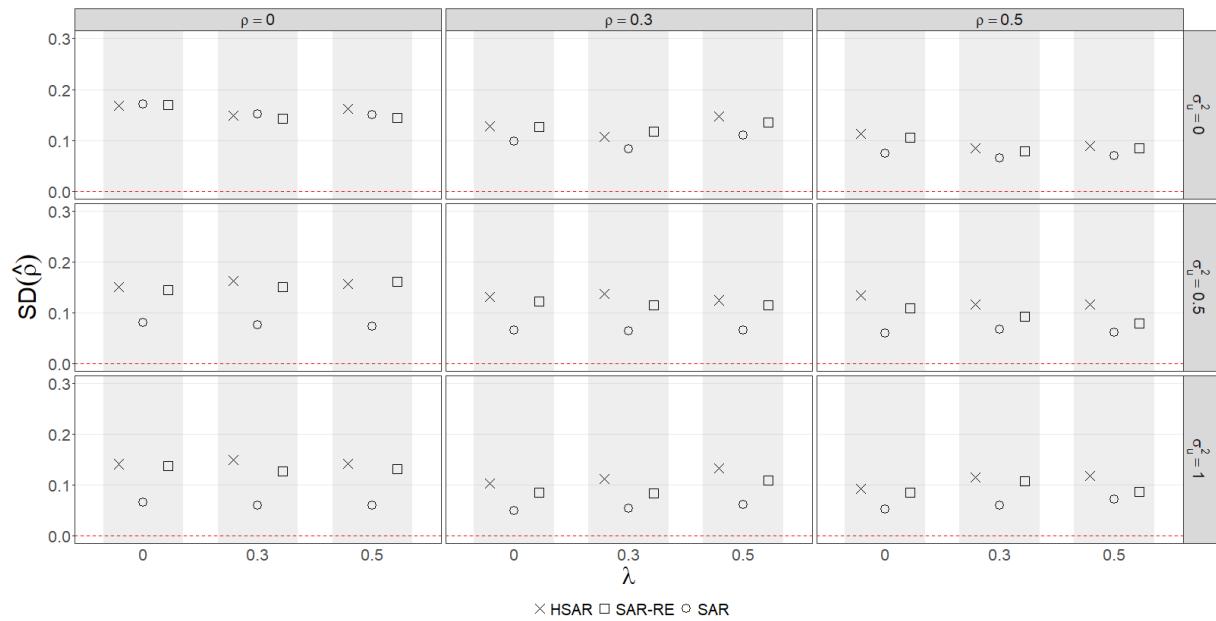


Figure R64: RMSE in $\hat{\rho}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

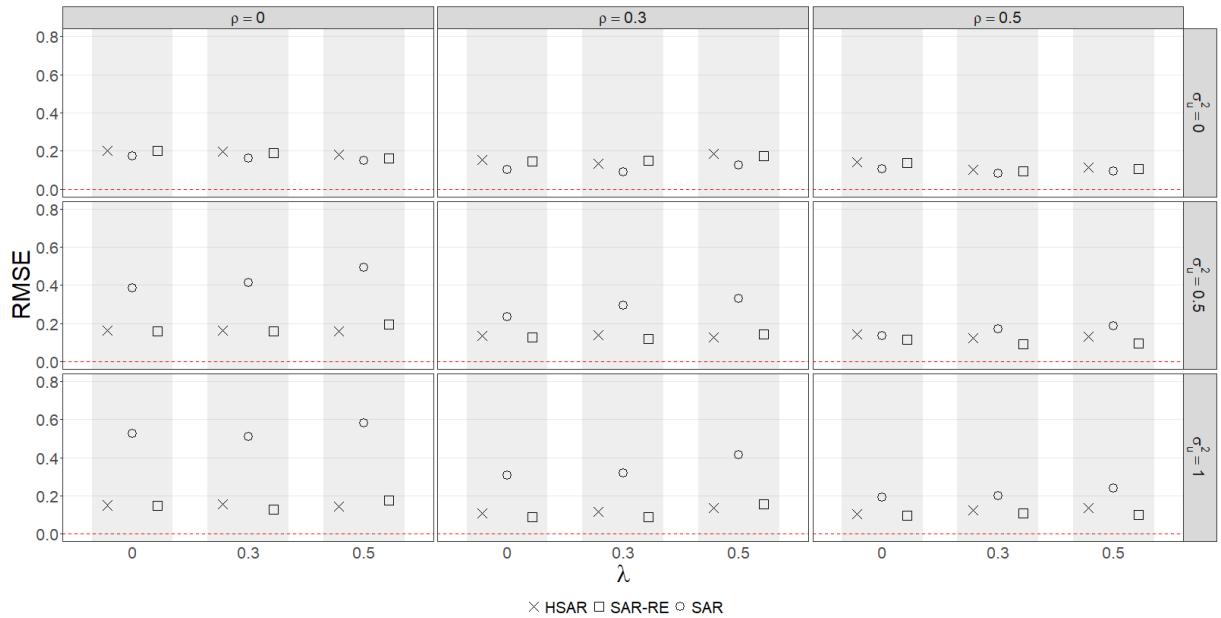


Figure R65: Bias in $\hat{\lambda}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

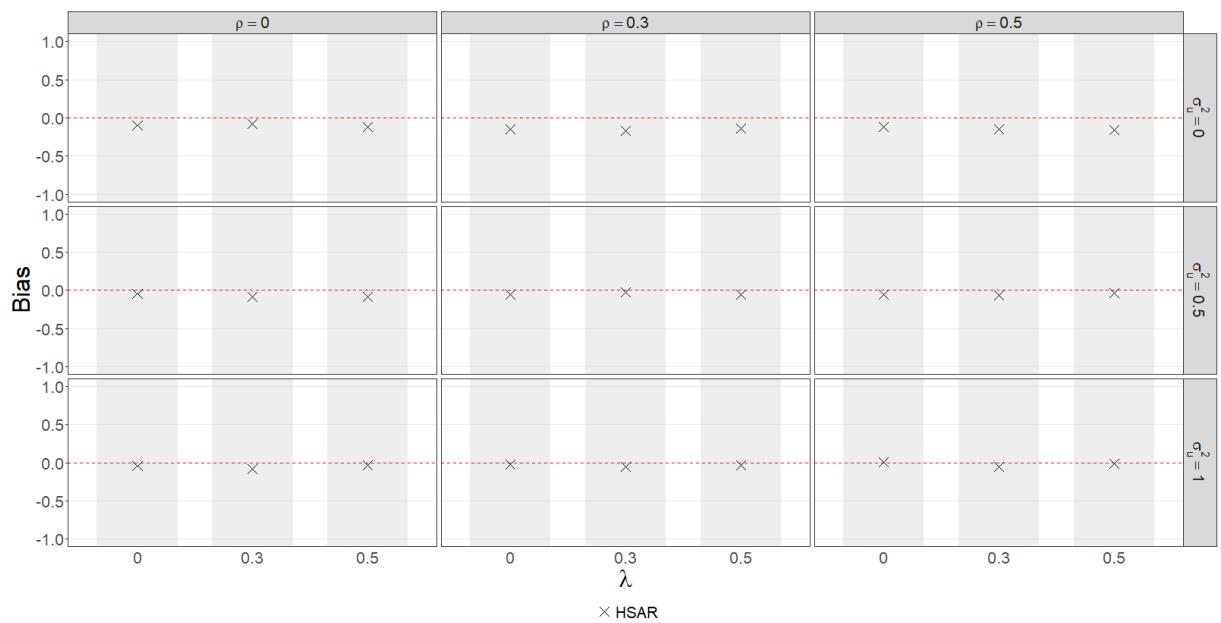


Figure R66: SD in $\hat{\lambda}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$

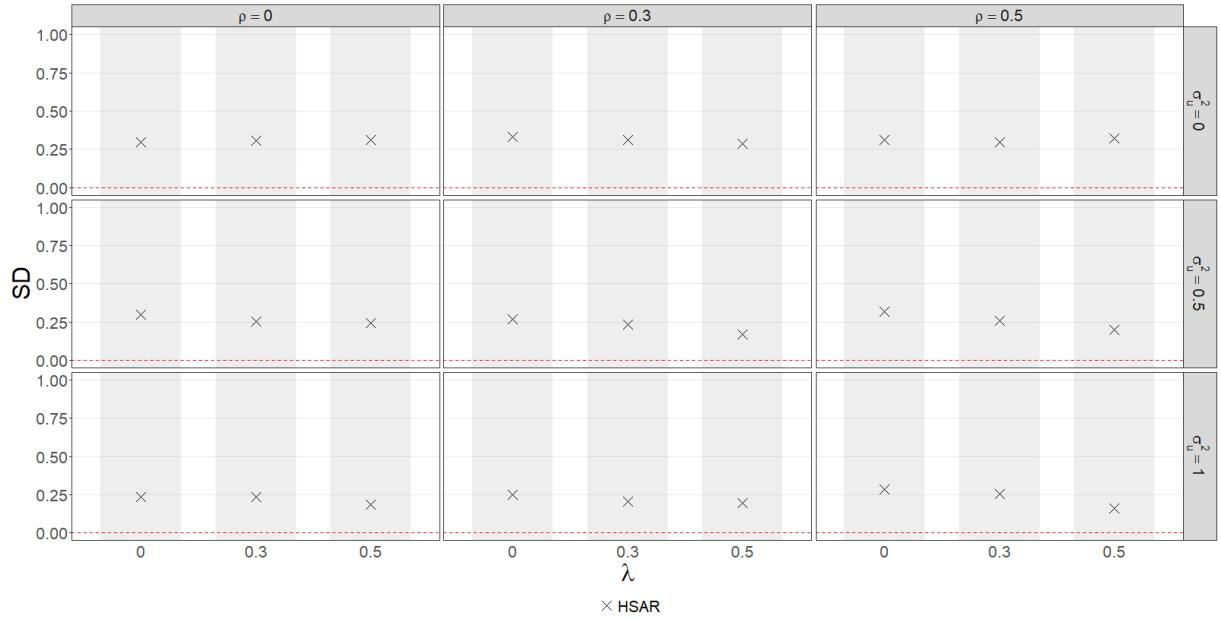
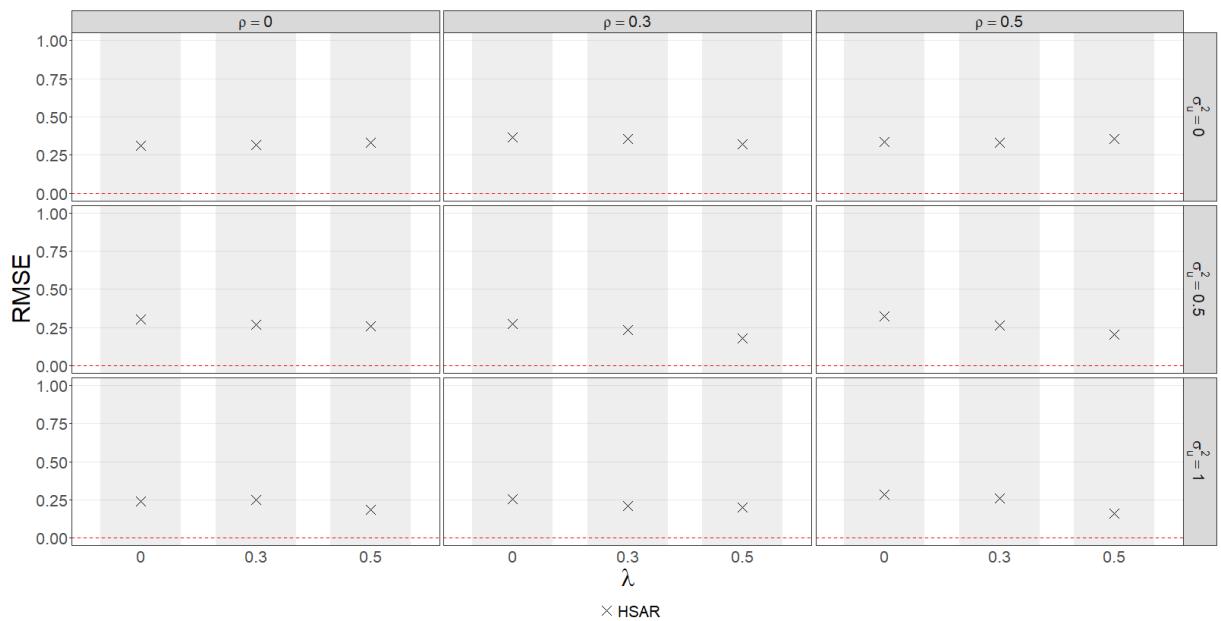


