

Sparse signal detection and fingerprint feature recognition based on fast 2D DFRFT

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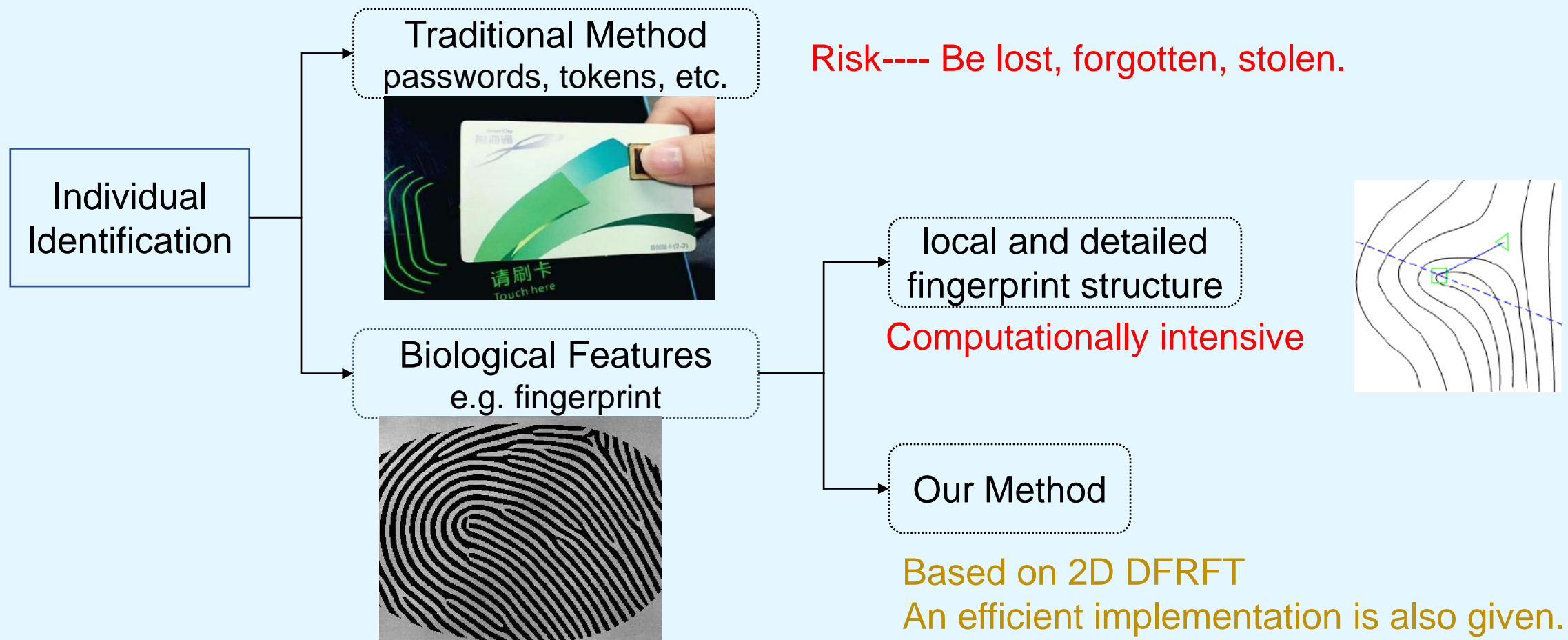
Outline

- 1. Research Motivation
 - 1.1 Fingerprint feature recognition
 - 1.2 Fractional Fourier transform
 - 1.3 Numerical calculation of 2D FRFT
- 2. The Sparse 2D Fractional Fourier Transform Algorithm
 - 2.1 Algorithm flow
 - 2.2 Comparison with traditional methods
 - 2.3 Algorithm convergence analysis
- 3. Fingerprint Feature Recognition Simulation
 - 3.1 2D chirp signal detection
 - 3.2 Fingerprint feature recognition

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1.1 Fingerprint feature recognition



