

$$L = \text{MAPE}(C_1(\mathbf{X} \cdot \mathbf{X}) + C_2(\mathbf{X} \cdot \mathbf{X}), (\mathbf{C}_3 \mathbf{X}) \cdot (\mathbf{C}_4 \mathbf{X})) \quad (16)$$

As expressed in (16), \mathbf{X} represents the actual value of $[\hat{\mathbf{V}}_{\sin,pu} \ \hat{\mathbf{V}}_{\text{squ},pu} \ \hat{\mathbf{V}}_{\cos,pu}]$ obtained in Fig. 2, and the physics loss is actually related to $(V_i V_j \cos \theta_{ij})^2 + (V_i V_j \sin \theta_{ij})^2 = V_i^2 \cdot V_j^2$ represented by (9) in the paper.

$$C_1 \begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \cdot \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix} + C_2 \begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \cdot \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix} \cdot \begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix} \quad (9)$$

Specifically, the variable correspondence is described as (1*)-(3*):

$$C_1 \begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \cdot \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix} = [(V_i V_j \cos \theta_{ij})^2]_{m \times 1} \quad (1^*)$$

$$C_2 \begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \cdot \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix} = [(V_i V_j \sin \theta_{ij})^2]_{m \times 1} \quad (2^*)$$

$$\begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix} \cdot \begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix} = [V_i^2 \cdot V_j^2]_{m \times 1} \quad (3^*)$$

To improve the training effect, the loss shown in (16) can also be further transformed. Let

$$\tilde{\mathbf{V}}_{\cos} = \sqrt{\begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix} \cdot \begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix}} - C_2 \begin{pmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \cdot \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \hat{\mathbf{V}}_{\cos} \end{bmatrix} \end{pmatrix}, \text{ and } \hat{\mathbf{P}}, \hat{\mathbf{Q}} \text{ can be obtained based on (4*) :}$$

$$\begin{bmatrix} \hat{\mathbf{P}} \\ \hat{\mathbf{Q}} \end{bmatrix} = \begin{bmatrix} \mathbf{B}^p & \mathbf{G}^\Lambda & \mathbf{G}^p \\ \mathbf{G}^Q & \mathbf{B}^\Lambda & \mathbf{B}^Q \end{bmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{\sin} \\ \hat{\mathbf{V}}_{\text{squ}} \\ \tilde{\mathbf{V}}_{\cos} \end{bmatrix} \quad (4^*)$$

Assigning $\mathbf{S}_{real} = [\mathbf{P} \ \mathbf{Q}]$, $\mathbf{S}_{pred} = [\hat{\mathbf{P}} \ \hat{\mathbf{Q}}]$, the improved loss function for network training can be given as (5*):

$$L = \text{MAPE}(\mathbf{S}_{real}, \mathbf{S}_{pred}) \quad (5^*)$$