Figure R67: RMSE in $\hat{\lambda}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$



S Binary Probit with Hierarchical X when $J = 111$,

$$N = 1117, \rho_x = 0.3, \rho \in \{0, 0.3, 0.5\}, \lambda \in \{0.3, 0.5\}, \sigma_u^2 \in \{0.5, 1.0\}, \lambda_x \in \{0.3, 0.5\}$$

The DGP is as follows:

$$\boldsymbol{\theta}_x = (\mathbf{I}_J - \lambda_x \mathbf{M})^{-1} \mathbf{u}_x$$

$$X_1 = (\mathbf{I} - \rho_x \mathbf{W} + \Delta \boldsymbol{\theta}_x)^{-1} \boldsymbol{\epsilon}_x$$

$$\boldsymbol{\theta} = (\mathbf{I}_J - \lambda \mathbf{M})^{-1} \mathbf{u}$$

$$\mathbf{y}^* = \rho \mathbf{W} \mathbf{y}^* + X \boldsymbol{\beta} + \Delta \boldsymbol{\theta} + \boldsymbol{\epsilon},$$

$$y_{ij} = 1 \iff y_{ij}^* \geq 0$$

$$y_{ij} = 0 \quad \text{otherwise}$$

$$\mathbf{u}_x, \mathbf{u} \sim \mathcal{N}(\mathbf{0}, \sigma_u^2 \mathbf{I}_J)$$

$$\boldsymbol{\epsilon}_x, \boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_N)$$

$$\boldsymbol{\theta}_x, \boldsymbol{\theta} \sim \mathcal{N}(0, \sigma_u^2 (\mathbf{B}' \mathbf{B})^{-1}) \text{ where } \mathbf{B} \equiv \mathbf{I}_J - \lambda \mathbf{M}$$

For simplicity, we assume that $\lambda = \lambda_x$ in the DGP.

Figure S68: Bias in $\hat{\beta}_1$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

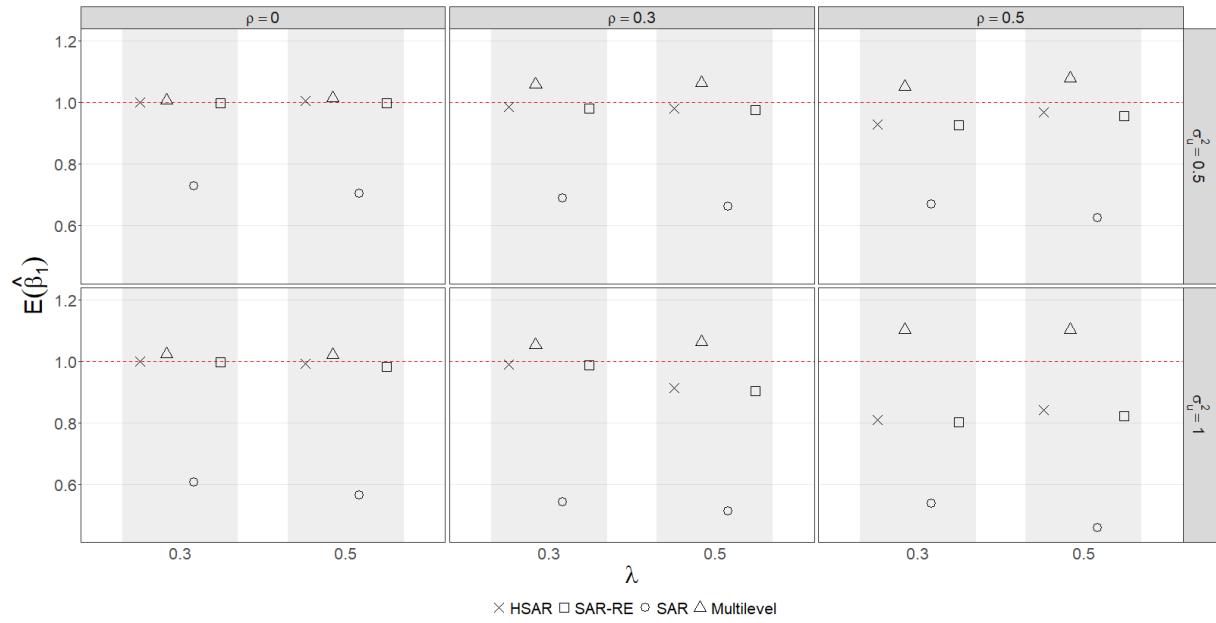


Figure S69: SD in $\hat{\beta}_1$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

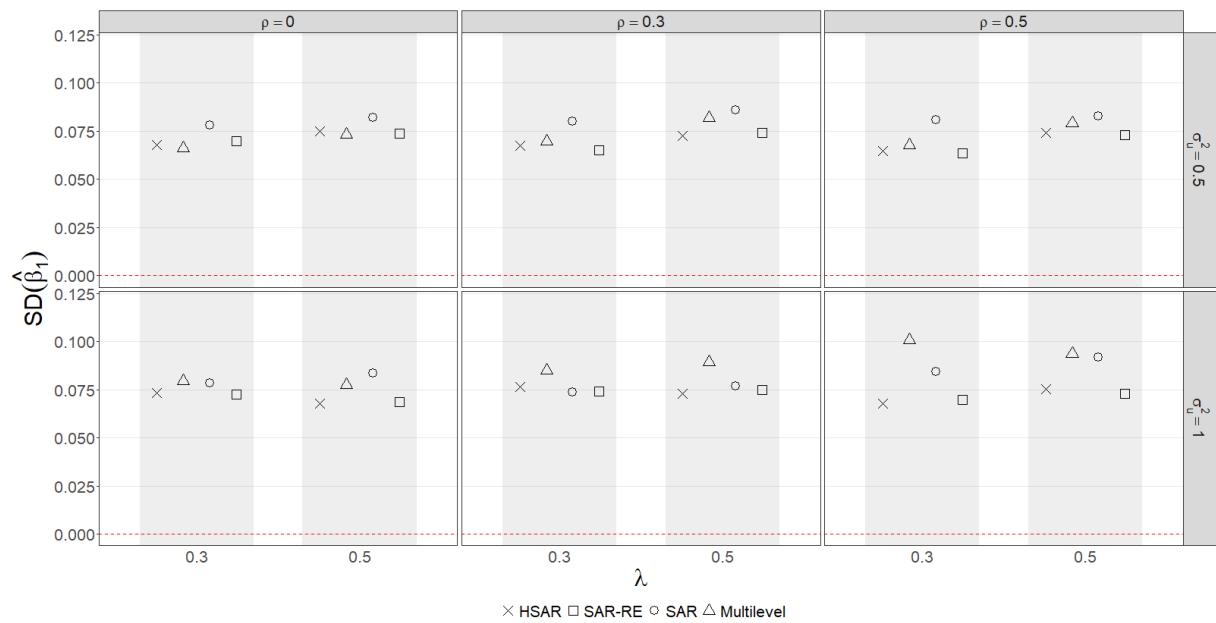


Figure S70: RMSE in $\hat{\beta}_1$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

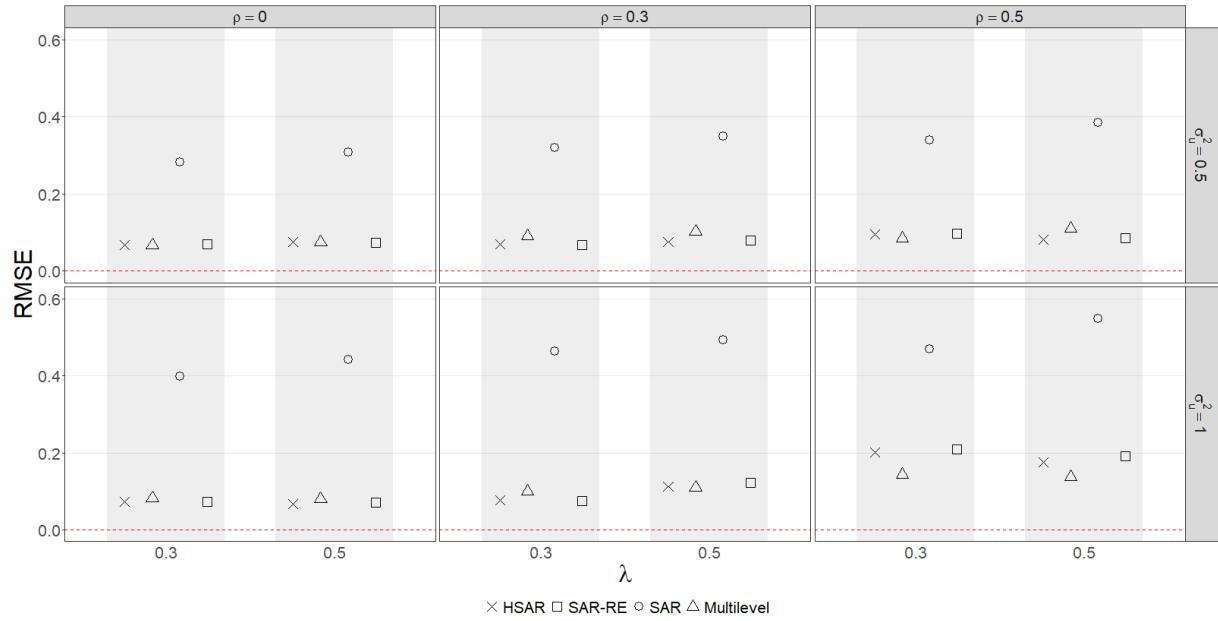


Figure S71: Bias in Direct Effect for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