1.2 Fractional Fourier transform

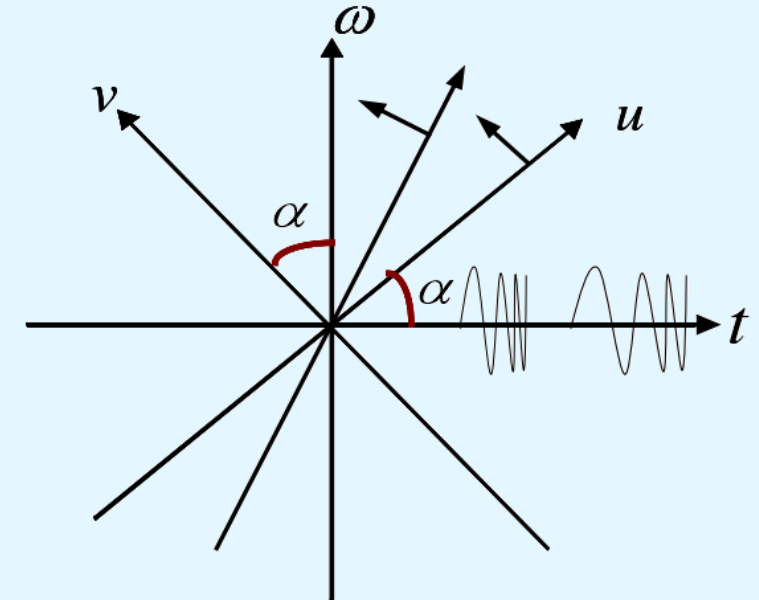
Definition of pei-type 2D DFRFT:

$$O_s^{\alpha,\beta}(u_1, u_2) = \begin{cases} AE_u \sum_{t_1=0}^{N_1-1} \sum_{t_2=0}^{N_2-1} s(t_1, t_2) E_t E_{ut} & \sin \alpha > 0, \sin \beta > 0 \\ AE_u \sum_{t_1=0}^{N_1-1} \sum_{t_2=0}^{N_2-1} s(t_1, t_2) E_t E_{ut}^{-1} & \sin \alpha < 0, \sin \beta < 0 \end{cases}$$

where

$$A = \sqrt{\frac{(\sin \alpha - j \cos \alpha)(\sin \beta - j \cos \beta)}{N_1 N_2}}, E_{ut} = \exp \left\{ -j 2\pi \left(\frac{t_1 u_1}{N_1} + \frac{t_2 u_2}{N_2} \right) \right\}$$

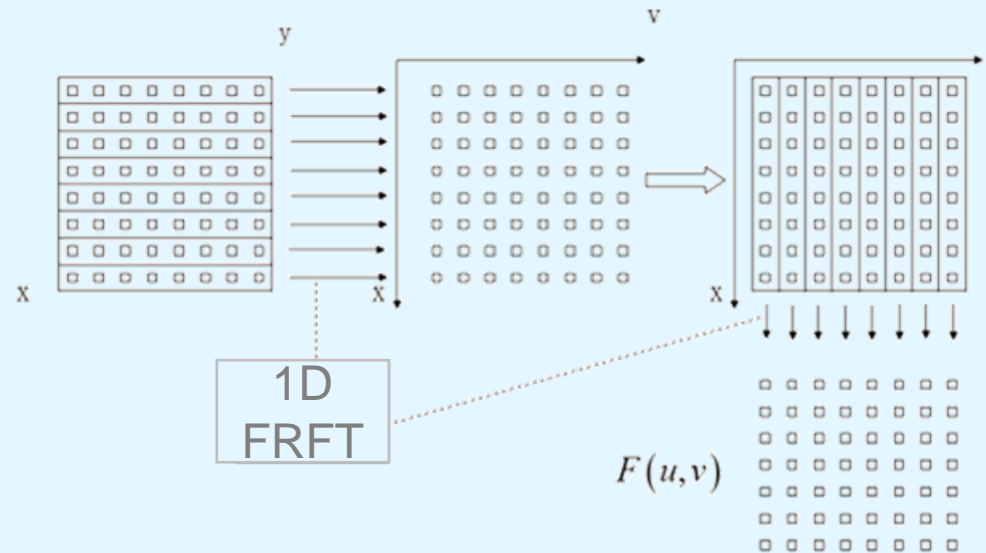
$$E_u = \exp \left\{ j \frac{u_1^2 \Delta u_1^2}{2 \tan \alpha} + j \frac{u_2^2 \Delta u_2^2}{2 \tan \beta} \right\}, E_t = \exp \left\{ j \frac{t_1^2 \Delta t_1^2}{2 \tan \alpha} + j \frac{t_2^2 \Delta t_2^2}{2 \tan \beta} \right\}$$



