Reconstruction of Uniform Sample Signals Related to High-Dimensional LCT

Deyun Wei, Jun Yang

Abstract—

Index Terms—LCT, uniform sample, nonuniform sample, high dimension

I. Introduction

THE linear canonical transform is one of the most powerful tools to deal with non-stationary signals.

II. Preliminaries

A. H-dimensional linear canonical transform

The LCT with parameters $A = [a, b; c, d]$ of the H-dimensional signal $f(\boldsymbol{t})$ can be expressed as

$$F_A(\boldsymbol{u}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(\boldsymbol{t}) \prod_{i=1}^{H} \mathcal{K}_A(u_i, t_i) dt_1 \cdots dt_H \tag{1}$$

where A is a parameter matrix of real numbers with $|A| = 1$ and H is a positive integer. Meanwhile, $\boldsymbol{t} = (t_1, t_2, \cdots, t_H) \in R^H$ and $\boldsymbol{u} = (u_1, u_2, \cdots, u_H) \in R^H$. In addition, $\mathcal{K}_A(u_i, t_i)$ is the kernel function of the one dimensional LCT, which is defined as

$$\mathcal{K}_A(u, t) = \begin{cases} \sqrt{\frac{1}{j2\pi b}} e^{\frac{j}{2b}(at^2 - 2ut + du^2)} & , b \neq 0 \\ \sqrt{d} e^{\frac{jcd}{2}u^2} \delta(du) & , b = 0 \end{cases} \tag{2}$$

and $\delta()$ is the unit impulse function. j is the imaginary unit with $j^2 = -1$.

The above HD LCT is an unitary transform. That is, its corresponding inverse transform can be expressed as

$$f(\boldsymbol{t}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} F_A(\boldsymbol{u}) \prod_{i=1}^{H} \mathcal{K}_{A^{-1}}(t_i, u_i) du_1 \cdots du_H \tag{3}$$

By definition, HD LCT degenerates to a chirp product when b = 0. Without loss of generality, we assume that $b > 0$ in subsequent sections. Then, HD LCT and HD ILCT are simplified to

$$F_A(\boldsymbol{u}) = \frac{1}{(j2\pi b)^{\frac{H}{2}}} \int_{R^H} f(\boldsymbol{t}) e^{\frac{j}{2b}[a\boldsymbol{t}^T\boldsymbol{t} - 2\boldsymbol{t}^T\boldsymbol{u} + d\boldsymbol{u}^T\boldsymbol{u}]} d\boldsymbol{t} \tag{4}$$

$$f(\boldsymbol{t}) = \frac{1}{(-j2\pi b)^{\frac{H}{2}}} \int_{R^H} F_A(\boldsymbol{u}) e^{-\frac{j}{2b}[a\boldsymbol{t}^T\boldsymbol{t} - 2\boldsymbol{t}^T\boldsymbol{u} + d\boldsymbol{u}^T\boldsymbol{u}]} d\boldsymbol{u} \tag{5}$$

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D. Wei and J. Yang are with the School of Mathematics and Statistics, Xidian University, Xi'an 710071, China. (e-mail: dywei@xidian.edu.cn).

where, T indicates transpose.

Bandlimited signal is one of the important basis of Fourier domain sampling theory. Similarly, the research on bandlimited signal in LCT domain is important. The H-dimensional signal $f(\boldsymbol{t})$ is bandlimited in the LCT domain with the parameter matrix A, if there exists a bounded open set $D \subset R^H$ and the HD LCT of $f(\boldsymbol{t})$ satisfies

$$F_A(\boldsymbol{u}) = 0, \boldsymbol{u} \notin D \tag{6}$$

B. Lattice sampled signal and its HD LCT

In this subsection, we will introduce the details of high-dimensional signals sampling at any grid. And the LCT of the sampled signals is described.