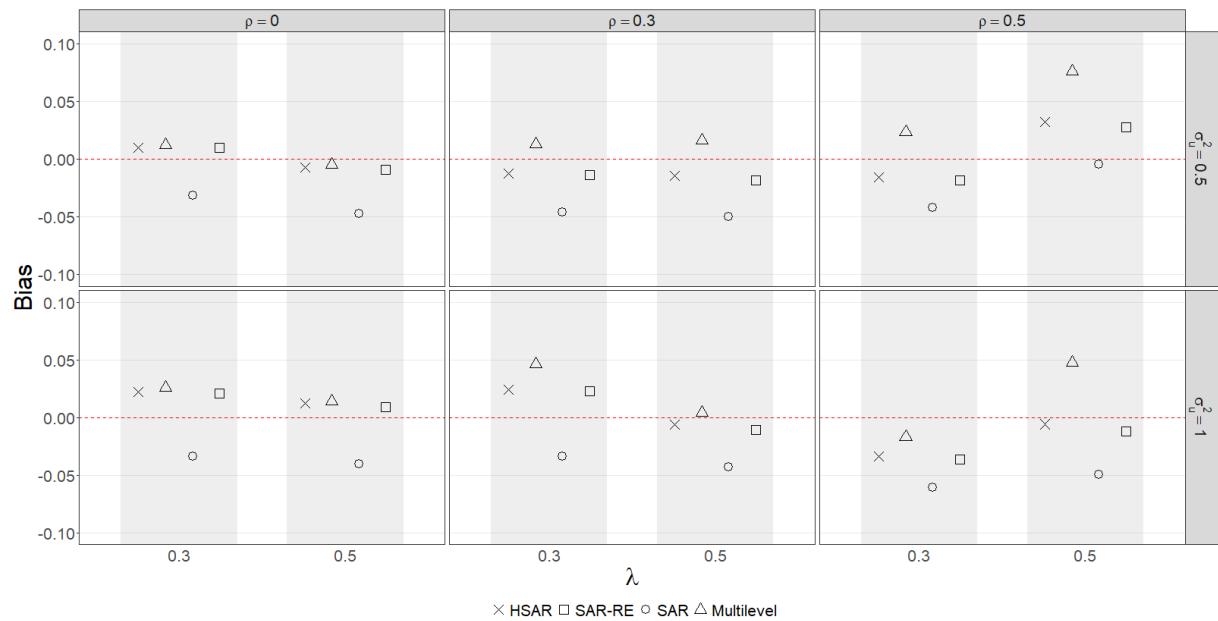


Figure S72: RMSE in Direct Effect for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

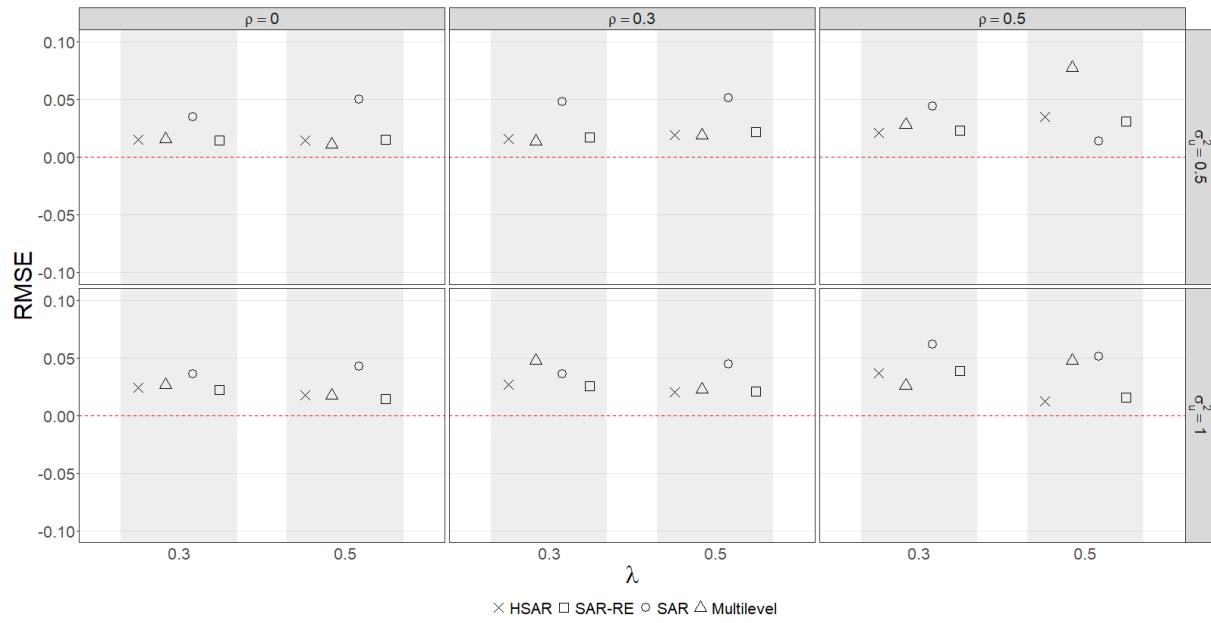


Figure S73: Bias in Indirect Effect for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

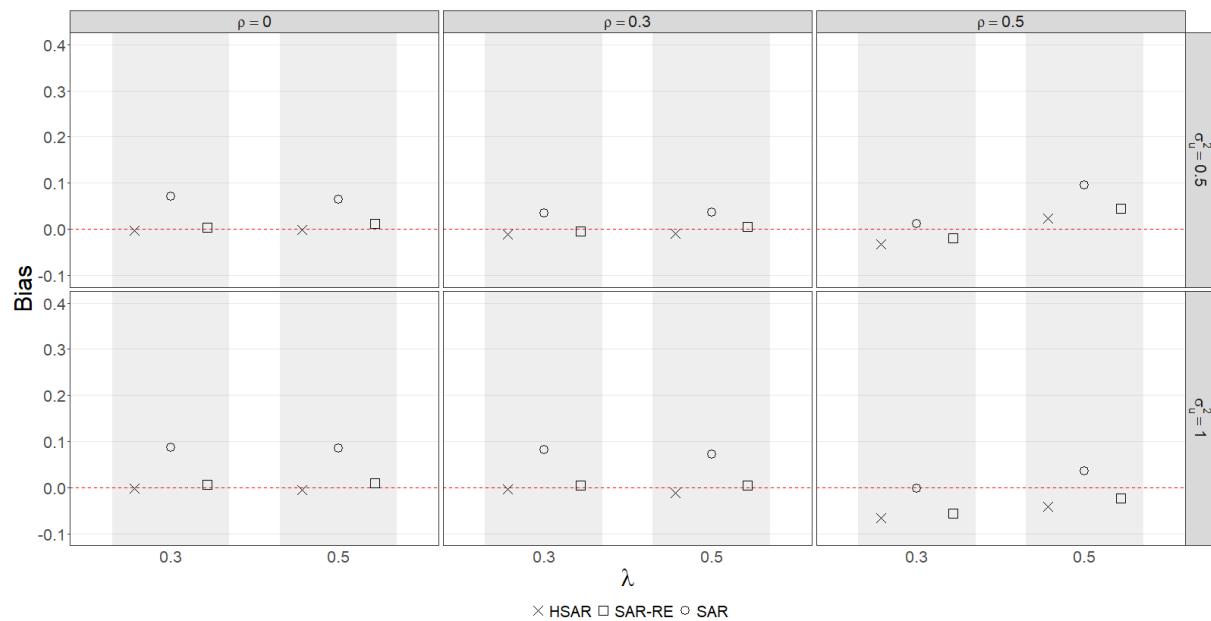


Figure S74: RMSE in Indirect Effect for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

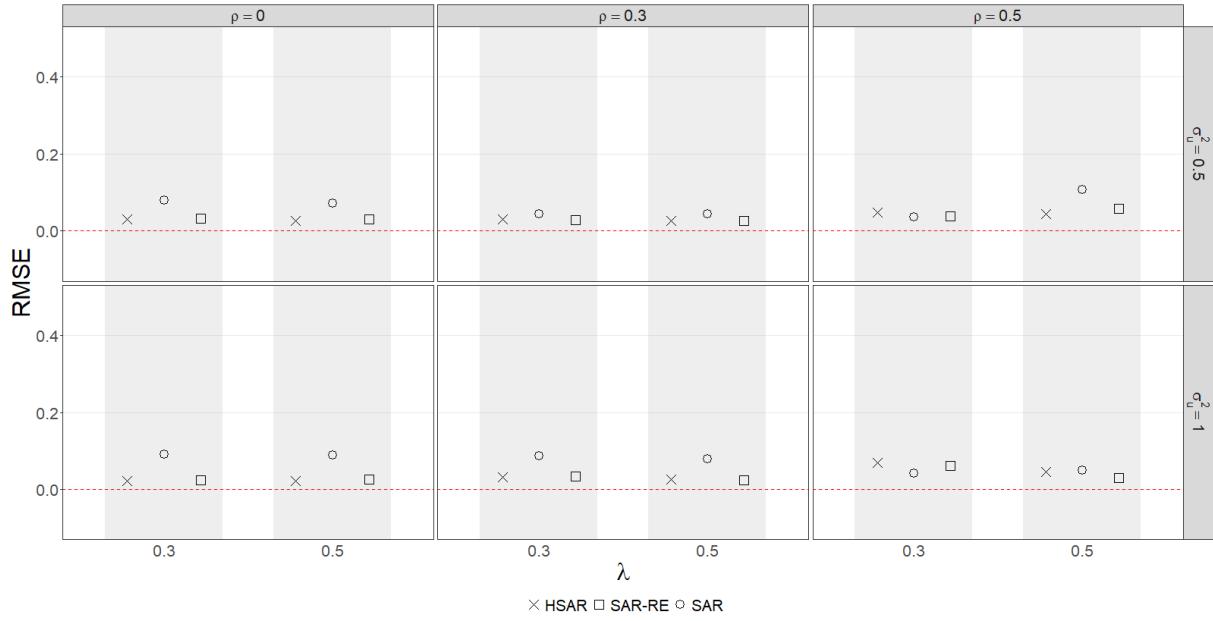


Figure S75: Bias in $\hat{\sigma}_u^2$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