Explanation: Signals rotate in the time-frequency plane with free-rotation parameters α, β . Therefore, fingerprint images can show more comprehensive features by FRFT.

1.3 Numerical calculation of 2D FRFT

Traditional Methods: Decomposing and fast 1D FRFT.



Runtime Complexity: $(N_1 + N_2)$ times of 1D FRFT.

- For applications with large amounts of data, this calculation is very expensive.
- It also cannot meet the needs of real-time processing.

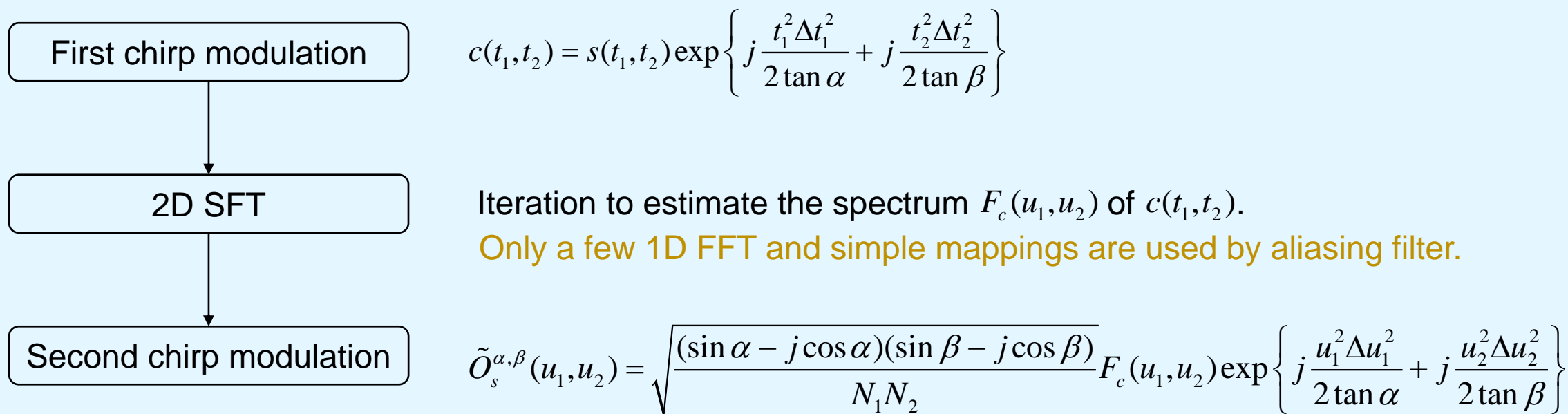
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2.1 STDFRFT algorithm flow

Suppose: $O_s^{\alpha,\beta}(u_1, u_2)$ is k -sparse. $\sin \alpha > 0, \sin \beta > 0$

Purpose: estimate the locations $(\xi_1^i, \xi_2^i)_{i=1}^k$ and amplitudes $[O_s^{\alpha,\beta}(\xi_1^i, \xi_2^i)]_{i=1}^k$ of the k significant frequencies.