Assuming that $\Delta_t \in \mathcal{R}^{H \times H}$ is a nonsingular real number matrix, the sampling lattice constructed by Δ_t is defined as

$$Lat(\Delta_t) = \{\Delta_t \boldsymbol{n}, \boldsymbol{n} \in \mathcal{Z}^{H \times 1}\} \tag{7}$$

and Δ_t is known as generator of lattices (7). The associated unit cell $\mathbb{U}(\Delta_t)$ is defined as

$$\mathbb{U}(\Delta_t) = \{\Delta_t \boldsymbol{t} : \boldsymbol{t} \in [0, 1)^H\} \tag{8}$$

Acting the lattice generator Δ_t on the H-dimensional signal $f(\boldsymbol{t})$, the sampled signal is

$$\widetilde{f}(\boldsymbol{t}) = f(\boldsymbol{t}) \sum_{\boldsymbol{n} \in \mathcal{Z}^{H \times 1}} \delta(\boldsymbol{t} - \Delta_t \boldsymbol{n}) \tag{9}$$

where $\delta()$ is the H-dimensional unit impulse function. According to the definition (4), HD LCT with parameter matrix A of sampled signal $\widetilde{f}(\boldsymbol{t})$ can be expressed as

$$\tilde{F}_A(\boldsymbol{u}) = \frac{1}{|\det \Delta_t|} e^{\frac{jd}{2b}\boldsymbol{u}^T\boldsymbol{u}} \sum_{\boldsymbol{n} \in \mathcal{Z}^{H \times 1}} F_A(\boldsymbol{u}') e^{-\frac{jd}{2b}(\boldsymbol{u}')^T\boldsymbol{u}'} \tag{10}$$

where $F_A(\boldsymbol{u})$ denotes the HD LCT of original continuous signal $f(\boldsymbol{t})$ and $\boldsymbol{u}' = \boldsymbol{u} - 2\pi b \Delta_t^{-T} \boldsymbol{n}$. Moreover, det represents the determinant.

It can be seen from (10) that sampling in time domain induces periodicity (phase transition) in HD LCT domain. And the spectrum replicates with reciprocal lattice $Lat(2\pi b \Delta_t^{-T})$. Moreover, if $f(\boldsymbol{t})$ is bandlimited to set $D \subset R^H$ in the LCT domain with parameter A, then the support of $\tilde{F}_A(\boldsymbol{u})$ is expressed as

$$\text{supp.}\tilde{F}_A(\boldsymbol{u}) = \bigcup_{\boldsymbol{l} \in Lat(2\pi b \Delta_t^{-T})} (D - \boldsymbol{l}) \tag{11}$$

Research shows that if (11) satisfies the following constraints,

$$D \cap (D - \boldsymbol{l}) = \emptyset, \boldsymbol{l} \in Lat(2\pi b \Delta_t^{-T}) \backslash \{0\} \tag{12}$$

$$R^H = \bigcup_{\boldsymbol{l} \in Lat(2\pi b \Delta_t^{-T})} (D - \boldsymbol{l}) \tag{13}$$

then the original signal can be reconstructed by the sampling point uniquely and $D = \mathbb{U}(2\pi b \Delta_t^{-T})$. Similarly, sampling by lattice generator Δ_u in HD LCT domain induces periodicity with lattice $Lat(2\pi b \Delta_u^{-T})$ in time domain.

By sampling $(N, N, \cdots, N)$ points uniformly in one period $2\pi b \Delta_t^{-T}$ in the HD LCT domain, the lattice generator in the LCT can be denoted as

$$\Delta_u = \frac{2\pi b \Delta_t^{-T}}{N} \tag{14}$$

Therefore, the HD DLCT with parameter matrix A of discrete signals

$$f(\boldsymbol{n}) = f(\Delta_t \boldsymbol{n}), \boldsymbol{n} \in [N]^H. \tag{15}$$

is defined as

$$F_A(\boldsymbol{m}) = C \sum_{\boldsymbol{n} \in [N]^H} f(\boldsymbol{n}) e^{\frac{j}{2b}\psi} e^{-j\frac{2\pi}{N}\boldsymbol{n}^T \boldsymbol{m}} \tag{16}$$

where $\boldsymbol{m} \in [N]^H$, $[N]$ denotes $\{\lambda \bmod N : \lambda \in \mathcal{Z}\}$ and

$$\psi = d\boldsymbol{m}^T \Delta_u^T \Delta_u \boldsymbol{m} + a\boldsymbol{n}^T \Delta_t^T \Delta_t \boldsymbol{n}. \tag{17}$$

Coefficient $C = 1/N^{\frac{H}{2}}$ to ensure reversibility. Likewise, HD ILCT with parameter matrix A is defined as

$$f(\boldsymbol{n}) = C \sum_{\boldsymbol{m} \in [N]^H} F_A(\boldsymbol{m}) e^{-\frac{j}{2b}\psi} e^{j\frac{2\pi}{N}\boldsymbol{n}^T \boldsymbol{m}} \tag{18}$$

III. Reconstruction of uniform signals from nonuniform samples

Due to system noise, AD converters acquire samples in non-uniform lattices for applications. In fact, these lattices can be regarded as random perturbations of uniform lattices. Applying the acquired samples as samples on uniform lattices will lead to errors. Subsequently, we will give how to reconstruct samples of uniform lattice points from samples of non-uniform lattice points.