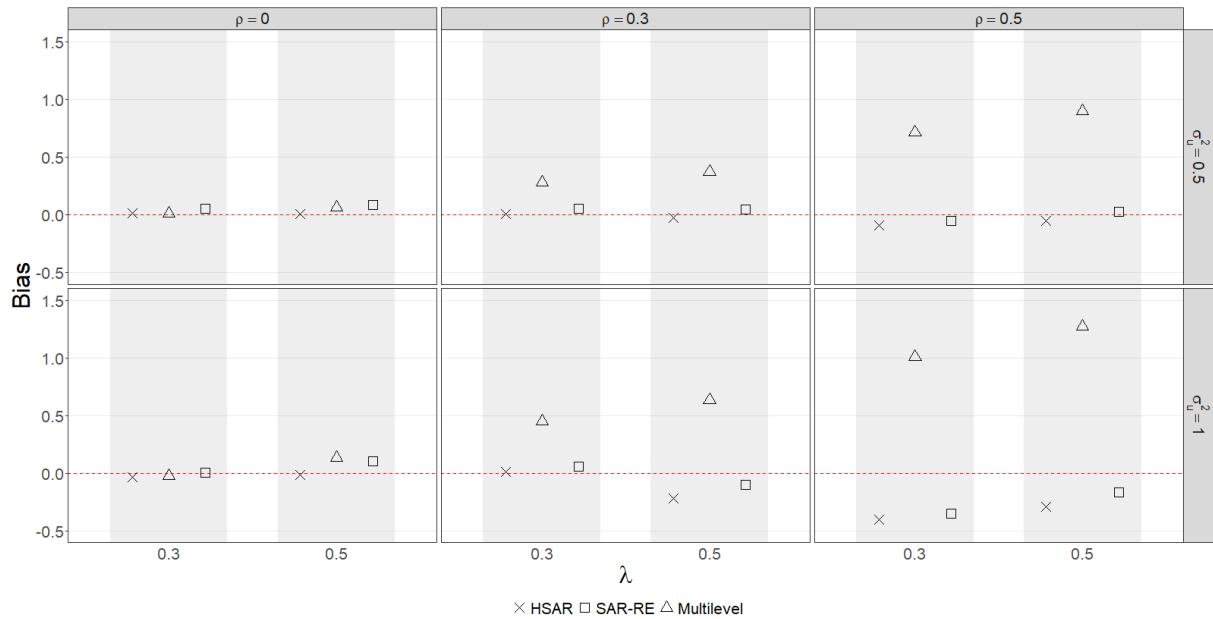


Figure S76: SD in $\hat{\sigma}_u^2$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

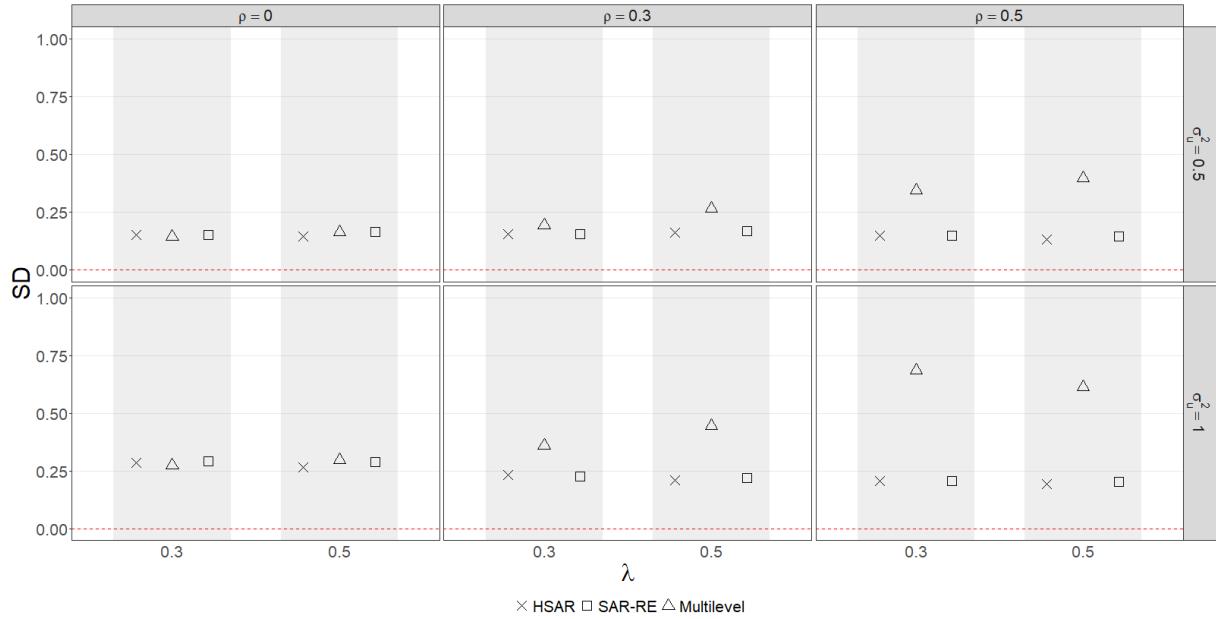


Figure S77: RMSE in $\hat{\sigma}_u^2$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

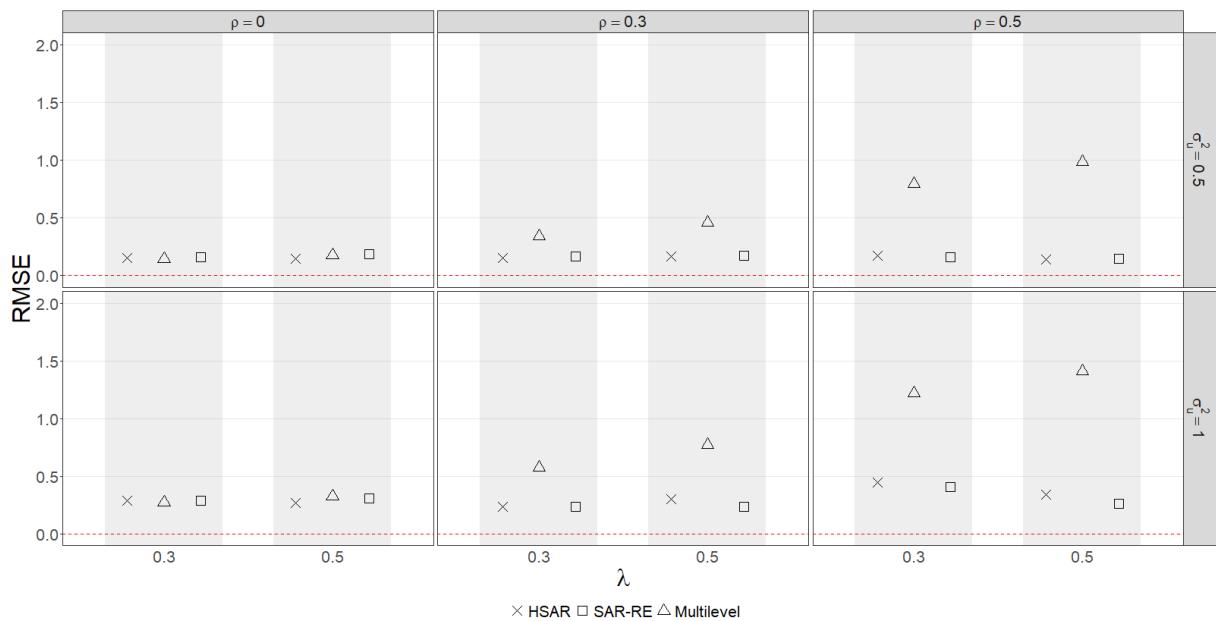


Figure S78: Bias in $\hat{\beta}_0$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

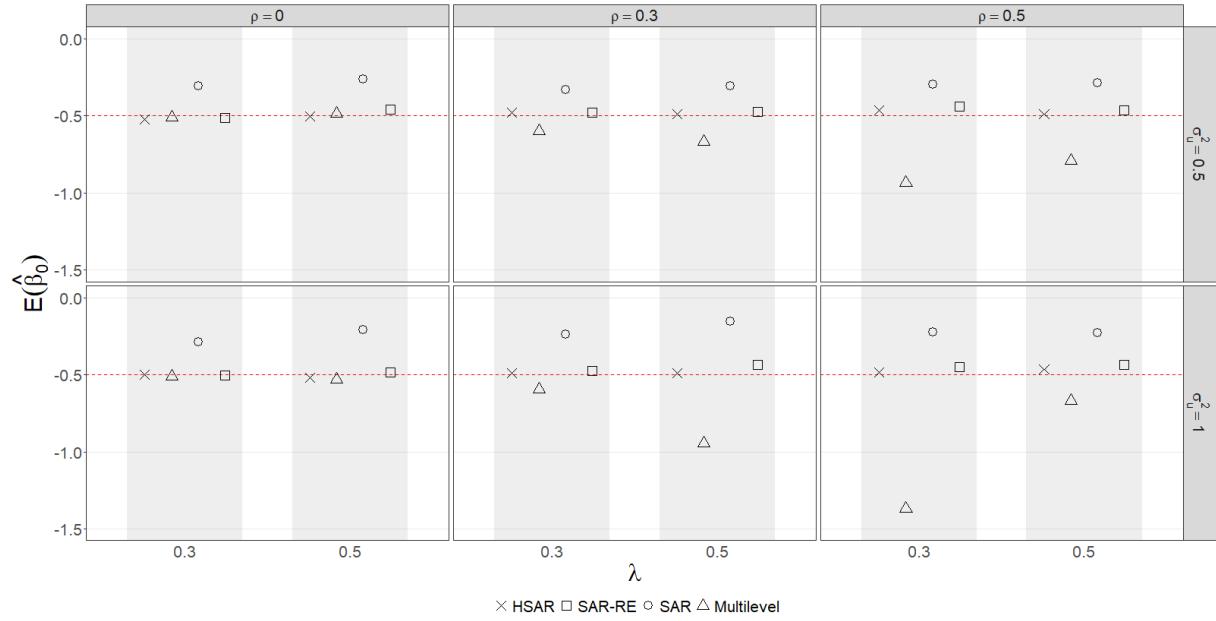


Figure S79: SD in $\hat{\beta}_0$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

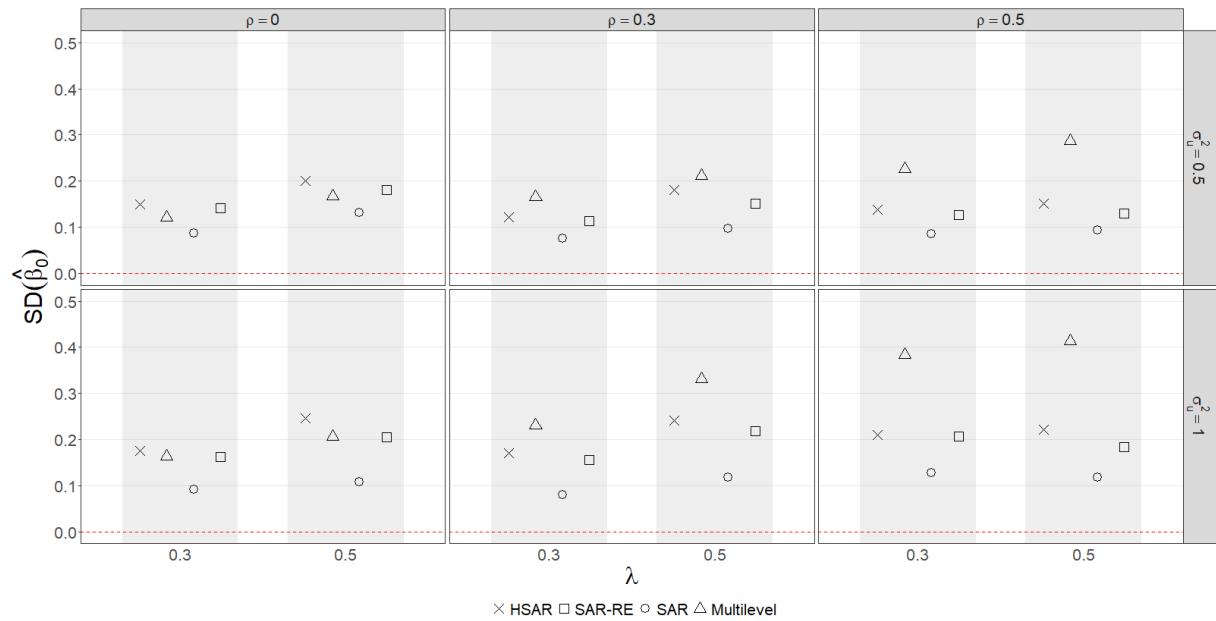


Figure S80: RMSE in $\hat{\beta}_0$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