2.1 STDFRFT algorithm flow

An iteration of 2D SFT

Bucketing:

extract random parallel slices $b_0(l), b_1(l), b_2(l)$ in $c(t_1, t_2)$, $l = 0, 1, \dots, LCM(N_1, N_2) - 1$
 perform FFT on slices to get $F_{b_0}(h), F_{b_1}(h), F_{b_2}(h)$
 update buckets $F_{b_0}(h), F_{b_1}(h), F_{b_2}(h)$

Estimating:

for each 1-sparse large-value bucket h , estimate the frequency:

$$\xi_1 = \left\lceil \frac{N_1}{2\pi} \phi\left(\frac{F_{b_1}(h)}{F_{b_0}(h)}\right) \right\rceil_{N_1}$$

$$\xi_2 = \left\lceil \frac{N_2}{2\pi} \phi\left(\frac{F_{b_2}(h)}{F_{b_0}(h)}\right) \right\rceil_{N_2}$$

$$F_c^r(\xi_1, \xi_2) = NF_{b_0}(h) \exp\left[-j2\pi\left(\frac{\xi_1\tau_1}{N_1} + \frac{\xi_2\tau_2}{N_2}\right)\right] / B$$

Noise-free: Significant frequencies can be estimated **accurately** with a high probability.

Noisy-case: Voting method to reduce the probability of decoding errors.

2.2 Comparison with traditional methods

Fractional Domain Sparse Random Signal Detection

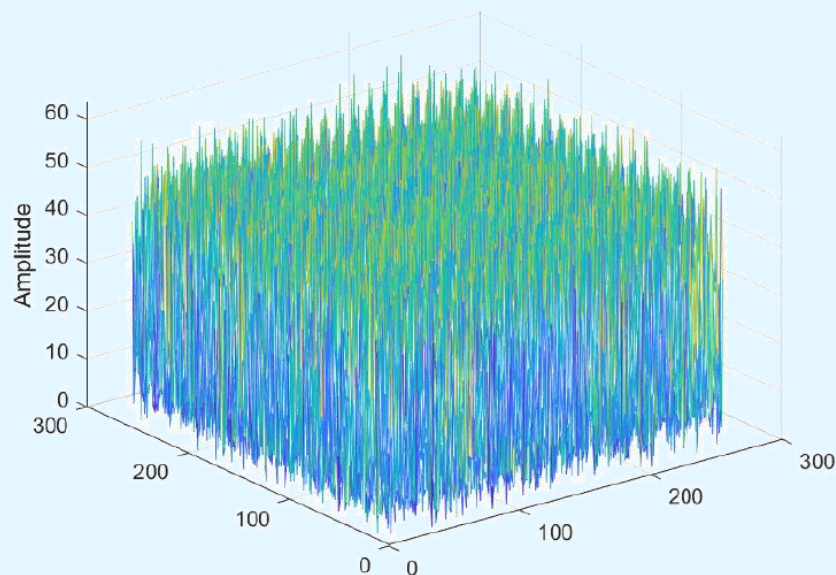
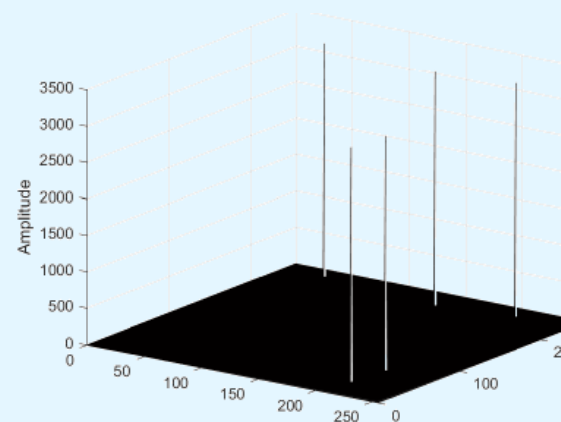
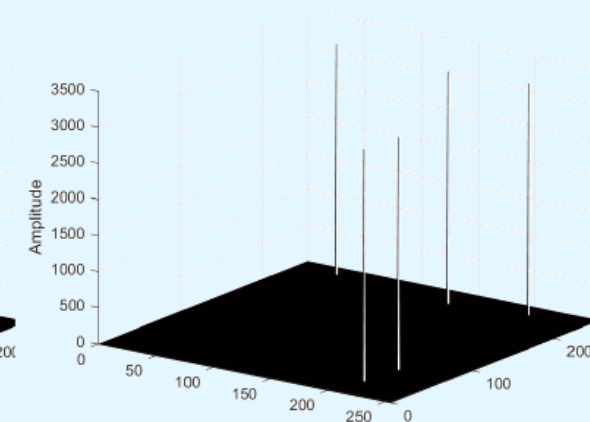


Fig. 1. The experimental signal.