Assume that the H-dimensional continuous signal $f(\boldsymbol{t})$ is bandlimited to set D in the HD LCT domain with parameter matrix A. The sampling sequence with size N^H obtained by the lattice generator Δ_t is denoted as $f(\boldsymbol{n}), \boldsymbol{n} \in [N]^H$. The HD DLCT is denoted as (16), and the corresponding inverse transform is (18).

Suppose the random perturbation is $\xi_{\boldsymbol{n}}$, then the non-uniform sampled signal is

$$f'(\boldsymbol{n}) = f(\Delta_t(\boldsymbol{n} + \xi_{\boldsymbol{n}})), \boldsymbol{n} \in [N]^H \tag{19}$$

And HD DLCT of $\xi_{\boldsymbol{n}}$ is represented as

$$F'_A(\mathbf{m}) = C \sum_{\mathbf{n} \in [N]^H} f'(\mathbf{n}) e^{\frac{j}{2b}\psi} e^{-j\frac{2\pi}{N}\mathbf{n}^T \mathbf{m}} \tag{20}$$

Besides, making use of (18), $f'(\boldsymbol{n})$ can be expressed as

$$f'(\mathbf{n}) = C \sum_{\mathbf{m} \in [N]^H} F_A(\mathbf{m}) e^{-\frac{j}{2b}\varphi} e^{j\frac{2\pi}{N}(\mathbf{n}+\xi_{\mathbf{n}})^T \mathbf{m}} \tag{21}$$

where

$$\varphi = d\mathbf{m}^T \Delta_u^T \Delta_u \mathbf{m} + a(\mathbf{n} + \xi_{\mathbf{n}})^T \Delta_t^T \Delta_t (\mathbf{n} + \xi_{\mathbf{n}}) \tag{22}$$

Insert (21) into (20), it can be obtained that

$$\begin{aligned} F'_A(\mathbf{m}) =& C^2 \sum_{\mathbf{n} \in [N]^H} [\sum_{\boldsymbol{k} \in [N]^H} F_A(\boldsymbol{k}) e^{-\frac{jd}{2b}\boldsymbol{k}^T \Delta_u^T \Delta_u k} \\ & e^{-\frac{ja}{2b}(\mathbf{n}+\xi_{\mathbf{n}})^T \Delta_t^T \Delta_t (\mathbf{n}+\xi_{\mathbf{n}})} e^{j\frac{2\pi}{N}(\mathbf{n}+\xi_{\mathbf{n}})^T \boldsymbol{k}}] \\ & e^{\frac{j}{2b}[d\mathbf{m}^T \Delta_u^T \Delta_u \mathbf{m} + a\mathbf{n}^T \Delta_t^T \Delta_t \mathbf{n}]} e^{-j\frac{2\pi}{N}\mathbf{n}^T \mathbf{m}} \\ =& N^{-H} e^{\frac{jd}{2b}\mathbf{m}^T \Delta_u^T \Delta_u \mathbf{m}} \sum_{k \in [N]^H} [\sum_{n \in [N]^H} e^{j\frac{2\pi}{N}\xi_n^T k} \\ & e^{-\frac{ja}{2b}[\xi_n^T \Delta_t^T \Delta_t (\mathbf{n}+\xi_{\mathbf{n}}) + \mathbf{n}^T \Delta_t^T \Delta_t \xi_{\mathbf{n}}] - j\frac{2\pi}{N}\mathbf{n}^T(\mathbf{m}-k)}] \\ & F_A(k) e^{-\frac{jd}{2b}k^T \Delta_u^T \Delta_u k} \\ =& N^{-H} e^{\frac{jd}{2b}\mathbf{m}^T \Delta_u^T \Delta_u \mathbf{m}} \sum_{k \in [N]^H} B(k, \mathbf{m} - k) F_A(k) e^{-\frac{jd}{2b}k^T \Delta_u^T \Delta_u k} \end{aligned} \tag{23}$$

where

$$B(k, m) = \sum_{n \in [N]^H} e^{-\frac{ja}{2b}[\xi_n^T \Delta_t^T \Delta_t (\mathbf{n}+\xi_{\mathbf{n}}) + \mathbf{n}^T \Delta_t^T \Delta_t \xi_{\mathbf{n}}] + j\frac{2\pi}{N}[\xi_n^T k - \mathbf{n}^T(\mathbf{m}-k)]} \tag{24}$$

IV. Guidelines For Manuscript Preparation

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V. Guidelines for Graphics Preparation and Submission

A. Types of Graphics

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1) Color/Grayscale figures: Figures that are meant to appear in color, or shades of black/gray. Such figures may include photographs, illustrations, multicolor graphs, and flowcharts.