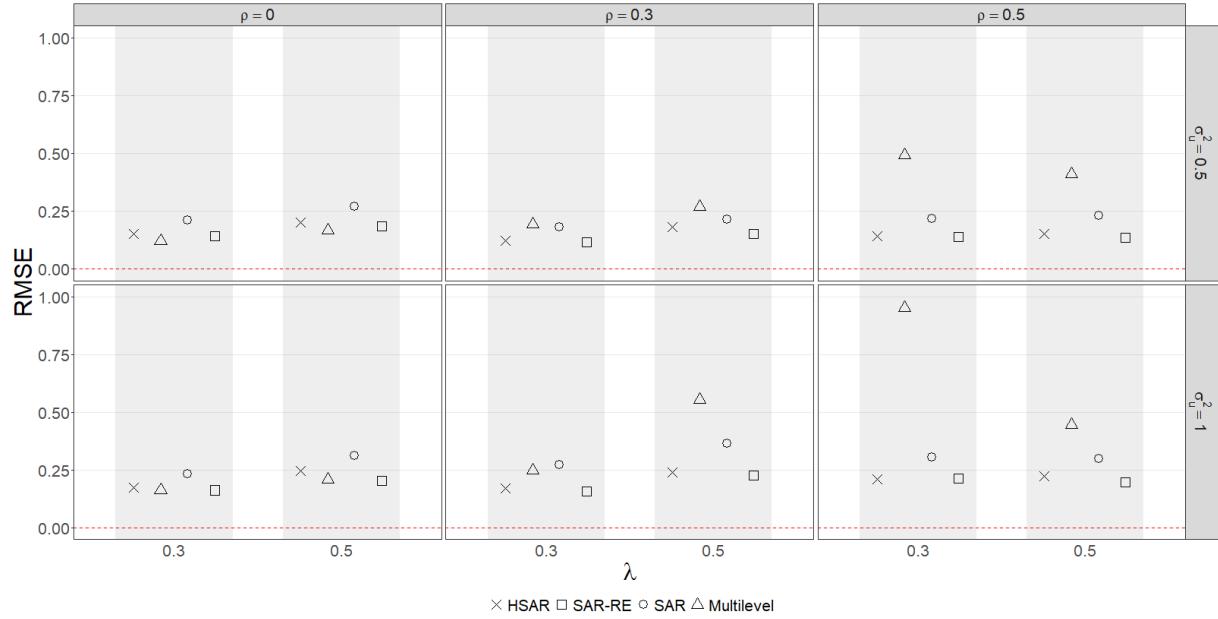


Figure S81: Bias in $\hat{\rho}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

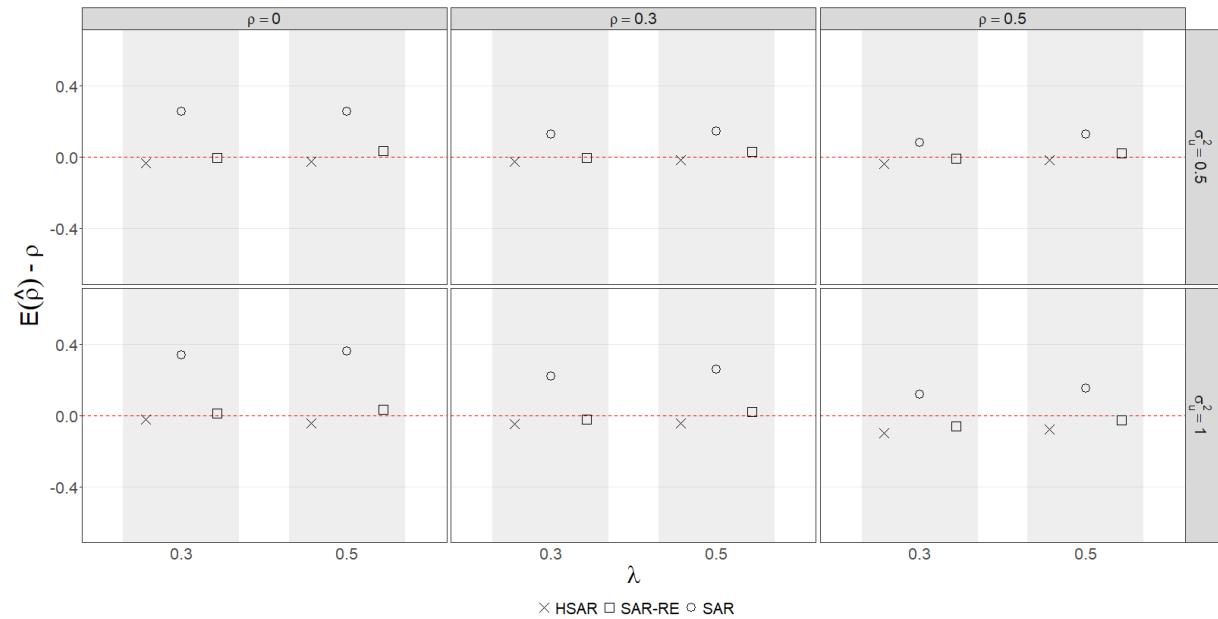


Figure S82: SD in $\hat{\rho}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

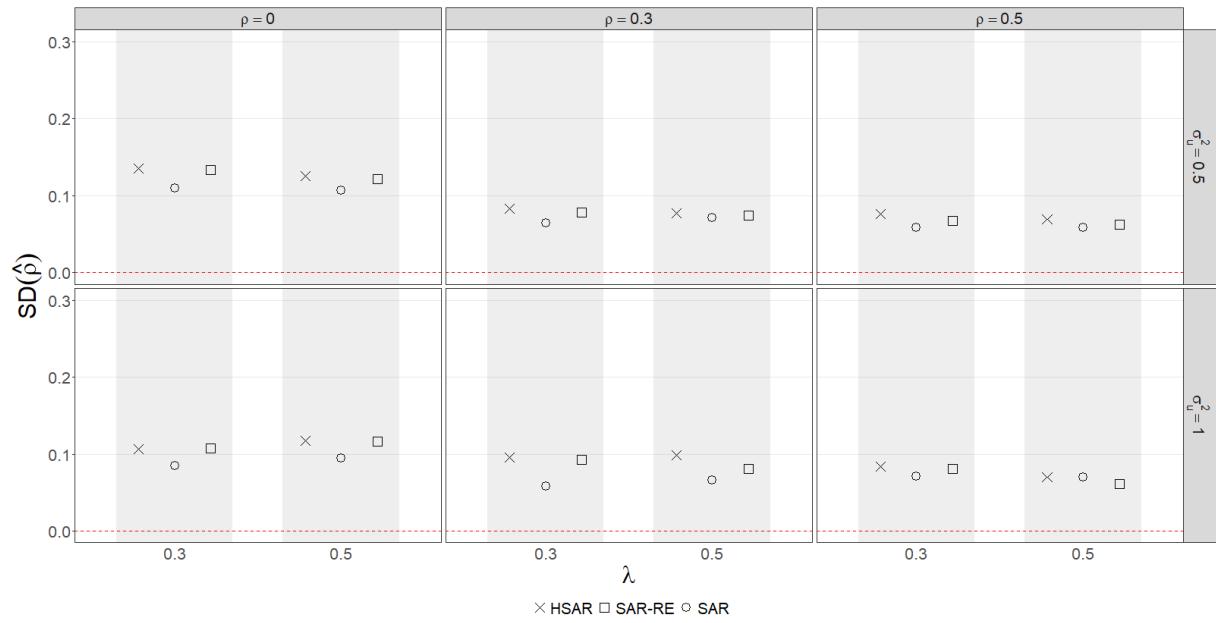


Figure S83: RMSE in $\hat{\rho}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

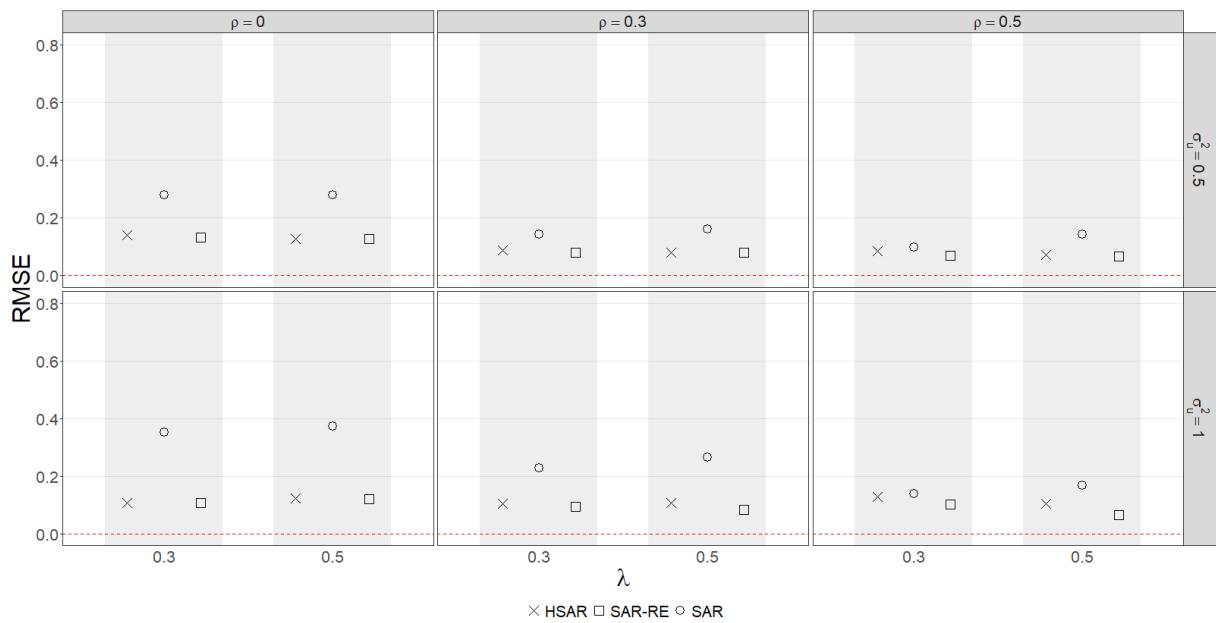


Figure S84: Bias in $\hat{\lambda}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