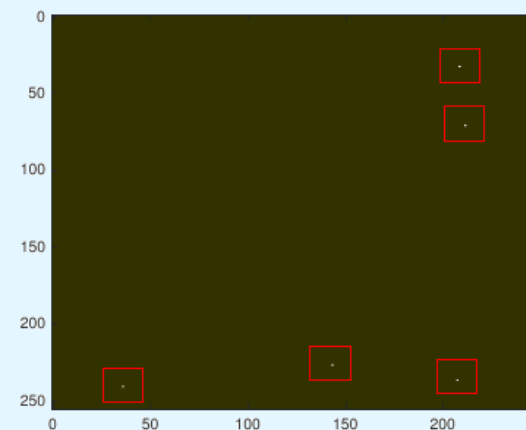
$$k = 5, \alpha = \beta = 1.2566, \text{SNR} = 26.9825\text{dB}$$



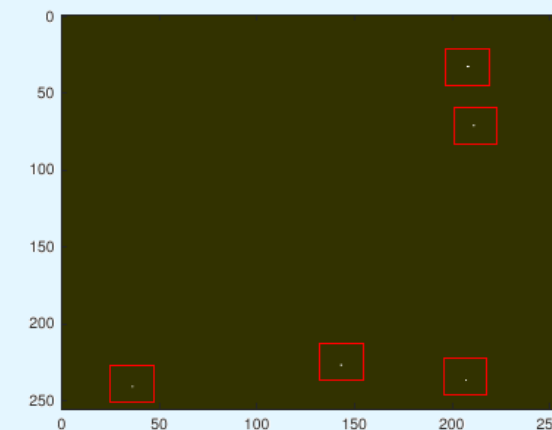
(a) Result by decomposing.



(b) Our result.



(c) Result by decomposing.



(d) Our result.

Fig. 2. Detection Results of Sparse Random Signals in Fractional Domain.

2.2 Comparison with traditional methods

	<i>Time/s</i>	<i>L₂ error</i>	<i>samples number</i>
Direct method	13.3246	31.5184	65536
Decomposing and [10] method	0.25001	42.1781	65536
Decomposing and [7] method	0.01396	31.7074	65536
Decomposing and [11] method	0.45293	25.4828	65536
Our result	0.00630	6.9803	2304

- [7] S.-C. Pei and J.-J. Ding, "Closed-form discrete fractional and affine Fourier transforms," IEEE Transactions on Signal Processing, vol. 48, no. 5, pp. 1338–1353, 2000.
- [10] H. M. Ozaktas, O. Arikan, M. A. Kutay, and G. Bozdagt, "Digital computation of the fractional Fourier transform," IEEE Transactions on signal processing, vol. 44, no. 9, pp. 2141–2150, 1996.
- [11] J. R. de Oliveira Neto, J. B. Lima, G. J. da Silva Jr, and R. M. C.de Souza, "Computation of an eigendecomposition-based discrete fractional Fourier transform with reduced arithmetic complexity," Signal Processing, vol. 165, pp. 72–82, 2019.