2) Line Art figures: Figures that are composed of only black lines and shapes. These figures should have no shades or half-tones of gray, only black and white.

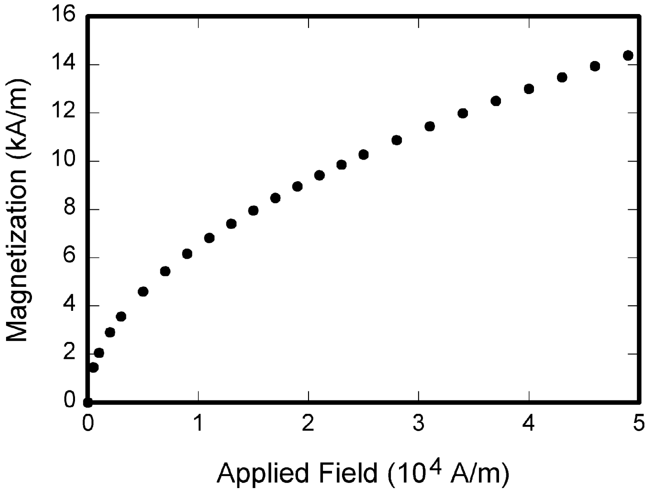

Fig. 1: Magnetization as a function of applied field. Note that "Fig." is abbreviated. There is a period after the figure number, followed by two spaces. It is good practice to explain the significance of the figure in the caption.

3) Tables: Data charts which are typically black and white, but sometimes include color.

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Figures compiled of more than one sub-figure presented side-by-side, or stacked. If a multipart figure is made up of multiple figure types (one part is lineart, and another is grayscale or color) the figure should meet the stricter guidelines.

C. File Formats For Graphics

Format and save your graphics using a suitable graphics processing program that will allow you to create the images as PostScript (PS), Encapsulated PostScript (.EPS), Tagged Image File Format (.TIFF), Portable Document Format (.PDF), Portable Network Graphics (.PNG), or Metapost (.MPS), sizes them, and adjusts the resolution settings. When submitting your final paper, your graphics should all be submitted individually in one of these formats along with the manuscript.

D. Sizing of Graphics

Most charts, graphs, and tables are one column wide (3.5 inches/88 millimeters/21 picas) or page wide (7.16 inches/181 millimeters/43 picas). The maximum depth a graphic can be is 8.5 inches (216 millimeters/54 picas). When choosing the depth of a graphic, please allow space for a caption. Figures can be sized between column and page widths if the author chooses, however it is recommended that figures are not sized less than column width unless when necessary.

TABLE I: Units for Magnetic Properties

Symbol	Quantity	Conversion from Gaussian and CGS EMU to SI[a]
Φ	Magnetic flux	1 Mx $\rightarrow 10^{-8}$ Wb $= 10^{-8}$ V $\cdot$ s
B	Magnetic flux density, magnetic induction	1 G $\rightarrow 10^{-4}$ T $= 10^{-4}$ Wb/m^2
H	Magnetic field strength	1 Oe $\rightarrow 10^{-3}/(4\pi)$ A/m
m	Magnetic moment	1 erg/G $= 1$ emu $\rightarrow 10^{-3}$ A $\cdot$ m$^2 = 10^{-3}$ J/T
M	Magnetization	1 erg/(G $\cdot$ cm^3) $= 1$ emu/cm^3 $\rightarrow 10^{-3}$ A/m
$4\pi M$	Magnetization	1 G $\rightarrow 10^{-3}/(4\pi)$ A/m
σ	Specific magnetization	1 erg/(G $\cdot$ g) $= 1$ emu/g $\rightarrow 1$ A $\cdot$ m^2/kg
j	Magnetic dipole moment	1 erg/G $= 1$ emu $\rightarrow 4\pi \times 10^{-10}$ Wb $\cdot$ m
J	Magnetic polarization	1 erg/(G $\cdot$ cm^3) $= 1$ emu/cm^3 $\rightarrow 4\pi \times 10^{-4}$ T
χ, κ	Susceptibility	$1 \rightarrow 4\pi$
χ_ρ	Mass susceptibility	1 cm^3/g $\rightarrow 4\pi \times 10^{-3}$ m^3/kg
μ	Permeability	$1 \rightarrow 4\pi \times 10^{-7}$ H/m $= 4\pi \times 10^{-7}$ Wb/(A $\cdot$ m)
μ_r	Relative permeability	$\mu \rightarrow \mu_r$
w, W	Energy density	1 erg/cm$^3 \rightarrow 10^{-1}$ J/m^3
N, D	Demagnetizing factor	$1 \rightarrow 1/(4\pi)$