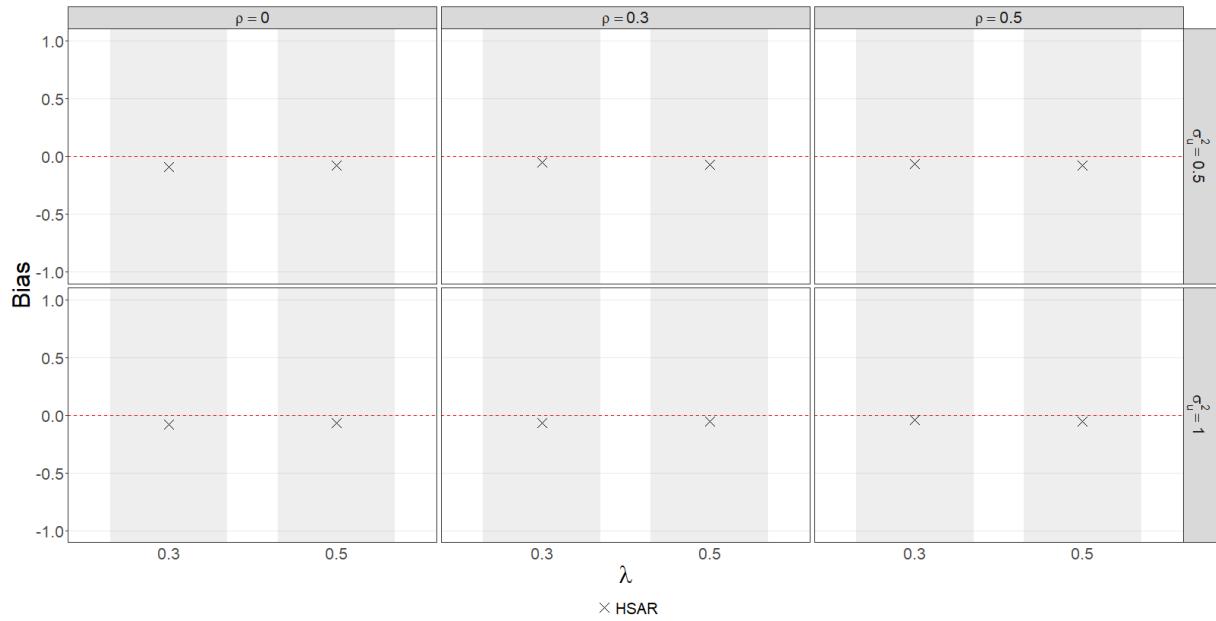


Figure S85: SD in $\hat{\lambda}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$

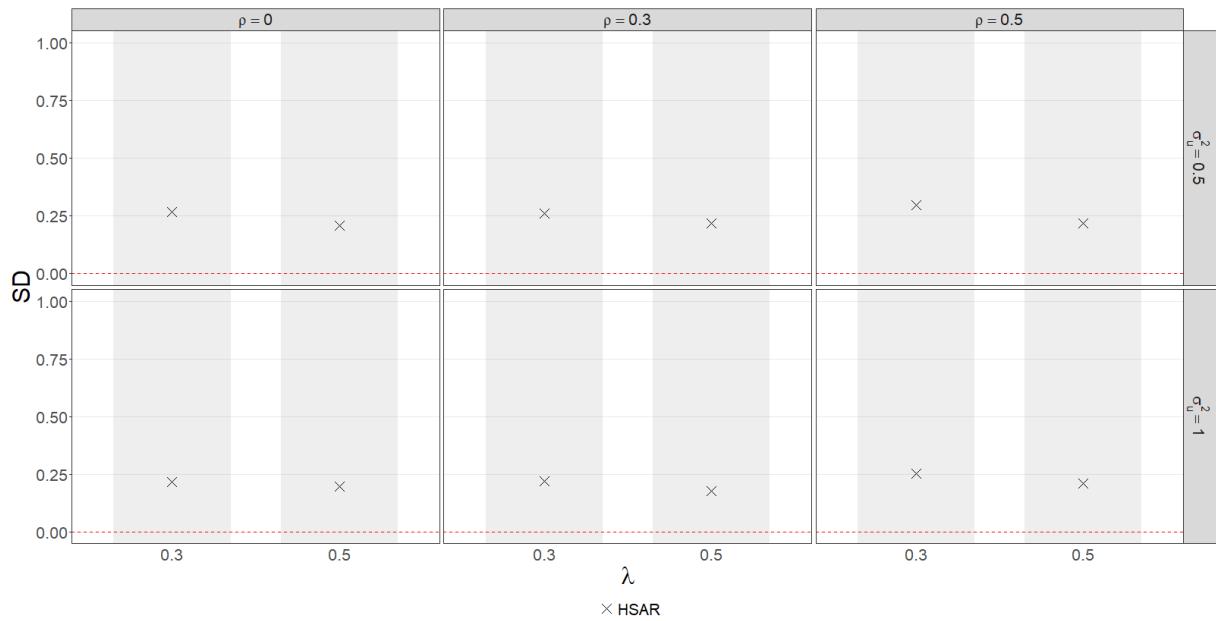
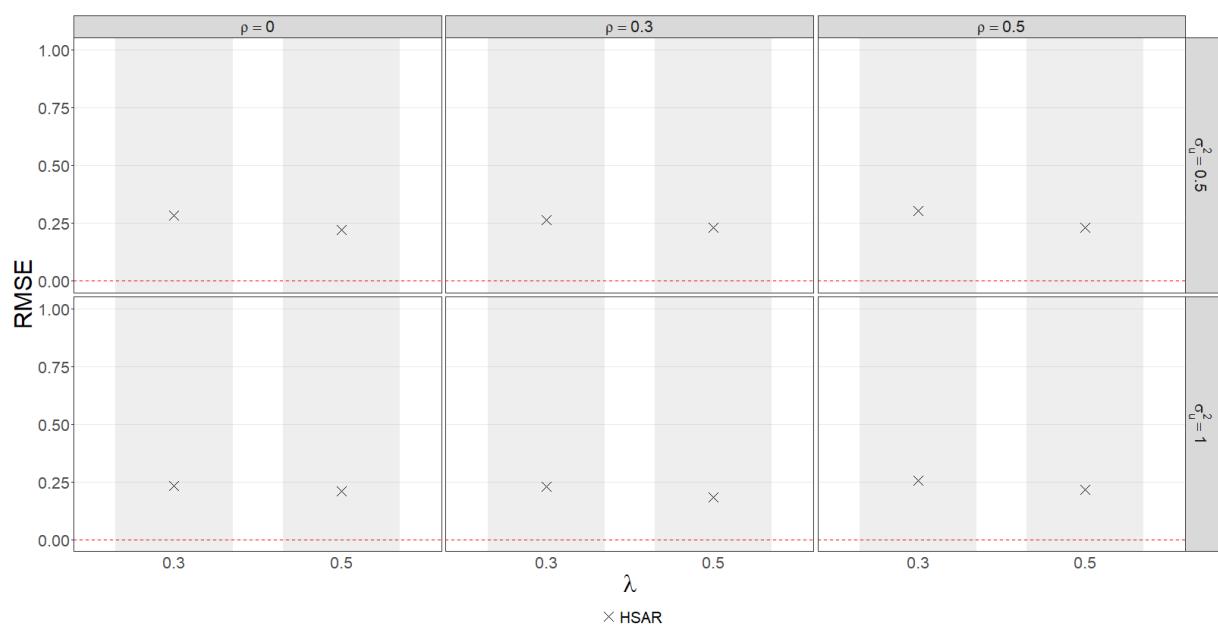


Table S109: Area Under the Curve (AUC) Comparison for $\rho_x = 0.3$

ρ	$\lambda = \lambda_x$	σ_u^2	HSAR better	SAR-RE better	No Difference	No. of Trials
0	0.3	0.5	3	16	81	100
0	0.5	0.5	3	15	82	100
0	0.3	1.0	2	5	93	100
0	0.5	1.0	2	3	95	100
0.3	0.3	0.5	23	15	62	100
0.3	0.5	0.5	36	14	40	100
0.3	0.3	1.0	38	18	44	100
0.3	0.5	1.0	41	7	52	100
0.5	0.3	0.5	54	19	27	100
0.5	0.5	0.5	59	10	31	100
0.5	0.3	1.0	53	18	29	100
0.5	0.5	1.0	65	6	29	100
			389	146	642	1200

Note: One hundred trials were conducted for each combination of parameters. The performance comparison is based on DeLong's test ([DeLong, DeLong and Clarke-Pearson, 1988](#)) implemented with the **pROC** package ([Robin et al., 2021](#)). For each trial, we used the test to determine which of the two models was better based on the z-statistic. The fourth and fifth columns denote the total number of trials in which the HSAR (SAR with random effects) model outperformed the other while the sixth column denotes the total number of trials in which there was no statistical difference between the two models. The SAR with random effects model seems to perform slightly better on average when $\rho = 0$ in the data-generating process. On the other hand, the HSAR model outperforms the SAR with random effects model when there are both spatial diffusion in the lower-level units and spatial diffusion in the unobserved group effects.

Figure S86: RMSE in $\hat{\lambda}$ for $J = 111$, $N = 1117$, $\rho_x = 0.3$, $\lambda_x \in \{0.3, 0.5\}$



T Conditional Posterior Distributions

In this section, we reproduce some of the other derivations in [Dong and Harris \(2015\)](#) for the convenience of the readers.

We employ diffuse priors to allow the likelihood to be dominant in estimating the parameters. The priors may be specified as follows ([Dong and Harris, 2015](#)):

$$\begin{aligned}\pi(\boldsymbol{\beta}) &\sim \mathcal{N}(\mathbf{c}_0, \mathbf{T}_0) \\ \pi(\rho) &\sim \mathcal{U}\left[\frac{1}{\nu_{\min}}, 1\right] \propto 1 \\ \pi(\lambda) &\sim \mathcal{U}\left[\frac{1}{\nu_{\min}^*}, 1\right] \propto 1 \\ \pi(\sigma_u^2) &\sim \mathcal{IG}(a_0, b_0) \\ \pi(\boldsymbol{\theta} | \lambda, \sigma_u^2) &\sim \mathcal{N}\left(\mathbf{0}, \sigma_u^2((\mathbf{I} - \lambda \mathbf{M})'(\mathbf{I} - \lambda \mathbf{M}))^{-1}\right)\end{aligned}$$

where ν_{\min} is the minimum eigenvalue of the spatial weights matrix for the lower-level units, \mathbf{W} , and ν_{\min}^* is the minimum eigenvalue of the spatial weights matrix of high-level units, \mathbf{M} , and a_0 and b_0 are user-specified prior parameters.