2.3 Algorithm convergence analysis

$$N = N_1 N_2 = 65536$$

$$SNR = 34.1541dB$$

$$B = LCM(N_1, N_2)$$

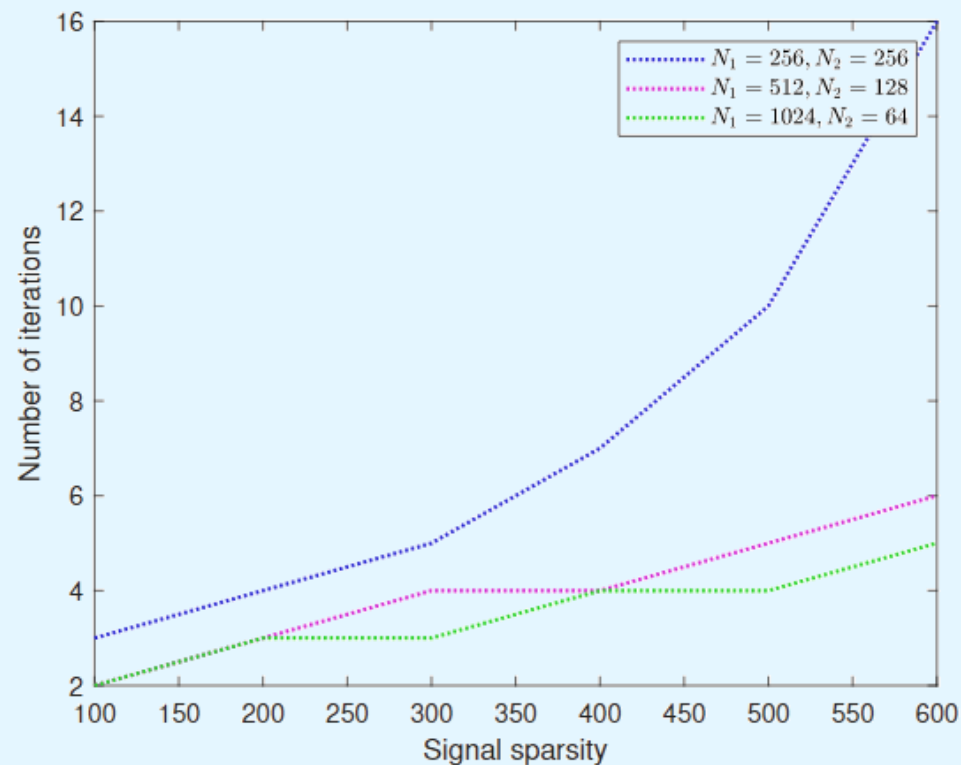


Fig. 3. Number of iterations of STDFRFT algorithm vs. sparsity of signal.

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3.1 2D chirp signal detection