Vertical lines are optional in tables. Statements that serve as captions for the entire table do not need footnote letters.
[a]Gaussian units are the same as cg emu for magnetostatics; Mx = maxwell, G = gauss, Oe = oersted; Wb = weber, V = volt, s = second, T = tesla, m = meter, A = ampere, J = joule, kg = kilogram, H = henry.

E. Resolution

The proper resolution of your figures will depend on the type of figure it is as defined in the "Types of Figures" section. Author photographs, color, and grayscale figures should be at least 300dpi. Line art, including tables should be a minimum of 600dpi.

F. Vector Art

In order to preserve the figures' integrity across multiple computer platforms, we accept files in the following formats: .EPS/.PDF/.PS. All fonts must be embedded or text converted to outlines in order to achieve the best-quality results.

G. Accepted Fonts Within Figures

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Multipliers can be especially confusing. Write "Magnetization (kA/m)" or "Magnetization (10^3 A/m)." Do not write "Magnetization (A/m) $\times$ 1000" because the reader would not know whether the top axis label in Fig. 1 meant 16000 A/m or 0.016 A/m. Figure labels should be legible, approximately 8 to 10 point type.

2) Subfigure Labels in Multipart Figures and Tables: Multipart figures should be combined and labeled before final submission. Labels should appear centered below each subfigure in 8 point Times New Roman font in the format of (a) (b) (c).

I. File Naming

Figures (line artwork or photographs) should be named starting with the first 5 letters of the author's last name. The next characters in the filename should be the number that represents the sequential location of this image in your article. For example, in author "Anderson's" paper, the first three figures would be named ander1.tif, ander2.tif, and ander3.ps.

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Acknowledgment

The preferred spelling of the word "acknowledgment" in American English is without an "e" after the "g." Use the singular heading even if you have many acknowledgments. Avoid expressions such as "One of us (S.B.A.) would like to thank . . . ." Instead, write "F. A. Author thanks ... ." In most cases, sponsor and financial support acknowledgments are placed in the unnumbered footnote on the first page, not here.

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[3] J. U. Duncombe, "Infrared navigation Part I: An assessment of feasibility," IEEE Trans. Electron Devices, vol. ED-11, no. 1, pp. 34–39, Jan. 1959,10.1109/TED.2016.2628402.
[4] E. P. Wigner, "Theory of traveling-wave optical laser," Phys. Rev., vol. 134, pp. A635–A646, Dec. 1965.
[5] E. H. Miller, "A note on reflector arrays," IEEE Trans. Antennas Propagat., to be published.

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[6] E. E. Reber, R. L. Michell, and C. J. Carter, "Oxygen absorption in the earth's atmosphere," Aerospace Corp., Los Angeles, CA, USA, Tech. Rep. TR-0200 (4230-46)-3, Nov. 1988.
[7] J. H. Davis and J. R. Cogdell, "Calibration program for the 16-foot antenna," Elect. Eng. Res. Lab., Univ. Texas, Austin, TX, USA, Tech. Memo. NGL-006-69-3, Nov. 15, 1987.

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[14] J. S. Turner, “New directions in communications,” IEEE J. Sel. Areas Commun., vol. 13, no. 1, pp. 11–23, Jan. 1995.
[15] W. P. Risk, G. S. Kino, and H. J. Shaw, “Fiber-optic frequency shifter using a surface acoustic wave incident at an oblique angle,” Opt. Lett., vol. 11, no. 2, pp. 115–117, Feb. 1986.
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Examples:

[18] R. J. Hijmans and J. van Etten, “Raster: Geographic analysis and modeling with raster data,” R Package Version 2.0-12, Jan. 12, 2012. [Online]. Available: http://CRAN.R-project.org/package=raster
[19] Teralyzer. Lytera UG, Kirchhain, Germany [Online]. Available: http://www.lytera.de/Terahertz_THz_Spectroscopy.php?id=home, Accessed on: Jun. 5, 2014.