Conditional Posterior Distribution for β

$$p(\beta | \mathbf{y}^*, \rho, \lambda, \boldsymbol{\theta}, \sigma_u^2) \propto \mathcal{L}(\mathbf{y}^* | \rho, \lambda, \beta, \boldsymbol{\theta}, \sigma_u^2) \cdot \pi(\beta)$$

$$\propto \exp \left\{ -\frac{1}{2} (\mathbf{A}\mathbf{y}^* - \mathbf{X}\beta - \Delta\boldsymbol{\theta})' (\mathbf{A}\mathbf{y}^* - \mathbf{X}\beta - \Delta\boldsymbol{\theta}) \right\} \times \exp \left\{ -\frac{1}{2} (\beta - \mathbf{M}_0)' \mathbf{T}_0^{-1} (\beta - \mathbf{M}_0) \right\}$$
(5)

$$\propto \exp \left\{ -\frac{1}{2} \beta' [\mathbf{X}'\mathbf{X} + \mathbf{T}_0^{-1}] \beta + [(\mathbf{A}\mathbf{y}^* - \Delta\boldsymbol{\theta})' \mathbf{X} + \mathbf{T}_0^{-1} \mathbf{M}_0] \beta + C \right\}$$
(6)

The basic logic behind deriving the full conditional distributions is to treat priors that do not contain the parameters of interest as constants. This allows us to simplify the original expression summarizing the relationship between the posterior on the one hand and the likelihood and the prior on the other as shown above. We can then just work with the kernel of the distributions by omitting any constants that do not affect the proportionality: equations 5 and 6 show how the conditional distribution for β is derived after omitting the priors for the parameters that are not of interest and working with the kernels. It is well-known that this form can be simplified further by making use of the properties of the normal distribution ([Smith and LeSage, 2004](#)):

$$p(\beta | \mathbf{y}^*, \rho, \lambda, \boldsymbol{\theta}, \sigma_u^2) \sim \mathcal{MVN}(\mathbf{M}_\beta, \Sigma_\beta) \quad (7)$$

where $\Sigma_\beta \equiv [\mathbf{X}'\mathbf{X} + \mathbf{T}_0^{-1}]^{-1}$ and $\mathbf{M}_\beta \equiv \Sigma_\beta [\mathbf{X}'(\mathbf{A}\mathbf{y}^* - \Delta\boldsymbol{\theta}) + \mathbf{T}_0^{-1} \mathbf{M}_0]$. Equation 7 shows that we can use multivariate normal distribution with mean \mathbf{M}_β and variance Σ_β to draw

updated values of β .

Conditional Posterior Distribution for θ

The logic for updating θ is very similar to the logic for updating β ([Dong and Harris, 2015](#)):

$$\pi(\theta | \lambda, \sigma_u^2) \cdot \mathcal{L}(y^* | \rho, \lambda, \beta, \theta, \sigma_u^2)$$

$$\theta | \lambda, \sigma_u^2 \sim \mathcal{MVN}(\mathbf{M}_\theta, \Sigma_\theta)$$

where $\Sigma_\theta \equiv [\Delta' \Delta + (\sigma_u^2)^{-1} \mathbf{B}' \mathbf{B}]^{-1}$ and $\mathbf{M}_\theta \equiv \Sigma_\theta [\Delta' (AY - X\beta)]$.

Conditional Posterior Distribution for σ_u^2

The conditional posterior distribution for σ_u^2 is ([Dong and Harris, 2015](#)):

$$\sigma_u^2 \sim \mathcal{IV}\left(\frac{J}{2} + a_0, \frac{\theta' \mathbf{B}' \mathbf{B} \theta}{2} + b_0\right)$$

where \mathcal{IV} denotes the inverse-gamma distribution and a_0 and b_0 are parameters set a priori by the researcher.³⁰

³⁰We set these to 0.01 similar to [Dong and Harris \(2015\)](#).

Conditional Posterior Distribution for ρ

$$\begin{aligned}
p(\rho | \lambda, \beta, \theta, \mathbf{y}^*, \mathbf{y}) &= \frac{p(\rho, \lambda, \beta, \theta, \mathbf{y}^* | \mathbf{y})}{p(\lambda, \beta, \theta, \mathbf{y}^* | \mathbf{y})} \\
&\propto p(\rho, \lambda, \beta, \theta, \mathbf{y}^* | \mathbf{y}) \\
&\propto \pi(\rho) \cdot \pi(\mathbf{y}^* | \rho, \lambda, \beta, \theta, \sigma_u^2) \\
&\propto \det |\mathbf{A}| \times \exp \left\{ -\frac{1}{2} (\mathbf{A}\mathbf{y}^* - \mathbf{X}\beta - \Delta\theta)' (\mathbf{A}\mathbf{y}^* - \mathbf{X}\beta - \Delta\theta) \right\} \tag{8}
\end{aligned}$$

where $\mathbf{A} = \mathbf{I} - \rho\mathbf{W}$. Equation 8 is not a distribution of a known form and we have to use the Metropolis-Hastings algorithm. We do not work with the acceptance ratio directly but instead use a logged-transformed version for the purposes of numerical stability (Hoff, 2009).

Conditional Posterior Distribution for λ

The conditional posterior distribution for λ is (Dong and Harris, 2015):

$$\begin{aligned}
p(\lambda | \mathbf{y}^*, \rho, \beta, \sigma_u^2, \theta) &\propto \pi(\theta | \lambda, \sigma_u^2) \cdot \pi(\lambda) \\
&\propto |\mathbf{B}| \times \exp \left\{ -\frac{1}{2\sigma_u^2} \theta' \mathbf{B}' \mathbf{B} \theta \right\} \tag{9}
\end{aligned}$$

where $\mathbf{B} = \mathbf{I}_J - \lambda\mathbf{M}$. Similar to the conditional distribution of ρ , equation 9 is not a distribution of a known form and we will have to use the Metropolis-Hastings sampling algorithm. Once again, we use a logged-transformed version of the ratio for numerical stability purposes (Hoff, 2009).

U Ordered Probit

Similar to the binary outcome case, the ordered outcomes are first conceptualized as a latent variable and are then categorized into C different outcomes depending on the threshold cutpoints. We once again find that our proposed HSAR ordered probit model performs favorably compared to the ordered spatial probit model and is comparable to the multilevel probit model in terms of recovering the estimates of β .

Extending the algorithm applied above to ordered outcomes is relatively straightforward. In the case of regular spatial ordered probit with three categories, [LeSage and Pace \(2009\)](#) shows that the spatial ordered probit is a straightforward extension of the spatial binary probit model. We can apply the same algorithm as the binary probit case other than minute changes needed for generating \mathbf{y}^* and estimating ϕ_c which represent the cutoff thresholds for categorizing the latent values into different discrete outcomes. Once again, if we conceptualize the outcome as a continuous latent variable, the observation y_{ij} is of category c if

$$y_{ij} = c \quad \text{if} \quad \phi_{c-1} < y_{ij}^* \leq \phi_c$$

[LeSage and Pace \(2009, 297\)](#) notes that for an ordered case of C alternatives, three values of ϕ are fixed, namely $\phi_0 = -\infty$, $\phi_1 = 0$ and $\phi_C = +\infty$ while the thresholds ϕ_c for $c = 2, \dots, C - 1$ are to be estimated. The details for estimating these parameters are explained in [LeSage and Pace \(2009, 297-299\)](#). We use the codes from the **spatialprobit** package ([Wilhelm and de Matos, 2013](#)) for estimating these cutpoints for our hierarchical

ordered spatial probit model.

We show the results for the additional simulations where $J = 49$ and $N = 980$, $\sigma_u^2 = 1.0$ similar to the binary case. We compare the results from our hierarchical ordered spatial probit model to the ordered spatial probit model implemented with the **spatialprobit** package (Wilhelm and de Matos, 2013) and the multilevel probit model implemented with the **ordinal** package (Christensen, 2019).

The results for the bias in $\hat{\beta}_1$ are presented in Figure U87. We see that the hierarchical spatial ordered probit model again performs favorably compared to the ordered spatial probit model and is comparable to the multilevel probit model in terms of recovering the estimates. The results for $\hat{\rho}$ are similar to those of the binary probit case: we see that the ordered spatial probit model consistently overestimates ρ . We present other simulation results as tables in Appendices V, W, X, and Y.

Figure U87: Bias in $\hat{\beta}_1$ for $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$ for Ordered Outcomes

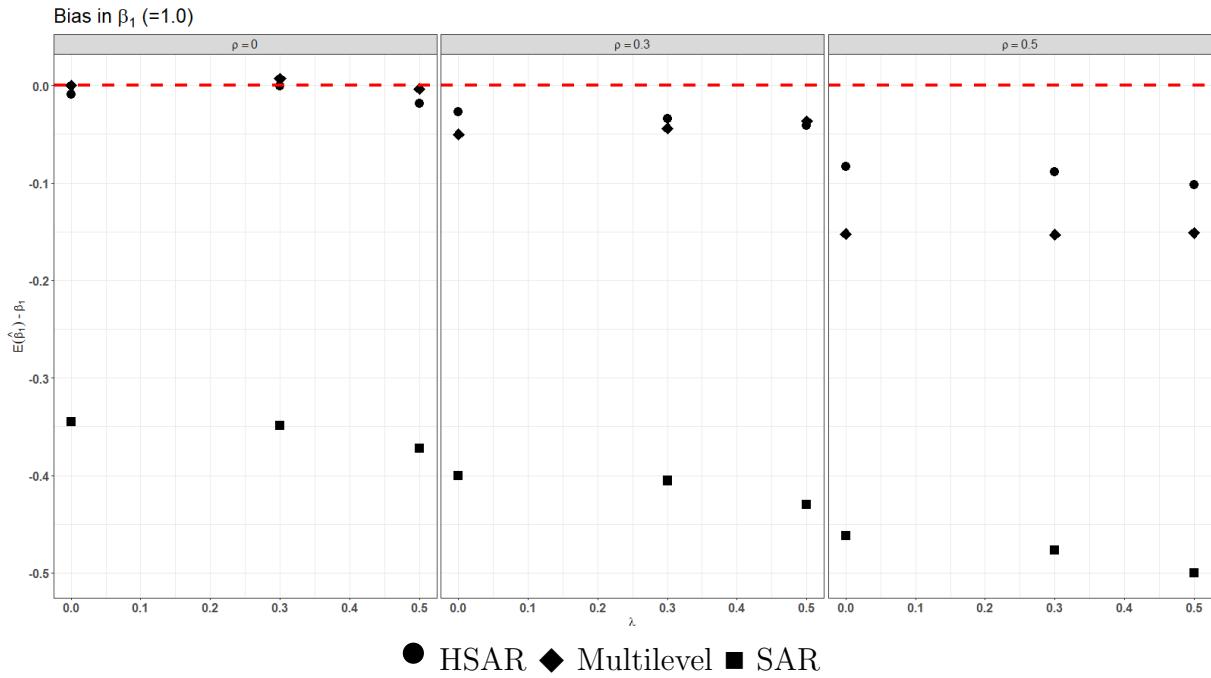
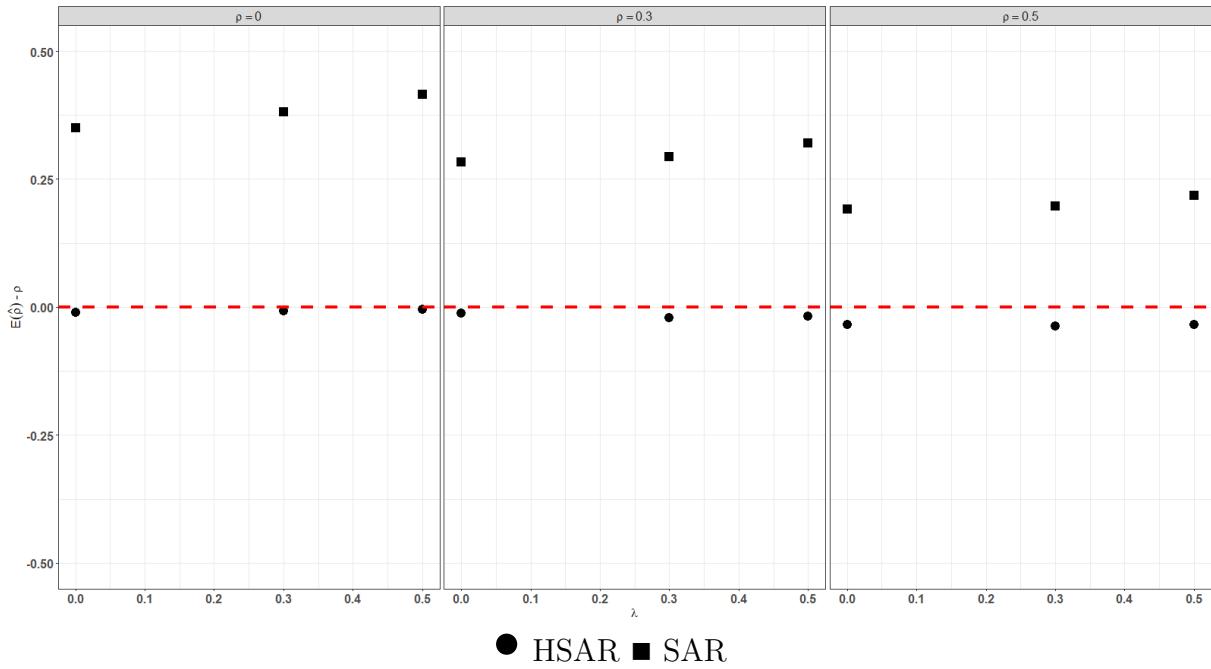