parameters

sample rate 4096Hz/s

pulse duration 2^{-6} s

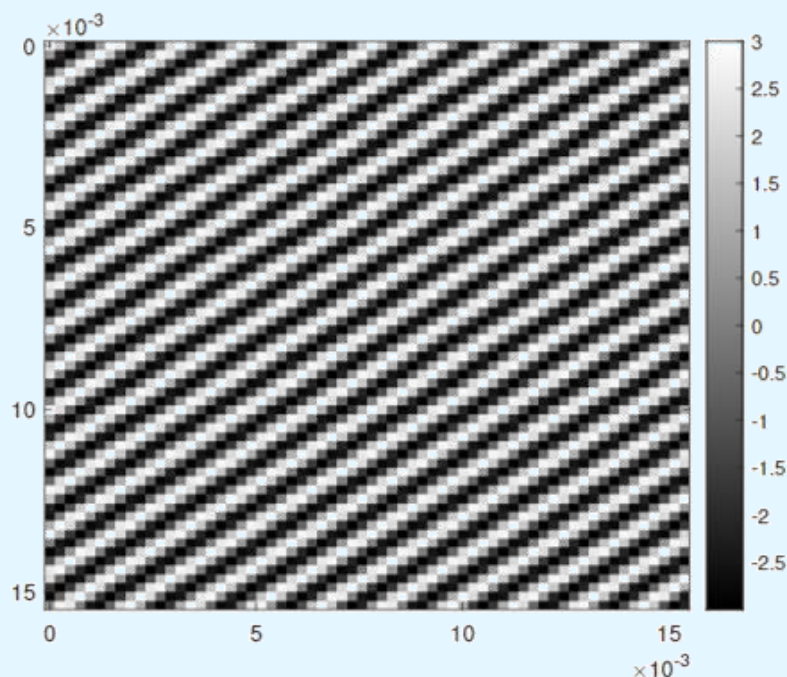


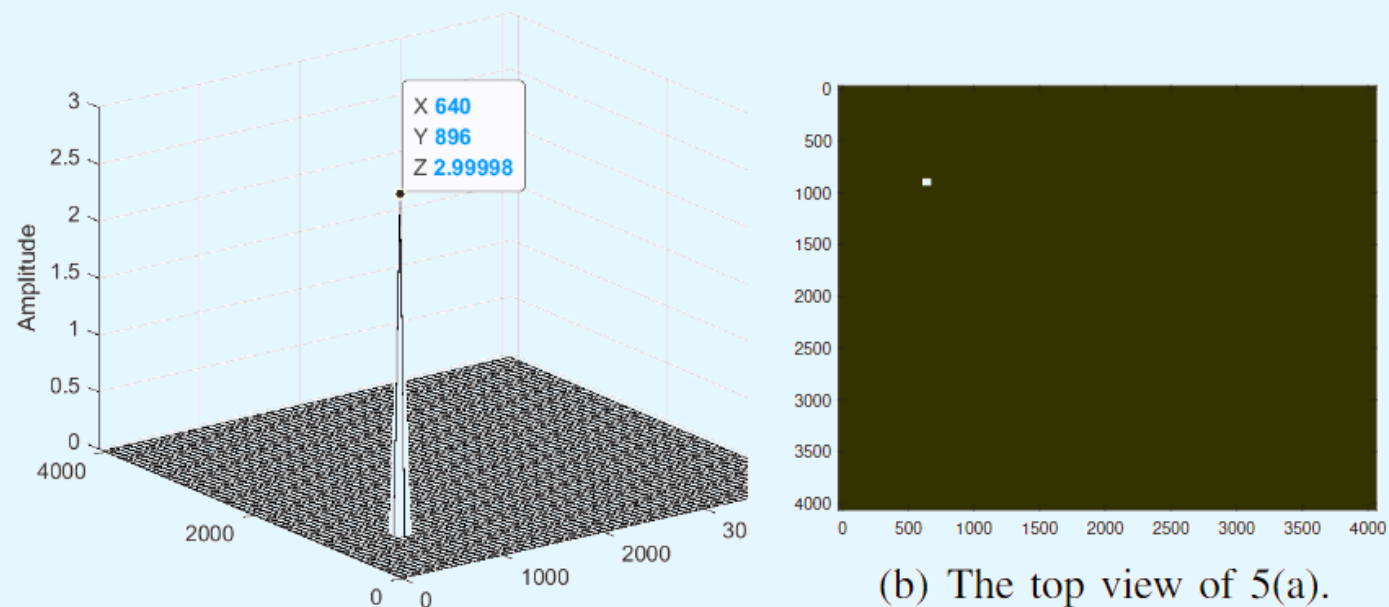
Fig. 4. Two-dimensional chirp signal

parameters

pulse amplitude 3

modulation frequency (-16.4Hz/s, -7.8Hz/s)

center frequency (640Hz, 896Hz)



(a) Amplitude spectrum in fractional Fourier domain.

Fig. 5. Chirp signal detection results.

3.2 Fingerprint feature recognition

1. Gao [12] proposed that the phase of FRFT has relative time-shift invariance.
2. The texture of the fingerprint can be approximated as 2D chirp signals.



(a) Fingerprint image.