Basic format for computer programs and electronic documents (when available online):

Legislative body. Number of Congress, Session. (year, month day). Number of bill or resolution, Title. [Type of medium]. Available: site/path/file NOTE: ISO recommends that capitalization follow the accepted practice for the language or script in which the information is given.

Example:

[20] U. S. House. 102nd Congress, 1st Session. (1991, Jan. 11). H. Con. Res. 1, Sense of the Congress on Approval of Military Action. [Online]. Available: LEXIS Library: GENFED File: BILLS

Basic format for patents (when available online):

Name of the invention, by inventor’s name. (year, month day). Patent Number [Type of medium]. Available:site/path/file

Example:

[21] Musical tooth brush with mirror, by L. M. R. Brooks. (1992, May 19). Patent D 326 189 [Online]. Available: NEXIS Library: LEXPAT File: DES

Basic format for conference proceedings (published):

J. K. Author, “Title of paper,” in Abbreviated Name of Conf., City of Conf., Abbrev. State (if given), Country, year, pp. xxx–xxx.

Example:

[22] D. B. Payne and J. R. Stern, “Wavelength-switched passively coupled single-mode optical network,” in Proc. IOOC-ECOC, Boston, MA, USA, 1985, pp. 585–590.

Example for papers presented at conferences (unpublished):

[23] D. E behard and E. Voges, “Digital single sideband detection for inter ferometric sensors,” presented at the 2nd Int. Conf. Optical Fiber Sensors, Stuttgart, Germany, Jan. 2–5, 1984.

Basic formatfor patents:

J. K. Author, “Title of patent,” U. S. Patent x xxx xxx, Abbrev. Month, day, year.

Example:

[24] G. Brandli and M. Dick, “Alternating current fed power supply,” U. S. Patent 4 084 217, Nov. 4, 1978.

Basic format for theses (M.S.) and dissertations (Ph.D.):

a) J. K. Author, "Title of thesis," M. S. thesis, Abbrev. Dept., Abbrev. Univ., City of Univ., Abbrev. State, year.

b) J. K. Author, "Title of dissertation," Ph.D. dissertation, Abbrev. Dept., Abbrev. Univ., City of Univ., Abbrev. State, year.

Examples:

[25] J. O. Williams, "Narrow-band analyzer," Ph.D. dissertation, Dept. Elect. Eng., Harvard Univ., Cambridge, MA, USA, 1993.
[26] N. Kawasaki, "Parametric study of thermal and chemical nonequilibrium nozzle flow," M.S. thesis, Dept. Electron. Eng., Osaka Univ., Osaka, Japan, 1993.

Basic format for the most common types of unpublished references:

a) J. K. Author, private communication, Abbrev. Month, year.

b) J. K. Author, "Title of paper," unpublished.

c) J. K. Author, "Title of paper," to be published.

Examples:

[27] A. Harrison, private communication, May 1995.
[28] B. Smith, "An approach to graphs of linear forms," unpublished.
[29] A. Brahms, "Representation error for real numbers in binary computer arithmetic," IEEE Computer Group Repository, Paper R-67-85.

Basic formats for standards:

a) Title of Standard, Standard number, date.

b) Title of Standard, Standard number, Corporate author, location, date.

Examples:

[30] IEEE Criteria for Class IE Electric Systems, IEEE Standard 308, 1969.
[31] Letter Symbols for Quantities, ANSI Standard Y10.5-1968.

Article number in reference examples:

[32] R. Fardel, M. Nagel, F. Nuesch, T. Lippert, and A. Wokaun, "Fabrication of organic light emitting diode pixels by laser-assisted forward transfer," Appl. Phys. Lett., vol. 91, no. 6, Aug. 2007, Art. no. 061103.
[33] J. Zhang and N. Tansu, "Optical gain and laser characteristics of InGaN quantum wells on ternary InGaN substrates," IEEE Photon. J., vol. 5, no. 2, Apr. 2013, Art. no. 2600111

Example when using et al.:

[34] S. Azodolmolky et al., Experimental demonstration of an impairment aware network planning and operation tool for transparent/translucent optical networks," J. Lightw. Technol., vol. 29, no. 4, pp. 439–448, Sep. 2011.