Figure U88: Bias in $\hat{\rho}$ for $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$ for Ordered Outcomes



V Ordered Probit $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$,

$\rho \in \{0, 0.3, 0.5\}$ and $\lambda \in \{0, 0.3, 0.5\}$

Table V110: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.011	-0.067	-0.009	0.35	-0.345	0
SD	0.064	0.228	0.066	0.065	0.045	0.065
RMSE	0.065	0.238	0.067	0.356	0.348	0.065

Table V111: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.008	-0.009	-0.001	0.381	-0.349	0.007
SD	0.063	0.176	0.065	0.068	0.047	0.067
RMSE	0.064	0.176	0.065	0.387	0.352	0.067

Table V112: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.005	-0.083	-0.019	0.416	-0.372	-0.004
SD	0.059	0.19	0.079	0.069	0.052	0.078
RMSE	0.059	0.207	0.081	0.421	0.376	0.079

Table V113: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.011	-0.015	-0.027	0.283	-0.4	-0.05
SD	0.046	0.217	0.068	0.042	0.046	0.064
RMSE	0.048	0.218	0.073	0.286	0.403	0.082

Table V114: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.022	-0.074	-0.034	0.294	-0.405	-0.044
SD	0.048	0.231	0.067	0.052	0.056	0.065
RMSE	0.053	0.242	0.075	0.298	0.409	0.078

Table V115: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.017	-0.048	-0.041	0.32	-0.43	-0.037
SD	0.047	0.167	0.059	0.042	0.049	0.064
RMSE	0.05	0.174	0.072	0.323	0.433	0.073

Table V116: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.034	-0.011	-0.083	0.192	-0.462	-0.152
SD	0.043	0.259	0.067	0.025	0.048	0.067
RMSE	0.055	0.259	0.107	0.194	0.464	0.167

Table V117: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.038	-0.09	-0.089	0.198	-0.477	-0.153
SD	0.042	0.256	0.075	0.032	0.063	0.066
RMSE	0.056	0.271	0.116	0.201	0.481	0.167

Table V118: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.035	-0.079	-0.102	0.217	-0.5	-0.151
SD	0.037	0.192	0.074	0.029	0.061	0.068
RMSE	0.051	0.208	0.126	0.219	0.504	0.166

W Ordered Probit $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$,

$\rho \in \{0, 0.3, 0.5\}$ and $\lambda \in \{0, 0.3, 0.5\}$

Table W119: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.008	-0.061	-0.002	0.347	-0.402	0.005
SD	0.052	0.24	0.066	0.058	0.042	0.066
RMSE	0.052	0.248	0.066	0.352	0.404	0.066

Table W120: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.008	-0.072	-0.012	0.364	-0.41	0.001
SD	0.057	0.212	0.069	0.066	0.057	0.072
RMSE	0.058	0.224	0.07	0.37	0.414	0.072

Table W121: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.007	-0.066	-0.004	0.407	-0.428	0.017
SD	0.056	0.2	0.072	0.077	0.054	0.07
RMSE	0.056	0.211	0.072	0.414	0.432	0.072

Table W122: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.014	-0.001	-0.022	0.265	-0.446	0.015
SD	0.045	0.235	0.07	0.04	0.055	0.071
RMSE	0.047	0.235	0.073	0.268	0.449	0.072

Table W123: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.012	-0.096	-0.025	0.276	-0.454	0.017
SD	0.046	0.207	0.072	0.043	0.049	0.07
RMSE	0.047	0.228	0.077	0.279	0.456	0.073

Table W124: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.018	-0.078	-0.047	0.304	-0.476	0.011
SD	0.053	0.173	0.061	0.049	0.051	0.066
RMSE	0.057	0.19	0.077	0.308	0.478	0.067

Table W125: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.036	-0.021	-0.075	0.179	-0.51	-0.049
SD	0.032	0.227	0.067	0.026	0.045	0.068
RMSE	0.048	0.228	0.1	0.181	0.512	0.084

Table W126: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.041	-0.061	-0.101	0.19	-0.521	-0.073
SD	0.039	0.21	0.073	0.028	0.056	0.075
RMSE	0.056	0.218	0.125	0.192	0.524	0.105

Table W127: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 1.0$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.041	-0.088	-0.106	0.21	-0.552	-0.056
SD	0.038	0.18	0.075	0.029	0.056	0.071
RMSE	0.056	0.2	0.13	0.211	0.554	0.091

X Ordered Probit $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$,

$\rho \in \{0, 0.3, 0.5\}$ and $\lambda \in \{0, 0.3, 0.5\}$

Table X128: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.017	-0.027	0.014	0.217	-0.273	0.013
SD	0.062	0.246	0.069	0.058	0.043	0.066
RMSE	0.064	0.247	0.071	0.224	0.276	0.067

Table X129: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.008	-0.049	0.01	0.241	-0.275	0.009
SD	0.065	0.215	0.076	0.066	0.05	0.072
RMSE	0.065	0.221	0.076	0.249	0.28	0.073

Table X130: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.02	-0.051	0.011	0.27	-0.284	0.012
SD	0.064	0.184	0.064	0.066	0.049	0.06
RMSE	0.067	0.191	0.065	0.278	0.288	0.061

Table X131: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.014	-0.01	0.002	0.177	-0.312	-0.034
SD	0.051	0.23	0.079	0.046	0.051	0.073
RMSE	0.053	0.23	0.079	0.183	0.316	0.08

Table X132: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.009	-0.051	-0.004	0.196	-0.322	-0.039
SD	0.053	0.209	0.071	0.052	0.047	0.067
RMSE	0.054	0.215	0.071	0.203	0.325	0.077

Table X133: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.018	-0.098	-0.021	0.213	-0.34	-0.049
SD	0.051	0.202	0.058	0.051	0.05	0.055
RMSE	0.054	0.224	0.062	0.219	0.343	0.073

Table X134: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.024	-0.014	-0.038	0.128	-0.364	-0.129
SD	0.039	0.263	0.066	0.034	0.049	0.059
RMSE	0.046	0.263	0.076	0.133	0.367	0.141

Table X135: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.026	-0.04	-0.045	0.139	-0.377	-0.131
SD	0.042	0.225	0.068	0.029	0.045	0.06
RMSE	0.049	0.228	0.081	0.142	0.379	0.144

Table X136: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.0$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.029	-0.078	-0.058	0.157	-0.395	-0.133
SD	0.046	0.208	0.072	0.039	0.057	0.063
RMSE	0.054	0.222	0.092	0.162	0.399	0.147

Y Ordered Probit $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$,

$\rho \in \{0, 0.3, 0.5\}$ and $\lambda \in \{0, 0.3, 0.5\}$

Table Y137: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.006	-0.076	0.009	0.221	-0.315	0.009
SD	0.054	0.252	0.067	0.058	0.052	0.065
RMSE	0.054	0.263	0.068	0.228	0.319	0.066

Table Y138: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.012	-0.044	0.003	0.224	-0.321	0
SD	0.061	0.207	0.065	0.063	0.044	0.061
RMSE	0.062	0.212	0.065	0.232	0.324	0.061

Table Y139: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.0$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.007	-0.084	0.013	0.255	-0.328	0.015
SD	0.06	0.187	0.063	0.066	0.052	0.061
RMSE	0.06	0.205	0.064	0.263	0.332	0.062

Table Y140: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.011	-0.029	0.001	0.163	-0.348	0.021
SD	0.046	0.263	0.065	0.046	0.045	0.063
RMSE	0.047	0.265	0.065	0.17	0.351	0.066

Table Y141: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.009	-0.067	0.003	0.185	-0.358	0.027
SD	0.047	0.225	0.069	0.05	0.048	0.071
RMSE	0.048	0.235	0.069	0.191	0.361	0.076

Table Y142: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.3$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.018	-0.081	-0.021	0.213	-0.392	0.013
SD	0.054	0.203	0.065	0.053	0.049	0.067
RMSE	0.057	0.219	0.068	0.22	0.395	0.068

Table Y143: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.0$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.024	-0.033	-0.052	0.117	-0.415	-0.051
SD	0.043	0.236	0.064	0.035	0.046	0.066
RMSE	0.049	0.238	0.082	0.122	0.417	0.084

Table Y144: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.3$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.024	-0.049	-0.059	0.129	-0.422	-0.047
SD	0.041	0.212	0.07	0.032	0.048	0.067
RMSE	0.048	0.217	0.092	0.133	0.425	0.082

Table Y145: Ordered Probit: $J = 49$, $N = 980$, $\sigma_u^2 = 0.5$, $\rho_x = 0.3$, $\rho = 0.5$, $\lambda = 0.5$

	HSAR			SAR		Multilevel
	ρ	λ	β_1	ρ	β_1	β_1
Bias	-0.032	-0.057	-0.071	0.149	-0.456	-0.045
SD	0.04	0.186	0.058	0.038	0.053	0.056
RMSE	0.051	0.195	0.091	0.154	0.459	0.072