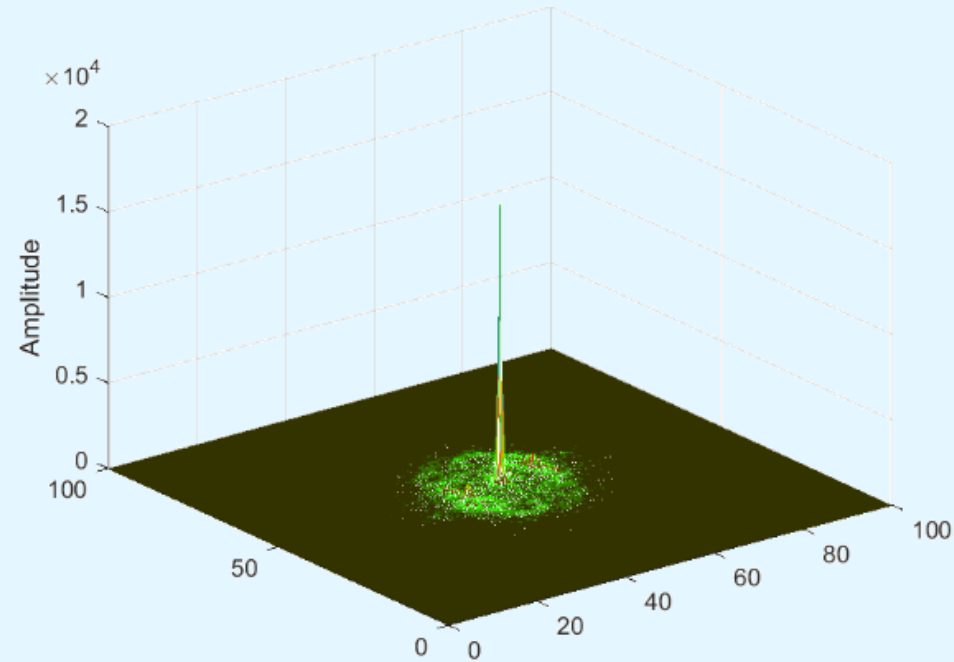


Fig. 7. Recognition result of fingerprint in fractional domain by direct method.

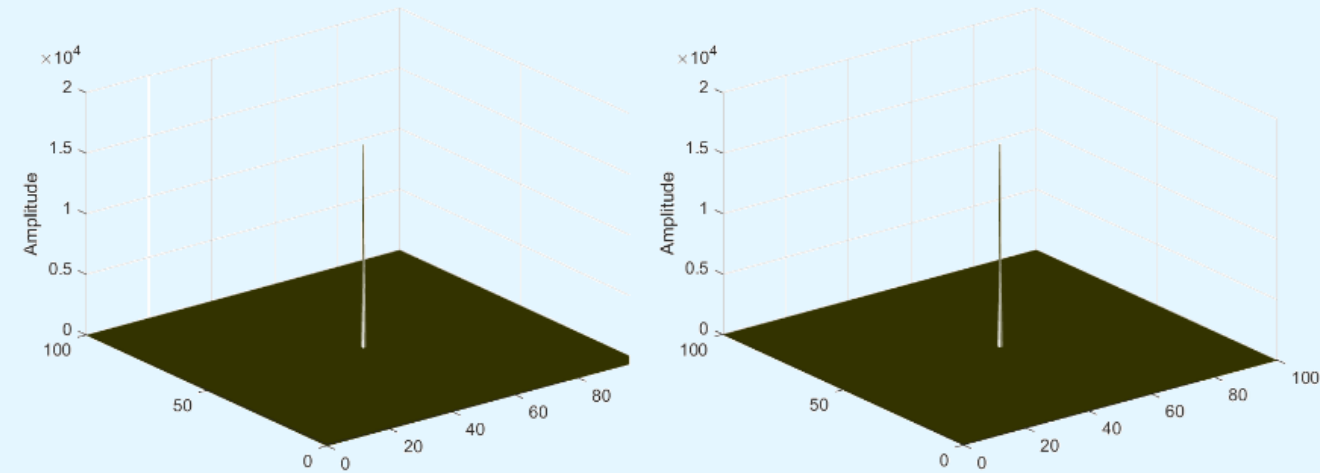
3.2 Fingerprint feature recognition



(a) Fingerprint image.

(b) Incomplete fingerprint image.

Fig. 6. Fingerprint images after preprocessing.



(a) Our result for 6(a).

(b) Our result for 6(b).

Fig. 8. Recognition result of fingerprints in fractional domain by our method.

Our algorithm can extract the main features even for incomplete fingerprints.

**Thanks for listening and feel free to ask
questions!**

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